# CAT 2024 Slot 1 Question Paper

## Quant

**47.** Consider two sets $A = \{2, 3, 5, 7, 11, 13\}$ and $B = \{1, 8, 27\}$. Let f be a function from A to B such that for every element in B, there is at least one element a in A such that $f(a) = b$. Then, the total number of such functions f is

**A** 665

**B** 667

**C** 537

**D** 540

**48.** Let $x, y$, and $z$ be real numbers satisfying
$4(x^2 + y^2 + z^2) = a,$
$4(x - y - z) = 3 + a$
The a equals

**A** $3$

**B** $1\frac{1}{3}$

**C** $4$

**D** $1$

**49.** If the equations $x^2 + mx + 9 = 0, x^2 + nx + 17 = 0$ and $x^2 + (m + n)x + 35 = 0$ have a common negative root, then the value of $(2m + 3n)$ is

**50.** Suppose $x_1, x_2, x_3, ..., x_{100}$ are in arithmetic progression such that $x_5 = -4$ and $2x_6 + 2x_9 = x_{11} + x_{13}$, Then,$x_{100}$ equals

**A** -194

**B** -196

**C** 204

**D** 206

**51.** Renu would take 15 days working 4 hours per day to complete a certain task whereas Seema would take 8 days working 5 hours per day to complete the same task. They decide to work together to complete this task. Seema agrees to work for double the number of hours per day as Renu, while Renu agrees to work for double the number of days as Seema. If Renu works 2 hours per day, then the number of days Seema will work, is

**52.** When $10^{100}$ is divided by 7, the remainder is

A 3

B 4

C 1

D 6

**53.**

The sum of all real values of k for which $\left(\frac{1}{8}\right)^{k} \times \left(\frac{1}{32768}\right)^{\frac{1}{3}} = \frac{1}{8} \times \left(\frac{1}{32768}\right)^{\frac{1}{k}}$, is

A $\frac{2}{3}$

B $\frac{4}{3}$

C $-\frac{2}{3}$

D $-\frac{4}{3}$

**54.** For any natural number $n$ let $a_n$ be the largest integer not exceeding $\sqrt{n}$. Then the value of $a_1 + a_2 + \dots\dots + a_{50}$ is

**55.** In September, the incomes of Kamal, Amal and Vimal are in the ratio 8 : 6 : 5. They rent a house together, and Kamal pays 15%, Amal pays 12% and Vimal pays 18% of their respective incomes to cover the total house rent in that month. In October, the house rent remains unchanged while their incomes increase by 10%, 12% and 15%, respectively. In October, the percentage of their total income that will be paid as house rent, is nearest to

A 15.18

B 13.26

C 14.84

D 12.75

**56.** The sum of all four-digit numbers that can be formed with the distinct non-zero digits a, b, c, and d, with each digit appearing exactly once in every number, is 153310 + n, where n is a single digit natural number. Then, the value of (a + b + c + d + n) is

**57.** ABCD is a rectangle with sides AB = 56 cm and BC = 45 cm, and E is the midpoint of side CD. Then, the length, in cm, of radius of incircle of $\triangle ADE$ is

**58.** In the XY-plane, the area, in sq. units, of the region defined by the inequalities $y \geq x + 4$ and $-4 \leq x^2 + y^2 + 4(x - y) \leq 0$ is

**A** $2\pi$

**B** $4\pi$

**C** $\pi$

**D** $3\pi$

**59.** If x is a positive real number such that $4 \log_{10} x + 4 \log_{100} x + 8 \log_{1000} x = 13$, then the greatest integer not exceeding x, is

**60.** The selling price of a product is fixed to ensure 40% profit. If the product had cost 40% less and had been sold for 5 rupees less, then the resulting profit would have been 50%. The original selling price, in rupees, of the product is

**A** 15

**B** 14

**C** 10

**D** 20

**61.** A glass is filled with milk. Two-thirds of its content is poured out and replaced with water. If this process of pouring out two-thirds the content and replacing with water is repeated three more times, then the final ratio of milk to water in the glass, is

**A** 1 : 27

**B** 1 : 80

**C** 1 : 81

**D** 1 : 26

**62.** A fruit seller has a total of 187 fruits consisting of apples, mangoes and oranges. The number of apples and mangoes are in the ratio 5 : 2. After she sells 75 apples, 26 mangoes and half of the oranges, the ratio of number of unsold apples to number of unsold oranges becomes 3 : 2. The total number of unsold fruits is

**63.** Two places A and B are 45 kms apart and connected by a straight road. Anil goes from A to B while Sunil goes from B to A. Starting at the same time, they cross each other in exactly 1 hour 30 minutes. If Anil reaches B exactly 1 hour 15 minutes after Sunil reaches A, the speed of Anil, in km per hour, is

**A** 18

**B** 16

**C** 14

**D** 12

**64.** There are four numbers such that average of first two numbers is 1 more than the first number, average of first three numbers is 2 more than average of first two numbers, and average of first four numbers is 3 more than average of first three numbers. Then, the difference between the largest and the smallest numbers, is

**65.** An amount of Rs 10000 is deposited in bank A for a certain number of years at a simple interest of 5% per annum. On maturity, the total amount received is deposited in bank B for another 5 years at a simple interest of 6% per annum. If the interests received from bank A and bank B are in the ratio 10 : 13, then the investment period, in years, in bank A is

**A** 4

**B** 5

**C** 3

**D** 6

**66.** A shop wants to sell a certain quantity (in kg) of grains. It sells half the quantity and an additional 3 kg of these grains to the first customer. Then, it sells half of the remaining quantity and an additional 3 kg of these grains to the second customer. Finally, when the shop sells half of the remaining quantity and an additional 3 kg of these grains to the third customer, there are no grains left. The initial quantity, in kg, of grains is

**A** 50

**B** 36

**C** 42

**D** 18

**67.** If $(a+b\sqrt{n})$ is the positive square root of $(29-12\sqrt{5})$, where a and b are integers, and n is a natural number, then the maximum possible value of $(a+b+n)$ is

A 18

B 22

C 4

D 6

**68.** The surface area of a closed rectangular box, which is inscribed in a sphere, is 846 sq cm, and the sum of the lengths of all its edges is 144 cm. The volume, in cubic cm, of the sphere is

A 1125 $\pi$

B 750 $\pi$

C 1125 $\pi\sqrt{2}$

D 750 $\pi\sqrt{2}$

# Answers

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 47.**D** | 48.**A** | 49.**38** | 50.**A** | 51.**6** | 52.**B** | 53.**C** | 54.**217** |
| 55.**B** | 56.**31** | 57.**10** | 58.**A** | 59.**31** | 60.**B** | 61.**B** | 62.**66** |
| 63.**D** | 64.**15** | 65.**D** | 66.**C** | 67.**A** | 68.**C** | | |

# Explanations

47. **D**
Set A={2,3,5,7,11,13} so |A|=6
Set B={1, 8, 27} so |B|=3

Without any restrictions, each element in A can map to any of the 3 elements in B. Thus, the total number of functions is: $3^6 = 729$

Excluding Functions That Miss One Element in B: If a function does not map to an element in B, there are 2 elements in B left for mapping. The total number of such functions (for each specific element not mapped) is: $2^6 = 64$

Since there are 3 elements in B, the total number of such functions is:3x64=192

Adding Back Functions That Miss Two Elements in B**:** If a function misses two elements in B, there is only 1 element left for mapping. The total number of such functions is: 1^6=1.

Since there are ${}^3C_2$ ways to choose which two elements are missed, the total number of such functions is: 3

Using the inclusion-exclusion principle, the number of functions where all elements of B are mapped by at least one element of A is:
729-192+3=540.

48. **A**
We have two equations,

$4(x^2 + y^2 + z^2) = a$ ---(1)

$4(x - y - z) = 3 + a$ ---(2)

Substituting the value of a from equation 1 in equation 2, we get,

$4\,(x - y - z) = 3 + 4(x^2 + y^2 + z^2)$

$3 + 4(x^2 + y^2 + z^2) - 4\,(x - y - z) = 0$

$3 + 4x^2 + 4y^2 + 4z^2 - 4x + 4y + 4z = 0$

It can be written as,

$4x^2 - 4x + 1 + 4y^2 + 4y + 1 + 4z^2 + 4z + 1 = 0$

$(2x - 1)^2 + (2y + 1)^2 + (2z + 1)^2 = 0$

We know that if the sum of the squares of terms is 0, then all the terms must be equal to 0

2x - 1 = 0

x = $\frac{1}{2}$

2y + 1 = 0

y = $-\frac{1}{2}$

2z + 1 = 0

z = $-\frac{1}{2}$

Substituting the values in equation 2, we get,

$$4\left(\frac{1}{2} - \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)\right) = 3 + a$$

$$4\left(\frac{3}{2}\right) = 3 + a$$

$$6 = 3 + a$$

$$a = 3$$

Therefore, the correct answer is option A.

49. **38**

When given more than one equations, stating the fact that there is a common root,
We need to equate the two equations to get discernible values for $x$

Here, we are given three equations with the values of $m, n$

$$x^2 + mx + 9 = x^2 + (m+n)x + 35$$
$$mx + 9 = mx + nx + 35$$
$$nx = -26$$

Similarly, we can do it for the other equation as well,

$$x^2 + nx + 17 = x^2 + (m+n)x + 35$$
$$mx = -18$$

Substituting the value of either $mx$ or $nx$ in the original equations, we get

$$x^2 - 18 + 9 = 0$$
$$x^2 = 9$$
$$x = \pm 3$$

Since we are given that the root is negative, $x = -3$

$$n = -\frac{26}{-3}$$
$$m = -\frac{18}{-3}$$

$$3n = 26$$
$$2m = 12$$

$$2m + 3n = 38$$

50. **A**

Using the arithmetic progression formula for the nth term, where

$x_n = a + (n-1)\,d$

Substituting the value for n and using that in the equation that is given,

$2x_6 + 2x_9 = x_{11} + x_{13}$, Then,$x_{100}$ equals

We get, $2\,(a+5d) + 2\,(a+8d) = a + 10d + a + 12d$

$4a + 26d = 2a + 22d$

$2a = -4d$

$a = -2d$

We are given, $x_5 = -4$

$a + 4d = -4$

Substituting the value for a in terms of d,

$2d = -4$

$d = -2$

$a = 4$

$x_{100} = a + 99d$

$x_{100} = 4 - 198 = -194$

51. **6**

Let us assign the amount of work done by Renu in one hour is R

And the amount of work done by Seema in one hour is S

We are told that a certain task with different time durations for Seema and Renu

Renu=15 days with 4 hours each day, that is total work done by Renu is $15 \times\ 4 \times\ R = 60R$

Similarly for Seema, 8 days and 5 hours each day, total work done by Seema is $40S$

We know that $60R = 40S$ or $S = 1.5R$

For this task, let us assume that the amount of days that Seema decides to work is X and the hours per day that Renu decides to work is Y

We are then told that, Seema works for 2Y hours per day and Renu works for 2X days,

Work done by them will be $2XYR$ and $2XYS$

We are told that Y=2, Making this $4XR$ and $4XS$

$S = 1.5R$

Total work will be, $4XR$+$6XR$=$60R$

We get the value of $X = 6$

X is the number of days Seema will work, which is 6.

52. **B**

To find the value of $10^{100} mod\,(7)$

When 10 is divided by 7, it leaves a remainder 3, so the above equation can be written as,

$3^{100} mod\,(7)$

Now looking at the cyclicality of powers of 3 when divided by 7,

$3^1 mod\ 7 = 3$

$3^2 mod\ 7 = 2$

$3^3 mod\ 7 = 6$

$3^4 mod\ 7 = 4$

$3^5 mod\ 7 = 5$

$3^6 mod\ 7 = 1$

From this calculation, it is evident that the powers of 3 modulo 7 repeat every 6 steps. This forms a cycle: 3, 2, 6, 4, 5, 1

$3^{100} = \left(3^6\right)^{16} \times \ \left(3^4\right)$

Since $3^6 mod 7 = 1$

We just need to consider $3^4 mod 7$ which equals 4

Hence the answer is 4.

53. **C**

To solve this question, we need to immediately recognise the fact that, $32768 = 8^5$

Substituting this in the above given equation,

$$\left(\frac{1}{8}\right)^k \times \left(\frac{1}{8}\right)^{5\times\frac{1}{3}} = \left(\frac{1}{8}\right) \times \left(\frac{1}{8}\right)^{5\times\frac{1}{k}}$$

Since the bases are equal, we can equate the powers on either side of the equation,

$k + \frac{5}{3} = 1 + \frac{5}{k}$

$\frac{(3k+5)}{3} = \frac{(k+5)}{k}$

$3k^2 + 5k = 3k + 15$

$3k^2 + 2k - 15 = 0$

Here in the given quadratic equation, the Discriminant is greater than 0, $2^2 - (4)\,(3)\,(-15) > 0$

That means both the roots are real, hence we can simply take the sum of the roots of the quadratic equation in k,

Which in a standard quadratic equation of the form $ax^2 + bx + c$ is $-\frac{b}{a}$

Here, the sum of the real values of k is $-\frac{2}{3}$

54. **217**

We are told that, for any natural number$n_1$ let $a_n$ be the largest integer not exceeding$\sqrt{n}$

So for n=1, the largest integer not exceeding$\sqrt{1}$ will be 1

For n=2, the largest integer not exceeding$\sqrt{2}$ will be 1

For n=3, the largest integer not exceeding$\sqrt{3}$ will be 1

For n=4, the largest integer not exceeding$\sqrt{4}$ will be 2

We see a pattern here regarding the squares of the numbers,
Listing down all the perfect squares,
1, 4, 9, 16, 25, 36, 49, 64, ...
We see that the difference between 4 and 1 is 3 and there were three natural numbers in the given pattern with the value as 1,
So we can write for the rest of the numbers as well,
3 numbers will have value 1, giving a total value of 3
5 numbers will have value 2, giving a total value of 10
7 numbers will have value 3, giving a total value of 21
9 numbers will have value 4, giving a total value of 36
11 numbers will have value 5, giving a total value of 55
13 numbers will have value 6, giving a total value of 78
Now, only the values of $a_{49},\ a_{50}$ will have the value of 7, total value of 14.

Adding these values, we get the total sum as 217, which is the answer.

55. **B**
We are told that, the incomes of Kamal, Amal and Vimal are in the ratio 86 : 5
Lets assume them to be 80X, 60X, 50X respectively.

Money that each one of them pays towards rent,
Kamal 15% which comes to 12X
Amal 12% which comes to 7.2X
Vimal 18% which comes to 9X
Total Rental expenditure will be, 28.2X

Their incomes increase by 10%, 12% and 15% respectively,
Kamal 10% which comes to 88X
Amal 12% which comes to 67.2X
Vimal 15% which comes to 57.5X
Total income will be 212.7X

We are told the rent is the same, so it will be 28.2X

Percentage of income going towards rent in October will be $\frac{28.2X}{212.7X} = 0.13258$

Hence, the answer is 13.26%

56. **31**
When gives n distinct numbers, and asked to find the sum of the possible n distinct numbers that can be formed,
We use the formula, $\left(10^{n-1} + 10^{n-2} + 10^{n-3} + ...\right)\left((n-1)!\right)\left(Sum\ of\ numbers\right)$

Inserting n=4, $(1000 + 100 + 10 + 1)\,(3!)\,(a + b + c + d)$
$(6666)\,(a + b + c + d)$
We are told that this value equals, $153310 + n$
Since we are told that n is a single digit natural number, the total value cannot be that much greater and 6666 should perfectly divide $153310 + n$

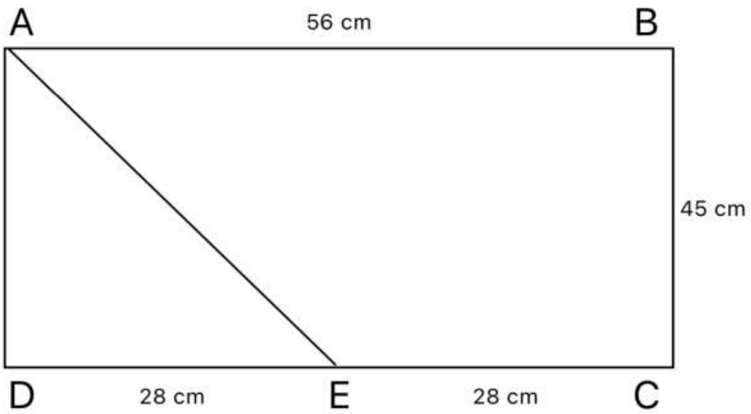
Upon dividing 153310 by 6666 we get the quotient as 22.99
Nearest value being 23, We take 6666x23 giving us the value 153318

Hence the value of n=8 and the value of a+b+c+d=23

Value of a+b+c+d+n=31.

57. **10**

Drawing the figure described in the question, we get the following representation,



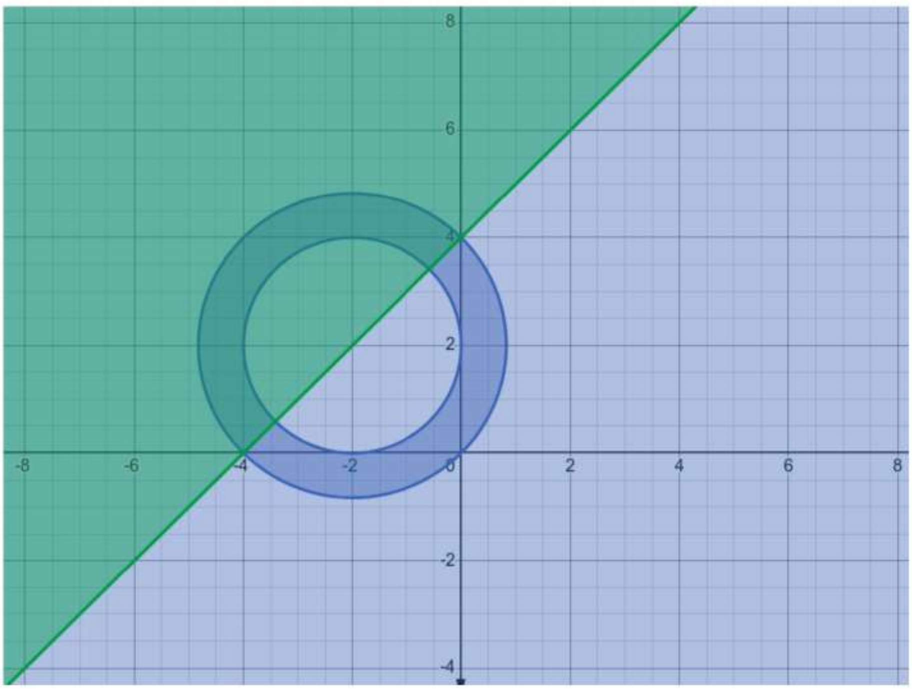

Since E is the midpoint of CD, the length of each half will be 28cm.

We need to find the incircle of the triangle ADE, which is a right angled triangle, we can do that with the formula,

$$\frac{(a+b-h)}{2}$$

$$h^2 = 28^2 + 45^2$$
$$h^2 = 784 + 2025 = 2809$$
$$h = 53$$
$$r = \frac{(28+45-53)}{2} = 10$$

58. **A**



We have two inequalities,

$y \geq x + 4$

$-4 \leq x^2 + y^2 + 4(x - y) \leq 0$

The second inequality can be written as two separate inequalities,

$-4 \leq x^2 + y^2 + 4(x - y)$ and $x^2 + y^2 + 4(x - y) \leq 0$

The first inequality can be written as,

$x^2 + y^2 + 4x - 4y + 4 \geq 0$

$x^2 + 4x + 4 + y^2 - 4y + 4 - 4 \geq 0$

$(x + 2)^2 + (y - 2)^2 \geq 4$

The second inequality can be written as,

$x^2 + y^2 + 4x - 4y \leq 0$

$x^2 + 4x + 4 + y^2 - 4y + 4 - 8 \leq 0$

$(x + 2)^2 + (y - 2)^2 \leq 8$

Representing all three inequalities in the graph, we get the graph as shown above, and we must calculate the intersection of all three inequalities,

We can see that the line passes through the centre of both circles (-2, 2), and the area obtained from the second inequality is the area between the two circles. So, the area of intersection of all three graphs is half of the area between both the circles as the line divides the circle in half, and we must only consider the area above the line as per the given inequality. We know that the area of the bigger circle is $\sqrt{8}$ and the area of the smaller circle is $\sqrt{4}$ from the equations of the circles as we know that equation of circle as $(x - a)^2 + (y - b)^2 = (radius)^2$ where (a,b) is the centre of the circle.

The area of intersection

= $\frac{1}{2}(Area\ of\ bigger\ circle - Area\ of\ smaller\ circle)$

= $\frac{1}{2}\left(\pi\ (\sqrt{8})^2 - \pi\ (\sqrt{4})^2\right)$

= $\frac{1}{2}(8\pi - 4\pi)$

= $\frac{1}{2}(4\pi)$

= $2\pi$

Therefore, the correct answer is option A.

59. **31**

Using the logarithmic property that $\log_{a^p} b = \frac{1}{p}\log_a b$

$4\log_{10} x + 4\log_{100} x + 8\log_{1000} x = 13$

Can be written as

$4\log_{10} x + 2\log_{10} x + \frac{8}{3}\log_{10} x = 13$

$\frac{26}{3}\log_{10} x = 13$

$\log_{10} x = 1.5$
$x = 10^{1.5}$
$x = \sqrt{1000}$

$[\sqrt{1000}] = 31$

Where [.] is Greatest Integer Function since that is what is asked in the question,
31 is the greatest integer that does not exceed x.

60. **B**
Let us fix the Cost Price of the product to be X, and the Selling Price of the product to be 1.4X, since it is given that it is fixed to have a profit of 40%.
If the CP has been 40% less, making the CP 0.6X,
And the selling price is 5 rupees less, making it 1.4X-5
Profit will be 50%,
So, $1.5\,(0.6X) = 1.4X - 5$
$0.9X = 1.4X - 5$
$0.5X = 5$
$X = 10$

Original selling price will be 14.

61. **B**
Let us say the capacity of the glass is X, and it is completely filled with milk,
If two-thirds of its content is poured out and replaced with water, the remaining fraction of the milk will be one third.
And this is said to be done three more times, that means a total of 4 times.
So the contents of Milk initially being X,
And after 4 times the contents will be, $X\left(1 - \frac{2}{3}\right)^4 = \frac{X}{81}$

Since the total contents is X, and the milk contents is X/81, the water contents will be 80X/81.

Ratio of milk to water will be $\frac{X}{81} : \frac{80X}{81}$

Answer is $1 : 80$

62. **66**
The number of apples and mangoes are in the ratio 5 : 2.
Let us write the number of apples as 5X and number of mangoes as 2X
This means oranges will be 187-7X.

After selling the remaining fruits,
Apples: 5X-75
Mangoes: 2X-26
Oranges: (187-7X)/2

Unsold Apples to Unsold oranges is 3:2

$\frac{2(5x-75)}{187-7x} = \frac{3}{2}$

$20x - 300 = 561 - 21x$
$41x = 861$
$x = 21$

Total number of unsold fruits will be,
Apples: 30
Mangoes: 16
Oranges: 20

Total is 66.

63. **D**
We can use the formula for when two people moving towards each other and meeting in a straight line. $t^2 = t_1 \times\ t_2$
Where $t$ is time taken for them to meet each other. $t_1$ is the time taken by person 1 to reach the destination after meeting and
$t_2$ is the time taken by person 2 to reach the destination after meeting

We are told they meet each other in 1 and a half hours, that is 90 minutes.
And if Sunil takes X minutes to reach A, Anil will take X+75 minutes
Since it is given that, Anil reaches B exactly 1 hour 15 minutes after Sunil reaches A

$$90^2 = x\left(x+75\right)$$
$$8100 = x^2 + 75x$$
$$x^2 + 75x - 8100 = 0$$
$$\frac{\left(-75 \pm \sqrt{5625+32400}\right)}{2}$$
$$\frac{\left(-75+195\right)}{2}$$
$$x = 60$$

Total time of travel of Anil is, $90\ \min\ +60\ \min\ +75\ \min$
Total of 225 minutes.
Time in hours will be 3.75 hours.
Speed is $\frac{45}{3.75} = 12\ \frac{km}{hr}$

64. **15**
Let us assume the four numbers to be a, b, c and d in ascending order.

Average of first two numbers is 1 more than the first number
$$\frac{(a+b)}{2} = a+1$$
$$b-a = 2$$
$$b = a+2$$

Average of first three numbers is 2 more than average of first two numbers
$$\frac{(a+b+c)}{3} = \frac{(a+b)}{2} + 2$$
$$2c = a+b+12$$
Substituting the value for b
$$2c = a+a+2+12$$
$$2c = 2a+14$$
$$c = a+7$$

Average of first four numbers is 3 more than average of first three numbers.
$\frac{(a+b+c+d)}{4} = \frac{(a+b+c)}{3} + 3$
$3d = a + b + c + 36$
Substituting the value of b and c
$3d = a + a + 2 + a + 7 + 36$
$3d = 3a + 45$
$d = a + 15$

d is the largest and a is the smallest and we know that d=a+15

Hence the difference between the smallest and the largest values is 15.

65. **D**
We are told that, 10000 is deposited in bank A for a certain number of years at a simple interest of 5% per annum.
Let us say that the number of years is x
Total value of the deposit after x years is, $10000\left(1 + x\left(0.05\right)\right)$

On maturity, the total amount received is deposited in bank B for another 5 years at a simple interest of 6% per annum
Here we know the years and the interest rate,
$10000\left(1 + x\left(0.05\right)\right)\left(1 + 5\left(0.06\right)\right)$
$10000\left(1 + \left(0.05\right)x\right)\left(1.3\right)$

Interest received from Bank A is $\left(x\left(0.05\right)\right)10000$
Interest received from Bank B is $0.3\left(10000\left(1 + x\left(0.05\right)\right)\right)$

This ratio is given to be 10:13.

$$\frac{x\left(0.05\right)}{0.3\left(1 + x\left(0.05\right)\right)} = \frac{10}{13}$$

$0.65x = 3 + 0.15x$
$0.5x = 3$
$x = 6$

Hence the number of years the money was invested in Bank A is 6 years.

66. **C**
Let us say the quantity of grains is X
For the first customer he sells $\frac{X}{2} + 3$
Remaining is $\frac{X}{2} - 3$

Second customer he sells: $\frac{X}{4} - \frac{3}{2} + 3 = \frac{X}{4} + \frac{3}{2}$
Remaining will be $\frac{X}{4} - \frac{9}{2}$

Third customer he sells: $\frac{X}{8} - \frac{9}{4} + 3$ = $\frac{X}{8} + \frac{3}{4}$
Remaining will be $\frac{X}{8} - \frac{21}{4}$
Now, this is said to be 0,
$\frac{X}{8} - \frac{21}{4} = 0$
X=42

67. **A**

$(a+b\sqrt{n})$ is the positive square root of $(29-12\sqrt{5})$

So $29-12\sqrt{5}=(a+b\sqrt{n})^2$

$29-12\sqrt{5}=a^2+b^2n+2ab\sqrt{n}$

$a^2+b^2n=29$ and

$ab\sqrt{n}=-6\sqrt{5}$

$a^2b^2n=180$

$b^2n=\frac{180}{a^2}$

Substituting this in the above equation,

$a^2+\frac{180}{a^2}=29$

$a^4-29a^2+180=0$

$a^2=\frac{\left(29\pm\sqrt{29^2-4(180)}\right)}{2}$

$a^2=\frac{\left(29+\sqrt{841-720}\right)}{2}$

$a^2=9\ or\ 20$

That means, one of $a^2\ or\ b^2n$ is 9 and 20.

We also have, $ab\sqrt{n}=-6\sqrt{5}$ that means one of a or b should be negative

And also the fact that this is a positive square root,
And we need to maximise the value of a, b and n.

We can have a=-3, b=1 and n=20.
This satisfies all the above equations, and the value of a+b+n=18.

68. **C**

Given that, The surface area of a closed rectangular box, which is inscribed in a sphere, is 846 sq cm
So, $2(lb+bh+hl)=846$.

And $4(l+b+h)=144$

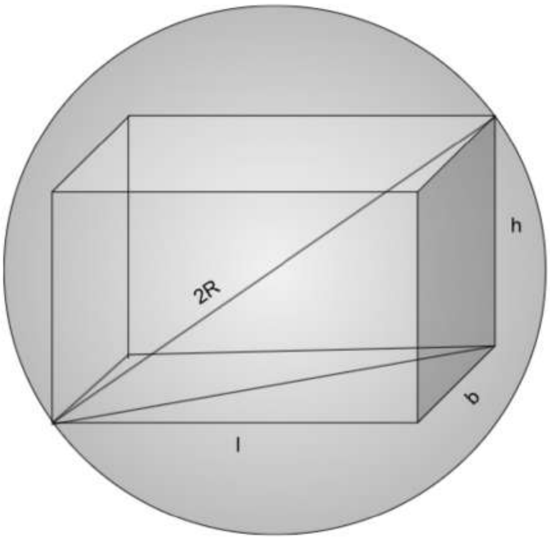
$(l+b+h)=36$

$(l+b+h)^2=l^2+b^2+h^2+2\,(lb+bh+hl)$

$1296=\left(l^2+b^2+h^2\right)+846$

$450=l^2+b^2+h^2$

We are told that this cuboid is inscribed in a sphere, the body diagonal of the cuboid equals the diameter of the sphere, this can be visualised as:




This is nothing but, $\sqrt{l^2+b^2+h^2}=2R$
$l^2+b^2+h^2=4R^2$
$450=4R^2$
$R^2=\frac{225}{2}$
$R=\frac{15}{\sqrt{2}}$

Volume of sphere will be $\dfrac{4}{3}\times\ \pi\ \times\ \left(\dfrac{15}{\sqrt{2}}\right)^3$

$\dfrac{4}{3}\pi\ \left(\dfrac{3375}{2\sqrt{2}}\right)$

$\pi\ \times\ 1125\sqrt{2}$