CBSE Test Paper 03
Chapter 7 System of Particles & Rotational

1. A fan of moment of inertia 0.3 kg m^2 is to run up to a working speed of 0.5 revolution per second. Indicate the correct value of the angular momentum of the fan **1**

    a. (π/6) (kg $\times$ m^2)/sec
    b. 3(kg $\times$ m^2)/sec
    c. 0.3 π kg $\times$ m^2/sec
    d. 6 kg $\times$ m^2/sec

2. Centre of gravity can be defined **1**

    a. as that point where the total gravitational torque on the body is greater than zero
    b. as the center of mass
    c. as that point where the total gravitational torque on the body is zero
    d. as that point where the total gravitational force on the body is zero

3. The radius of gyration of a circular loop of radius r and mass m rotating about its diameter as axis is **1**

    a. R/2
    b. $\sqrt{2}R$
    c. R/$\sqrt{2}$
    d. R

4. If a body is rotating about z axis with a speed ω and a point is at a distance of r in the x-y plane then the velocity of the point is **1**

    a. $3r\omega$
    b. $r\omega$
    c. $2r\omega$
    d. $r\omega/2$

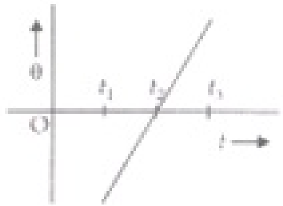
5. A body having moment of inertia about its axis equal to 3 kg m^2 is rotating with

angular velocity equal to 3 rad/s. The kinetic energy of this rotating body is the same as that of a body of mass 27 kg moving with a speed of **1**

a. 0.5 m/s
b. 1.0 m/s
c. 1.5 m/s
d. 2.0 m/s

6. Is centre of mass a reality? **1**

7. The variation of angular position θ , of a point on a rotating rigid body, with time t is shown in Figure. Is the body rotating clock-wise or anti-clockwise? **1**



8. What is the value of instantaneous speed of the point of contact during pure rolling? **1**

9. If ice on poles melts, then what is the change in duration of day? **2**

10. How can you define moment of inertia of a rotating body in terms of its **2**

i. angular momentum
ii. torque?

11. The speed of a whirlwind in a tornado is alarmingly high. Why? **2**

12. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis? **3**

13. The oxygen molecule has a mass of 5.30×10^{-26} kg and a moment of inertia of $1.94 \times 10^{-46} \mathrm{kgm}^2$ about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation.

Find the average angular velocity of the molecule. **3**

14. A flywheel in the shape of a uniform disc has a mass of 40 kg and is of radius 0.5 m. It is revolving around its own axis at the rate of 360 rpm. What torque is needed to bring it to rest in 10 s? If the torque is on account of a force applied tangentially on the rim of flywheel, what is the magnitude of force? **3**

15. Two particles each of mass m and speed v travel in opposite direction along parallel lines, separated by a distance d. Show that vector angular momentum of the two particles system is same whatever be the point about which angular momentum is taken. **5**

Answer

1. c. 0.3 π kg $\times$ m^2 / sec

   Explanation: n = 0.5 revolution per second
   angular velocity $\omega = 2\pi n = 2\pi \times 0.5 = \pi rad/s$
   moment of inertia $I = 0.3Kgm^2$
   angular momentum $L = I\omega = 0.3 \times \pi = 0.3\pi Kgm^2/s$

2. c. as that point where the total gravitational torque on the body is zero

   Explanation: The point at which the entire weight of a body may be thought of as centered so that if supported at this point the body would balance perfectly, so it can also be defined as as that point where the total gravitational torque on the body is zero.

3. c. R/$\sqrt{2}$

   Explanation: Moment of inertia of ring or circular loop about axis passing through centre of mass and perpendicular to plane is MR^2 ,
   Applying perpendicular axis theorm
   $I_z = I_x + I_y$
   $MR^2 = I + I$
   $I = \frac{1}{2}MR^2$
   Here I = Moment of inertia about axis passing through diameter
   As moment of inertia in terms of radius of gyration (k) is Mk^2
   $I = Mk^2$
   $\frac{1}{2}MR^2 = Mk^2$
   $k = R/\sqrt{2}$

4. b. $r\omega$

   Explanation: angular displacement of particle about z axis
   $d\theta = \frac{ds}{r}$
   differentiate with respect to time

$\frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt}$
$\frac{d\theta}{dt} = \omega$
$\frac{ds}{dt} = v$
$\omega = \frac{v}{r}$
$v = r\omega$

5. b. 1.0 m/s

Explanation: $K_{rot} = \frac{1}{2}I\omega^2$

$K_{trans} = \frac{1}{2}mv^2$

given that

$K_{rot} = K_{trans}$

$\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$

I = 3 Kgm^2

$\omega = 3rad/s$

m = 27Kg

v = ?

$I\omega^2 = mv^2$

$v = \sqrt{\frac{I\omega^2}{m}} = \sqrt{\frac{3\times3\times3}{27}}$ = 1.0 m/s

6. No, the centre of mass of a system is a hypothetical point which acts as a single mass particle of the system for an external force. Its a concept developed so that laws of mechanics can be applied to bulk bodies.
7. As the $\theta - t$ graph has +ve slope so $\frac{d\theta}{dt} = \omega$ is +ve so the rotation is clockwise.
8. Zero
9. If ice on poles melts, then the molten ice from poles goes into ocean and which results in the mass going away from axis of rotation. Therefore, moment of inertia of earth increases which results in the decrease of angular velocity (ω) to conserve angular momentum, So, time period of rotation increases (T = $2\pi/\omega$). So, net effect of global warming is increasing in the duration of day.
10. i. We know that, L = Iω and if ω= 1, then L = I. Hence, the moment of inertia (I) of a rotating body is numerically equal to its angular momentum (L) when rotating

with uniform unit angular velocity (ω about the given axis.

ii. We know that $\tau = I\alpha$ and if α = 1, then τ = I. Hence, the moment of inertia (I) of a rotating body may be numerically considered as the torque (τ) needed to produce a unit angular acceleration (α) in that body.

11. In a whirlwind, the air from nearby region gets concentrated in a small space thereby decreasing the value of the moment of inertia considerably. Since, $I\omega = constant$, the angular speed becomes quite high due to the decrease in moment of inertia,

12. Mass of the cylinder, m = 20 kg

Angular speed, ω = 100 rad s^{-1}
Radius of the cylinder, r = 0.25 m
The moment of inertia of the solid cylinder:
$I = \frac{mr^2}{2}$
$= \frac{1}{2} \times 20 \times (0.25)^2$
= 0.625 kg m^2
$\therefore$ Kinetic energy $= \frac{1}{2} I\omega^2$
$= \frac{1}{2} \times 6.25 \times (100)^2 = 3125J$
$\therefore$ Angular momentum, L= Iω
= 6.25 $\times$ 100
= 62.5 Js

13. Mass of an oxygen molecule, m = 5.30×10^{-26} kg
Moment of inertia, $l = 1.94 \times 10^{-46} kgm^2$
Velocity of the oxygen molecule, v = 500 m/s
The separation between the two atoms of the oxygen molecule = 2r
Mass of each oxygen atom = $\frac{m}{2}$
Hence, moment of inertia I, is calculated as:
$\left(\frac{m}{2}\right) r^2 + \left(\frac{m}{2}\right) r^2 = mr^2$
$r = \sqrt{\frac{I}{m}}$
$\sqrt{\frac{1.94 \times 10^{-45}}{5.36 \times 10^{-28}}} = 0.60 \times 10^{-10} \mathrm{m}$
It is given that:
$KE_{rot} = \frac{2}{3} KE_{trave}$

$\frac{1}{2}I\omega^2 = \frac{2}{3} \times \frac{1}{2} \times mv^2$
$mr^2\omega^2 = \frac{2}{3}mv^2$
$\omega = \sqrt{\frac{2}{3}\frac{v}{r}}$
$= \sqrt{\frac{2}{3}} \times \frac{500}{0.6\times10^{-10}}$
$= 6.80 \times 10^{12}\text{rad/s}$

14. Given: Mass of the flywheel, m = 40 kg and radius, r = 0.5 m, $\omega_0 = 360\text{rpm}$ = 6 rps = $12\pi\ \text{rad/s}$, ω = 0 and t = 10 seconds

    Now, Moment of inertia of flywheel, I = $\frac{1}{2}mr^2 = \frac{1}{2} \times 40 \times (0.5)^2$ = 5 kg m^2

    Angular acceleration, $\alpha = \frac{\omega - \omega_0}{t}$ = $\frac{0-12\pi}{10}$ = -1.2π rad/s^2

    Therefore, Torque needed, $\tau = I\alpha = 5 \times (-1.2\pi) = -18.85\ \text{Nm}$

    Negative sign of torque means that torque is opposing the rotational motion.

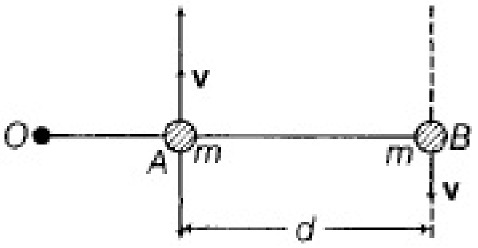
    If a force F is applied on the rim of flywheel, then,

    F $\times$ r = τ

    F = $\frac{\tau}{r} = \frac{-18.85}{0.5}$ = -37.7 N

    Negative sign means that the force is a resistive force opposing the motion.

15. Suppose, O be the origin chosen.



Then, angular momentum of particle at A is

$I_1 = OA \times p = OA \times mv$

$= m(OA \times v)$

and angular momentum of particle at B is

$I_2 = OB \times p = OB \times (-mv)$

$= -m(OB \times v)$

so, total angular momentum of the system of particles is

$L = I_1 + I_2$

$= m(OA \times v) - m(OB \times v)$

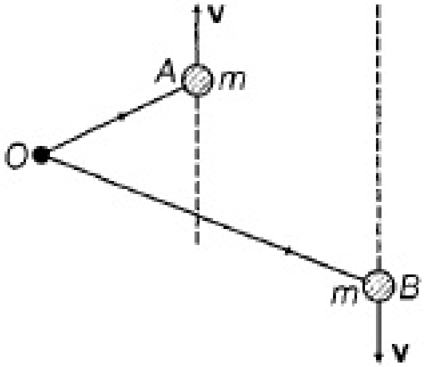
$= m(OA - OB) \times v$

$= m(BA) \times v$

$= m(BA) \times v$

{As, BA = position vector of A - position vector of B}

Above expression is independent of choice of origin.



This is true even when particles are not in a straight line.

$L_i(I_1 + I_2 = m(OA \times v - OB \times v)$

$= m(BA) \times v$

Which is the same as a previous result. So, the angular momentum of the system is independent of the choice of origin.