CBSE Test Paper 05

Chapter 12 Linear Programming

1. Maximize Z = –x + 2y, subject to the constraints: $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.

    a. Z has no maximum value
    b. Maximum Z = 14 at (2, 6)
    c. Maximum Z = 12 at (2, 6)
    d. Maximum Z = 10 at (2, 6)

2. Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4. Find $P(A \cup B)$

    a. 0.62
    b. 0.58
    c. 0.51
    d. 0.55

3. Maximize Z = x + y, subject to $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$.

    a. Maximum Z = 14 at (2, 6)
    b. Maximum Z = 12 at (2, 6)
    c. Z has no maximum value
    d. Maximum Z = 8 at (2, 6)a

4. In linear programming, optimal solution

    a. satisfies all the constraints only
    b. maximizes the objective function only
    c. is not unique
    d. satisfies all the constraints as well as the objective function

5. Minimize Z = x + 2y subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$.

    a. Minimum Z=6 at all the points on the line segment joining the points (6,0) and (0,3)
    b. Minimum Z=7 at all the points on the line segment joining the points (6,0) and (0,3)
    c. Minimum Z=8 at all the points on the line segment joining the points (6,0) and (0,3)

d. Minimum Z=9 at all the points (6, 0) and (0, 3)

6. A feasible region of a system of linear inequalities is said to be ________ , if it can be enclosed within line(s).
7. The process of obtaining the optimal solution of the linear programming problem is called ________.
8. In a LPP, the objective function is always ________.

9. Maximise the function Z = 11x + 7y, subject to the constraints:
$x \leqslant 3, y \leqslant 2, x \geqslant 0, y \geqslant 0.$

10. Determine the maximum value of Z = 11x + 7y subject to the constraints:
$2x + y \leqslant 6, x \leqslant 2, x \geqslant 0, y \geqslant 0.$

11. If a young man rides his motor-cycle at 25 km per hour, he has to spend of Rs 2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per h, the petrol cost increases to Rs 5 per km and rate of pollution also increases. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as an LPP. Solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question?

12. A housewife wishes to mix together two kinds of food, X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of 1 kg of food is given below

	Vitamin A	**Vitamin B**	**Vitamin C**
Food X	1	2	3
Food Y	2	2	1

1 kg of food X costs Rs 6 and 1 kg of food Y costs Rs 10. Formulate the above problem as a linear programming problem and find the least cost of the mixture which will produce the diet graphically. What value will you like to attach with this problem?

13. Minimise and maximise Z = x + 2y
subject to $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200\ x, y \geq 0$

14. A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as Rs. 10500 and Rs. 9000 respectively. To control weeds, a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit. Form an L.P.P. from the above and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment?

15. Minimize Z=400x + 200y subject to
    $5x+2y\geq 30$
    $2x+y\geq 15$
    $x\leq y$, $x \geq 0, y \geq 0$

16. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs.17.50 per package on nuts and Rs.7.00 per package on bolts. How many packages of each should be produced each day to maximise his profit, if he operates his machines for at most 12 hours a day?

17. One kind of cake requires 200g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cake which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

18. Maximize Z = 3x + 2y subject to $x + 2y \leqslant 10, 3x + y \leqslant 15, x, y \geqslant 0$.

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Solution

1. a. Z has no maximum value

   Explanation: Objective function is Z = - x + 2 y ……………………(1).

   The given constraints are : $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.

Corner points	Z = - x + 2y
D(6,0)	-6
A(4,1)	-2
B(3,2)	1

   Here, the open half plane has points in common with the feasible region.

   Therefore, Z has no maximum value.

2. b. 0.58

   Explanation: Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4

   Since the events are independent, $P(A \cap B) = P(A).P(B)$

   Therefore $P(A \cup B) = P(A) + P(B) = 0.3 + 0.4 - 0.12 = 0.58$

3. c. Z has no maximum value

   Explanation: Objective function is Z = x + y ……………………(1).

   The given constraints are : $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$.

   Here , there is no common feasible region between the lines $x - y = -1$ and $-x + y = 0$.

   Therefore , it has no solution. Thus , Z has no maximum value.

4. d. satisfies all the constraints as well as the objective function

   Explanation: In linear programming, any point in the feasible region which gives that gives the optimal value (maximum or minimum) of the objective function is called optimal solution. In other words, it satisfies all the constraints as well as the objective function.

5. a. Minimum Z = 6 at all the points on the line segment joining the points (6, 0) and (0, 3)

Explanation: Objective function is Z = x + 2 y ....(1).

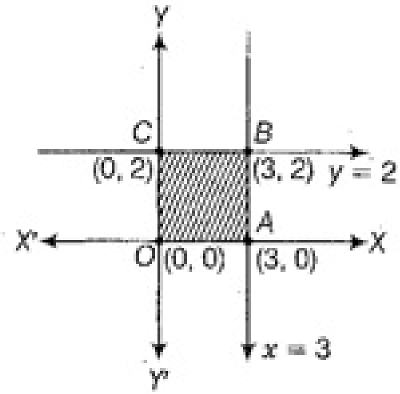
The given constraints are : 2x + y ≥ 3, x + 2y ≥ 6, x, y ≥ 0.

Corner points	Z = x + 2y
A(0 ,3)	6..(Minimum)
B(6,0)	6(Minimum)

Here, Z = 18 is minimum at (0, 3) and (6 , 0).

Minimum Z = 6 at all the points on the line segment joining the points (6, 0) and (0, 3).

6. bounded
7. optimisation technique
8. Linear
9. Maximise Z = 11x + 7y, subject to the constraints $x \leqslant 3, y \leqslant 2, x \geqslant 0, y \geqslant 0$.



The shaded region as shown in the figure as OABC is bounded and the coordinates of corner points are (0, 0), (3, 0), (3, 2), and (0, 2), respectively.

Corner Points	**Corresponding value of Z**
(0, 0)	0
(3, 0)	33
(3, 2)	47 (Maximum)
(0, 2)	14

Hence, Z is maximise at (3, 2) and its maximum value is 47.

10. We have, maximise Z = 11x + 7y ...... (i)

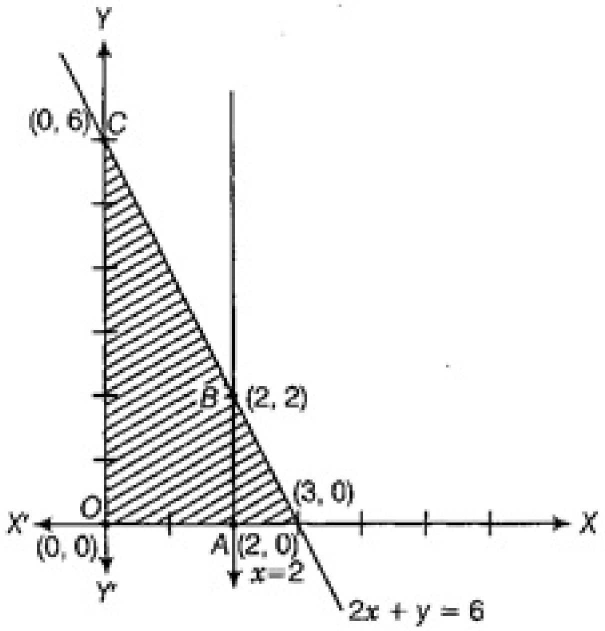
Subject to the constraints

$2x + y \leqslant 6$ ..... (ii)

$x \leqslant 2$ .... (iii)

$x \geqslant 0, y \geqslant 0$ ..... (iv)

We see that, the feasible region as shaded determined by the system of constraint (ii) to (iv) is OABC and is bounded. So, now we shall use corner point method to determine the maximum value of Z.



Corner Points	Corresponding value of Z
(0, 0)	0
(2, 0)	22
(2, 2)	36
(0, 6)	42 (Maximum)

Hence, the maximum value of Z is 42 at (0, 6).

11. Let the young man covers x km at the speed of 25 km/h and y km at the speed of 40 km/h. The total distance travelled is $x + y$,

Here, objective function is max $(Z) = x + y$

Cost constraints

According to the question, the cost of 1 km at the speed of 25 km/h = Rs 2

The cost of x km at the speed of 25 km/h = 2x

Also, the cost of 1 km at the speed of 40 km =Rs 5

∴ The cost of y km at the speed of 40 km/h = 5y

So, the total cost of travel (x + y) km= $2x + 5y$

Given, the driver has Rs 100 to spend.

Hence, cost constraints is $2x + 5y \leq 100$

Time constraints

According to the question, Total available time = 1 h

Time to travel a distance of 25 km = 1 h

∴Time to travel a distance of x km = $\frac{x}{25} h$

Also, time to travel a distance of 40 km = 1 h

Time to travel a distance of y km = $\frac{y}{40}$ h

∴The inequation representing time constraint is

$\frac{x}{25} + \frac{y}{40} \leq 1$

$\Rightarrow \quad 8x + 5y \leq 200$

The linear programming problem

is max $(Z) = x + y$

Subject to constraints

2x + 5y$\leq$ 100

8x + 5y$\leq$ 200

x, y $\geq$0

Consider the inequalities as equations,

$2x + 5y = 100$............(i)

$8x + 5y = 200$.....(ii)

$x, y = 0$..............(iii)

Table for line $2x + 5y = 100$ is

x	0	50
y	20	0

So, it passes through the points (0, 20) and (50, 0).

Putting (0,0) in the inequality 2x + 5y $\leq$ 100, we get

2(0) + 5(0) $\leq$ 100 $\Rightarrow$ 0 $\leq$ 100 [which is true]

The half plane is towards the origin.

Table for line 8x + 5y = 200 is

x	0	25
y	40	0

So, it passes through the points (0, 40) and (25, 0)

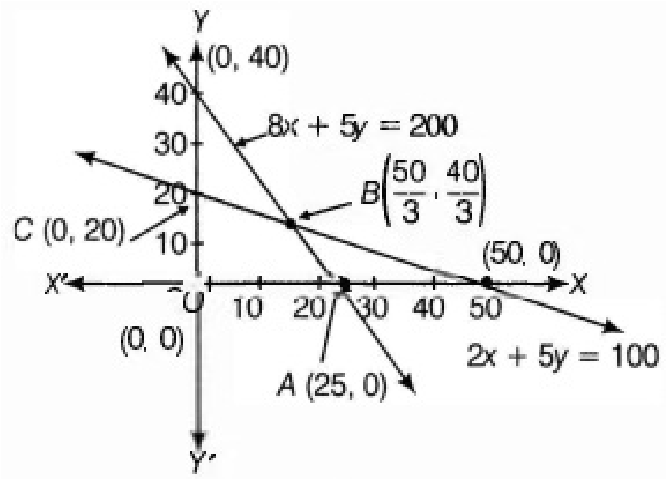
On putting (0,0) in the inequality 8x + 5y$\leq$ 200, we get

8(0 + 5(0) $\leq$ 200 = 0 $\leq$ 200 [which is true]

The half plane is towards the origin

Since, x, y $\geq$ 0, so the feasible region lies in the first quadrant.

The point of intersection of Equations (i) and (ii) is B $\left(\frac{50}{3}, \frac{4}{3}\right)$



The corner points of the feasible region $OABC$ are

$O(0,0), A(25,0), B\left(\frac{50}{3}, \frac{4}{3}\right)$ *and* $C(0,20)$.

Corner Points	Value of Z = x + y
$O(0,0)$	$Z = 0 + 0 = 0$
$A(25,0)$	$Z = 25 + 0 = 25$
B$\left(\frac{50}{3}, \frac{4}{3}\right)$	Z $= \frac{50}{3} + \frac{40}{3}$ = 30(maximum)
$C(0,20)$	$Z = 0 + 20 = 20$

The maximum value of Z is 30 at point B

Hence, young man cover $\frac{50}{3}$ km at the speed of 25 km/h and $\frac{40}{3}$ km at the speed of 210 km/h.

He can travel maximum 30 km in 1 hour.

Here, On increasing the speed of motor-cycle, air pollution and expenditure on petrol also increases.

12. Let the quantity of food X be x kg and the quantity of food Y be y kg

The objective function is to be minimised, $Z = 6x + 10y$.

Subject to the constraints

$x + 2y \geq 10$

$2x + 2y \geq 12$

$3x + y \geq 8$

$x, y \geq 0$

Considering the constraints as equations,

$x + 2y = 10$.....(i)

$2x + 2y = 12$......(ii)

$3x + y = 8$.......(iii)

and $x = 0, y = 0$...(iv)

Table for line $x + 2y = 10$ is

x	0	10
y	5	0

It passes through the points $(0, 5)\ and\ (0, 0)$.

On putting (0, 0) in the inequality x + 2y$\geq$10, we get

$(0 + 20) \geq 10 \Rightarrow 0 \geq 10$ [which is False]

The half plane is away from the origin.

Table for line $2x + 2y = 12$ is

x	0	6
y	6	0

It passes through the points $(0, 6)\ and\ (6, 0)$.

On putting (0,0) in the inequality 2x + 2y$\geq$12, we get

$2(0) + 2(0) \geq 12 \Rightarrow 0 \geq 12$ [which is False]

The half plane is away from the origin.

Table for line $3x + y = 8$ is

x	0	8/3
y	8	0

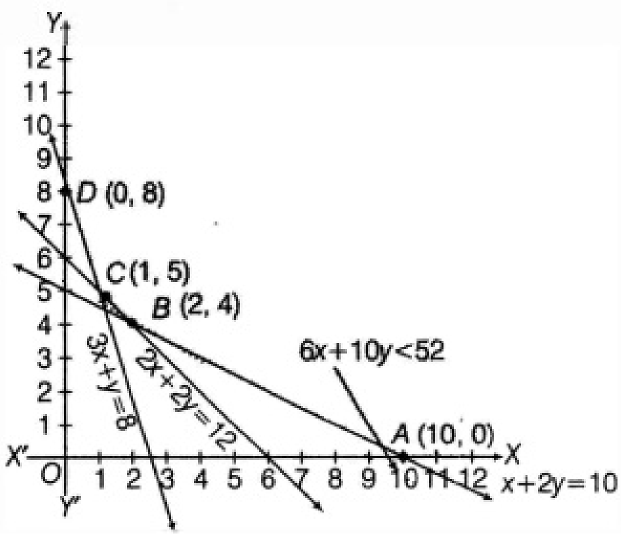
It passes through the points (0,8) and (8/ 3,0).

On putting (0, 0) in the inequality 3x + y$\geq$ 8, we get

$3(0) + 0 \geq 8 \Rightarrow 0 \geq 8$ (which is false)

The half plane is away from the origin.

The points of intersection of lines (i), (ii) and (iii) are $B(2, 4)\ and\ C(1, 5)$.



The extreme points are $A(0,0), B(2,4), C(0,5)\ and\ D(0,8)$

Corner points	Value of Z = 6x + 10y
$A(0,0)$	$Z=6(10)+10(0)=60$
$B(2,4)$	$Z=6(2)+10(4)=52$ (Minimum)
$C(1,5)$	$Z=6(1)+10(5)=56$
$D(0,8)$	$Z=6(0)+10(8)=80$

As the feasible region is unbounded, therefore 52 may or may not be the minimum value of Z.

For this, we draw a dotted graph of the inequality 6x + 10y < 52 and check, whether the resulting half plane has point in common with the feasible region or not. It can be seen that the feasible region has no common point with 3x + 5y< 26.

Therefore, the minimum value of Z is 52 at B(2, 4). Hence, the mixture should contain 2 kg of food X and 4 kg of food Y. The minimum cost of the mixture is Rs 52.

This problem attaches the value of taking a healthy and well-balanced diet which has all the important vitamins in the right proportion.

13. We have linear constraints as

$x+2y \geq 100$

$2x-y \leq 0$

$2x+y \leq 200$

$x \geq 0, y \geq 0$

and objective function is min or max $(Z)=x+2y$

Now, reducing the above inequations into equations and finding their point of intersections i.e.,

$x+2y=100$ ...(i)

$2x-y=0$ ...(ii)

$2x+y=200$ ...(iii)

$x=0, y=0$...(iv)

Equations	Point of Intersection
(i) and (ii)	$x=20$ and $y=40$
	$\Rightarrow$ point is$(20,40)$
(i) and (iii)	$x=100$ and $y=0$
	$\Rightarrow$ point is $(100,0)$
(i) and (iv)	when$x=0 \Rightarrow y=50$
	$\Rightarrow$ point is $(0,50)$
	when $y=0 \Rightarrow x=100$
	$\Rightarrow$ point $(100,0)$
(i) and (iii)	$x=50, y=100$
	$\Rightarrow$ point is $(50,100)$
(i) and (iv)	$x=0, y=0$
	$\Rightarrow$ point is $(0,0)$
(i) and (iv)	when x = 0 ⇒ y = 200
	⇒ point is (0, 200)
	when $y=0, x=100$
	$\Rightarrow$ point is $(100,0)$

Now for feasible region, using origin testing method for each constraint

For $x+2y \geq 100$, let $x=0, y=0$

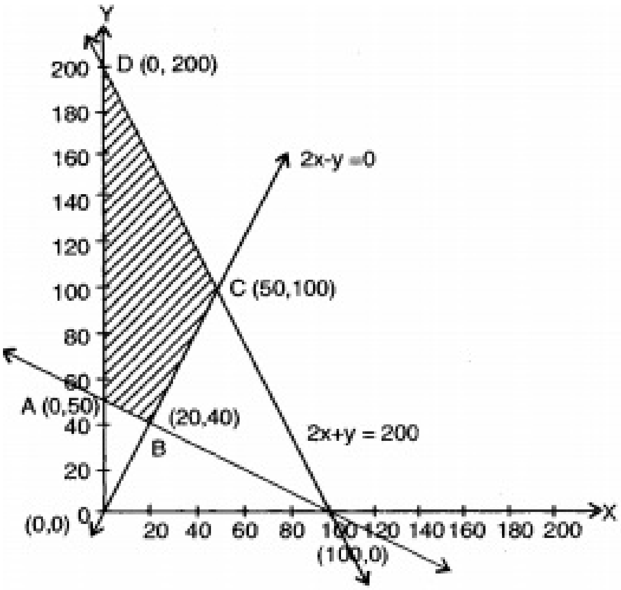
$\Rightarrow 0 \geq 100$ i.e., true $\Rightarrow$ the shaded region will be away from the origin

For $2x+y \leq 200$, let $x=0, y=0$

$\Rightarrow 0 \leq 200$ i.e., true $\Rightarrow$ The shaded region will be toward the origin.

Also, non-negative restrictions $x \geq 0, y \geq 0$ indicates that the feasible region will be exist in first quadrant.

Now, corner points are $A(0,50), B(20,40), C(50,100)\ and\ D(0,200)$



For optimal solution substituting the value of all comer points in $Z = x + 2y$

Corner points	$Z = x + 2y$	
$A(0,50)$	$\text{Z} = 0 + 2 \times 50 = 100$	Minimum
$B(20,40)$	$\text{Z} = 20 + 2 \times 40 = 100$	
$C(50,100)$	$Z = 50 + 2 \times 100 = 250$	
$D(0,200)$	$Z = 0 + 2 \times 200 = 400$	$\rightarrow$ Maximum

Hence, minimum (Z) = 100 at all points on the line segment joining the points A(0, 50) and B(20, 40) : Maximum (Z) = 400 at (0, 200)

14. Let x hectares of land be used for crop A and y hectares of land be used for crop B.

Given LPP is

$Z = 10500x + 9000y$

Subject to constraints

$x + y \leq 50$ ...(i)

$20x + 10y \leq 800$

$\Rightarrow \quad 2x + y \leq 80$ ...(ii)

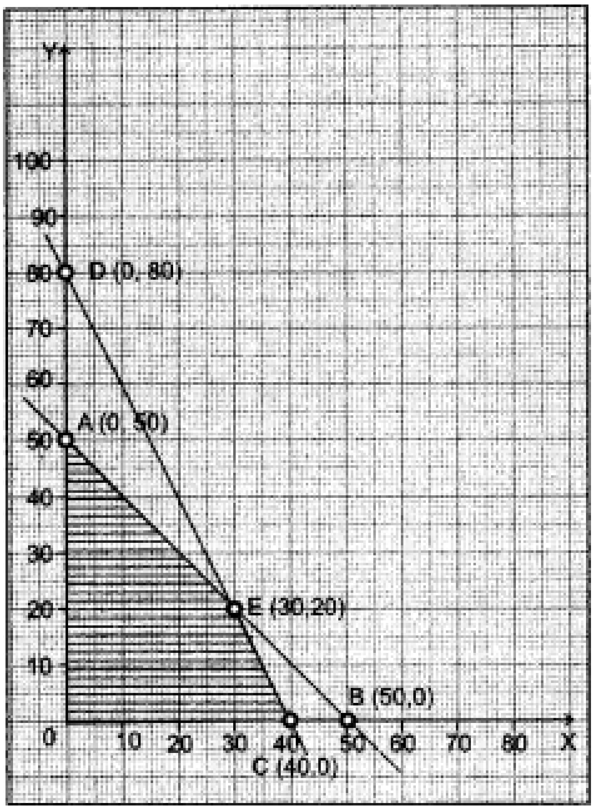
Consider the line

$x + y = 50$

when $x = 0, y = 50$ and when $y = 0, x = 50$.

Line (i) passes through the points $A(0,50)\ and\ B(50,0)$.

Consider the line $2x+y=80$.
When $x=0, y=80$ and $y=0, x=40$.
Line (ii) passes through the points $C(40,0)\ and\ D(0,80)$
Lines (i) and (ii) intersect each other at $E(30,20)\ and\ O(0,0)$ satisfy the inequalities (i) and (ii).
The vertices of the bounded feasible region are $C(40,0), E(30,20), A(0,50)$.

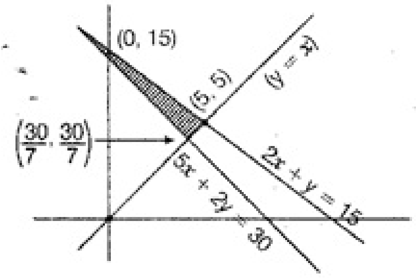


At $C(40,0)$
$Z=40\times 10500+0\times 9000=420000$
At $E(30,20)$
$Z=30\times 10500+20\times 9000=495000$
At $A(0,50)$
$Z=0\times 10500+50\times 9000=450000$
Z appears to maximum at E(30, 20) Maximum profit is $Rs. 495000$, when x= 30 and y= 20. 30 hectares and 20 hectares of land should be allocated for the crops of type A and B respectively.
Yes, the protection of wildlife is utmost necessary to preserve the balance in environment.

15. we have minimise Z=400x + 200y subject to 5x + 2y$\geqslant$30.
$2x+y\geqslant 15, x\leqslant y, x\geqslant 0, y\geqslant 0$

On solving x-y=0 and 5x+2y=30, we get

$y=\frac{30}{7}, x=\frac{30}{7}$



On solving x - y = 0 and 2x + y = 15 we get x = 5, y = 5

So, from the shaded feasible region it is clear that coordinates of corner points are (0,15) , (5,5) and $\left(\frac{30}{7}, \frac{30}{7}\right)$.

Corner Points	**Corresponding value of X = 400x + 200y**
(0, 15)	3000
(5, 5)	3000
$\left(\frac{30}{7}, \frac{30}{7}\right)$	$400 \times \frac{30}{7}+200 \times \frac{30}{7}=\frac{18000}{7}$
	= 2571.43 (minimum)

Hence, the minimum is Rs 2571.43.

16. Let x be the number of packages of nuts and y be number of packages of bolts produced.

Item	**Number**	**Machine A**	**Machine B**	**Profit**
Nuts	x	1 hour	3 hours	Rs.17.50
Bolts	y	3 hours	1 hour	Rs.7.00

We have to maximise $Z=17.50x+7y$

i.e., $Z=\frac{35}{2}x+7y$ .... (i)

Subject to constraints

$x+3y \leq 12$ .... (ii)

$3x+y \leq 12$ ..... (iii)

$x, y \geq 0$

Consider the line

$x+3y=12$

when $x=0, y=4$ and when $y=0, x=12$

(iv) passes through $A(0,4)$ *and* $B(12,0)$ consider the line

$3x+y=12$ ..... (v)

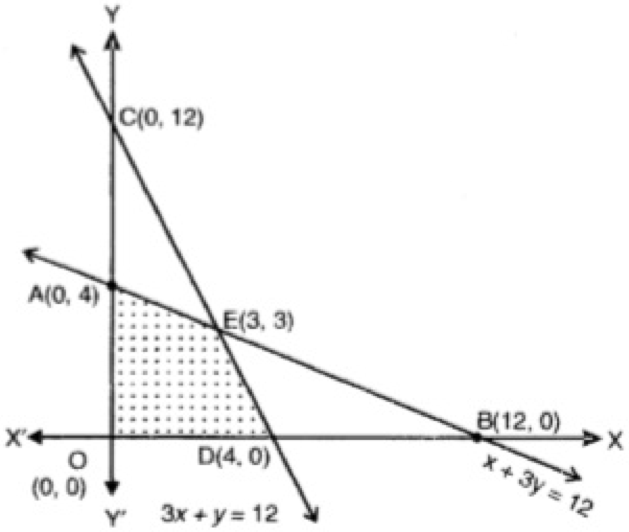
when $x=0, y=12$ and $y=0, x=4$

(v) passes through $C(0,12)$ *and* $D(4,0)$. Further (iv) and (i;) intersect at E (3, 3).

Also, 0(0, 0) satisfies (ii) and (iii)

Vertices of feasible region are $O(0,0), D(4,0), E(3,3)$ *and* $A(0,4)$

At $O(0,0), Z=0+0=0$



At $D(4,0)$, $Z=\frac{35}{2}\times 4+0=70$

At $E(3,3), Z=\frac{35}{2}\times 3+7\times 3=73.50$

At $A(0,4)$, $\mathrm{Z}=0+7\times 4=28$

Maximum Z = 73.50 at E(3, 3)

∴ 3 packages of nuts and 3 packages of bolts should be produces to maximise profit Rs.73.50.

17. Let number of cakes made of first kind are x and that of second kind is y.

∴ we have to maximize Z = x + y

According to question $200x+100y\leqslant 5000$ and $25x+50y\leqslant 1000$

$x\geqslant 0, y\geqslant 0$

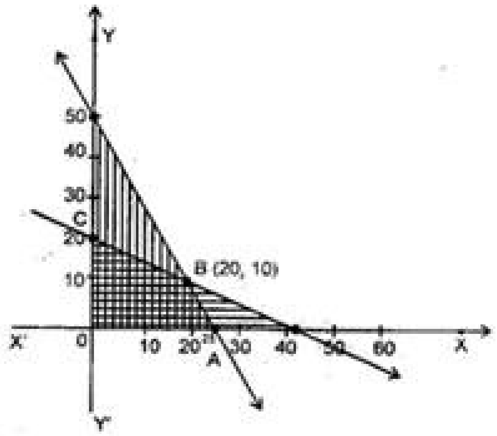
Consider $200x+100y\leqslant 5000$

Let 200x + 100y = 5000

⇒2x + y = 50

$\Rightarrow \frac{x}{25}+\frac{y}{50}=1$

Here, (0, 0) satisfies this inequation, therefore the required half plane contains (0, 0).



Again consider $25x + 50y \leqslant 1000$

Let 25x + 50y = 1000

$\Rightarrow x + 2y = 40$

$\Rightarrow \frac{x}{40} + \frac{y}{20} = 1$

Here, again (0, 0) satisfies this inequation, therefore the required half plane contains (0, 0).

The double shaded region is the feasible region which is solution set.

The corner points of this region are O (0, 0), A (25, 0), B (20, 10) and C (0, 20).

$\therefore$ Z = x + y

At O(0, 0) Z = 0 + 0 = 0

At A(25, 0) Z = 25 + 0 = 25

At B(20, 10) Z = 20 + 10 = 30

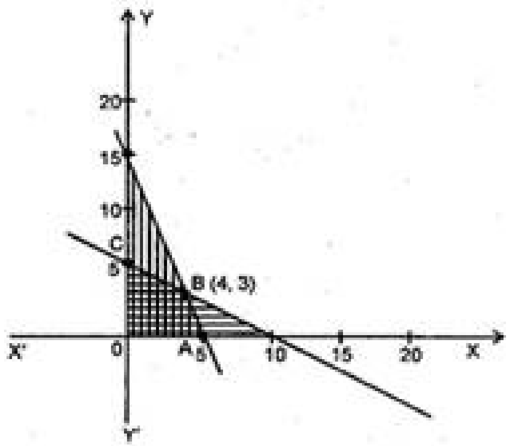
At C(0, 20) Z = 0 + 20 = 20

Hence, maximum number of cakes Z = 30 when x = 20, y = 10.

18. Consider $x + 2y \leqslant 10$

Let x + 2y = 10

$\Rightarrow \frac{x}{10} + \frac{y}{5} = 1$



Since, (0, 0) satisfies the inequation, therefore the half plane containing (0, 0) is the required plane.

Again $3x + 2y \leqslant 15$

Let 3x + y = 15

$\Rightarrow \frac{x}{5} + \frac{y}{15} = 1$

It also satisfies by (0, 0) and its required half plane contains (0, 0).

Now double shaded region in the first quadrant contains the solution.

Now OABC represents the feasible region.

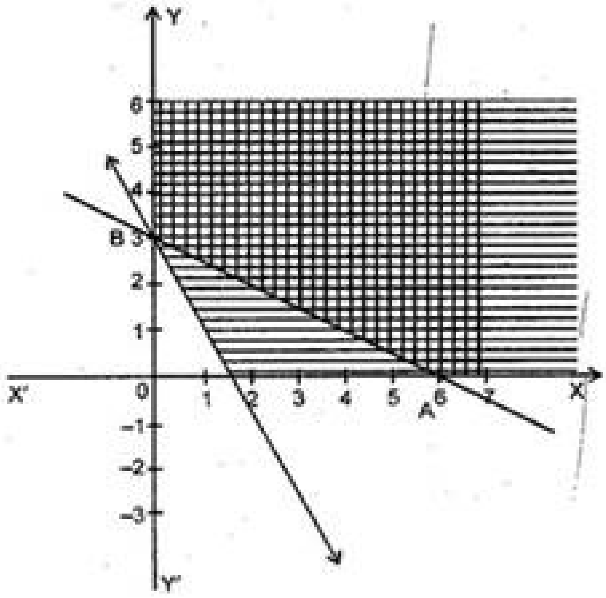
Z = 3x + 2y

At O (0, 0) Z = 3 $\times$ 0 + 2 $\times$ 0 = 0

At A (5, 0) Z = 3 $\times$ 5 + 2 $\times$ 0 = 15

At B (4, 3) Z = 3 $\times$ 4 + 2 $\times$ 3 = 18

At C (0, 5) Z = 3 $\times$ 0 + 2 $\times$ 5 = 10



Hence, Z is maximum i.e., 18 at x = 4, y = 3.