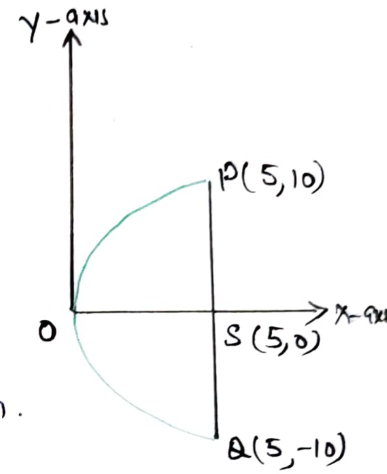
MISCELLANEOUS EXERCISE

Q No 1 If a parabolic reflector is 20cm in diameter and 5 cm deep, find the focus.

Sol Let vertex of parabolic reflector be at origin and x-axis be the axis of Parabola.



Let equation of Parabola be

$$y^2 = 4ax \quad \text{---} (1)$$

Since depth = 5 cm, diameter = 20m.

$\therefore$ $P(5,10)$ lies on Parabola (1)

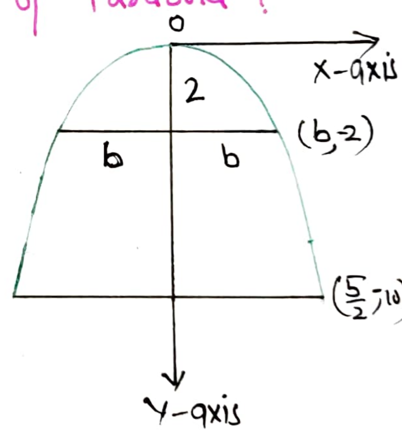
$\therefore$ $100 = 4a \times 5 \Rightarrow a = 5$

$\therefore$ Focus S is $(a,0)$ i.e $(5,0)$

Which is mid-point of diameter.

Q No 2. An arch is in form of Parabola with its axis vertical. The arch is 10 m high and 5 m wide at the base. How wide is it 2 m from vertex of Parabola?

Sol. Since axis of parabola is vertical and 5 m wide at base



$\therefore$ Let equation of Parabola be

$$x^2 = -4ay \quad \text{--} (1)$$

Since arch is 10 m high and 5 m wide at base.

$\therefore$ Point $\left(\frac{5}{2}, -10\right)$ lies on parabola.

$\therefore$ $\frac{25}{4} = -4a(-10) \Rightarrow 4a = \frac{5}{8}$

$\therefore$ Eqn. (1) becomes $x^2 = -\frac{5}{8}y$ ... (2)

Let width of arch 2 m from vertex be $2b$.

$\therefore$ Point $(b,-2)$ lies on (2)

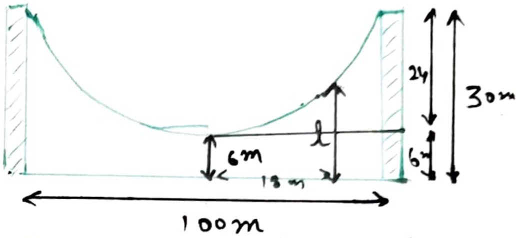
$\therefore$ $b^2 = -\frac{5}{8}(-2) = \frac{5}{4} \Rightarrow b = \frac{\sqrt{5}}{2}$

$\therefore$ Width of Arch $= 2b = 2 \times \frac{\sqrt{5}}{2} = \sqrt{5}$ m ≈ 2.23 m (approx

Q.No 3 The cable of a uniformly loaded suspension bridge hangs in the form of parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from middle.

Sol The cable is in form of parabola

$$x^2 = 4ay \quad \text{---} \quad (1)$$



Focus is at the middle of the cable and shortest and longest vertical wire supports are 6 m and 30 m and the roadway is 100 m wide.

$\therefore$ Pt $(50, 24)$ lies on parabola (1)

$\therefore$ $(50)^2 = 4a(24)$ or $4a = \frac{625}{6}$

$\therefore$ Eqn of parabola is $x^2 = \frac{625}{6}y$;

Let the support at 18 m from middle be l

Then the point $(18, l-6)$ lies on parabola.

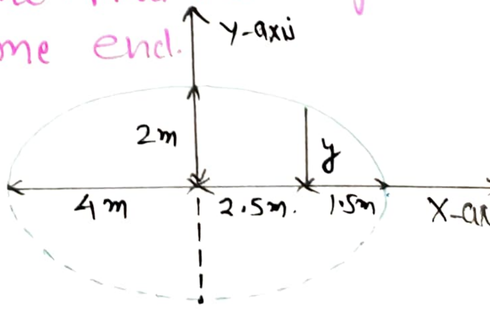
$\therefore$ $(18)^2 = \frac{625}{6}(l-6)$

$\Rightarrow$ $l-6 = \frac{18 \times 18 \times 6}{625}$

$\Rightarrow$ $l = \frac{18 \times 18 \times 6}{625} + 6 = 3.11 + 6 = 9.11$ (approx.)

$\therefore$ Length of support $= 9.11$ m (approx)

Q.No. 4. An arch is in the form of a semi-ellipse. It is 8 m wide and 2 m high at centre. Find the height of arch at a point 1.5 m from one end.



Sol. Since the arch is 8 m wide and 2 m high at centre in the form of a semi-ellipse

$\therefore$ Arch is part of ellipse whose major axis is 8 m and minor axis is 2 m.

i.e. $a = 4$ and $b = 2$.

$\therefore$ Eqn of ellipse is $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$

or $x^2 + 4y^2 = 16$

A point 1.5 m from one end of arch is $4 - 1.5 = 2.5$ m from origin

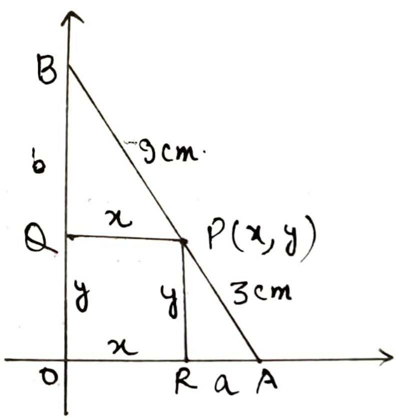
$\therefore$ Height 'y' of the arch at this point will be

$$(2.5)^2 + 4y^2 = 16$$

or $4y^2 = 16 - 6.25 = 9.75$

$\therefore y^2 = \dfrac{9.75}{4} \Rightarrow y = \dfrac{\sqrt{9.75}}{2} = \dfrac{3.1}{2} = 1.56$ (approx.)

Q.No 5 A rod of length 12 cm moves with its ends always touching the coordinate axes. Determine the eqn. of locus of a point P on the rod which is 3 cm from the end in contact with x-axis.



Sol Let $AB = 12$ cm be the rod and $P(x, y)$ is point on rod such that

$PA = 3$ cm.

$\therefore PB = AB - PA = 12 - 3 = 9$ cm.

Draw $PQ \perp OB$ and $PR \perp OA$

Let $BQ = b$ and $AR = a$.

Then as $\Delta BQP \approx \Delta PRA$

$\Rightarrow \dfrac{b}{9} = \dfrac{y}{3}$ and $\dfrac{a}{3} = \dfrac{x}{9}$

$\therefore b = 3y$ and $a = \dfrac{1}{3}x$.

$\therefore OA = x + a = x + \dfrac{1}{3}x = \dfrac{4}{3}x$

$OB = y + b = y + 3y = 4y$.

Since $OA^2 + OB^2 = AB^2$

$\therefore \frac{16}{9}x^2 + 16y^2 = 144.$

or $\frac{x^2}{81} + \frac{y^2}{9} = 1.$

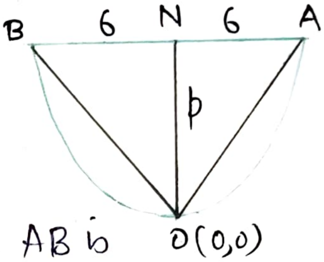
Which is locus of P and is an ellipse.

Q No. 6: Find the area of triangle formed by the lines joining the vertex of parabola $x^2 = 12y$ to the ends of its latus rectum.

Sol The parabola is $x^2 = 12y$ --- (1)

$\therefore$ Length of latus-rectum $= 4a$

$= 4a = 12$



Also the distance $p = ON$ of vertex O from latus rectum AB is

$(6)^2 = 12p$ or $p = 3.$

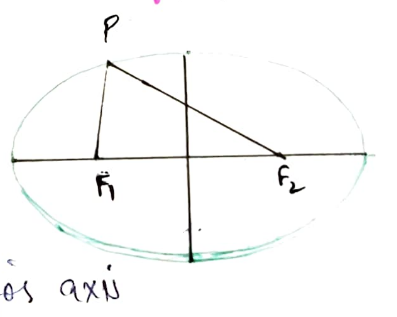
$\therefore$ Area of Δ formed by joining vertex O to the ends A and B of latus rectum

$= ar(\Delta AOB) = \frac{1}{2} \times AB \times ON$

$= \frac{1}{2} \times 12 \times 3 = 18$ sq. units.

Q No 7. A man running a racecourse notes that the sum of distances from two flag posts from him is always 10m and the distance between the flag posts is 8 m. Find the equation of the posts traced by the man.

Sol. The path traced by man will be an ellipse whose foci F_1 and F_2 will be flag posts and sum of distances of man P from F_1 and F_2 is equal to major axis



Let eqn of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ --- (1)

Now $PF_1 + PF_2 = 2a = 10$

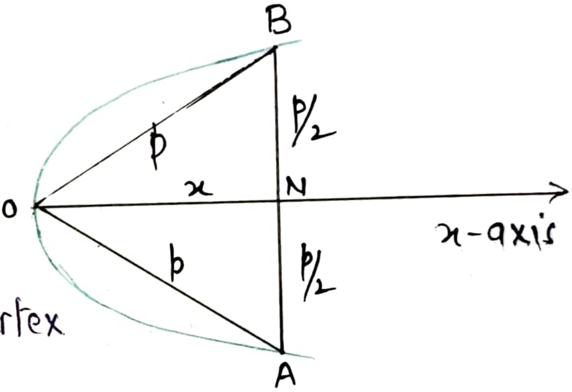
$\therefore a = 5$

$F_1F_2 = 2c = 8 \Rightarrow c = 4.$

$\therefore c^2 = a^2 - b^2 \Rightarrow (4)^2 = (5)^2 - b^2 \Rightarrow b^2 = 9.$

$\therefore$ from (1) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ which is required eqn.

Q No 8. A equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex is at the vertex of the parabola. Find the length of the side of the triangle.



Sol. The equation of parabola is

$$y^2 = 4ax \quad \text{---} (1)$$

Let p be the side of the equilateral ΔOAB whose one vertex of parabola.

$\therefore$ By symmetry AB is perpendicular to the axis ON of parabola.

Let $ON = x$, then $BN = \frac{p}{2}$

$\therefore \left(x, \frac{p}{2}\right)$ lies on parabola (1)

$$\therefore \frac{p^2}{4} = 4ax.$$

$$\therefore x = \frac{p^2}{16a}.$$

Now from ΔONB

$$OB^2 = ON^2 + NB^2$$

or $$p^2 = \frac{p^4}{(16a)^2} + \frac{p^2}{4}$$

$$\Rightarrow 1 = \frac{p^2}{256a^2} + \frac{1}{4}$$

$\therefore \quad \frac{p^2}{256\,a^2} = \frac{3}{4}$

$\therefore \quad p^2 = 256 \times a^2 \times \frac{3}{4}$

or. $p^2 = 16 \times 4 \times 3 \times a^2$

$p = 4 \times 2 \times \sqrt{3}\,a. = 8\sqrt{3}\,a.$

$\therefore$ Side of the tsiangle $= 8\sqrt{3}\,a$.

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