Long Answer Type Questions

[4 MARKS]

Que 1. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

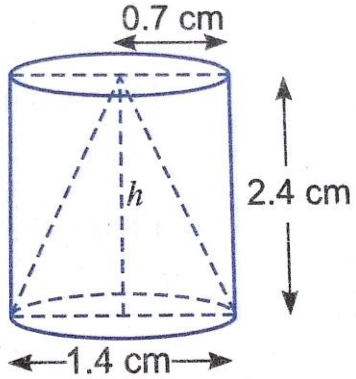

Fig. 13.23

Sol. We have,

Radius of the cylinder = $\frac{1.4}{2} = 0.7\ cm$

Height of the cylinder = 2.4 cm

Also. Radius of the cone = 0.7 cm
and height of the cone = 2.4 cm

Now, slant height of the cone = $\sqrt{(0.7)^2 + (2.4)^2}$

$\Rightarrow \quad l = \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5\ cm$

$\therefore$ Total surface area of the remaining solid

= Curved surface area of cylinder + Curved surface area of the cone + Area of upper circular base of cylinder

$= 2\pi rh + \pi rl + \pi r^2 = \pi r\ (2h + l + r)$

$= \frac{22}{7} \times 0.7 \times [2 \times 2.4 + 2.5 + 0.7] = 22 \times 0.1 \times (4.8 + 2.5 + 0.7)$

$= 2.2 \times 8.0 = 17.6\ cm^2 \approx 18\ cm^2$

Que 2. The decorative block shown in figure is made of two solids – a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block. $\left(\textbf{Take}\ \pi = \frac{22}{7}\right)$.

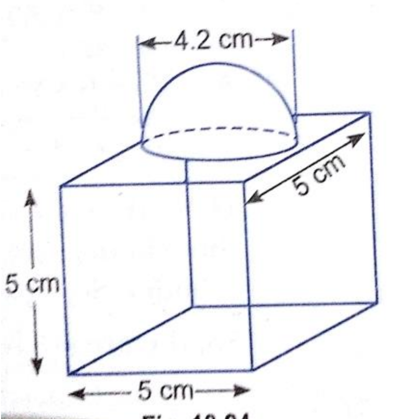

Fig. 13.24

Sol. The total surface area of the cube = 6 × (edge)^2

= $6 \times 5 \times 5$ cm^2 =150 cm^2

∴ Total surface area of the block

= Total surface area of cube – Base area of hemisphere + Curved surface area of hemisphere

$$= 150 - \pi r^2 + 2\pi r^2 = (150 + \pi r^2)\text{cm}^2$$

$$= \left(150 + \frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2}\right) cm^2 = (150 + 13.86)\ cm^2 = 163.86\ cm^2$$

Que 3. Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere (Fig. 13.25). The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. $\left(\textbf{Take}\pi = \frac{22}{7}\right)$

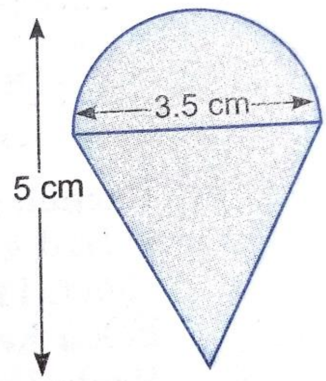

Fig. 13.25

Sol. Total surface area of the top

= Curved surface area of hemisphere + Curved surface area of cone.

Now, the curved surface area of hemisphere = $2\pi r^2$

$$= \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) cm^2$$

Also, the height of the cone = Height of the top – Height (radius) of the hemispherical part

$$= \left(5 - \frac{3.5}{2}\right) cm = 3.25\ cm$$

So, the slant height of the cone (l)

$$= \sqrt{r^2 + h^2} = \sqrt{\frac{3.5}{2}^2 + (3.25)^2}cm = 3.7\ cm\ (approx)$$

Therefore, curved surface area of cone = πrl = $\left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right) \text{cm}^2$

Thus, the surface area of the top

$$= \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) cm^2 + \left(\frac{22}{7} \times \frac{3.5}{2} 3.7\right) cm^2$$

$$= \frac{22}{7} \times \frac{3.5}{2}(3.5 + 3.7)\ cm^2 = \frac{22}{7} \times \frac{3.5}{2} \times 7.2\ cm^2$$

$= 39.6\ \text{cm}^2$ (approx.).

Que 4. A wooden toy rocket is in the shape of a mounted on a cylinder, in Fig. 13.26. The height of the entire rocket is 26 cm, while the of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take π = 3.14)

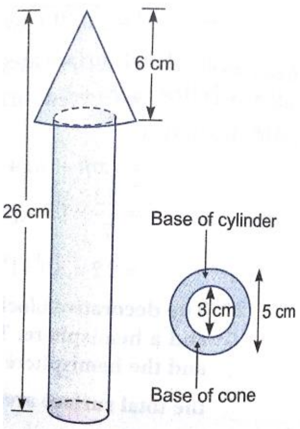

Fig. 13.26

Sol. Denote radius, slant height and height of cone by r, l and h, respectively, and radius and height of cylinder by r' and h', respectively. Then r = 2.5 cm, h = 6 cm, r' = 1.5 cm, h' = 26 – 6 = 20 cm and

$$l = \sqrt{r^2 + h^2} = \sqrt{(2.5)^2 + (6)^2} = 6.5\ cm$$

Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder. So, a part of the base of the cone (a ring) is to be painted.

So, the area to be painted orange

= Curved surface area of the cone + Base area of the cone – Base area of the cylinder

$$= \pi rl + \pi r^2 - \pi(r')^2$$

$$= \pi[(2.5 \times 6.5) + (2.5)^2 - (1.5)^2]\text{cm}^2$$

$$= \pi[20.25]\text{cm}^2 = 3.14 \times 20.25\ \text{cm}^2 = 63.585\ \text{cm}^2$$

Now, the area to be painted yellow

= Curved surface area of the cylinder + Area of one base of the cylinder

$= 2\pi r' h' + \pi(r')^2 = \pi r'(2h' + r')$

$= (3.14 \times 1.5)(2 \times 20 + 1.5)\ cm^2 = 4.71 \times 41.5\ cm^2 = 195.465\ cm^2$

Que 5. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

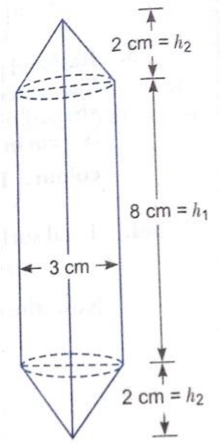

Fig. 13..27

Sol. Here, radius of cylindrical portion = $\frac{3}{2}\ cm$

Height of each cone = 2 cm

Height of cylindrical portion = 12 – 2 – 2 = 8 cm

Volume of the air contained in the model

= Volume of the cylindrical portion of the model + Volume of two conical ends.

$= \pi r^2 h^1 + 2 \times \frac{1}{3}\pi r^2 h^2 = \pi r^2 \left(h^1 + \frac{2}{3}h^2\right)$

$= \pi \times \left(\frac{3}{2}\right)^2 \times \left(8 + \frac{2}{3} \times 2\right) = \frac{22}{7} \times \frac{9}{4} \times \frac{28}{3} = 66\ \text{cm}^3$

Que 6. A gulab jamun, contains sugar syrup about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (Fig. 13.28).

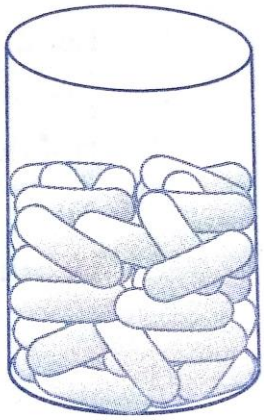

Fig. 13.28

Sol. We have,

Radius of cylindrical portion and hemispherical portion of a gulab jamun

$$= \frac{2.8}{2} = 1.4\ cm$$

Length of cylindrical portion = 5 – 1.4 – 1.4 = 2.2 cm

Volume of one gulab jamun

= Volume of the cylindrical portion + Volume of the hemispherical ends

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 \ = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi r^2 \left(h + \frac{4}{3} r\right) \ = \frac{22}{7} \times (1.4)^2 \left(22 + \frac{4}{3} \times 1.4\right)$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times \left(\frac{6.6+5.6}{3}\right) \ = \frac{22}{7} \times 1.96 \times \frac{12.2}{3}$$

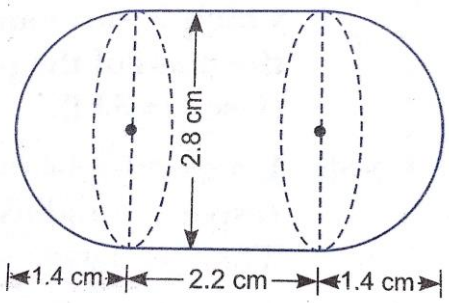

Fig. 13.29

∴ Volume of 45 gulab jamuns $= 45 \times \frac{22}{7} \times 1.96 \times \frac{12.2}{3}$

∴ Quantity of syrup in 45 gulab jamuns = 30% of their Volume

$$= \frac{30}{100} \times 45 \times \frac{22}{7} \times 1.96 \times \frac{12.2}{3}$$

= 338.184 cm^3 = 338 cm^3 (approx.)

Que 7. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the

volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$)

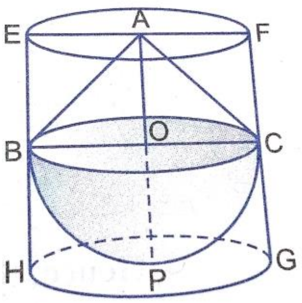

Fig. 13.30

Sol. Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere (see Fig. 13.30).

The radius BO of the hemisphere (as well as of the cone) = $\frac{1}{2} \times 4\ cm = 2\ cm$

So, Volume of the toy = $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$$= \left[\frac{2}{3} \times 3.14 \times (2)^3 + \frac{1}{2} \times 3.14 \times (2)^2 \times 2\right] cm^3 = 25.12\ cm^3$$

Now, let the right circular cylinder EFGH circumscribe the given solid. The radius of the base of the right circular cylinder = HP = BO = 2 cm and its height is
EH = AO + OP = (2 + 2) cm = 4 cm
So, the required Volume
= Volume of the right circular cylinder – Volume of the toy
= $(3.14 \times 2^2 \times 4 - 25.12)\ cm^3 = 25.12\ cm^3$
Hence, the required difference of the two volumes = 25.12 cm^3.

Que 8. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (Fig. 13.31).

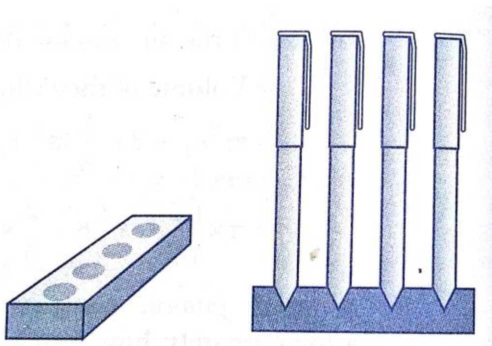

Fig. 13.31

Sol. We have,

Length of cuboid = l = 15 cm

Breadth of cuboid = b = 10 cm
Height of cuboid = h = 3.5 cm
And radius of conical depression = 0.5 cm
Depth of conical depression = 1.4 cm
Now, Volume of wood in the entire pen stand

= Volume of cuboid – 4 × Volume of a conical depression

$= lbh - 4 \times \frac{1}{3}\pi r^2 h \;= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

$= (525 - 1.47)\ cm^3 \;= 523.53\ cm^2$

Que 9. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. ($\boldsymbol{Use\ \pi = 3.14}$).

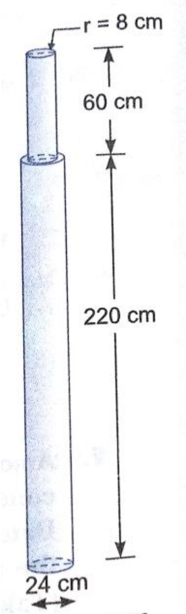

Fig. 13.32

Sol. Let r_1 and h_1 be the radius and height of longer cylinder, respectively, and r_2, h_2 be the respective radius and height of smaller cylinder mounted on the longer cylinder.
Then we have,

r_1 = 12 cm, h_1 = 220 cm
r_2 = 8 cm, h_2 = 60cm

Now, Volume of solid iron pole

= Volume of the longer cylinder + Volume of smaller cylinder
$= \pi {r_1}^2 h_1 \;+\; \pi {r_2}^2$
$= 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$
$= 3.14 \times 144 \times 220 + 3.14 \times 64 \times 60$
= 99475.2 + 12057.6 = 111532.8 cm_3

Hence, the mass of the pole = (111532.8 x 8) grams

$$= \frac{111532.8 \times 8}{1000}\ kg \;=\; 892.2624\ kg$$

Que 10. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

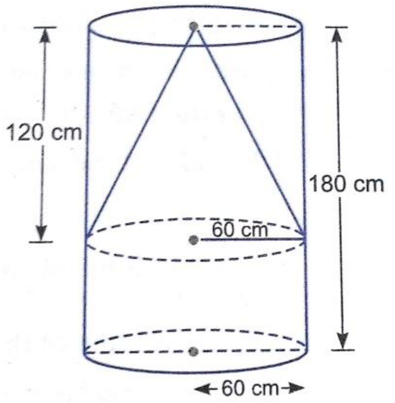

Fig. 13.33

Sol. We have,

Radius of cylinder = Radius of cone = Radius of hemisphere = 60 cm

Height of cone = 120 cm

∴ Height of cylindrical vessel = 120 + 60 = 180 cm

Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times (60)^2 \times 180$

Now, volume of the solid

= Volume of cone + Volume of hemisphere

$= \frac{1}{3} \times \frac{22}{7} \times (60)^2 \times 120 + \frac{2}{3} \times \frac{22}{7} \times (60)^3$

Volume of water left in the cylinder

= Volume of the cylinder – Volume of the solid

$= \frac{22}{7} \times (60)^2 \times 180 - \frac{1}{3} \times \frac{22}{7} (60)^2 \times 120 - \frac{2}{3} \times \frac{22}{7} \times (60)^3$

$= \frac{22}{7} (60)^2 \left[180 - \frac{1}{3} \times 120 - \frac{2}{3} \times 60\right]$

$= \frac{22}{7} \times 3600[180 - 40 - 40] \ = \frac{22 \times 3600 \times 100}{7}$

$= \frac{7920000}{7} = \frac{792}{700} m^3 \ = 1.131\ m^3$ (approx.)