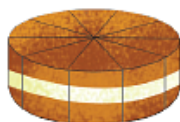# Decimals

Tenths

There are ten children attending Pooja's birthday party. Her mother cuts the cake into 10 equal parts. Each part is one-tenth of the whole cake.

Look at the following figures.
Each of the rectangles has been divided into ten equal parts. How many parts are shaded? The fraction is written below each figure.

One-tenth

Four-tenths

Nine-tenths

$$\frac{1}{10}$$

$$\frac{4}{10}$$

$$\frac{9}{10}$$

The denominator of these fractions is 10. These are special types of fractions called **decimal fractions**.

A fraction with denominator 10, 100, 1000, … is called a decimal number or simply **decimal**.

The fraction 1 / 10 (one-tenth) can be written as 0.1. Similarly, 4 / 10 is written as **0.4** and 910 is written as **0.9**. The numerals 0.1, 0.4 and 0.9 are called **decimal numerals** or simply **decimals**. The dot in 0.1, 0.4 and 0.9 is called a **decimal point**. 0.1 can be read in two ways as under:

0.1 is read as → one-tenth.
→ zero point one.

This system of writing numbers with a decimal point is called the **decimal system**.

Tips: The convention is to write a decimal numeral for a fraction like 5 / 10 as 0.5 and not .5. A zero is put before the point in 0.5. Thus, 1 / 10 is 0.1, 2 / 10 is 0.2, 3 / 10 is 0.3 and so on.
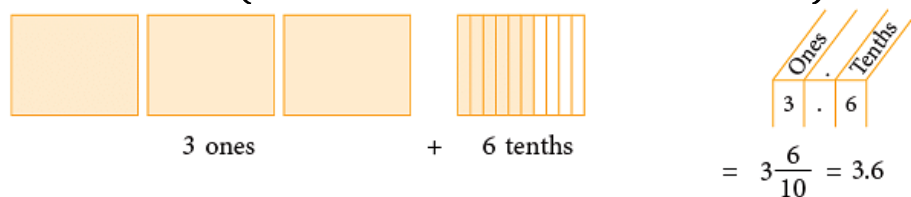
Decimals and Whole Numbers

Whole numbers can also be renamed using a decimal point.

For example: $1 = 1.0; 3 = 3.0; 5 = 5.0; 7 = 7.0; 9 = 9.0$
The zero after the decimal point means there are 0 tenths.

More than One (Mixed Numbers and Mixed Decimals)

3 ones   +   6 tenths

$$= \ 3\frac{6}{10} = 3.6$$

You can use a mixed decimal to name a mixed number. $3\frac{6}{10}$ and 3.6 are both read as "three and six-tenths."
The table below illustrates different ways of reading and writing tenths. The zero after the decimal point means there are 0 tenths.

Read	Write as Fraction	Write as Decimal	Read the Decimal as
One-tenth	$\frac{1}{10}$	0.1	Zero point one
Three-tenths	$\frac{3}{10}$	0.3	Zero point three
Ten-tenths	$\frac{10}{10}$	1.0 = 1	One
Six and zero tenths	$6\frac{0}{10}$	6.0 = 6	Six
One and one-tenth	$1\frac{1}{10}$	1.1	One point one
Eight and nine-tenths	$8\frac{9}{10}$	8.9	Eight point nine
Twenty-six and five-tenths	$26\frac{5}{10}$	26.5	Twenty-six point five

The decimal numbers can also be shown on a number line. The following picture shows a number line which has been divided into tenths.
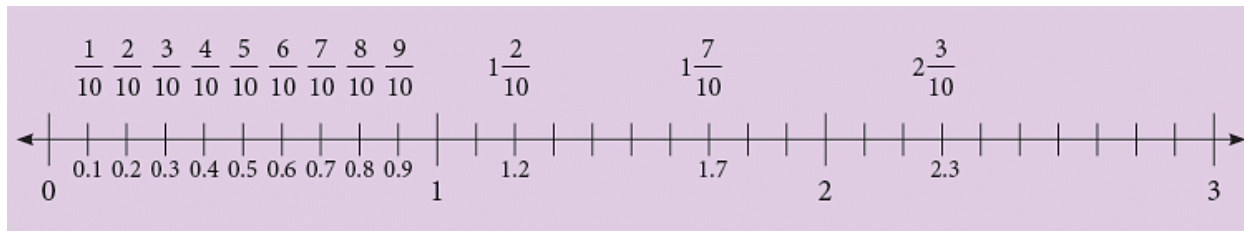
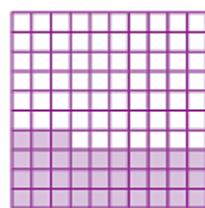## Hundredths

There are 100 small squares in the large square. 1 small square is one -hundredth or 1 / 100 of the large square. The shaded part consists of 9 small squares. 9 small squares make "**nine-hundredths**" of the large square. We say that the shaded part is **9 / 100** or **0.09**.
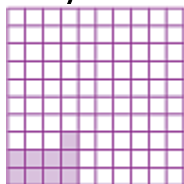
Since there are two zeros in the denominator of 9 / 100, therefore, there are two digits to the right of the decimal point in 0.09.

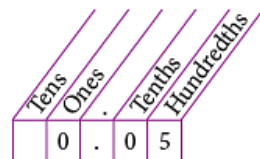**Now, look at the adjacent figure:**

$$\frac{33}{100} = 0.33$$

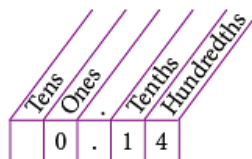$$\frac{33}{100} = \frac{30}{100} + \frac{3}{100}$$

The shaded part is "thirty-three" hundredths of the large square.
Notice that there are two digits after the decimal point when writing a decimal to show hundredths.

$$\frac{5}{100} = 0.05$$

Five-hundredths

$$\frac{14}{100} = 0.14$$

Fourteen-hundredths
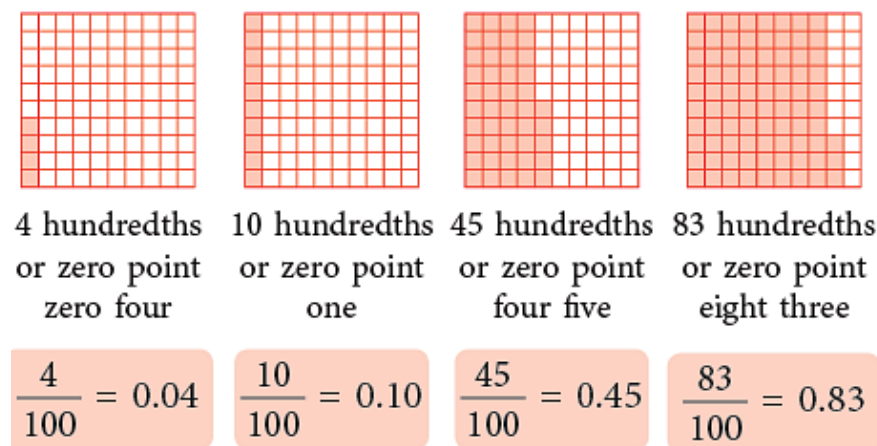
Reading a Decimal Number

0.01 is read as
- one-hundredth.
- or
- zero point zero one.

0.08 is read as
- eight-hundredths.
- or
- zero point zero eight.

0.10 is read as
- ten-hundredths.
- or
- zero point one zero.

0.33 is read as
- thirty-three hundredths.
- or
- zero point three three.

The following diagrams are drawn to help you gain complete understanding:

4 hundredths or zero point zero four	10 hundredths or zero point one	45 hundredths or zero point four five	83 hundredths or zero point eight three
$\dfrac{4}{100} = 0.04$	$\dfrac{10}{100} = 0.10$	$\dfrac{45}{100} = 0.45$	$\dfrac{83}{100} = 0.83$

0.10 = 1 tenth and 0 hundredths
0.45 = 4 tenths and 5 hundredths
0.83 = 8 tenths and 3 hundredths

Writing Mixed Decimals for a Mixed Number

A mixed decimal can be written for a mixed number as under:

Fraction Name	Mixed Number	Decimal Number	Read the Decimal Number as
Four and five-hundredths	$4\dfrac{5}{100}$	4.05	Four point zero five
Twelve and eighteen-hundredths	$12\dfrac{18}{100}$	12.18	Twelve point one eight
Three hundred twenty-five and thirty-eight hundredths	$325\dfrac{38}{100}$	325.38	Three hundred twenty-five point three eight

From the above examples, we can see that:
A decimal number is made up of two parts: a **whole number** part and a **decimal** part separated by a **decimal point**. The number of digits after the decimal point is known as the number of **decimal places**.

For example: 31.9 has one decimal place.
583.28 has two decimal places.

31.9 ———— One decimal place
583.28
———— Two decimal places
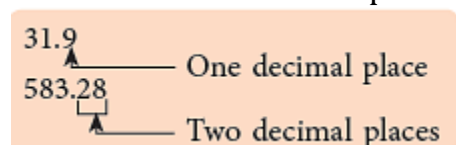
Thousandths

One-thousandth means 1 / 1000. If you divide a square into 1000 small squares, then one small square denotes **one-thousandth** part or 1 / 1000, which is written in decimal form as **0.001**.

1 / 10 = 0.1 meaning one-tenth
1 / 100 = 0.01 meaning one-hundredth
1 / 1000 = 0.001 meaning one-thousandth

The decimal number 0.001 is read as zero point zero zero one. Similarly, we have,

$$\frac{5}{1000} = 0.005; \quad \frac{37}{1000} = 0.037; \quad \frac{288}{1000} = 0.288; \quad \frac{625}{1000} = 0.625$$

Tips: Since there are three zeros in the denominator, therefore, there are three digits to the right of the decimal point.

You can read and write thousandths in different ways as given below in the table.

Fraction Name	Fraction	Decimal	Decimal Number
Seven-thousandths	$\frac{7}{1000}$	0.007	Zero point zero zero seven
Sixty-five thousandths	$\frac{65}{1000}$	0.065	Zero point zero six five
Three hundred sixty-thousandths	$\frac{360}{1000}$	0.360	Zero point three six-zero
Fifteen and seven-thousandths	$15\frac{7}{1000}$	15.007	Fifteen point zero zero seven
Four hundred ninety-one and seven hundred fifty-thousandths	$491\frac{750}{1000}$	491.750	Four hundred ninety-one point seven five zero

Thus, the number of digits after the decimal point is the same as the number of zeros in the denominator of the fraction.

Tips: The digits after the decimal point are read one by one. For example, 17.035 is read as seventeen point zero three five and not as sevaenteen point thirty-five.

Decimal Fractions in the Place Value Chart

The place value chart that you have studied till now can be shown as under:

Ten thousands	Thousands	Hundreds	Tens	Ones
10000	1000	100	10	1

$\div 10 \qquad \div 10 \qquad \div 10 \qquad \div 10$

Starting from the left and moving towards the right,

$1000 = \dfrac{1}{10}$ of 10000; $100 = \dfrac{1}{10}$ of 1000; $10 = \dfrac{1}{10}$ of 100; $1 = \dfrac{1}{10}$ of 10.

As we move towards the right, each place value becomes one-tenth of the previous one.

Observing the above pattern, you can now guess that to show one-tenth, one-hundredth and one-thousandth on the place value chart, we have to extend it to the right of ones.
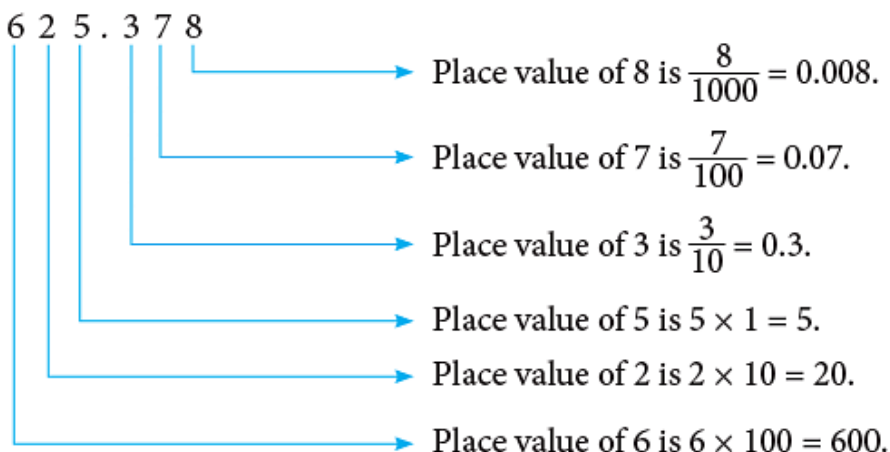
Thus, the place value chart extended beyond ones to the right is as given below.

TTh	Th	H	T	O	Decimal Point	Tenths	Hundredths	Thousandths
10000	1000	100	10	1	.	$\dfrac{1}{10}$	$\dfrac{1}{100}$	$\dfrac{1}{1000}$
						(0.1)	(0.01)	(0.001)

For example, the decimal number 683.459 can be shown in the place value chart as:

	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
Digit	6	8	3	4	5	9
Place value	600	80	3	$\dfrac{4}{10}$	$\dfrac{5}{100}$	$\dfrac{9}{1000}$

Example 1: Find the place value of each digit in 625.378.

6 2 5 . 3 7 8

Place value of 8 is $\dfrac{8}{1000} = 0.008$.

Place value of 7 is $\dfrac{7}{100} = 0.07$.

Place value of 3 is $\dfrac{3}{10} = 0.3$.

Place value of 5 is $5 \times 1 = 5$.

Place value of 2 is $2 \times 10 = 20$.

Place value of 6 is $6 \times 100 = 600$.

Expanded Form

Let us write the expanded form for the number 634.957.

634.957 = 6 hundreds + 3 tens + 4 ones + 9 tenths + 5 hundredths + 7 thousandths

$$= 600 + 30 + 4 + \frac{9}{10} + \frac{5}{100} + \frac{7}{1000} = 600 + 30 + 4 + 0.9 + 0.05 + 0.007$$

This is the **expanded form** of the number 634.957.
We give some more examples to help you get better understanding.

Example 2: Write each decimal numeral in expanded form.
(a) 5.86
(b) 0 .476
(c) 6 5.26
(d) 3 92.067

(a) 5.86 = 5 ones + 8 tenths + 6 hundredths

$$= 5 + \frac{8}{10} + \frac{6}{100}$$

(b) 0.476 = 4 tenths + 7 hundredths + 6 thousandths

$$= \frac{4}{10} + \frac{7}{100} + \frac{6}{1000}$$

(c) $65.26 = 6$ tens $+ 5$ ones $+ 2$ tenths $+ 6$ hundredths

$$= 60 + 5 + \frac{2}{10} + \frac{6}{100}$$

(d) $392.067 = 3$ hundreds $+ 9$ tens $+ 2$ ones $+ 0$ tenths $+ 6$ hundredths $+ 7$ thousandths

$$= 300 + 90 + 2 + \frac{0}{10} + \frac{6}{100} + \frac{7}{1000}$$

Equivalent Decimals

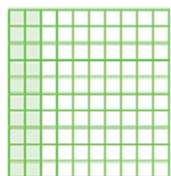The shaded square on the right shows that:
$0.2 = 0.20$

  2 tenths   20 hundredths

$= \frac{2}{10} = 0.2$ $= \frac{20}{100} = 0.20$

Thus, we can rename a decimal by writing as many zeros as we like after the last digit in a decimal number.
We have,
$0.5 = 0.50 = 0.500 = 0.5000$ etc.

$\boxed{0.5 = 0.50 = 0.500 = 0.5000}$ ← **Equivalent decimals**

Tenths can be renamed as hundredths as well as thousandths: $0.1 = 0.10 = 0.100$.
Hundredths can be renamed as thousandths: $0.08 = 0.080$; $0.37 = 0.370$.

Money and Decimals
You know that 100 paise make 1 rupee, i.e., $100 \text{ p} = 1$ rupee or ₹ 1.
Therefore,

- $1 \text{ p} = \frac{1}{100}$ rupee $= 0.01$ rupee $=$ ₹ 0.01;

- $5 \text{ p} = \frac{5}{100}$ rupee $= 0.05$ rupee $=$ ₹ 0.05;

- $10 \text{ p} = \dfrac{10}{100} \text{ rupee} = 0.10 \text{ rupee} = ₹\, 0.10;$

- $15 \text{ p} = \dfrac{15}{100} \text{ rupee} = 0.15 \text{ rupee} = ₹\, 0.15.$

Hence, 3 rupees 50 paise is written as ₹ 3.50 and read as rupees three and fifty paise. In the same way, rupees six and forty paise is written as ₹ 6.40.
Also, one hundred eighty-four rupees and seventy-five paise in figures is written as ₹ 184.75. This shows that we use a decimal point to separate rupees from paise.

Conversions of Rupees to Paise
To convert rupees into paise, we multiply the amount in rupees by 100.

Examples:
₹ 7 = 7 × 100 p = 700 p; ₹ 69 = 69 × 100 p = 6900 p;
₹ 225 = 225 × 100 p = 22500 p

To convert rupees and paise into paise, multiply the amount in rupees by 100 and add it to the number of paise.

Example: ₹ 25.30 = 25 rupees 30 paise
= 25 × 100 p + 30 p
= 2500 p + 30 p = 2530 p;
₹ 20.55 = 2055 p; ₹ 89.60 = 8960 p;
₹ 128.25 = 12825 p

Tips: In fact, you can just remove the decimal point to get the number of paise.
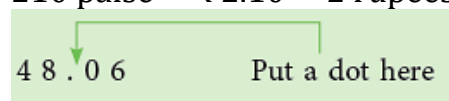
Conversion of Paise to Rupees
To convert paise into rupees or rupees and paise, we divide the amount by 100.

To convert paise into rupees and paise, we put a dot (.) after 2 digits from the right of the given number showing paise. The numeral on the left of the dot indicates the number of rupees and that on the right of the dot indicates the number of paise.

Examples: 4806 paise = ₹ 48.06 = 48 rupees 6 paise;
89 paise = ₹ 0.89;
210 paise = ₹ 2.10 = 2 rupees 10 paise

4 8 . 0 6   Put a dot here

Addition and Subtraction of Money

While adding and subtracting money, take care to place the decimal (point) exactly one below the other and then add or subtract like you add or subtract any other numbers.

Remember to put the decimal in the same place (two digits from the right) in the answer.

Example 3: Shyam purchased a pencil worth ₹ 3.40 and an eraser worth ₹ 2.10. How much did he spend in all?

Step 1: Arrange the amount in proper columns.

Step 2: Add as ordinary numbers.

Step 3: Place the decimal point to separate rupees from paise.

Ones	.	Tenths	Hundredths
3	.	4	0
+ 2	.	1	0
5	.	5	0

₹ 3.40
+ ₹ 2.10
₹ 5.50

Hence, Shyam spent ₹ **5.50** in all.

Example 4: Amit carried ₹ 5.40 to a shop. He bought a candy worth ₹ 1.20. How much money did he take back home?

Ones	.	Tenths	Hundredths
5	.	4	0
− 1	.	2	0
4	.	2	0

₹ 5.40
− ₹ 1.20
₹ 4.20

Hence, the amount that Amit took back home = ₹ **4.20.**

Example 5: Find the total money due in the bill given alongside. If a 100-rupee note and a 50-rupee note were paid to the shopkeeper, then what was the amount received as change?

Total amount of money $= ₹\,100 + ₹\,50$
$= ₹\,150$

Bill	
Items	Amount
Sketch pens	₹ 10.50
Water bottle	₹ 42.25
Tiffin box	₹ 42.25
Pencil box	₹ 15.75

Bill	
Items	Amount
Sketch pens	₹ 10.50
Water bottle	₹ 42.25
Tiffin box	₹ 42.25
Pencil box	₹ 15.75
Total	₹ 110.75

Amount paid to the shopkeeper = 110.75
Amount received as a change
$= ₹\,150 – ₹\,110.75 = ₹\,39.25$

```
    ₹ 150.00
 –  ₹ 110.75
    ₹  39.25
```

Hence, the amount received as change = **₹ 39.25.**

Multiplication of Money
Multiplication of money is done in the same way as you would multiply whole numbers. However, we must not forget to put the decimal point after two digits from the right in the product.

Example 6: How much will 3 ice creams cost if each costs ₹ 16.50?

Step 1: Arrange the numbers properly and multiply as ordinary numbers.

Step 2: Put a decimal point two places from the right in the answer.

```
  1 6 . 5 0
      × 3
  4 9 . 5 0
```

Decimal point

Thus, 3 ice creams will cost ₹ **49.50**.

Example 7: Heena bought 15 pairs of earrings, each pair costing ₹ 8.50. How much change did she get back from ₹ 150?

Cost of 1 pair of earrings = ₹ 8.50
Cost of 15 pairs of earrings = ₹ 8.50 × 15 = ₹ 127.50

Amount she got back = ₹ 150.00 – ₹ 127.50
= ₹ 22.50

```
    8 . 5 0
     × 1 5
    4 2 5 0
  + 8 5 0 0
  1 2 7 . 5 0
```

Hence, Heena got ₹ **22.50** back as change.

Division of Money by a Whole Number

- Divide as ordinary numbers.
- Place the decimal point in the quotient above the point in the dividend.

Example 8: Aisha paid ₹ 513.18 for 9 books. How much did she pay for each book?

```
          5 7 . 0 2
  9 ) 5 1 3 . 1 8
    – 4 5
        6 3
      – 6 3
              1 8
            – 1 8
                 0
```

Hence, Aisha paid ₹ **57.02** for each book.

Estimating Total Cost

Rule: For a quick mental estimate, round off each addend to its greatest place value and then add the rounded numbers.

Example 9: Harish and Ritesh went to a CD store to buy the following DVDs.

DVD's Name	Quantity	Price of one DVD
Superman	2	₹ 390.50
Home Alone	3	₹ 212.85
Harry Potter	1	₹ 325.60

Estimate the total amount needed to purchase the above DVDs.

DVD's Name	Quantity	Actual Price of 1 DVD	Estimated Price of (1 DVD)	Estimated Price (as per the given quantity)
Superman	2	₹ 390.50	₹ 400	₹ 400 × 2 = ₹ 800
Home Alone	3	₹ 212.85	₹ 200	₹ 200 × 3 = ₹ 600
Harry Potter	1	₹ 325.60	₹ 300	₹ 300 × 1 = ₹ 300
				Total = ₹ 1700

Thus, the estimated total cost for the DVD's is ₹ **1700.**