Fundamental Concepts (Including Fundamental Operations)

POINTS TO REMEMBER

1. **Constants and Variables :** The numbers which has fixed value is called constant and same at English alphabet which can be assigned any value according to the requirement is called variables.
2. **Term :** A term is a number, (constant), a variable or a combination of numbers and variables.
3. **Algebraic Expression :** An algebraic expression is a collection of one or more terms, which are separated from each other by addition (+) or subtraction (-) signs.
4. **Types of algebraic expressions :**
   (i) Monomial : It has only one term
   (ii) Binomial : It has two terms
   (iii) Trinomial : It has three terms
   (iv) Multinomial : It has more than three terms
   (v) Polynomial : It has two or more than two terms.

   Note : An expression of the type $\frac{2}{5}$ does not form a monomial unless JC is not equal to zero.
5. **Product:** When two or more quantities are multiplied together, the result is called their product.
6. **Factors :** Each of the quantities (numbers or variables) multiplied together to form a term is called a factor of the given term.
7. **Co-efficient:** In a monomial, any factor or group of factors of a term is called the co-efficient of the remaining part of the monomial.
8. **Degree of a monomial:** The degree of a monomial is the exponent of its variable or the sum of the exponents of its variables.
9. **Degree of a polynomial:** The degree of a polynomial is the degree of its highest degree term.
10. **Like and unlike terms** : Terms having the same literal co-efficients or alphabetic letters are called like terms ; whereas the terms with different literal co-efficients are called unlike terms.
11. **Addition and subtraction** : Addition and subtraction of only like terms is possible by adding or subtracting the numerical co-efficients.
12. **Multiplication and division** :
    (A) Multiplication :
    (i) Multiplications of monomials.
    (a) Multiply the numerical co-efficient together
    (ii) Multiply the literal co-efficients separately together.
    (iii) Combine the like terms.
    (B) Division :
    (i) Dividing a polynomial by a monomial Divide each term of the polynomial by monomial and simplify each fractions.

(ii) While dividing one polynomial by another polynomial ; arrange the terms of both the dividend and the divisior both in descending or in ascending order of their powers and then divide.

SOME IMPORTANT POINTS

TYPES OF BRACKETS:
The name of different types of brackets and the order in which they are removed is shown below:
(a) ____ ; Bar (Vinculum) bracket
(b) ( ); Circular bracket .
(c) { } ; Curly bracket and then
(d) [ ]; square bracket

EXERCISE 11 (A)

Question 1.
Separate constant terms and variable terms from tile following :

$$(i)\ 8, x, 6xy, 6+x, -5xy^2, 15az^2, \frac{32z}{xy}, \frac{y^2}{3x}$$

Solution:

Constant is only 8 others are variables

Question 2.
Constant is only 8 others are variables
(i) 2x ÷ 15
(ii) ax+ 9
(iii) $3x^2 \times 5x$
(iv) 5 + 2a-3b
(v) $2y - \frac{7}{3} z \div x$
(vi) 3p x q ÷ z
(vii) 12z ÷ 5x + 4
(viii) 12 – 5z – 4
(ix) $a^3 - 3ab^2 \times c$

Answer:

$(i)\ 2x \div 15 = \dfrac{2x}{15}$

It is a monomial as it has one term.

$(ii)\ ax + 9$: It is binomial

(∵ It has two terms)

$(iii)\ 3x^2 \times 5x = 15x^3$: It is monomial

(∵ It has one term)

$(iv)\ 5 + 2a - 3b$: It is trinomial

(∵ It has three terms)

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$(v)\ 2y - \dfrac{7}{3}z \div x = 2y - \dfrac{7z}{3x}$: It is binomial

(∵ It has two terms)

$(vi)\ 3p \times q \div z = \dfrac{3pq}{z}$: It is monomial

(∵ It has one term)

$(vii)\ 12z \div 5x + 4 = \dfrac{12z}{5x} + 4$: It is binomial

(∵ It has two terms)

$(viii)\ 12 - 5z - 4 = 8 - 5z$: It is binomial

(∵ It has two terms)

$(ix)\ a^3 - 3ab^2 \times c = a^3 - 3ab^2c$: It is binomial

(∵ It has two terms)

Question 3.

Write the coefficient of:

(i) xy in – 3axy

(ii) z^2 in p^2yz^2

(iii) mn in -mn

(iv) 15 in – $15p^2$

Solution:

(i) Co-efficient of xy in – 3 axy = – 3a

(ii) Co-efficient of z^2 in p^2yz^2 = p^2y

(iii) Co-efficient of mn in – mn = – 1
(iv) Co-efficient of 15 in – $15p^2$ is $-p^2$

Question 4.

For each of the following monomials, write its degree :
(i) 7y
(ii) – x^2y
(iii) xy^2z
(iv) – $9y^2z^3$
(v) 3 m^3n^4
(vi) – $2p^2q^3r^4$

Solution:

(i) Degree of 7y = 1
(ii) Degree of – x^2y = 2+1=3
(iii) Degree of xy^2z = 1 + 2 + 1 = 4
(iv) Degree of – $9y^2z^3$ = 2 + 3 = 5
(v) Degree of $3m^3n^4$ = 3 + 4 = 7
(vi) Degree of – $2p^2q^3r^4$ = 2 + 3 + 4 = 9

Question 5.

Write the degree of each of the following polynomials :
(i) $3y^3$-x^2y^2 + 4x
(ii) p^3q^2 – $6p^2q^5$ + p^4q^4
(iii) – $8mn^6$+ $5m^3n$
(iv) 7 – $3x^2y$ + y^2
(v) 3x – 15
(vi) $2y^2z$ + $9yz^3$

Solution:

(i) The degree of $3y^3$ – x^2y^2+ 4x is 4 as x^2 y^2 is the term which has highest degree.
(ii) The degree of p^3q^2 – $6p^2q^5$-p^4q^4 is 8 as p^4 q^4 is the term which has highest degree.
(iii) The degree of- $8mn^6$ + $5m^3n$ is 7 as – $8mx^6$ is the term which has the highest degree.
(iv) The degree of 7 – $3x^2$ y + y^2 is 3 as – $3x^2y$ is the term which has the highest degree.
(v) The degree of 3x – 15 is 1 as 3x is the term which is highest degree.
(vi) The degree of $2y^2$ z + 9y z^3 is 4 as $9yz^3$ has the highest degree.

Question 6.

Group the like term together :
(i) $9x^2$, xy, – $3x^2$, x^2 and – 2xy
(ii) ab, – a^2b, – 3ab, $5a^2b$ and – $8a^2b$
(iii) 7p, 8pq, – 5pq – 2p and 3p

Solution:
(i) $9x^2$, $-3x^2$ and x^2 are like terms
xy and – 2xy are like terms
(ii) ab, – 3ab, are like terms,
$-a^2b$, $5a^2b$, $-8a^2b$ are like terms
(iii) 7p, – 2p and 3p are like terms,
8pq, – 5pq are like terms.

Question 7.
Write numerical co-efficient of each of the followings :
(i) y
(ii) -y
(iii) $2x^2y$
(iv) $-8xy^3$
(v) $3py^2$
(vi) $-9a^2b^3$

Solution:
(i) Co-efficient of y = 1
(ii) Co-efficient of-y = – 1
(iii) Co-efficient of 2x2y is = 2
(iv) Co-efficient of – 8xy3 is = – 8
(v) Co-efficient of Ipy2 is = 3
(vi) Co-efficient of – 9a2b3 is = – 9

Question 8.
In $-5x^3y^2z^4$; write the coefficient of:
(i) z^2
(ii) y^2
(iii) yz^2
(iv) x^3y
(v) $-xy^2$
(vi) $-5xy^2z$
Also, write the degree of the given algebraic expression.

Solution:
$-5x^3y^2z^4$
(i) Co-efficient of z2 is $-5x^3y^2z^2$
(ii) Co-efficient of y2 is $-5x^3z^4$
(iii) Co-efficient of yz^2 is $-5x^3yz^2$
(iv) Co-efficient of x^3y is $-5yz^4$
(v) Co-efficient of $-xy^2$ is $5x^2z^4$
(vi) Co-efficient of $-5xy^2z$ is x^2z^3
Degree of the given expression is 3 + 2 + 4 = 9

EXERCISE 11 (B)

Question 1.

Fill in the blanks :

(i) $8x + 5x$ = ……

(ii) $8x – 5x$ =……..

(iii) $6xy^2 + 9xy^2$ =……..

(iv) $6xy^2 – 9xy^2$ = ……

(v) The sum of 8a, 6a and 5b = ……..

(vi) The addition of 5, 7xy, 6 and 3xy = ……

(vii) $4a + 3b – 7a + 4b$ = …….

(viii) $– 15x + 13x + 8$ = ……

(ix) $6x^2y + 13xy^2 – 4x^2y + 2xy^2$ = ……..

(x) $16x^2 – 9x^2$ = and $25xy^2 – 17xy^2$=……

Solution :

(*i*) $8x + 5x = 13x$

(*ii*) $8x - 5x = 3x$

(*iii*) $6xy^2 + 9xy^2 = 15xy^2$

(*iv*) $6xy^2 - 9xy^2 = -3xy^2$

(*v*) The sum of $8a$, $6a$ and $5b$

$= 8a + 6a + 5b = 14a + 5b$

(*vi*) The addition of 5, $7xy$, 6 and $3xy$

$= 5 + 6 + 7xy + 3xy = 11 + 10xy$

(*vii*) $4a + 3b - 7a + 4b$

$= 4a - 7a + 3b + 4b = -3a + 7b = 7b - 3a$

(*viii*) $-15x + 13x + 8$

$= -2x + 8 = 8 - 2x$

(*ix*) $6x^2y + 13xy^2 - 4x^2y + 2xy^2$

$= 6x^2y - 4x^2y + 13xy^2 + 2xy^2 = 2x^2y + 15xy^2$

(*x*) $16x^2 - 9x^2 = 7x^2$ and

$25xy^2 - 17xy^2 = 8xy^2$

Question 2.

Add :

(i)- 9x, 3x and 4x

(ii) $23y^2$, $8y^2$ and $-12y^2$

(iii) 18pq – 15pq and 3pq

Solution:

$$(i) -9x + 3x + 4x$$
$$= -9x + 7x = -2x$$
$$(ii)\ 23y^2 + 8y^2 - 12y^2$$
$$= 31y^2 - 12y^2 = 19y^2$$
$$(iii)\ 18pq - 15pq + 3pq$$
$$= 18pq + 3pq - 15pq = 21pq - 15pq = 6pq$$

Question 3.

Simplify :

(i) 3m + 12m – 5m

(ii) $7n^2 - 9n^2 + 3n^2$

(iii) 25zy—8zy—6zy

(iv) $-5ax^2 + 7ax^2 - 12ax^2$

(v) – 16am + 4mx + 4am – 15mx + 5am

Solution:

$$(i)\ 3m + 12m - 5m = 15m - 5m = 10m$$
$$(ii)\ 7n^2 - 9n^2 + 3n^2$$
$$= 7n^2 + 3n^2 - 9n^2 = 10n^2 - 9n^2 = n^2$$
$$(iii)\ 25zy - 8zy - 6zy$$
$$= 25zy - 14zy = 11zy$$
$$(iv)\ -5ax^2 + 7ax^2 - 12ax^2$$
$$= -5ax^2 - 12ax^2 + 7ax^2$$
$$= -17ax^2 + 7ax^2 = -10ax^2$$
$$(v)\ -16am + 4mx + 4am - 15mx + 5am$$
$$= -16am + 4am + 5am + 4mx - 15mx$$
$$= -16am + 9am + 4mx - 15mx = -7am - 11mx$$

Question 4.

Add :

(i) a + i and 2a + 3b

(ii) 2x + y and 3x – 4y

(iii)- 3a + 2b and 3a + b

(iv) 4 + x, 5 – 2x and 6x

Solution:

(*i*) $a + b$ and $2a + 3b$ (*ii*) $2x + y$ and $3x - 4y$

(*iii*) $-3a + 2b$ and $3a + b$ (*iv*) $4 + x$, $5 - 2x$ and $6x$

(*i*) $a + b + 2a + 3b$

$= a + 2a + b + 3b = 3a + 4b$

(*ii*) $2x + y + 3x - 4y$

$= 2x + 3x + y - 4y = 5x - 3y$

(*iii*) $-3a + 2b + 3a + b$

$= -3a + 3a + 2b + b = 0 + 3b = 3b$

(*iv*) $4 + x + 5 - 2x + 6x$

$= x - 2x + 6x + 4 + 5$

$= 7x - 2x + 9 = 5x + 9$

Question 5.

Find the sum of:

(i) 3x + 8y + 7z, 6y + 4z- 2x and 3y – 4x + 6z

(ii) 3a + 5b + 2c, 2a + 3b-c and a + b + c.

(iii) $4x^2 + 8xy - 2y^2$ and $8xy - 5y^2 + x^2$

(iv) $9x^2 - 6x + 7$, $5 - 4x$ and $6 - 3x^2$

(v) $5x^2 - 2xy + 3y^2$ and $-2x^2 + 5xy + 9y^2$ and $3x^2 - xy - 4y^2$

(vi) $a^2 + b^2 + 2ab$, $2b^2 + c^2 + 2bc$ and $4c^2 - a^2 + 2ac$

(vii) 9ax – 6bx + 8, 4ax + 8bx – 7 and – 6ax – 46x – 3

(viii) abc + 2 ba + 3 ac, 4ca – 4ab + 2 bca and 2ab – 3abc – 6ac

(ix) $4a^2 + 5b^2 - 6ab$, $3ab$, $6a^2 - 2b^2$ and $4b^2 - 5ab$

(x) $x^2 + x - 2$, $2x - 3x^2 + 5$ and $2x^2 - 5x + 7$

(xi) $4x^3 + 2x^2 - x + 1$, $2x^3 - 5x^2 - 3x + 6$, $x^2 + 8$ and $5x^3 - 7x$

Solution:

(*i*) $3x + 8y + 7z + 6y + 4z - 2x$

$+ 3y - 4x + 6z$

$= 3x - 2x - 4x + 8y + 6y + 3y + 7z + 4z + 6z$

$= 3x - 6x + 17y + 17z$

$= -3x + 17y + 17z$

(*ii*) $3a + 5b + 2c + 2a + 3b - c + a + b + c$

$= 3a + 2a + a + 5b + 3b + b + 2c - c + c$

$= 6a + 9b + 3c - c = 6a + 9b + 2c$

(*iii*) $4x^2 + 8xy - 2y^2$ and $8xy - 5y^2 + x^2$

$4x^2 + 8xy - 2y^2 + 8xy - 5y^2 + x^2$

$= 4x^2 + x^2 + 8xy + 8xy - 2y^2 - 5y^2$

$= 5x^2 + 16xy - 7y^2$

(*iv*) $9x^2 - 6x + 7$, $5 - 4x$ and $6 - 3x^2$

$9x^2 - 6x + 7 + 5 - 4x + 6 - 3x^2$

$= 9x^2 - 3x^2 - 6x - 4x + 7 + 5 + 6$

$= 6x^2 - 10x + 18$

(*v*) $5x^2 - 2xy + 3y^2$, $-2x^2 + 5xy + 9y^2$

and $3x^2 - xy - 4y^2$

$5x^2 - 2xy + 3y^2 - 2x^2 + 5xy + 9y^2$

$+ 3x^2 - xy - 4y^2$

$= 5x^2 - 2x^2 + 3x^2 - 2xy + 5xy - xy$

$+ 3y^2 + 9y^2 - 4y^2$

$= 8x^2 - 2x^2 + 5xy - 3xy + 12y^2 - 4y^2$

$= 6x^2 + 2xy + 8y^2$

(*vi*) $a^2 + b^2 + 2ab$, $2b^2 + c^2 + 2bc$

and $4c^2 - a^2 + 2ac$

$a^2 + b^2 + 2ab + 2b^2 + c^2 + 2bc$

$+ 4c^2 - a^2 + 2ac$

$= a^2 - a^2 + b^2 + 2b^2 + c^2 + 4c^2 + 2ab$

$+ 2bc + 2ac$

$= 3b^2 + 5c^2 + 2ab + 2bc + 2ac$

(*vii*) $9ax - 6bx + 8$, $4ax + 8bx - 7$

and $-6ax - 4bx - 3$

$9ax - 6bx + 8 + 4ax + 8bx - 7$

$- 6ax - 4bx - 3$

$= 9ax + 4ax - 6ax - 6bx + 8bx - 4bx$

$+ 8 - 7 - 3$

$= 13ax - 6ax + 8bx - 10bx + 8 - 10$

$= 7ax - 2bx - 2$

(*viii*) $abc + 2ba + 3ac$, $4ca - 4ab + 2bca$

and $2ab - 3abc - 6ac$

$abc + 2ab + 3ca + 4ca - 4ab + 2abc$

$+ 2ab - 3abc - 6ca$

$= abc + 2abc - 3abc + 2ab - 4ab + 2ab$

$+ 3ca + 4ca - 6ca$

$= 3abc - 3abc + 4ab - 4ab + 7ca - 6ca$

$= 0 + 0 + ca = ca$

(ix) $4a^2 + 5b^2 - 6ab$, $3ab$, $6a^2 - 2b^2$ and $4b^2 - 5ab$

$4a^2 + 5b^2 - 6ab + 3ab + 6a^2 - 2b^2 + 4b^2 - 5ab$

$= 4a^2 + 6a^2 + 5b^2 - 2b^2 + 4b^2 - 6ab + 3ab - 5ab$

$= 10a^2 + 9b^2 - 2b^2 - 11ab + 3ab$

$= 10a^2 + 7b^2 - 8ab$

(x) $x^2 + x - 2$, $2x - 3x^2 + 5$ and $2x^2 - 5x + 7$

$= x^2 + x - 2 + 2x - 3x^2 + 5 + 2x^2 - 5x + 7$

$= x^2 - 3x^2 + 2x^2 + x + 2x - 5x - 2 + 5 + 7$

$= 3x^2 - 3x^2 + 3x - 5x - 2 + 12$

$= 0 - 2x + 10$

$= -2x + 10$

(xi) $4x^3 + 2x^2 - x + 1$, $2x^3 - 5x^2 - 3x + 6$, $x^2 + 8$ and $5x^3 - 7x$

$4x^3 + 2x^2 - x + 1 + 2x^3 - 5x^2 - 3x + 6 + x^2 + 8 + 5x^3 - 7x$

$= 4x^3 + 2x^3 + 5x^3 + 2x^2 - 5x^2 + x^2 - x - 3x - 7x + 1 + 6 + 8$

$= 11x^3 + 3x^2 - 5x^2 - 11x + 15$

$= 11x^3 - 2x^2 - 11x + 15$

Question 6.

Find the sum of:

(i) x and 3y

(ii) -2a and +5

(iii) – $4x^2$ and +7x

(iv) +4a and -7b

(v) x^3+3x^2y and $2y^2$

(vi) 11 and -by

Solution:

(i) $x + 3y$ (ii) $-2a + 5$

(iii) $-4x^2 + 7x$ (iv) $4a - 7b$

(v) $x^3 + 3x^2y + 2y^2$ (vi) $11 - by$

Question 7.

The sides of a triangle are 2x + 3y, x + 5y and 7x – 2y, find its perimeter.

Solution:

Sides of a triangle are $2x + 3y, x + 5y, 7x - 2y$

$\therefore$ Perimeter = sum of three sides of the triangle

$= 2x + 3y + x + 5y + 7x - 2y$

$= 2x + x + 7x + 3y + 5y - 2y$

$= 10x + 8y - 2x = 10x + 6y$.

Question 8.

The two adjacent sides of a rectangle are 6a + 96 and 8a – 46. Find its, perimeter.

Solution

Sides of a rectangle are $6a + 9b$

and $8a - 4b$

Let, length $= 6a + 9b$

and breadth $= 8a - 4b$

$\therefore$ Perimeter = 2 (length + breadth)

$= 2(6a + 9b + 8a - 4b)$

$= 2(14a + 5b) = 28a + 10b$

Question 9.

Subtract the second expression from the first:

(*i*) $2a + b, a + b$ (*ii*) $-2b + 2c, b + 3c$

(*iii*) $5a + b, -6b + 2a$ (*iv*) $a^3 - 1 + a, 3a - 2a^2$

(*v*) $p + 2, 1$

(*vi*) $x + 2y + z, -x - y - 3z$

(*vii*) $3a^2 - 8ab - 2b^2, 3a^2 - 4ab + 6b^2$

(*viii*) $4pq - 6p^2 - 2q^2, 9p^2$

(*ix*) $10abc, 2a^2 + 2abc - 4b^2$

(*x*) $a^2 + ab + c^2, a^2 - d^2$

Solution:

(*i*) $(2a + b) - (a + b)$

$= 2a + b - a - b = 2a - a + b - b$

$= a + 0 = a$.

(*ii*) $(-2b + 2c) - (b + 3c)$

$= -2b + 2c - b - 3c$

$= -2b - b + 2c - 3c$

$= -3b - c$

(*iii*) $(5a + b) - (-6b + 2a)$

$= 5a + b + 6b - 2a$

$= 5a - 2a + b + 6b$

$= 3a + 7b$

(*iv*) $(a^3 - 1 + a) - (3a - 2a^2)$

$= a^3 - 1 + a - 3a + 2a^2$

$= a^3 + 2a^2 - 2a - 1$

(*v*) $(p + 2) - 1 = p + 2 - 1 = p + 1$

(*vi*) $(x + 2y + z) - (-x - y - 3z)$

$= x + 2y + z + x + y + 3z$

$= x + x + 2y + y + z + 3z$

$= 2x + 3y + 4z$

(*vii*) $(3a^2 - 8ab - 2b^2) - (3a^2 - 4ab + 6b^2)$

$= 3a^2 - 8ab - 2b^2 - 3a^2 + 4ab - 6b^2$

$= 3a^2 - 3a^2 - 2b^2 - 6b^2 - 8ab + 4ab$

$= 0 - 8b^2 - 4ab$

$= -4ab - 8b^2$

(*viii*) $(4pq - 6p^2 - 2q^2) - (9p^2)$

$= 4pq - 6p^2 - 2q^2 - 9p^2$

$= 4pq - 15p^2 - 2q^2$

(ix) $10abc - (2a^2 + 2abc - 4b^2)$

$= 10abc - 2a^2 - 2abc + 4b^2$

$= 10abc - 2abc - 2a^2 + 4b^2$

$= 8abc - 2a^2 + 4b^2$

(x) $(a^2 + ab + c^2) - (a^2 - d^2)$

$= a^2 + ab + c^2 - a^2 + d^2$

$= a^2 - a^2 + ab + c^2 + d^2$

$= ab + c^2 + d^2$

Question 10.

Subtract:

(i) $4x$ from $8 - x$

(ii) $-8c$ from $c + 3d$

(iii) $-5a - 2b$ from $b + 6c$

(iv) $4p + p^2$ from $3p^2 - 8p$

(v) $5a - 3b + 2c$ from $4a - b - 2c$

(vi) $-xy + yz - zx$ from $xy - yz + xz$

(vii) $2x^2 - 7xy - y^2$ from $3x^2 - 5xy + 3y^2$

(viii) $a^2 - 3ab - 6b^2$ from $2b^2 - a^2 + 2ab$

(ix) $4x^2 - 5x^2y + y^2$ from $-3y^2 + 5xy^2 - 7x^2 - 9x^2y$

(x) $6m^3 + 4m^2 + 7m - 3$ from $3m^3 + 4$

Solution:

(*i*) $4x$ from $8 - x$

$(8 - x) - 4x = 8 - x - 4x = 8 - 5x$

(*ii*) $-8c$ from $c + 3d$

$(c + 3d) - (-8c)$

$c + 3d + 8c = 9c + 3d$

(*iii*) $-5a - 2b$ from $b + 6c$

$(b + 6c) - (-5a - 2b)$

$= b + 6c + 5a + 2b = 5a + 3b + 6c$

(*iv*) $4p + p^2$ from $3p^2 - 8p$

$(3p^2 - 8p) - (4p + p^2)$

$= 3p^2 - 8p - 4p - p^2 = 2p^2 - 12p$

(*v*) $5a - 3b + 2c$ from $4a - b - 2c$

$(4a - b - 2c) - (5a - 3b + 2c)$

$= 4a - b - 2c - 5a + 3b - 2c$

$= 4a - 5a - b + 3b - 2c - 2c$

$= -a + 2b - 4c$

(*vi*) $-xy + yz - zx$ from $xy - yz + xz$

$(xy - yz + zx) - (-xy + yz - xz)$

$= xy - yz + zx + xy - yz + xz$

$= xy + xy - yz - yz + zx + xz$

$= 2(xy - yz + zx)$

(*vii*) $2x^2 - 7xy - y^2$ from $3x^2 - 5xy + 3y^2$

$(3x^2 - 5xy + 3y^2) - (2x^2 - 7xy - y^2)$

$= 3x^2 - 5xy + 3y^2 - 2x^2 + 7xy + y^2$

$= 3x^2 - 2x^2 - 5xy + 7xy + 3y^2 + y^2$

$= x^2 + 2xy + 4y^2$

(*viii*) $a^2 - 3ab - 6b^2$ from $2b^2 - a^2 + 2ab$

$(2b^2 - a^2 + 2ab) - (a^2 - 3ab - 6b^2)$

$= 2b^2 - a^2 + 2ab - a^2 + 3ab + 6b^2$

$= -a^2 - a^2 + 2b^2 + 6b^2 + 2ab + 3ab$

$= -2a^2 + 8b^2 + 5ab$

$= 8b^2 + 5ab - 2a^2$

(*ix*) $4x^2 - 5x^2y + y^2$ from $-3y^2 + 5xy^2 - 7x^2 - 9x^2y$

$(-3y^2 + 5xy^2 - 7x^2 - 9x^2y)$

$- (4x^2 - 5x^2y + y^2)$

$= -3y^2 + 5xy^2 - 7x^2 - 9x^2y - 4x^2$

$+ 5x^2y - y^2$

$= -3y^2 - y^2 + 5xy^2 - 7x^2 - 4x^2$

$- 9x^2y + 5x^2y$

$= -4y^2 + 5xy^2 - 11x^2 - 4x^2y$

(*x*) $6m^3 + 4m^2 + 7m - 3$ from $3m^3 + 4$

$(3m^3 + 4) - (6m^3 + 4m^2 + 7m - 3)$

$= 3m^3 + 4 - 6m^3 - 4m^2 - 7m + 3$

$= 3m^3 - 6m^3 - 4m^2 - 7m + 4 + 3$

$= -3m^3 - 4m^2 - 7m + 7$

Question 11.

Subtract $-5a^2 - 3a + 1$ from the sum of $4a^2 + 3 - 8a$ and $9a - 7$.

Solution:

Sum of $4a^2 + 3 - 8a$ and $9a - 7$

$= 4a^2 + 3 - 8a + 9a - 7 \quad = 4a^2 + a - 4$

$\therefore (4a^2 + a - 4) - (-5a^2 - 3a + 1)$

$= 4a^2 + a - 4 + 5a^2 + 3a - 1$

$= 4a^2 + 5a^2 + a + 3a - 4 - 1$

$= 9a^2 + 4a - 5$

Question 12.

By how much does $8x^3 - 6x^2 + 9x - 10$ exceed $4x^3 + 2x^2 + 7x - 3$?

Solution:

$8x^3 - 6x^2 + 9x - 10$ exceeds $4x^3 + 2x^2 + 7x - 3$

$= (8x^3 - 6x^2 + 9x - 10) - (4x^3 + 2x^2 + 7x - 3)$

$= 8x^3 - 6x^2 + 9x - 10 - 4x^3 - 2x^2 - 7x + 3$

$= 8x^3 - 4x^3 - 6x^2 - 2x^2 + 9x - 7x - 10 + 3$

$= 4x^3 - 8x^2 + 2x - 7$

Question 13.

What must be added to $2a^3 + 5a - a^2 - 6$ to get $a^2 - a - a^3 + 1$?

Solution:

We get, the required result by subtracting

$2a^3 - a^2 + 5a - 6$ from $-a^3 + a^2 - a + 1$

$= (-a^3 + a^2 - a + 1) - (2a^3 - a^2 + 5a - 6)$

$= -a^3 + a^2 - a + 1 - 2a^3 + a^2 - 5a + 6$

$= -a^3 - 2a^3 + a^2 + a^2 - a - 5a + 1 + 6$

$= -3a^3 + 2a^2 - 6a + 7$

Question 14.

What must be subtracted from $a^2 + b^2 + lab$ to get $-4ab + 2b^2$?

Solution:

We get, the required result by subtracting $-4ab + 2b^2$ from $a^2 + b^2 + 2ab$.

$$= a^2 + b^2 + 2ab - (-4ab + 2b^2)$$
$$= a^2 + b^2 + 2ab + 4ab - 2b^2$$
$$= a^2 + b^2 - 2b^2 + 2ab + 4ab$$
$$= a^2 - b^2 + 6ab$$

Question 15.

Find the excess of $4m^2 + 4n^2 + 4p^2$ over $m^2 + 3n^2 - 5p^2$

Solution:

The required result will be by subtracting $m^2 + 3n^2 - 5p^2$ from $4m^2 + 4n^2 + 4p^2$

$$= 4m^2 + 4n^2 + 4p^2 - (m^2 + 3n^2 - 5p^2)$$
$$= 4m^2 + 4n^2 + 4p^2 - m^2 - 3n^2 + 5p^2$$
$$= 4m^2 - m^2 + 4n^2 - 3n^2 + 4p^2 + 5p^2$$
$$= 3m^2 + n^2 + 9p^2$$

Question 16.

By how much is $3x^3 - 2x^2y + xy^2 - y^3$ less than $4x^3 - 3x^2y - 7xy^2 + 2y^3$

Solution:

We can get the required result by subtracting

$3x^3 - 2x^2y + xy^2 - y^3$ from $4x^3 - 3x^2y - 7xy^2 + 2y^3$

$$= (4x^3 - 3x^2y - 7xy^2 + 2y^3) - (3x^3 - 2x^2y + xy^2 - y^3)$$
$$= 4x^3 - 3x^2y - 7xy^2 + 2y^3 - 3x^3 + 2x^2y - xy^2 + y^3$$
$$= 4x^3 - 3x^3 - 3x^2y + 2x^2y - 7xy^2 - xy^2 + 2y^3 + y^3$$
$$= x^3 - x^2y - 8xy^2 + 3y^3$$

Question 17.

Subtract the sum of $3a^2 - 2a + 5$ and $a^2 - 5a - 7$ from the sum of $5a^2 - 9a + 3$ and $2a - a^2 - 1$

Solution:

Sum of $3a^2 - 2a + 5$ and $a^2 - 5a - 7$

$= 3a^2 - 2a + 5 + a^2 - 5a - 7$

$= 3a^2 + a^2 - 2a - 5a + 5 - 7$

$= 4a^2 - 7a - 2$

and sum of $5a^2 - 9a + 3$ and $2a - a^2 - 1$

$= 5a^2 - 9a + 3 + 2a - a^2 - 1$

$= 5a^2 - a^2 - 9a + 2a + 3 - 1$

$= 4a^2 - 7a + 2$

Now $(4a^2 - 7a + 2) - (4a^2 - 7a - 2)$

$= 4a^2 - 7a + 2 - 4a^2 + 7a + 2$

$= 4a^2 - 4a^2 - 7a + 7a + 2 + 2$

$= 0 + 0 + 4 = 4.$

Question 18.

The perimeter of a rectangle is $28x^3 + 16x^2 + 8x + 4$. One of its sides is $8x^2 + 4x$. Find the other side

Solution:

Perimeter of a rectangle $(2l + 2b)$

$= 28x^3 + 16x^2 + 8x + 4$

Let one side $(l) = 8x^2 + 4x$

$\therefore\ 2l = 2(8x^2 + 4x) = 16x^2 + 8x$

$\therefore\ 2b = (28x^3 + 16x^2 + 8x + 4)$

$- (16x^2 + 8x)$

$= 28x^3 + 16x^2 + 8x + 4 - 16x^2 - 8x$

$= 28x^3 + 4$

$$\therefore \text{ Other side } (b) = \frac{28x^3 + 4}{2}$$

$$= 14x^3 + 2$$

Question 19.

The perimeter of a triangle is $14a^2 + 20a + 13$. Two of its sides are $3a^2 + 5a + 1$ and $a^2 + 10a - 6$. Find its third side.

Solution:

Perimeter of a triangle $= 14a^2 + 20a + 13$

Sum of two sides

$= 3a^2 + 5a + 1 + a^2 + 10a - 6$

$= 3a^2 + a^2 + 5a + 10a + 1 - 6$

$= 4a^2 + 15a - 5$

$\therefore$ Third side $= (14a^2 + 20a + 13) - (4a^2 + 15a - 5)$

$= 14a^2 + 20a + 13 - 4a^2 - 15a + 5$

$= 14a^2 - 4a^2 + 20a - 15a + 13 + 5$

$= 10a^2 + 5a + 18$

Question 20.

$x = 4a^2 + b^2 - 6ab$

$y = 3b^2 - 2a^2 + 8ab$

$z = 6a^2 + 8b^2 - 6ab$

(*i*) $x + y + z = 4a^2 + b^2 - 6ab + 3b^2 - 2a^2 + 8ab + 6a^2 + 8b^2 - 6ab$

$= 4a^2 - 2a^2 + 6a^2 + b^2 + 3b^2 + 8b^2 - 6ab + 8ab - 6ab$

$= 10a^2 - 2a^2 + 12b^2 - 12ab + 8ab$

$= 8a^2 + 12b^2 - 4ab$

(*ii*) $x - y - z = (4a^2 + b^2 - 6ab) - (3b^2 - 2a^2 + 8ab) - (6a^2 + 8b^2 - 6ab)$

$= 4a^2 + b^2 - 6ab - 3b^2 + 2a^2 - 8ab - 6a^2 - 8b^2 + 6ab$

$= 4a^2 + 2a^2 - 6a^2 + b^2 - 3b^2 - 8b^2 - 6ab - 8ab + 6ab$

$= 6a^2 - 6a^2 + b^2 - 11b^2 - 14ab + 6ab$

$= -10b^2 - 8ab$

Solution:

If $x = 4a^2 + b^2 - 6ab$. $y = 3b^2 - 2a^2 + 8ab$ and $z = 6a^2 + 8b^2 - 6ab$ find :

(*i*) $x + y + z$ (*ii*) $x - y - z$

Question 21.

If $m = 9x^2 - 4xy + 5y^2$ and $n = -3x^2 + 2xy - y^2$ find :

(*i*) $2m - n$

(*ii*) $m + 2n$

(*iii*) $m - 3n$.

Solution:

$m = 9x^2 - 4xy + 5y^2$

$n = -3x^2 + 2xy - y^2$

(*i*) $2m - n = 2(9x^2 - 4xy + 5y^2) - (-3x^2 + 2xy - y^2)$

$= 18x^2 - 8xy + 10y^2 + 3x^2 - 2xy + y^2$

$= 18x^2 + 3x^2 - 8xy - 2xy + 10y^2 + y^2$

$= 21x^2 - 10xy + 11y^2$

(*ii*) $m + 2n = (9x^2 - 4xy + 5y^2) + 2(-3x^2 + 2xy - y^2)$

$= 9x^2 - 4xy + 5y^2 - 6x^2 + 4xy - 2y^2$

$= 9x^2 - 6x^2 - 4xy + 4xy + 5y^2 - 2y^2$

$= 3x^2 + 3y^2$

(*iii*) $m = 9x^2 - 4xy + 5y^2$

$n = -3x^2 + 2xy - y^2$

Now,

$m - 3n = 9x^2 - 4xy + 5y^2 - 3(-3x^2 + 2xy - y^2)$

$= 9x^2 - 4xy + 5y^2 + 9x^2 - 6xy + 3y^2$

$= 18x^2 - 10xy + 8y^2$

Question 22.

Simplify:

(*i*) $3x + 5(2x + 6) - 7x$

(*ii*) $3(4y - 10) + 2(y - 1)$

(*iii*) $-(7 + 6x) - 7(x + 2)$

(*iv*) $x - (x - y) - y - (y - x)$

(*v*) $4x + 7y - [5y - 8] - 2x$

(*vi*) $-2m + 5 + 4(m - 3)$

(*vii*) $2x - y + 5 - (x - y)$

(*viii*) $2(x - y) - (x - 8)$

(*ix*) $4(3x - 8) - 3(5x + 3) - 2(6x - 8)$

(*x*) $5(x - 4) - 3(x - 4) + 7(x - 4)$

Solution:

(*i*) $3x + 5(2x + 6) - 7x$

$\Rightarrow 3x + 10x + 30 - 7x$

$\Rightarrow 3x + 10x - 7x + 30$

$\Rightarrow 13x - 7x + 30$

$\Rightarrow 6x + 30$

(*ii*) $3(4y - 10) + 2(y - 1)$

$\Rightarrow 12y - 30 + 2y - 2$

$\Rightarrow 12y + 2y - 30 - 2$

$\Rightarrow 14y - 32$

(*iii*) $-(7 + 6x) - 7(x + 2)$

$\Rightarrow -7 - 6x - 7x - 14$

$\Rightarrow -7x - 6x - 7 - 14$

$\Rightarrow -13x - 21$

(*iv*) $x - (x - y) - y - (y - x)$

$\Rightarrow x - x + y - y - y + x$

$\Rightarrow 2x - x - 2y + y$

$\Rightarrow x - y$

(*v*) $4x + 7y - [5y - 8] - 2x$

$\Rightarrow 4x + 7y - 5y + 8 - 2x$

$\Rightarrow 4x - 2x + 7y - 5y + 8$

$\Rightarrow 2x + 2y + 8$

(*vi*) $-2m + 5 + 4(m - 3)$

$\Rightarrow -2m + 5 + 4m - 12$

$\Rightarrow -2m + 4m + 5 - 12$

$\Rightarrow 2m - 7$

(vii) $2x - y + 5 - (x - y)$

$\Rightarrow 2x - y + 5 - x + y$

$\Rightarrow 2x - x + 5$

$\Rightarrow x + 5$

(viii) $2(x - y) - (x - 8)$

$\Rightarrow 2x - 2y - x + 8$

$\Rightarrow 2x - x - 2y + 8$

$\Rightarrow x - 2y + 8$

(ix) $4(3x - 8) - 3(5x + 3) - 2(6x - 8)$

$\Rightarrow 12x - 32 - 15x - 9 - 12x + 16$

$\Rightarrow 12x - 15x - 12x - 32 - 9 + 16$

$\Rightarrow 12x - 27x - 41 + 16$

$\Rightarrow -15x - 25$

(x) $5(x - 4) - 3(x - 4) + 7(x - 4)$

$\Rightarrow 5x - 20 - 3x + 12 + 7x - 28$

$\Rightarrow 5x + 7x - 3x - 20 - 28 + 12$

$\Rightarrow 12x - 3x - 48 + 12$

$\Rightarrow 9x - 36$

EXERCISE 11 (C)

Question 1.

Multiply:

(i) $3x$, $5x^2y$ and $2y$

(ii) 5, $3a$ and $2ab^2$

(iii) $5x + 2y$ and $3xy$

(iv) $6a - 5b$ and $-2a$

(v) $4a + 5b$ and $4a - 5b$

(vi) $9xy + 2y^2$ and $2x - 3y$

(vii) $-3m^2n + 5mn - 4mn^2$ and $6m^2n$

(viii) $6xy^2 - 7x^2y^2 + 10x^3$ and $-3x^2y^3$

Solution:

(*i*) Product of $3x$, $5x^2y$ and $2y$

$= 3x + 5x^2y \times 2y$

$= 3 \times 5 \times 2 \times x \times x^2 \times y \times y$

$= 30x^3y^2$

(*ii*) Product of 5, $3a$ and $2ab^2$

$= 5 \times 3a \times 2ab^2$

$= 5 \times 3 \times 2 \times a \times ab^2$

$= 30a^2b^2$

(*iii*) Product of $5x + 2y$ and $3xy$

$= 3xy\,(5x + 2y)$

$= 3xy \times 5x + 3xy \times 2y$

$= 15x^2y + 6xy^2$

(*iv*) Product of $6a - 5b$ and $-2a$

$= -2a\,(6a - 5b)$

$= -2a \times 6a + (-2a)(-5b)$

$= -12a^2 + 10ab$

(*v*) Product of $4a + 5b$ and $4a - 5b$

$= 16a^2 - 25b^2$

$$\begin{array}{r} 4a + 5b \\ \times\, 4a - 5b \\ \hline 16a^2 + 20ab \\ \quad - 20ab - 25b^2 \\ \hline 16a^2 \qquad - 25b^2 \\ \hline \end{array}$$

(*vi*) Product of $9xy + 2y^2$ and $2x - 3y$

$= 18x^2y - 23xy^2 - 6y^3$

$$\begin{array}{r} 9xy + 2y^2 \\ \times\; 2x \;\; - 3y \\ \hline 18x^2y + 4xy^2 \\ \quad - 27xy^2 - 6y^3 \\ \hline 18x^2y - 23xy^2 - 6y^3 \\ \hline \end{array}$$

(*vii*) Proudct of $-3m^2n + 5mn - 4mn^2$ and $6m^2n$

$= 6m^2n(-3m^2n + 5mn - 4mn^2)$

$= 6m^2n \times (-3m^2n) + 6m^2n \times 5mn + 6m^2n \times (-4mn^2)$

$= -18m^4n^2 + 30m^3n^2 - 24m^3n^3$

(*viii*) Product of $6xy^2 - 7x^2y^2 + 10x^3$ and $-3x^2y^3$

$= -3x^2y^3(6xy^2 - 7x^2y^2 + 10x^3)$

$= -3x^2y^3 \times 6xy^2 + (-3x^2y^3)(-7x^2y^2) + (-3x^2y^3) \times 10x^3$

$= -18x^3y^5 + 21x^4y^5 - 30x^5y^3$

Question 2.

Copy and complete the following multi-plications :

(*i*) $3a + 2b$ × $-3xy$

(*ii*) $9x + 5y$ × $-3xy$

(*iii*) $3xy - 2x^2 - 6x$ × $-5x^2y$

(*iv*) $a + b$ × $a + b$

(*v*) $ax - b$ × $2ax + 2b^2$

(*vi*) $2a - b + 3c$ × $2a - 4b$

(*vii*) $3m^2 + 5m - 2n$ × $5n - 3m$

(*viii*) $6 - 3x + 2x^2$ × $1 + 5x - x^2$

(*ix*) $4x^3 - 10x^2 + 6x - 8$ × $3 + 2x - x^2$

Solution:

(*i*)
$$\begin{array}{r} 3a + 2b \\ \times \quad -3xy \\ \hline -9axy - 6bxy \end{array}$$

(*ii*)
$$\begin{array}{r} 9x - 5y \\ \times \quad -3xy \\ \hline -27x^2y + 15xy^2 \end{array}$$

(*iii*)
$$\begin{array}{r} 3xy - 2x^2 - 6x \\ \times \quad -5x^2y \\ \hline -15x^3y^2 + 10x^4y + 30x^3y \end{array}$$

(*iv*)
$$\begin{array}{r} a + b \\ \times a + b \\ \hline a^2 + ab \\ ab + b^2 \\ \hline a^2 + 2ab + b^2 \end{array}$$

(*v*)
$$\begin{array}{r} ax - b \\ \times 2ax + 2b^2 \\ \hline 2a^2x^2 - 2abx + 2ab^2x - 2b^3 \end{array}$$

(*vi*)
$$\begin{array}{r} 2a - b + 3c \\ \times \quad 2a - 4b \\ \hline 4a^2 - 2ab + 6ac \\ \quad - 8ab + 4b^2 - 12bc \\ \hline 4a^2 - 10ab + 6ac + 4b^2 - 12bc \\ \hline \end{array}$$

(*vii*)
$$\begin{array}{r} 3m^2 + 6m - 2n \\ \times \quad 5n - 3m \\ \hline 15m^2n + 30mn - 10n^2 - 9m^3 - 18m^2 \\ \quad + 6mn \\ \hline 15m^2n + 36mn - 10n^2 - 9m^3 - 18m^2 \\ \hline \end{array}$$

(*viii*)
$$\begin{array}{r} 6 - 3x + 2x^2 \\ \times 1 + 5x - x^2 \\ \hline 6 - 3x + 2x^2 \\ + 30x - 15x^2 + 10x^3 \\ - 6x^2 + 3x^3 - 2x^4 \\ \hline 6 + 27x - 19x^2 + 13x^3 - 2x^4 \\ \hline \end{array}$$

(*ix*)
$$\begin{array}{r} 4x^3 - 10x^2 + 6x - 8 \\ \times \quad 3 + 2x - x^2 \\ \hline 12x^3 - 30x^2 + 18x - 24 \\ 8x^4 - 20x^3 + 12x^2 - 16x \\ 4x^5 + 10x^4 - 6x^3 + 8x^2 \\ \hline 4x^5 + 18x^4 - 14x^3 - 10x^2 + 2x - 24 \\ \hline \end{array}$$

Question 3.

Evaluate :

(*i*) $(c + 5)(c - 3)$ (*ii*) $(3c - 5d)(4c - 6d)$

(*iii*) $\left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a - \frac{1}{2}b\right)$

(*iv*) $(a^2 + 2ab + b^2)(a + b)$

(*v*) $(3x - 1)(4x^3 - 2x^2 + 6x - 3)$

(*vi*) $(4m - 2)(m^2 + 5m - 6)$

(*vii*) $(8 - 12x + 7x^2 - 6x^3)(5 - 2x)$

(*viii*) $(4x^2 - 4x + 1)(2x^3 - 3x^2 + 2)$

(*ix*) $(6p^2 - 8pq + 2q^2)(-5p)$

(*x*) $-4y(15x + 12y - 8z)(x - 2y)$

(*xi*) $(a^2 + b^2 + c^2 - ab - bc - ca)(a + b + c)$

Solution:

(*i*) $(c + 5)(c - 3) = c(c - 3) + 5(c - 3)$

$= c^2 - 3c + 5c - 15$

$= c^2 + 2c - 15$

(*ii*) $(3c - 5d)(4c - 6d)$

$= 3c(4c - 6d) - 5d(4c - 6d)$

$= 12c^2 - 18cd - 20cd + 30d^2$

$= 12c^2 - 38cd + 30d^2$

(*iii*) $\left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a - \frac{1}{2}b\right)$

$= \frac{1}{2a}\left(\frac{1}{2}a - \frac{1}{2}b\right) + \frac{1}{2}b\left(\frac{1}{2}a - \frac{1}{2}b\right)$

$= \frac{1}{4}a^2 - \frac{1}{4}ab + \frac{1}{4}ab - \frac{1}{4}b^2$

$= \frac{1}{4}a^2 - \frac{1}{4}b^2$

(*iv*) $(a^2 + 2ab + b^2)(a + b)$

$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$

$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$

(*v*) $(3x - 1)(4x^3 - 2x^2 + 6x - 3)$

$= 3x(4x^3 - 2x^2 + 6x - 3) - 1(4x^3 - 2x^2 + 6x - 3)$

$= 12x^4 - 6x^3 + 18x^2 - 9x - 4x^3 + 2x^2 - 6x + 3$

$= 12x^4 - 6x^3 - 4x^3 + 18x^2 + 2x^2 - 9x - 6x + 3$

$= 12x^4 - 10x^3 + 20x^2 - 15x + 3$

(*vi*) $(4m - 2)(m^2 + 5m - 6)$

$= 4m(m^2 + 5m - 6) - 2(m^2 + 5m - 6)$

$= 4m^3 + 20m^2 - 24m - 2m^2 - 10m + 12$

$= 4m^3 + 20m^2 - 2m^2 - 24m - 10m + 12$

$= 4m^3 + 18m^2 - 34m + 12$ **Ans.**

(*vii*) $(8 - 12x + 7x^2 - 6x^3)(5 - 2x)$

$= 5(8 - 12x + 7x^2 - 6x^3)$

$- 2x(8 - 12x + 7x^2 - 6x^3)$

$= 40 - 60x + 35x^2 - 30x^3 - 16x + 24x^2$

$- 14x^3 + 12x^4$

$= 40 - 60x - 16x + 35x^2 + 24x^2 - 30x^3$

$- 14x^3 + 12x^4$

$= 40 - 76x + 59x^2 - 44x^3 + 12x^4$

(*viii*) $(4x^2 - 4x + 1)(2x^3 - 3x^2 + 2)$

$= 4x^2(2x^3 - 3x^2 + 2) - 4x(2x^3 - 3x^2 + 2)$

$+ 1(2x^3 - 3x^2 + 2)$

$= 8x^5 - 12x^4 + 8x^2 - 8x^4 + 12x^3 - 8x + 2x^3$

$- 3x^2 + 2$

$= 8x^5 - 12x^4 - 8x^4 + 12x^3 + 2x^3 + 8x^2$

$- 3x^2 - 8x + 2$

$= 8x^5 - 20x^4 + 14x^3 + 5x^2 - 8x + 2$

(*ix*) $(6p^2 - 8pq + 2q^2)(-5p)$

$= -5p \times 6p^2 - 5p \times (-8pq) - 5p(2q^2)$

$= -30p^3 + 40p^2q - 10pq^2$

(*x*) $-4y(15 + 12y - 8z)(x - 2y)$

$= -4y(x - 2y)(15x + 12y - 8z)$

$= (-4xy + 8y^2)(15x + 12y - 8z)$

$= -4xy(15x + 12y - 8z) + 8y^2$

$(15x + 12y - 8z)$

$= -60x^2y - 48xy^2 + 32xyz + 120xy^2$

$+ 96y^3 - 64y^2z$

$= -60x^2y - 48xy^2 + 120xy^2 - 64y^2z + 96y^3$

$+ 32xyz$

$= -60x^2y + 72xy^2 - 64y^2z + 96y^3 + 32xyz$

(xi) $(a^2 + b^2 + c^2 - ab - bc - ca)(a + b + c)$

$= a(a^2 + b^2 + c^2 - ab - bc - ca)$

$+ b(a^2 + b^2 + c^2 - ab - bc - ca)$

$+ c(a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + ab^2 + ac^2 - a^2b - abc - ca^2 + a^2b$

$+ b^3 + bc^2 - ab^2 - b^2c$

$- abc + a^2c + b^2c + c^3 - abc - bc^2 - c^2a$

$= a^3 + b^3 + c^3 - a^2b + a^2b - ca^2 + a^2c$

$+ bc^2 - bc^2 - ab^2 + ab^2$

$- abc - abc - abc + ac^2 - ac^2 + b^2c - b^2c$

$= a^3 + b^3 + c^3 - 3abc$

Question 4.

Evaluate:

(i) $(a + b)(a - b)$

(ii) $(a^2 + b^2)(a + b)(a - b)$; using the result of (i).

(iii) $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$; using the result of (ii).

Solution:

(i) $(a + b)(a - b)$

$= a(a - b) + b(a - b)$

$= a^2 - ab + ab - b^2 = a^2 - b^2$

(ii) $(a^2 + b^2)(a + b)(a - b)$

$= (a^2 + b^2)(a^2 - b^2)$ {from (i)}

$= a^2(a^2 - b^2) + b^2(a^2 - b^2)$

$= a^4 - a^2b^2 + a^2b^2 - b^4 = a^4 - b^4$

(iii) $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$

$= (a^4 + b^4)(a^4 - b^4)$ {from (ii)}

$= a^4(a^4 - b^4) + b^4(a^4 - b^4)$

$= a^8 - a^4b^4 + a^4b^4 - b^8 = a^8 - b^8$

Question 5.

Evaluate :

(*i*) $(3x - 2y)(4x + 3y)$

(*ii*) $(3x - 2y)(4x + 3y)(8x - 5y)$

(*iii*) $(a + 5)(3a - 2)(5a + 1)$

(*iv*) $(a + 1)(a^2 - a + 1)$ and $(a - 1)(a^2 + a + 1)$;
and then : $(a + 1)(a^2 - a + 1) + (a - 1)(a^2 + a + 1)$

(*v*) $(5m - 2n)(5m + 2n)(25m^2 + 4n^2)$

Solution:

(*i*) $(3x - 2y)(4x + 3y)$

$= 3x(4x + 3y) - 2y(4x + 3y)$

$= 12x^2 + 9xy - 8xy - 6y^2$

$= 12x^2 + xy - 6y^2$

(*ii*) $(3x - 2y)(4x + 3y)(8x - 5y)$

$= (12x^2 + xy - 6y^2)(8x - 5y)$ {from (*i*)}

$= 8x(12x^2 + xy - 6y^2) - 5y(12x^2 + xy - 6y^2)$

$= 96x^3 + 8x^2y - 48xy^2 - 60x^2y - 5xy^2 + 30y^3$

$= 96x^3 + 8x^2y - 60x^2y - 48xy^2 - 5xy^2 + 30y^3$

$= 96x^3 - 52x^2y - 53xy^2 + 30y^3$

(*iii*) $(a + 5)(3a - 2)(5a + 1)$

$= \{a(3a - 2) + 5(3a - 2)(5a + 1)$

$= (3a^2 - 2a + 15a - 10)(5a + 1)$

$= (3a^2 + 13a - 10)(5a + 1)$

$= 5a(3a^2 + 13a - 10) + 1(3a^2 + 13a - 10)$

$= 15a^3 + 65a^2 - 50a + 3a^2 + 13a - 10$

$= 15a^3 + 68a^2 - 37a - 10$

(*iv*) $(a+1)(a^2 - a + 1)$ and $(a-1)(a^2 + a + 1)$;

$= a(a^2 - a + 1) + 1(a^2 - a + 1)$

$= a^3 - a^2 + a + a^2 - a + 1 = a^3 + 1$

$(a - 1)(a^2 + a + 1)$

$= a(a^2 + a + 1) - 1(a^2 + a + 1)$

$= a^3 + a^2 + a - a^2 - a - 1 = a^3 - 1$

Now, $(a+1)(a^2 - a + 1) + (a-1)(a^2 + a + 1)$

$= a^3 + 1 + a^3 - 1 = 2a^3$

(*v*) $(5m - 2n)(5m + 2n)(25m^2 + 4n^2)$

$= \{5m(5m + 2n) - 2n(5m + 2n)\}(25m^2 + 4n^2)$

$= (25m^2 + 10mn - 10mn - 4n^2)(25m^2 + 4n^2)$

$= (25m^2 - 4n^2)(25m^2 + 4n^2)$

$= 25m^2(25m^2 + 4n^2) - 4n^2(25m^2 + 4n^2)$

$= 625m^4 + 100m^2n^2 - 100m^2n^2 - 16n^4$

$= 625m^4 - 16n^4$

Question 6.

Multiply:

(*i*) mn^4, m^3n and $5m^2n^3$

(*ii*) $2mnpq$, $4mnpq$ and $5mnpq$

(*iii*) $pq - pm$ and p^2m

(*iv*) $x^3 - 3y^3$ and $4x^2y^2$

(*v*) $a^3 - 4ab$ and $2a^2b$

(*vi*) $x^2 + 5yx - 3y^2$ and $2x^2y$.

Solution:

(*i*) mn^4, m^3n and $5m^2n^3$

$\Rightarrow 5m^2n^3 \times mn^4 \times m^3n)$

$\Rightarrow 5m^{(2+1+3)}n^{(3+4+1)}$

$= 5m^6n^8$

(*ii*) $2mnpq$, $4mnpq$ and $5mnpq$

$\Rightarrow 5mnpq \times 2mnpq \times 4mnpq)$

$\Rightarrow 5 \times 2 \times 4\ m^{(1+1+1)}n^{(1+1+1)}p^{(1+1+1)}q^{(1+1+1)}$

$\Rightarrow 40m^3n^3p^3q^3$

(*iii*) $pq - pm$ and $p^2m \Rightarrow p^2m \times (pq - pm)$

$\Rightarrow p^3qm - p^3m^2$

(*iv*) $x^3 - 3y^3$ and $4x^2y^2$

$\Rightarrow 4x^2y^2 \times (x^3 - 3y^3)$

$\Rightarrow 4x^5y^2 - 12x^2y^5$

(*v*) $a^3 - 4ab$ and $2a^2b$

$\Rightarrow 2a^2b \times (a^3 - 4ab)$

$\Rightarrow 2a^5b - 8a^3b^2$

(*vi*) $x^2 + 5yx - 3y^2$ and $2x^2y$

$\Rightarrow 2x^2y \times (x^2 + 5yx - 3y^2)$

$\Rightarrow 2x^4y + 10x^3y^2 - 6x^2y^3$

Question 7.

Multiply:

(*i*) $(2x + 3y)(2x + 3y)$ (*ii*) $(2x - 3y)(2x + 3y)$

(*iii*) $(2x + 3y)(2x - 3y)$ (*iv*) $(2x - 3y)(2x - 3y)$

(*v*) $(-2x + 3y)(2x - 3y)$ (*vi*) $(xy + 2b)(xy - 2b)$

(*vii*) $(x - a)(x + 3b)$

(*viii*) $(2x + 5y + 6)(3x + y - 8)$

(*ix*) $(3x - 5y + 2)(5x - 4y - 3)$

(*x*) $(6x - 2y)(3x - y)$

(*xi*) $(1 + 6x^2 - 4x^3)(-1 + 3x - 3x^2)$

Solution:

$(i)\ (2x + 3y)\ (2x + 3y)$

$\Rightarrow 2x\ (2x + 3y) + 3y\ (2x + 3y)$

$\Rightarrow 4x^2 + 6xy + 6xy + 9y^2$

$\Rightarrow 4x^2 + 12xy + 9y^2$

$(ii)\ (2x - 3y)\ (2x + 3y)$

$\Rightarrow 2x\ (2x + 3y) - 3y\ (2x + 3y)$

$\Rightarrow 2x \times 2x + 2x \times 3y - 3y \times 2x - 3y \times 3y$

$\Rightarrow 4x^2 + 6xy - 6xy - 9y^2$

$\Rightarrow 4x^2 + 0 - 9y^2$

$\Rightarrow 4x^2 - 9y$

$(iii)\ (2x + 3y)\ (2x - 3y)$

$\Rightarrow 2x\ (2x - 3y) + 3y\ (2x - 3y)$

$\Rightarrow 2x \times 2x - 2x \times 3y + 3y \times 2x - 3y \times 3y$

$\Rightarrow 4x^2 - 6xy + 6xy - 9y^2$

$\Rightarrow 4x^2 - 0 - 9y^2$

$\Rightarrow 4x^2 - 9y^2$

$(iv)\ (2x - 3y)\ (2x - 3y)$

$\Rightarrow 2x\ (2x - 3y) - 3y\ (2x - 3y)$

$\Rightarrow 2x \times 2x - 2x \times 3y - 3y \times 2x + 3y \times 3y$

$\Rightarrow 4x^2 - 6xy - 6xy + 9y^2$

$\Rightarrow 4x^2 - 12xy + 9y^2$

$(v)\ (-2x + 3y)\ (2x - 3y)$

$\Rightarrow -2x\ (2x - 3y) + 3y\ (2x - 3y)$

$\Rightarrow -4x^2 + 6xy + 6xy - 9y^2$

$\Rightarrow -4x^2 + 12xy - 9y^2$

(*vi*) $(xy + 2b)(xy - 2b)$

$\Rightarrow xy(xy - 2b) + 2b(xy - 2b)$

$\Rightarrow x^2y^2 - 2bxy + 2bxy - 4b^2$

$\Rightarrow x^2y^2 - 4b^2$

(*vii*) $(x - a)(x + 3b)$

$\Rightarrow x(x + 3b) - a(x + 3b)$

$\Rightarrow x^2 + 3bx - ax - 3ab$

(*viii*) $(2x + 5y + 6)(3x + y - 8)$

$\Rightarrow 2x(3x + y - 8) + 5y(3x + y - 8) + 6(3x + y - 8)$

$\Rightarrow 6x^2 + 2xy - 16x + 15xy + 5y^2 - 40y + 18x + 6y - 48$

$\Rightarrow 6x^2 + 2xy + 15xy - 16x + 18x + 5y^2 - 40y + 6y - 48$

$\Rightarrow 6x^2 + 17xy + 2x + 5y^2 - 34y - 48$

(*ix*) $(3x - 5y + 2)(5x - 4y - 3)$

$\Rightarrow 3x(5x - 4y - 3) - 5y(5x - 4y - 3) + 2(5x - 4y - 3)$

$\Rightarrow 15x^2 - 12xy - 9x - 25xy + 20y^2 + 15y + 10x - 8y - 6$

$\Rightarrow 15x^2 - 12xy - 25xy - 9x + 10x + 20y^2 + 15y - 8y - 6$

$\Rightarrow 15x^2 - 37xy + x + 20y^2 + 7y - 6$

(*x*) $(6x - 2y)(3x - y)$

$\Rightarrow 6x(3x - y) - 2y(3x - 2y)$

$\Rightarrow 18x^2 - 6xy - 6xy + 2y^2$

$\Rightarrow 18x^2 - 12xy + 2y^2$

(*xi*) $(1 + 6x^2 - 4x^3)(-1 + 3x - 3x^2)$

$1(-1 + 3x - 3x^2) + 6x^2(-1 + 3x - 3x^2) - 4x^3(-1 + 3x - 3x^2)$

$-1 + 3x - 3x^2 - 6x^2 + 18x^3 - 18x^4 + 4x^3 - 12x^4 + 12x^5$

$-1 + 3x - 9x^2 + 22x^3 - 30x^4 + 12x^5$

EXERCISE 11 (D)

Question 1.

Divide:

(*i*) $-16ab^2c$ by $6abc$ (*ii*) $25x^2y$ by $-5y^2$

(*iii*) $8x + 24$ by 4 (*iv*) $4a^2 - a$ by $-a$

(*v*) $8m - 16$ by -8 (*vi*) $-50 + 40p$ by $10p$

(*vii*) $4x^3 - 2x^2$ by $-x$

(*viii*) $10a^3 - 15a^2b$ by $-5a^2$

(*ix*) $12x^3y - 8x^2y^2 + 4x^2y^3$ by $4xy$

(*x*) $9a^4b - 15a^3b^2 + 12a^2b^3$ by $-3a^2b$

Solution:

(*i*) $-16ab^2c$ by $6abc$

$$= -\frac{16ab^2c}{6abc} = -\frac{8}{3}b$$

(*ii*) $25x^2y$ by $-5y^2$

$$= \frac{25x^2y}{-5y^2} = -5\frac{x^2}{y}$$

(*iii*) $8x + 24$ by 4

$$= \frac{8x+24}{4} = \frac{8x}{4} + \frac{24}{4} = 2x + 6$$

(*iv*) $4a^2 - a$ by $-a$

$$= \frac{4a^2 - a}{-a} = \frac{4a^2}{-a} - \frac{a}{-a}$$

$$= -4a + 1$$ **Ans.**

(*v*) $8m - 16$ by -8

$$= \frac{8m-16}{-8} = \frac{8m}{-8} - \frac{16}{-8} = -m + 2$$

(*vi*) $-50 + 40p$ by $10p$

$$= \frac{-50+40p}{10p} = \frac{-50}{10p} + \frac{40p}{10p}$$

$$= -\frac{5}{p} + 4$$

(*vii*) $4x^3 - 2x^2$ by $-x$

$$= \frac{4x^3 - 2x^2}{-x} = \frac{4x^3}{-x} - \frac{2x^2}{-x}$$

$$= -4x^2 + 2x$$

(*viii*) $10a^3 - 15a^2b$ by $-5a^2$

$$= \frac{10a^3 - 15a^2b}{-5a^2} = \frac{10a^3}{-5a^2} - \frac{15a^2b}{-5a^2}$$

$= -2a + 3b$ **Ans.**

(*ix*) $12x^3y - 8x^2y^2 + 4x^2y^3$ by $4xy$

$$= \frac{12x^3y - 8x^2y^2 + 4x^2y^3}{4xy}$$

$$= \frac{12x^3y}{4xy} - \frac{8x^2y^2}{4xy} + \frac{4x^2y^3}{4xy}$$

$= 3x^2 - 2xy + xy^2$

(*x*) $9a^4b - 15a^3b^2 + 12a^2b^3$ by $-3a^2b$

$$= \frac{9a^4b - 15a^3b^2 + 12a^2b^3}{-3a^2b}$$

$$= \frac{9a^4b}{-3a^2b} - \frac{15a^3b^2}{-3a^2b} + \frac{12a^2b^3}{-3a^2b}$$

$= -3a^2 + 5ab - 4b^2$

Question 2.

Divide :

(*i*) $n^2 - 2n + 1$ by $n - 1$

(*ii*) $m^2 - 2mn + n^2$ by $m - n$

(*iii*) $4a^2 + 4a + 1$ by $2a + 1$

(*iv*) $p^2 + 4p + 4$ by $p + 2$

(*v*) $x^2 + 4xy + 4y^2$ by $x + 2y$

(*vi*) $2a^2 - 11a + 12$ by $a - 4$

(*vii*) $6x^2 + 5x - 6$ by $2x + 3$

(*viii*) $8a^2 + 4a - 60$ by $2a - 5$

(*ix*) $9x^2 - 24xy + 16y^2$ by $3x - 4y$

(*x*) $15x^2 + 31xy + 14y^2$ by $5x + 7y$

(*xi*) $35a^3 + 3a^2b - 2ab^2$ by $5a - b$

(*xii*) $6x^3 + 5x^2 - 21x + 10$ by $3x - 2$

Solution:

(*i*) $n^2 - 2n + 1$ by $n - 1$

$$
\begin{array}{r}
n - 1 \\
\hline
n - 1 \,\big)\, n^2 - 2n + 1 \,\big(\\
n^2 - n \\
\hline
- \quad + \\
\hline
-n + 1 \\
-n + 1 \\
\hline
+ \quad - \\
\hline
\times \\
\hline
\end{array}
$$

$= n - 1$

(*ii*) $m^2 - 2mn + n^2$ by $m - n$

$$
\begin{array}{r}
m - n \\
\hline
m - n \,\big)\, m^2 - 2mn + n^2 \,\big(\\
m^2 - mn \\
\hline
- \quad + \\
\hline
-mn + n^2 \\
-mn + n^2 \\
\hline
+ \quad - \\
\hline
\times \\
\hline
= m - n
\end{array}
$$

(*iii*) $4a^2 + 4a + 1$ by $2a + 1$

$$
\begin{array}{r}
2a + 1 \\
\hline
2a + 1 \,\big)\, 4a^2 + 4a + 1 \,\big(\\
4a^2 + 2a \\
\hline
- \quad - \\
\hline
2a + 1 \\
2a + 1 \\
\hline
- \quad - \\
\hline
\times \\
\hline
= 2a + 1
\end{array}
$$

(*iv*) $p^2 + 4p + 4$ by $p + 2$

$$
\begin{array}{r}
p + 2 \\
\hline
p+2 \;)\; p^2 + 4p + 4 \;(\\
p^2 + 2p \\
\hline
\quad 2p + 4 \\
\quad 2p + 4 \\
\hline
\quad\quad \times \\
\hline
= p + 2
\end{array}
$$

(*v*) $x^2 + 4xy + 4y^2$ by $x + 2y$

$$
\begin{array}{r}
x + 2y \\
\hline
x+2y \;)\; x^2 + 4xy + 4y^2 \;(\\
x^2 + 2xy \\
\hline
\quad 2xy + 4y^2 \\
\quad 2xy + 4y^2 \\
\hline
\quad\quad \times \\
\hline
= x + 2y
\end{array}
$$

(*vi*) $2a^2 - 11a + 12$ by $a - 4$

$$
\begin{array}{r}
2a - 3 \\
\hline
a-4 \;)\; 2a^2 - 11a + 12 \;(\\
2a^2 - 8a \\
\hline
\quad - 3a + 12 \\
\quad - 3a + 12 \\
\hline
\quad\quad \times \\
\hline
= 2a - 3
\end{array}
$$

(*vii*) $6x^2 + 5x - 6$ by $2x + 3$

$$
\begin{array}{r}
3x - 2 \\
\hline
2x + 3 \,\big)\, 6x^2 + 5x - 6 \,\big(\\
6x^2 + 9x \\
\hline
- \quad - \\
\hline
-4x - 6 \\
-4x - 6 \\
\hline
+ \quad + \\
\hline
\times \\
\hline
\end{array}
$$

$= 3x - 2$

(*viii*) $8a^2 + 4a - 60$ by $2a - 5$

$$
\begin{array}{r}
4a + 12 \\
\hline
2a - 5 \,\big)\, 8a^2 + 4a - 60 \,\big(\\
8a^2 - 20a \\
\hline
- \quad + \\
\hline
24a - 60 \\
24a - 60 \\
\hline
- \quad + \\
\hline
\times \\
\hline
\end{array}
$$

$= 4a + 12$

(*ix*) $9x^2 - 24xy + 16y^2$ by $3x - 4y$

$$
\begin{array}{r}
3x - 4y \\
\hline
3x - 4y \,\big)\, 9x^2 - 24xy + 16y^2 \,\big(\\
9x^2 - 12xy \\
\hline
- \quad + \\
\hline
-12xy + 16y^2 \\
-12xy + 16y^2 \\
\hline
+ \quad - \\
\hline
\times \\
\hline
\end{array}
$$

$= 3x - 4y$

(x) $15x^2 + 31xy + 14y^2$ by $5x + 7y$

$$
\begin{array}{r}
3x + 2y \\
\hline
5x + 7y \;)\; 15x^2 + 31xy + 14y^2 \;(\\
15x^2 + 21xy \\
\hline
- \quad - \\
\hline
10xy + 14y^2 \\
10xy + 14y^2 \\
\hline
- \quad - \\
\hline
\times \\
\hline
\end{array}
$$

$= 3x + 2y$ **Ans.**

(xi) $35a^3 + 3a^2b - 2ab^2$ by $5a - b$

$$
\begin{array}{r}
7a^2 + 2ab \\
\hline
5a - b \;)\; 35a^3 + 3a^2b - 2ab^2 \;(\\
35a^3 - 7a^2b \\
\hline
- \quad + \\
\hline
10a^2b - 2ab^2 \\
10a^2b - 2ab^2 \\
\hline
- \quad + \\
\hline
\times \\
\hline
\end{array}
$$

$= 7a^2 + 2ab$

(xii) $6x^3 + 5x^2 - 21x + 10$ by $3x - 2$

$$
\begin{array}{r}
2x^2 + 3x - 5 \\
\hline
3x - 2 \;)\; 6x^3 + 5x^2 - 21x + 10 \;(\\
6x^3 - 4x^2 \\
\hline
- \quad + \\
\hline
9x^2 - 21x \\
9x^2 - 6x \\
\hline
- \quad + \\
\hline
-15x + 10 \\
-15x + 10 \\
\hline
+ \quad - \\
\hline
\times \\
\hline
\end{array}
$$

$= 2x^2 + 3x - 5$

Question 3.

The area of a rectangle is $6x^2 - 4xy - 10y^2$ square unit and its length is $2x + 2y$ unit. Find its breadth

Solution:

Area of a rectangle

$= 6x^2 - 4xy - 10y^2$ sq. units

Length $= 2x + 2y$ units

$$\therefore \text{Breadth} = \frac{\text{Area}}{\text{Length}}$$

$$= \frac{6x^2 - 4xy - 10y^2}{2x + 2y}$$

$$\begin{array}{r|l} & 3x - 5y \\ \hline 2x + 2y & 6x^2 - 4xy - 10y^2 \\ & 6x^2 + 6xy \\ & - \quad - \\ \hline & -10xy - 10y^2 \\ & -10xy - 10y^2 \\ & + \quad + \\ \hline & \times \\ \hline \end{array}$$

$= 3x - 5y$ units

Hence breadth $= 3x - 5y$ units

Question 4.

The area of a rectangular field is $25x^2 + 20xy + 3y^2$ square unit. If its length is $5x + 3y$ unit, find its breadth, Hence find its perimeter.

Solution:

Area of a rectangular field

$$= 25x^2 + 20xy + 3y^2$$

Length $= (5x + 3y)$ unit

$$\therefore \quad \text{Breadth} = \frac{\text{Area}}{\text{Length}} = \frac{25x^2 + 20xy + 3y^2}{5x + 3y}$$

$$\begin{array}{r} 5x + y \\ \hline 5x + 3y \,)\; 25x^2 + 20xy + 3y^2 \;(\\ 25x^2 + 15xy \\ \hline - \quad - \\ \hline 5xy + 3y^2 \\ 5xy + 3y^2 \\ \hline - \quad - \\ \hline \times \\ \hline \end{array}$$

Hence Breadth $= 5x + y$

Hence perimeter of rectangular field

$$= 2\,(l + b)$$

$$= 2\,(5x + 3y + 5x + y)$$

$$= 2\,(10x + 4y)$$

$$= 20x + 8y$$

Question 5.

Divide:

(*i*) $2m^3n^5$ by $-mn$

(*ii*) $5x^2 - 3x$ by x

(*iii*) $10x^3y - 9xy^2 - 4x^2y^2$ by xy

(*iv*) $3y^3 - 9ay^2 - 6ab^2y$ by $-3y$

(*v*) $x^5 - 15x^4 - 10x^2$ by $-5x^2$

(*vi*) $12a^2 + ax - 6x^2$ by $3a - 2x$

(*vii*) $6x^2 - xy - 35y^2$ by $2x - 5y$

(*viii*) $x^3 - 6x^2 + 11x - 6$ by $x^2 - 4x + 3$

(*ix*) $m^3 - 4m^2 + m + 6$ by $m^2 - m - 2$

Solution:

(*i*) $2m^3n^5$ by $-mn$

$$= \frac{2m^3n^5}{-mn} = -2m^2n^4$$

(*ii*) $5x^2 - 3x$ by x

$$= \frac{5x^2 - 3x}{x} = \frac{5x^2}{x} - \frac{3x}{x} = 5x - 3$$

(*iii*) $10x^3y - 9xy^2 - 4x^2y^2$ by xy

$$= \frac{10x^3y - 9xy^2 - 4x^2y^2}{xy}$$

$$= \frac{10x^3y}{xy} - \frac{9xy^2}{xy} - \frac{4x^2y^2}{xy}$$

$$= 10x^2 - 9y - 4xy$$

(*iv*) $3y^3 - 9ay^2 - 6ab^2y$ by $-3y$

$$= \frac{3y^3 - 9ay^2 - 6aby^2}{-3y}$$

$$= \frac{3y^3}{-3y} - \frac{9ay^3}{-3y} - \frac{6ab^2y}{-3y}$$

$$= -y^2 + 3ay^2 + 2ab^2$$

(*v*) $x^5 - 15x^4 - 10x^2$ by $-5x^2$

$$= \frac{x^5 - 15x^4 - 10x^2}{-5x^2}$$

$$= \frac{x^5}{-5x^2} - \frac{15x^4}{-5x^2} - \frac{10x^2}{-5x^2}$$

$= \frac{-1}{5}x^3 + 3x^2 + 2$

(*vi*) $12a^2 + ax - 6x^2$ by $3a - 2x$

$$
\begin{array}{l}
3a - 2x\overline{)12a^2 + ax - 6x^2}(4a + 3x \\
\quad 12a^2 + 8ax \\
\quad - \quad - \\
\hline
\quad\quad 9ax - 6x^2 \\
\quad\quad 9ax - 6x^2 \\
\quad\quad - \quad + \\
\hline
\quad\quad\quad \times \\
\hline
\end{array}
$$

$= 4a + 3x$

(*vii*) $6x^2 - xy - 35y^2$ by $2x - 5y$

$$
\begin{array}{l}
2x - 5y\overline{)6x^2 - xy - 35y^2}(3x + 7y \\
\quad 6x^2 - 15xy \\
\quad - \quad + \\
\hline
\quad\quad 14xy - 35y^2 \\
\quad\quad 14xy - 35y^2 \\
\quad\quad - \quad + \\
\hline
\quad\quad\quad \times \\
\hline
\end{array}
$$

$= 3x + 7y$

(*viii*) $x^3 - 6x^2 + 11x - 6$ by $x^2 - 4x + 3$

$$
\begin{array}{l}
x^2 - 4x + 3\overline{)x^3 - 6x^2 + 11x - 6}(x - 2 \\
\quad x^3 - 4x^2 + 3x \\
\quad - \quad + \quad - \\
\hline
\quad\quad - 2x^2 + 8x - 6 \\
\quad\quad - 2x^2 + 8x - 6 \\
\quad\quad + \quad - \quad + \\
\hline
\quad\quad\quad\quad \times \\
\hline
\end{array}
$$

$= x - 2$

(ix) $m^3 - 4m^2 + m + 6$ by $m^2 - m - 2$

$$
\begin{array}{l}
m^2 - m - 2 \overline{)m^3 - 4m^2 + m + 6} \; (m - 3 \\
\quad\; m^3 - m^2 - 2m \\
\quad\; - \quad + \quad + \\
\hline
\quad\quad - 3m^2 + 3m + 6 \\
\quad\quad - 3m^2 + 3m + 6 \\
\quad\quad + \quad - \quad - \\
\hline
\quad\quad\quad\quad \times \\
\hline
\end{array}
$$

$= m - 3$

EXERCISE 11 (E)

Simplify

Question 1.

$$\frac{x}{2} + \frac{x}{4}$$

Solution:

$$\frac{x}{2} + \frac{x}{4}$$

$$\frac{2x + x}{4} = \frac{3x}{4}$$

Question 2.

$$\frac{a}{10} + \frac{2a}{5}$$

Solution:

$$\frac{a}{10} + \frac{2a}{5}$$

$$\frac{a + 4a}{10} = \frac{5a}{10} = \frac{a}{2}$$

Question 3.

$\frac{y}{4}+\frac{3y}{5}$

Solution:

$$\frac{y}{4}+\frac{3y}{5}$$

$$\frac{5y+12y}{20}=\frac{17y}{20}$$

Question 4.

$$\frac{x}{2}-\frac{x}{8}$$

Solution:

$$\frac{x}{2}-\frac{x}{8}$$

$$\frac{4x-x}{8}=\frac{3x}{8}$$

Question 5.

$$\frac{3y}{4}-\frac{y}{5}$$

Solution:

$$\frac{3y}{4}-\frac{y}{5}$$

$$\frac{15y-4y}{20}=\frac{11y}{20}$$

Question 6.

$$\frac{2p}{3}-\frac{3p}{5}$$

Solution:

$$\frac{2p}{3} - \frac{3p}{5}$$

$$\frac{10p - 9p}{15} = \frac{p}{15}$$

Question 7.

$$\frac{k}{2} + \frac{k}{3} + \frac{2k}{5}$$

Solution:

$$\frac{k}{2} + \frac{k}{3} + \frac{2k}{5}$$

$$\frac{15k + 10k + 12k}{30} = \frac{37k}{30}$$

(L.C.M. of 2, 3, 5 = 30)

Question 8.

$$\frac{2x}{5} + \frac{3x}{4} - \frac{3x}{5}$$

Solution:

$$\frac{2x}{5} + \frac{3x}{4} - \frac{3x}{5}$$ (LCM of 5, 4 = 20)

$$\frac{8x + 15x - 12x}{20} = \frac{23x - 12x}{20} = \frac{11x}{20}$$

Question 9.

$$\frac{4a}{7} - \frac{2a}{3} + \frac{a}{7}$$

Solution:

$$\frac{4a}{7} - \frac{2a}{3} + \frac{a}{7}$$

$$\frac{12a - 14a + 3a}{21} = \frac{15a - 14a}{21} = \frac{a}{21}$$

(LCM of 7, 3 = 21)

Question 10.

$$\frac{2b}{5} - \frac{7b}{15} + \frac{13b}{3}$$

Solution:

$$\frac{2b}{5} - \frac{7b}{15} + \frac{13b}{3}$$ (L.C.M. of 3, 5, 15 = 15)

$$\frac{6b - 7b + 65b}{15} = \frac{71b - 7b}{15} = \frac{64b}{15}$$

Question 11.

$$\frac{6k}{7} - \left(\frac{8k}{9} - \frac{k}{3}\right)$$

Solution:

$$\frac{6k}{7} - \left(\frac{8k}{9} - \frac{k}{3}\right) = \frac{54k - (56k - 21k)}{63}$$

(L.C.M. of 7, 9, 3 = 63)

$$= \frac{54k - (35k)}{63}$$

$$= \frac{54k - 35k}{63} = \frac{19k}{63}$$

Question 12.

$$\frac{3a}{8} + \frac{4a}{5} - \left(\frac{a}{2} + \frac{2a}{5}\right)$$

Solution:

$$\frac{3a}{8}+\frac{4a}{5}-\left(\frac{a}{2}+\frac{2a}{5}\right)$$

$$=\frac{15a+32a-(20a+16a)}{40}$$

(L.C.M. of 8, 5, 2 = 40)

$$=\frac{47a-36a}{40}=\frac{11a}{40}$$

Question 13.

$$x+\frac{x}{2}+\frac{x}{3}$$

Solution:

$$\frac{x}{1}+\frac{x}{2}+\frac{x}{3}$$

$$=\frac{6x+3x+2x}{6}=\frac{11x}{6}$$

Question 14.

$$\frac{y}{5}+y-\frac{19y}{15}$$

Solution:

$$\frac{y}{5}+\frac{y}{1}-\frac{19y}{15}$$

$$=\frac{3y+15y-19y}{15}=\frac{18y-19y}{15}$$

$$=\frac{-y}{15}$$

Question 15.

$$\frac{x}{5}+\frac{x+1}{2}$$

Solution:

$$\frac{x}{5}+\frac{x+1}{2}=\frac{2x+5x+5}{10}=\frac{7x+5}{10}.$$

(L.C.M. of 5, 2 = 10)

Question 16.

$$x+\frac{x+2}{3}$$

Solution:

$$\frac{x}{1}+\frac{x+2}{3}$$

$$=\frac{3x+x+2}{3}=\frac{4x+2}{3}$$

Question 17.

$$\frac{3y}{5}-\frac{y+2}{2}.$$

Solution:

$$\frac{3y}{5}-\frac{y+2}{2}$$

$$=\frac{6y-(5y+10)}{10}$$

$$=\frac{6y-5y-10}{10}=\frac{y-10}{10}.$$

Question 18.

$$\frac{2a+1}{3}+\frac{3a-1}{2}$$

Solution:

$$\frac{2a+1}{3}+\frac{3a-1}{2}$$

$$\frac{4a+2+9a-3}{6}=\frac{13a-1}{6}$$

(L.C.M. of 3, 2 = 6)

Question 19.

$$\frac{k+1}{2}+\frac{2k-1}{3}-\frac{k+3}{4}$$

Solution:

$$\frac{k+1}{2}+\frac{2k-1}{3}-\frac{k+3}{4}$$

$$\frac{6k+6+8k-4-3k-9}{12}$$

(L.C.M. of 2, 3, 4 = 12)

$$=\frac{14k-3k+6-13}{12}=\frac{11k-7}{12}$$

Question 20.

$$\frac{m}{5}-\frac{m-2}{3}+m$$

Solution:

$$\frac{m}{5}-\frac{m-2}{3}+\frac{m}{1}$$

$$\frac{3m-5(m-2)+15m}{15}$$

$$=\frac{3m-5m+10+15m}{15}$$

$$=\frac{18m-5m+10}{15}=\frac{13m+10}{15}$$

Question 21.

$$\frac{5(x-4)}{3}+\frac{2(5x-3)}{5}+\frac{6(x-4)}{7}$$

Solution:

$$\frac{5(x-4)}{3}+\frac{2(5x-3)}{5}+\frac{6(x-4)}{7}$$

$$\frac{175(x-4)+42(5x-3)+90(x-4)}{105}$$

(L.C.M. of 3, 5, 7 = 105)

$$=\frac{175x-700+210x-126+90x-360}{105}$$

$$=\frac{175x+210x+90x-700-126-360}{105}$$

$$=\frac{475x-1186}{105}$$

Question 22.

$$\left(p+\frac{p}{3}\right)\left(2p+\frac{p}{2}\right)\left(3p-\frac{2p}{3}\right)$$

Solution:

$$\left(p+\frac{p}{3}\right)\left(2p+\frac{p}{2}\right)\left(3p-\frac{2p}{3}\right)$$

$$=p\left(1+\frac{1}{3}\right)p\left(2+\frac{1}{2}\right)p\left(3-\frac{2}{3}\right)$$

$$=p^3\left(\frac{3+1}{3}\right)\left(\frac{4+1}{2}\right)\left(\frac{9-2}{3}\right)$$

$$=p^3\times\frac{4}{3}\times\frac{5}{2}\times\frac{7}{3}=p^3\times\frac{70}{9}=\frac{70p^3}{9}$$

Question 23.

$$\frac{7}{30} \text{ of } \left(\frac{p}{3} + \frac{7p}{15}\right)$$

Solution:

$$\frac{7}{30} \text{ of } \left(\frac{p}{3} + \frac{7p}{15}\right)$$

$$= \frac{7}{30} \text{ of } \left(\frac{5p + 7p}{15}\right) = \frac{7}{30} \times \frac{12}{15} p = \frac{14p}{75}$$

Question 24.

$$\left(2p + \frac{p}{7}\right) \div \left(\frac{9p}{10} + 4p\right)$$

Solution:

$$\left(2p + \frac{p}{7}\right) \div \left(\frac{9p}{10} + 4p\right)$$

$$= \frac{14p + p}{7} \div \frac{9p + 40p}{10} = \frac{15}{7} p \div \frac{49}{10} p$$

$$= \frac{15}{7} p \times \frac{10}{49p} = \frac{150}{343}$$

Question 25.

$$\left(\frac{5k}{8} - \frac{3k}{5}\right) \div \frac{k}{4}$$

Solution:

$$\left(\frac{5k}{8} - \frac{3k}{5}\right) \div \frac{k}{4}$$

$$= \frac{25k - 24k}{40} \div \frac{k}{4} = \frac{k}{40} \div \frac{k}{4}$$

$$= \frac{k}{40} \times \frac{4}{k} = \frac{1}{10}$$

Question 26.

$$\left(\frac{y}{6}+\frac{2y}{3}\right)\div\left(y+\frac{2y-1}{3}\right)$$

Solution:

$$\left(\frac{y}{6}+\frac{2y}{3}\right)\div\left(y+\frac{2y-1}{3}\right)$$

$$=\left(\frac{y+4y}{6}\right)\div\left(\frac{3y+2y-1}{3}\right)$$

$$=\frac{5y}{6}\div\frac{5y-1}{3}=\frac{5y}{6}\times\frac{3}{5y-1}$$

$$=\frac{5y}{2(5y-1)}=\frac{5y}{10y-2}$$

EXERCISE 11 (F)

Enclose the given terms in brackets as required :

Question 1.

x – y – z = x-{…….)

Solution:

x – y – z = x – (y + z)

Question 2.

$x^2 – xy^2 – 2xy – y^2 = x^2 – (……..)$

Solution:

$x^2 – xy^2 – 2xy – y^2$

$= x^2 – (xy^2 + 2xy + y^2)$

Question 3.

4a – 9 + 2b – 6 = 4a – (……..)

Solution:

4a – 9 + 2b – 6

= 4a – (9 – 2b + 6)

Question 4.
$x^2 - y^2 + z^2 + 3x - 2y = x^2 - (\ldots\ldots\ldots)$

Solution:
$x^2 - y^2 + z^2 + 3x - 2y$
$= x^2 - (y^2 - z^2 - 3x + 2y)$

Question 5.
$-2a^2 + 4ab - 6a^2b^2 + 8ab^2 = -2a (\ldots\ldots\ldots)$

Solution:
$-2a^2 + 4ab - 6a^2b^2 + 8ab^2$
$= -2a (a - 2b + 3ab^2 - 4b^2)$

Simplify :

Question 6.
2x – (x + 2y- z)

Solution:
2x-(x + 2y-z) = 2x – x – 2y + z
= x – 2y + z

Question 7.
p + q – (p – q) + (2p – 3q)

Solution:
p + q – (p – q) + (2p- 3q)
= p + q – p + q + 2p – 3q = 2p – q

Question 8.
9x – (-4x + 5)

Solution:
9x – (-4x + 5) = 9x + 4x – 5
= 13x- 5

Question 9.
6a – (- 5a – 8b) + (3a + b)

Solution:
6a – (- 5a – 8b) + (3a + b)

= 6a + 5a + 8b + 3a + b
= 6a + 5a + 3a + 8b + b
= 14a + 9b

Question 10.
(p – 2q) – (3q – r)

Solution:
(p-2q) – (3q – r) =p – 2q – 3q + r =p – 5q + r

Question 11.
9a (2b – 3a + 7c)

Solution:
9a (2b – 3a + 7c)
= 18ab – 27a^2 + 63ca

Question 12.
-5m (-2m + 3n – 7p)

Solution:
-5m (-2m + 3n- 7p)
= – 5m x (-2m) + (-5m) (3n) – (-5m) (7p)
= 10m^2 – 15mn + 35 mp.

Question 13.
-2x (x + y) + x^2

Solution:
– 2x (x + y) + x^2
= -2x x x + (-2x)y + x^2
= – 2x^2 – 2xy + x^2
= – 2x^2 + x^2 – 2xy = – x^2 – 2xy

Question 14.

$$b\left(2b-\frac{1}{b}\right)-2b\left(b-\frac{1}{b}\right)$$

Solution:

$$b\left(2b-\frac{1}{b}\right)-2b\left(b-\frac{1}{b}\right)$$

$$= b\times 2b - b\times\frac{1}{b} - 2b\times b + 2b\times\frac{1}{b}$$

$$= 2b^2 - 1 - 2b^2 + 2$$

$$= 2b^2 - 2b^2 - 1 + 2 = 1$$

Question 15.

8 (2a + 3b – c) – 10 (a + 2b + 3c)

Solution:

8 (2a + 3b -c)- 10 (a + 2b + 3c)
= 16a + 24b – 8c – 10a – 20b- 30c
= 16a – 10a + 24b – 20b – 8c – 30c
= 6a + 4b – 38c

Question 16.

$$a\left(a+\frac{1}{a}\right)-b\left(b-\frac{1}{b}\right)-c\left(c+\frac{1}{c}\right)$$

Solution:

$$a\left(a+\frac{1}{a}\right)-b\left(b-\frac{1}{b}\right)-c\left(c+\frac{1}{c}\right)$$

$$= a^2 + 1 - b^2 + 1 - c^2 - 1$$

$$= a^2 - b^2 - c^2 + 1$$

Question 17.

5 x (2x + 3y) – 2x (x – 9y)

Solution:

5x (2x + 3y) – 2x (x – 9y)
= $10x^2$ + 15xy – $2x^2$ + 18xy
= $10x^2$ – $2x^2$+ 15xy+ 18xy
= $8x^2$ + 33 xy

Question 18.
a + (b + c – d)

Solution:
a + (b + c – d) = a + (b + c – d)
= a + b + c – d

Question 19.
5 – 8x – 6 – x

Solution:
5 – 8x – 6 – x
= 5 – 6 – 8x – x
= -1 -7x

Question 20.
2a + (6- $\overline{a-b}$)

Solution:
2a + (6 – $\overline{a-b}$)
= 2a + (b – a + b)
= 2a + b – a + b
= a + 2b

Question 21.
3x + [4x – (6x – 3)]

Solution:
3x + [4x – (6x – 3)]
= 3x + [4x – 6x + 3]
= 3x + 4x – 6x + 3
= 3x + 4x – 6x + 3
= 7x – 6x + 3= x + 3

Question 22.
5b – {6a + (8 – b – a)}

Solution:
5b- {6a + 8- 6-a}
= 5b – 6a – 8 + b + a
= -6a + a + 5b +b – 8
= -5a + 6b-8

Question 23.
2x-[5y- (3x -y) + x]

Solution:
2x – [5y- (3x – y) + x]
= 2x – {5y – 3x +y + x}
= 2x – 5y + 3x -y – x
= 2x + 3x – x – 5y – y
= 4x – 6y

Question 24.
6a – 3 (a + b – 2)

Solution:
6a – *3* (a + b – *2)*
= 6a – 3a – 3b + 6
= 3a -3b + 6

Question 25.
8 [m + 2n-p – 7 (2m -n + 3p)]

Solution:
8 [m + 2n-p -1 (2m – n + 3p)]
8 [m + 2n-p- 14m + 7n-21p]
= 8m+ 16n -8p- 112m + 56n – 168p
= 8m – 112m + 16n + 56n -8p – 168p
= -104m + 72n – 176p

Question 26.
{9 – (4p – 6q)} – {3q – (5p – 10)}

Solution:
{9 – {4p – 6q)} – {3q – (5p – 10)}
{9 – 4p + 6q} – {3q -5p+ 10}
= *9* – 4p + 6q – 3q + 5p – *10*
= *9* – *4p* + 5p + 6q – 3q – *10*
= p + 3q – 1

Question 27.
2 [a – 3 {a + 5 {a – 2) + 7}]

Solution:
2 [a – 3 {a + 5 {a – 2) + 7}]

= 2 [a- 3 {a + 5a- 10 + 7}]
= 2 [a -3a- 15a + 30 -21]
= 2a-6a- 30a + 60-42
= 2a- 36a + 60-42
= -34a + 18

Question 28.
5a – [6a – {9a – (10a – $\overline{4a - 3a}$)}]

Solution:
5a – [6a – {9a – (10a – 4a + 3a)}]
= 5a – [6a – {9a – (10a – 4a + 3a)}]
= 5a – [6a – {9a – 10a + 4a – 3a}]
= 5a- [6a – 9a + 10a – 4a + 3a]
= 5a – 6a + 9a – 10a + 4a – 3a
= 5a + 9a + 4a – 6a – 10a – 3a
= 18a – 19a = – a

Question 29.
9x + 5 – [4x – {3x – 2 (4x – 3)}]

Solution:
9x + 5 – [4x – {3x – 2 (4x – 3)}]
= 9x + 5 – [4x – {3x – 8x + 6}]
= 9x + 5 – [4x – 3x + 8x – 6]
= 9x + 5-4x + 3x-8x + 6
= 9x + 3x-4x-8x + 5 + 6
= 12x- 12x+ 11 = 11

Question 30.
(x + y – z)x + (z + x – y)y – (x + y – z)z

Solution:
(x + y – z)x + (z + x -y)y – (x + y -z)z
= x^2 + xy – zx + yz + xy -y^2 – zx – yz + z^2
= x^2 -y^2 + z^2 + 2xy – 2zx

Question 31.
-1 [a-3 {b -4 (a-b-8) + 4a} + 10]

Solution:
– 1 [a – 3 {b – 4(a – b – 8) + 4a} + 10]
= -1 [a-3 {b-4{a-b-8) + 4a} + 10]

= -1[a-3 {b-4a + Ab +32 + 4a} + 10]
= -1 [a-3b+ 12a- 126-96- 12a + 10]
= -a + 3b – 12a + 12b + 96 + 12a – 10
= -a-12a + 12a+ 3b+ 12b-96-10
= – a + 15b – 106

Question 32.

$p^2 - [x^2 - \{x^2 - (q^2 - \overline{x^2 - q^2}) - 2y^2\}]$

Solution:

$p^2 - [x^2 - \{x^2 - (q^2 - \overline{x^2 - q^2}) - 2y^2\}]$

$= p^2 - [x^2 - \{x^2 - (q^2 - x^2 + q^2) - 2y^2\}]$

$= p^2 - [x^2 - \{x^2 - (2q^2 - x^2) - 2y^2\}]$

$= p^2 - [x^2 - \{x^2 - 2q^2 + x^2 - 2y^2\}]$

$= p^2 - x^2 + 2x^2 - 2q^2 - 2y^2$

$= p^2 + x^2 - 2q^2 - 2y^2$

Question 33.

10 – {4a – (7 – $\overline{a-5}$**) – (5a –** $\overline{1+a}$**)}**

Solution:

10 – {4a – (7 – $\overline{a-5}$) – (5a – $\overline{1+a}$)}
= 10 – {4a – (7 – a + 5) – (5a – 1 – a)}
= 10- {4a -(12 -a) -(4a- 1)}
= 10 – {4a – 12 + a- 4a + 1}
= 10 – 4a + 12 – a + 4a- 1
= 10 + 12 – 1 – 4a – a + 4a
= 21 -a

Question 34.

7a- [8a- (11a-(12a- $\overline{6a-5a}$**)}]**

Solution:

7a – [8a – {1 la – (12a **–**$\overline{6a-5a}$)}]
= 7a-[8a-{11a-(12a-6a + 5a)}]
= 7a -[8a -{11a -(17a -6a)}]
= 7a- [8a- {11a-(11a)}]

= 7a- [8a- {11a- 11a}]
= 7a – 8a = -a

Question 35.

$8x - [4y - \{4x + (2x - \overline{2y - 2x})\}]$

Solution:

$8x - [4y - \{4x + (2x - \overline{2y - 2x})\}]$

$= 8x - [4y - \{4x + (2x - 2y + 2x)\}]$

$= 8x - [4y - \{4x + (4x - 2y)\}]$

$= 8x - [4y - \{4x + 4x - 2y\}]$

$= 8x - [4y - 4x - 4x + 2y]$

$= 8x - [-8x + 6y]$

$= 8x + 8x - 6y$

$= 16x - 6y$

Question 36.

x-(3y- $\overline{4z - 3x}$ **+2z-** $\overline{5y - 7x}$**)**

Solution:

x-(3y- $\overline{4z - 3x}$ +2z- $\overline{5y - 7x}$)
= x – (3y – 4z + 3x + 2z -5y + 7x)
= x-(-2y-2z+10x)
= x + 2y + 2z- 10x
= -9x + 2y + 2z