

Chapter 9. Vector Algebra

Algebra of Vectors

1 Mark Questions

1. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.

Foreign 2014

? To find a vector in the direction of given vector, first of all we find unit vector in the direction of given vector and then multiply it with given magnitude.

$$\text{Let } \vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\begin{aligned} |\vec{a}| &= \sqrt{(2)^2 + (-3)^2 + (6)^2} \\ &= \sqrt{4 + 9 + 36} \\ &= \sqrt{49} = 7 \text{ units} \end{aligned} \quad (1/2)$$

The unit vector in the direction of the given vector \vec{a} is

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{7} (2\hat{i} - 3\hat{j} + 6\hat{k}) = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

Therefore, the vector of magnitude equal to 21 units and in the direction of \vec{a} is

$$\begin{aligned} 21\hat{a} &= 21 \left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k} \right) \\ &= 6\hat{i} - 9\hat{j} + 18\hat{k} \end{aligned} \quad (1/2)$$

2. Find a vector \vec{a} of magnitude $5\sqrt{2}$, making an angle of $\frac{\pi}{4}$ with X-axis, $\frac{\pi}{2}$ with Y-axis and an acute angle θ with Z-axis. All India 2014

Given, a vector \vec{a} makes an angle $\frac{\pi}{4}$ with

X-axis and $\frac{\pi}{2}$ with Y-axis.

$$\text{So, } l = \cos \frac{\pi}{4} \text{ and } m = \cos \frac{\pi}{2} \Rightarrow l = \frac{1}{\sqrt{2}}, m = 0$$

$$\text{We know that, } l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + (0)^2 + n^2 = 1 \Rightarrow \frac{1}{2} + n^2 = 1$$

$$\Rightarrow n^2 = 1 - \frac{1}{2} \Rightarrow n = \pm \frac{1}{\sqrt{2}} \Rightarrow n = \frac{1}{\sqrt{2}}$$

$$\therefore \cos \theta = \frac{1}{\sqrt{2}}$$

$[\because \theta \text{ is an acute angle with Z-axis}]$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Thus, direction cosines of a line are

$$\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \quad (1/2)$$

\therefore Vector \vec{a}

$$= |\vec{a}| \left(\cos \frac{\pi}{4} \hat{i} + \cos \frac{\pi}{2} \hat{j} + \cos \frac{\pi}{4} \hat{k} \right)$$

$$= 5\sqrt{2} \left(\frac{1}{\sqrt{2}} \hat{i} + (0) \hat{j} + \frac{1}{\sqrt{2}} \hat{k} \right) = 5\hat{i} + 5\hat{k} \quad (1/2)$$

3. Write a unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and

$$\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$$

Delhi 2014C

Given, $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$

$$\begin{aligned}\text{Now, } \vec{a} + \vec{b} &= 2\hat{i} + 2\hat{j} - 5\hat{k} + 2\hat{i} + \hat{j} - 7\hat{k} \\ &= 4\hat{i} + 3\hat{j} - 12\hat{k}\end{aligned}$$

$$\begin{aligned}\text{and } |\vec{a} + \vec{b}| &= \sqrt{(4)^2 + (3)^2 + (-12)^2} \\ &= \sqrt{16 + 9 + 144} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} = 13 \text{ units} \quad (1/2)\end{aligned}$$

\therefore Required unit vector = Unit vector along
the direction of $\vec{a} + \vec{b}$

$$= \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{4}{13}\hat{i} + \frac{3}{13}\hat{j} - \frac{12}{13}\hat{k} \quad (1/2)$$

4. Find the value of p for which the vectors
 $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel.

All India 2014

We have, $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are two
parallel vectors, so their direction ratios will be
proportional.

$$\begin{aligned}\therefore \quad \frac{3}{1} &= \frac{2}{-2p} = \frac{9}{3} \Rightarrow \frac{2}{-2p} = \frac{3}{1} \\ \Rightarrow -6p &= 2 \Rightarrow p = \frac{2}{-6} \Rightarrow p = -\frac{1}{3} \quad (1)\end{aligned}$$

5. Write the value of cosine of the angle which the
vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ makes with Y-axis.

Delhi 2014C

Given, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

Now, unit vector in the direction of \vec{a} is

$$\begin{aligned}\hat{a} &= \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{(1)^2 + (1)^2 + (1)^2}} \\ &= \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \frac{1}{\sqrt{3}} \hat{k}\end{aligned}$$

\therefore Cosine of angle which given vector makes with Y-axis is $\frac{1}{\sqrt{3}}$. (1)

6. Find the angle between X-axis and the vector $\hat{i} + \hat{j} + \hat{k}$.

All India 2014C

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

Now, unit vector in the direction of \vec{a} is

$$\begin{aligned}\hat{a} &= \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{1^2 + 1^2 + 1^2}} \Rightarrow \hat{a} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} \\ \Rightarrow \hat{a} &= \frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \frac{1}{\sqrt{3}} \hat{k}\end{aligned}$$

So, angle between X-axis and the vector

$\hat{i} + \hat{j} + \hat{k}$ is $\cos \alpha = \frac{1}{\sqrt{3}}$

$$\Rightarrow \alpha = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$[\because \hat{a} = l\hat{i} + m\hat{j} + n\hat{k} \text{ and } \cos \alpha = l \Rightarrow \alpha = \cos^{-1} l]$

(1)

7. Write a vector in the direction of the vector

$\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 units.

Delhi 2014C

Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Comparing with $X = x\hat{i} + y\hat{j} + z\hat{k}$, we get

$$x = 1, y = -2, z = 2$$

Magnitude $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

$$= \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{1 + 4 + 4} = 3 \text{ units}$$

\therefore Unit vector in the direction of given vector

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} \quad (1/2)$$

Hence, the vector in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$ which has magnitude 9 units is given by

$$9\hat{a} = 9 \left(\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} \right) = 3\hat{i} - 6\hat{j} + 6\hat{k} \quad (1/2)$$

8. Write a unit vector in the direction of vector

\vec{PQ} , where \vec{P} and \vec{Q} are the points (1, 3, 0) and (4, 5, 6) respectively.

Foreign 2014

💡 Firstly, find the vector \overrightarrow{PQ} by using the formula $(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$,
then required unit vector is given by $\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}$.

The given points are $\vec{P}(1, 3, 0)$ and $\vec{Q}(4, 5, 6)$.

Here, $x_1 = 1, y_1 = 3, z_1 = 0$

and $x_2 = 4, y_2 = 5, z_2 = 6$

So, vector \overrightarrow{PQ}

$$\begin{aligned} &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ &= (4 - 1)\hat{i} + (5 - 3)\hat{j} + (6 - 0)\hat{k} \\ &= 3\hat{i} + 2\hat{j} + 6\hat{k} \end{aligned} \quad (1/2)$$

∴ Magnitude of given vector

$$\begin{aligned} |\overrightarrow{PQ}| &= \sqrt{3^2 + 2^2 + 6^2} \\ &= \sqrt{9 + 4 + 36} \\ &= \sqrt{49} = 7 \text{ units} \end{aligned}$$

Hence, the unit vector in the direction of \overrightarrow{PQ}

$$\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{3\hat{i} + 2\hat{j} + 6\hat{k}}{7} = \frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k} \quad (1/2)$$

9. Write the value of the following:

$$\hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j}) \quad \text{Foreign 2014}$$

$$\begin{aligned}
& \hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j}) \\
&= \hat{i} \times \hat{j} + \hat{i} \times \hat{k} + \hat{j} \times \hat{k} + \hat{j} \times \hat{i} + \hat{k} \times \hat{i} + \hat{k} \times \hat{j} \\
&= \hat{k} - \hat{j} + \hat{i} - \hat{k} + \hat{j} - \hat{i} = 0 \\
& [\because \hat{i} \times \hat{j} = \hat{k}, \hat{i} \times \hat{k} = -\hat{j}, \hat{j} \times \hat{k} = \hat{i}, \hat{j} \times \hat{i} = -\hat{k}, \\
& \quad \hat{k} \times \hat{i} = \hat{j}, \hat{k} \times \hat{j} = -\hat{i}] \quad (1)
\end{aligned}$$

- 10.** Write a unit vector in the direction of the sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

Delhi 2013

$$\begin{aligned}
\text{Let } \vec{c} = \vec{a} + \vec{b} &= (2\hat{i} - \hat{j} + 2\hat{k}) + (-\hat{i} + \hat{j} + 3\hat{k}) \\
&= \hat{i} + 0\hat{j} + 5\hat{k}
\end{aligned}$$

$$\text{Now, } |\vec{c}| = |\vec{a} + \vec{b}| = \sqrt{1^2 + 5^2} = \sqrt{26}$$

$$\begin{aligned}
\therefore c &= \frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} \left[\because \hat{a} = \frac{\vec{a}}{|\vec{a}|} \right] \quad (1/2) \\
&= \frac{1}{\sqrt{26}} \hat{i} + \frac{5}{\sqrt{26}} \hat{k} \quad (1/2)
\end{aligned}$$

which is the required unit vector.

- 11.** If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of $x + y + z$.

HOTS; Delhi 2013

Two vectors are equal, if coefficients of their components are equal. (1/2)

$$\begin{aligned}
\text{Given, } \vec{a} &= \vec{b} \\
\Rightarrow x\hat{i} + 2\hat{j} - z\hat{k} &= 3\hat{i} - y\hat{j} + \hat{k}
\end{aligned}$$

$$\therefore x = 3, y = -2, z = -1$$

$$\text{Now, } x + y + z = 3 - 2 - 1 = 0 \quad (1/2)$$

- 14.** A and B are two points with position vectors $2\vec{a} - 3\vec{b}$ and $6\vec{b} - \vec{a}$, respectively. Write the position vector of a point P which divides the line segment AB internally in the ratio $1 : 2$.

All India 2013

Given, A and B are two points with position vectors $2\vec{a} - 3\vec{b}$ and $6\vec{b} - \vec{a}$, respectively. Also, point P divides the line segment AB in the ratio $1 : 2$ internally.

\therefore Position vector of a point P

$$= \frac{1 \times (6\vec{b} - \vec{a}) + 2 \times (2\vec{a} - 3\vec{b})}{1 + 2}$$

[by section formula]

$$= \frac{6\vec{b} - \vec{a} + 4\vec{a} - 6\vec{b}}{3} = \frac{3\vec{a}}{3} = \vec{a} \quad (1)$$

- 15.** Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Delhi 2012

Given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$,

$$\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k} \quad \text{and} \quad \vec{c} = \hat{i} - 6\hat{j} + 7\hat{k}$$

Sum of the vectors \vec{a} , \vec{b} and \vec{c} is

$$\begin{aligned} \vec{a} + \vec{b} + \vec{c} &= (\hat{i} - 2\hat{j} + \hat{k}) + (-2\hat{i} + 4\hat{j} + 5\hat{k}) \\ &\quad + (\hat{i} - 6\hat{j} - 7\hat{k}) \\ &= -4\hat{j} - \hat{k} \end{aligned} \quad (1)$$

- 16.** Find the scalar components of \overrightarrow{AB} with initial point $A(2, 1)$ and terminal point $B(-5, 7)$.

HOTS ; All India 2012



Scalar components of $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
 = Position vector of B – Position vector of A .

Given, initial point is $A(2, 1)$ and terminal point is $B(-5, 7)$ then scalar component of \overrightarrow{AB} are $x_2 - x_1$ and $y_2 - y_1$.

$$\Rightarrow -5 - 2 = -7 \text{ and } 7 - 1 = 6 \quad (1)$$

- 17.** For what values of a , the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?

HOTS ; Delhi 2011



If \vec{a} and \vec{b} are collinear, then $\vec{a} = \pm \lambda \vec{b}$ or
 $|\vec{a}| = \lambda \cdot |\vec{b}|$

Let given vectors are $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and
 $\vec{b} = a\hat{i} - 6\hat{j} - 8\hat{k}$

Vectors \vec{a} and \vec{b} are said to be collinear, if

$$\vec{a} = k \cdot \vec{b}, \text{ where } k \text{ is a scalar}$$

$$\therefore 2\hat{i} - 3\hat{j} + 4\hat{k} = k(a\hat{i} + 6\hat{j} - 8\hat{k})$$

Above equation is satisfied, when $a = -4$.

$$\therefore a = -4 \quad (1)$$

- 18.** Write the direction cosines of vector
 $-2\hat{i} + \hat{j} - 5\hat{k}$.

Delhi 2011



Direction cosines of the vector $a\hat{i} + b\hat{j} + c\hat{k}$ are

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Let $\vec{a} = -2\hat{i} + \hat{j} - 5\hat{k}$

∴ Direction cosines of \vec{a} are

$$\frac{-2}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}, \frac{1}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}$$

and $\frac{-5}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}$

i.e. $\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$ (1)

- 19.** Write the position vector of mid-point of the vector joining points $P(2, 3, 4)$ and $Q(4, 1, -2)$.

Foreign 2011



Mid-point of the position vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \text{ and}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} \text{ is } \frac{\vec{a} + \vec{b}}{2}$$

$$\text{or } \frac{(a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}}{2}$$

Given, points are $P(2, 3, 4)$ and $Q(4, 1, -2)$
whose position vectors $\vec{OP} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and
 $\vec{OQ} = 4\hat{i} + \hat{j} - 2\hat{k}$.

Now, position vector of mid-point of vector joining points $P(2, 3, 4)$ and $Q(4, 1, -2)$ is

$$\vec{OR} = \frac{\vec{OP} + \vec{OQ}}{2} = \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2}$$

$$\therefore \vec{OR} = \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k} \quad (1)$$

- 20.** Write a unit vector in the direction of vector

$$\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}.$$

Delhi 2009; All India 2011

We know that, unit vector in the direction of \vec{a} is $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$


\therefore Required unit vector in the direction of vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{(2)^2 + (1)^2 + (2)^2}}$

$$= \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k} \quad (1)$$

21. Find the magnitude of the vector

$$\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}. \quad \text{All India 2011C; Delhi 2008}$$

 Magnitude of a vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

Given, vector is $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

Magnitude of $\vec{a} = |\vec{a}| = \sqrt{(3)^2 + (-2)^2 + (6)^2}$

$$= \sqrt{9 + 4 + 36} = \sqrt{49} = 7 \text{ units} \quad (1)$$

22. Find a unit vector in the direction of vector

$$\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}. \quad \text{All India 2011C}$$

Given vector is $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Unit vector in the direction of $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

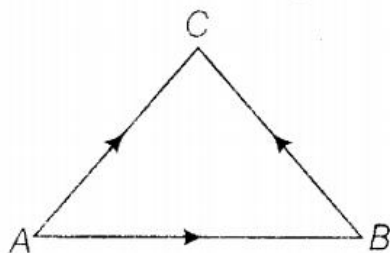
$$= \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{\sqrt{(2)^2 + (3)^2 + (6)^2}} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{\sqrt{49}}$$

$$= \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k} \quad (1)$$

- 23.** If A, B and C are the vertices of a $\triangle ABC$, then what is the value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$?

Delhi 2011C

Let $\triangle ABC$ be the given triangle.



Now, by triangle law of vector addition, we have $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{CA} + \overrightarrow{AC}$$

[adding \overrightarrow{CA} on both sides]

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{CA} - \overrightarrow{CA} [\because \overrightarrow{AC} = -\overrightarrow{CA}]$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0} \quad (1)$$

- 24.** Find a unit vector in the direction of

$$\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}.$$

Delhi 2011C

Given vector is $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$$\begin{aligned} \text{Unit vector in the direction of } \vec{a} &= \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{(2)^2 + (-3)^2 + (6)^2}} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{49}} \end{aligned}$$

$$= \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k} \quad (1)$$

- 25.** Find a vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, which has magnitude 6 units. Delhi 2010C

Do same as Que. 7. [Ans. $4\hat{i} - 2\hat{j} + 4\hat{k}$]

- 26.** Find a position vector of mid-point of the line segment AB , where A is point $(3, 4, -2)$ and B is point $(1, 2, 4)$. Delhi 2010

Do same as Que. 19. [Ans. $2\hat{i} - 3\hat{j} + \hat{k}$]

- 27.** Write a vector of magnitude 9 units in the direction of vector $-2\hat{i} + \hat{j} + 2\hat{k}$. **All India 2010**

Do same as Que. 7. [**Ans.** $-6\hat{i} - 3\hat{j} + 6\hat{k}$]

- 28.** Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$. **Delhi 2010**

Do same as Que. 7. [**Ans.** $5\hat{i} - 10\hat{j} + 10\hat{k}$]

- 29.** What is the cosine of angle which the vector $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with Y-axis? **HOTS; Delhi 2010**

Let $\vec{a} = \sqrt{2}\hat{i} + \hat{j} + \hat{k}$

Now, unit vector in the direction of \vec{a} is

$$\begin{aligned}\hat{a} &= \frac{\vec{a}}{|\vec{a}|} = \frac{\sqrt{2}\hat{i} + \hat{j} + \hat{k}}{\sqrt{(\sqrt{2})^2 + (1)^2 + (1)^2}} = \frac{\sqrt{2}\hat{i} + \hat{j} + \hat{k}}{2} \\ &= \frac{\sqrt{2}}{2}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}\end{aligned}$$

\therefore Cosine of angle which given vector makes with Y-axis is $1/2$. **(1)**

Alternate Method

 The direction cosines along Y-axis are $(0, 1, 0)$, then we use the following cosine formula

$$\begin{aligned}\theta &= \cos^{-1} \{l_1l_2 + m_1m_2 + n_1n_2\} \\ \Rightarrow \cos \theta &= l_1l_2 + m_1m_2 + n_1n_2\end{aligned}$$

Let $\vec{a} = \sqrt{2}\hat{i} + \hat{j} + \hat{k}$

Its direction cosines form

$$\vec{a} = \frac{\sqrt{2}}{2}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$$

$$\left[\begin{aligned} \therefore \text{Direction cosine of vector } a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \\ = \frac{a_1}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \frac{a_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \frac{a_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \end{aligned} \right]$$

$$\text{and } \sqrt{a_1^2 + a_2^2 + a_3^2}$$

and let the position along with Y-axis is

$$\vec{b} = 0\hat{i} + 1\hat{j} + 0\hat{k}$$

If θ is the angle between them.

$$\therefore \cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\Rightarrow \theta = \cos^{-1} \left\{ \frac{1}{\sqrt{2}} \cdot 0 + \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 \right\}$$

$$\therefore \cos \theta = \frac{1}{2}$$

So, cosine of angle which the given vector makes with Y-axis is $\frac{1}{2}$. (1)

30. Find a unit vector in the direction of vector

$$\vec{b} = 6\hat{i} - 2\hat{j} + 3\hat{k}.$$

All India 2009C

Do same as Que. 24. [Ans. $\frac{6}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}$]

31. Find a unit vector in the direction of vector

$$\vec{a} = -2\hat{i} + \hat{j} + 2\hat{k}.$$

Delhi 2009C

Do same as Que. 24. [Ans. $-\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$]

32. Write a unit vector in the direction of

$$\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}.$$

Delhi 2009

Do same as Que. 24. [Ans. $\frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}$]

33. Find the magnitude of the vector

$$\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}.$$

All India 2008C

Given, vector is $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$

Magnitude of $\vec{a} = |\vec{a}| = \sqrt{(2)^2 + (-6)^2 + (-3)^2}$

$$= \sqrt{4 + 36 + 9} = \sqrt{49} = 7 \text{ units} \quad (1)$$

34. Find a unit vector in the direction of

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}.$$

Delhi 2008C

Do same as Que. 24.

$$\left[\text{Ans. } \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k} \right]$$

35. If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$, then find a unit vector in the direction $\vec{a} - \vec{b}$.

All India 2008

4 Marks Questions



Firstly, find $\vec{a} - \vec{b}$ by subtracting the components \hat{i} , \hat{j} and \hat{k} of \vec{a} and \vec{b} simultaneously and then use the formula $= \frac{\vec{a} - \vec{b}}{|\vec{a} - \vec{b}|}$ to get required unit vector.

$$\text{Given, } \vec{a} = \hat{i} + 2\hat{j} - \hat{k} \text{ and } 3\hat{i} + \hat{j} - 5\hat{k}$$

$$\begin{aligned} \therefore \vec{a} - \vec{b} &= (\hat{i} + 2\hat{j} - \hat{k}) - (3\hat{i} + \hat{j} - 5\hat{k}) \\ &= -2\hat{i} + \hat{j} + 4\hat{k} \end{aligned}$$

$$\text{Let } \vec{a} - \vec{b} = \vec{c} = -2\hat{i} + \hat{j} + 4\hat{k}$$

Now, unit vector in the direction of \vec{c} is

$$\begin{aligned} \hat{c} &= \frac{\vec{c}}{|\vec{c}|} = \frac{-2\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{(-2)^2 + (1)^2 + (4)^2}} = \frac{-2\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{21}} \\ &= \frac{-2}{\sqrt{21}}\hat{i} + \frac{1}{\sqrt{21}}\hat{j} + \frac{4}{\sqrt{21}}\hat{k} \quad (1) \end{aligned}$$

which is the required unit vector.

36. Find a vector of magnitude 5 units and

parallel to the resultant of $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$

and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

HOTS ; Delhi 2011



Firstly, find resultant of the vectors \vec{a} and \vec{b} , which is $\vec{a} + \vec{b}$. Then, find a unit vector in the direction of $\vec{a} + \vec{b}$ i.e. the unit vector multiplying by 5.

Given $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Now, resultant of above vectors $= \vec{a} + \vec{b}$
 $= (2\hat{i} + 3\hat{j} - \hat{k}) + (\hat{i} - 2\hat{j} + \hat{k}) = 3\hat{i} + \hat{j}$ (1)

Let $\vec{a} + \vec{b} = \vec{c}$

$\therefore \vec{c} = 3\hat{i} + \hat{j}$

Now, unit vector \hat{c} in the direction of \vec{c} , i.e.

$$= \frac{\vec{c}}{|\vec{c}|} \quad (1)$$

$$= \frac{3\hat{i} + \hat{j}}{\sqrt{(3)^2 + (1)^2}} = \frac{3\hat{i} + \hat{j}}{\sqrt{10}} = \frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{j} \quad (1)$$

Hence, vector of magnitude 5 units and parallel to resultant of \vec{a} and \vec{b} is

$$5 \left(\frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{j} \right) \text{ or } \frac{15}{\sqrt{10}}\hat{i} + \frac{5}{\sqrt{10}}\hat{j} \quad (1)$$

- 37.** Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$. Find a vector of magnitude 6 units, which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

All India 2010

💡 Firstly, find the vector $2\vec{a} - \vec{b} + 3\vec{c}$, then find the vector in the direction of $2\vec{a} - \vec{b} + 3\vec{c}$, i.e. the unit vector multiplying by 6.

Given, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$

and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$$\begin{aligned} \therefore 2\vec{a} - \vec{b} + 3\vec{c} &= 2(\hat{i} + \hat{j} + \hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k}) \\ &= 2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k} \\ \Rightarrow 2\vec{a} - \vec{b} + 3\vec{c} &= \hat{i} - 2\hat{j} + 2\hat{k} \quad (1\frac{1}{2}) \end{aligned}$$

Now, a unit vector in the direction of vector

$$\begin{aligned} 2\vec{a} - \vec{b} + 3\vec{c} &= \frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} \\ &= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (2)^2}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{9}} \\ &= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} = \frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k} \quad (1\frac{1}{2}) \end{aligned}$$

Hence, vector of magnitude 6 units parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c} = 6\left(\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$

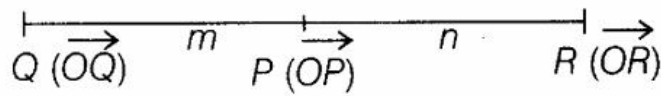
$$= 2\hat{i} - 4\hat{j} + 4\hat{k} \quad (1)$$

- 38.** Find the position vector of a point R , which divides the line joining two points P and Q whose position vectors are $2\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively, externally in the ratio $1:2$. Also, show that P is the mid-point of line segment RQ .

HOTS; Delhi 2010

? Here, we use the section formula for external division

$$\overrightarrow{OR} = \frac{m(\overrightarrow{OQ}) - n(\overrightarrow{OP})}{m - n}$$



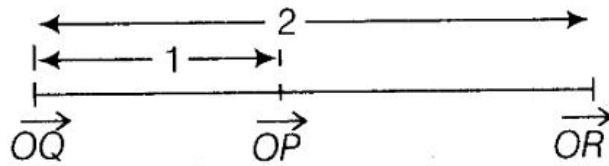
and then use the mid-point formula

$$\overrightarrow{OP} = \frac{\overrightarrow{OQ} + \overrightarrow{OR}}{2}$$

Given, \overrightarrow{OP} = Position vector of $P = 2\vec{a} + \vec{b}$

and \overrightarrow{OQ} = Position vector of $Q = \vec{a} - 3\vec{b}$

Let \overrightarrow{OR} be the position vector of point R , which divides PQ in the ratio $1 : 2$ externally



$$\therefore \overrightarrow{OR} = \frac{1(\vec{a} - 3\vec{b}) - 2(2\vec{a} + \vec{b})}{1 - 2}$$

$$\left[\because \overrightarrow{OR} = \frac{m(\overrightarrow{OQ}) - n(\overrightarrow{OP})}{m - n} \cdot \text{Here, } m = 1, n = 2 \right]$$

(1)

$$= \frac{\vec{a} - 3\vec{b} - 4\vec{a} - 2\vec{b}}{-1} = \frac{-3\vec{a} - 5\vec{b}}{-1} = 3\vec{a} + 5\vec{b}$$

$$\text{Hence, } \overrightarrow{OR} = 3\vec{a} + 5\vec{b} \quad (1\frac{1}{2})$$

Now, we have to show that P is the mid-point of RQ .

$$\text{i.e. } \overrightarrow{OP} = \frac{\overrightarrow{OR} + \overrightarrow{OQ}}{2}$$

$$\text{We have, } \overrightarrow{OR} = 3\vec{a} + 5\vec{b}, \overrightarrow{OQ} = \vec{a} - 3\vec{b}$$

$$\begin{aligned} \therefore \frac{\overrightarrow{OR} + \overrightarrow{OQ}}{2} &= \frac{(3\vec{a} + 5\vec{b}) + (\vec{a} - 3\vec{b})}{2} \\ &= \frac{4\vec{a} + 2\vec{b}}{2} \\ &= \frac{2(2\vec{a} + \vec{b})}{2} = 2\vec{a} + \vec{b} \\ &= \overrightarrow{OP} \quad [\because \overrightarrow{OP} = 2\vec{a} + \vec{b}, \text{ given}] \quad (1\frac{1}{2}) \end{aligned}$$

Hence, P is the mid-point of line segment RQ .

Dot and Cross Product of Two Vectors

1 Marks Questions

1. If \vec{a} and \vec{b} are perpendicular vectors,
 $|\vec{a} + \vec{b}| = 13$ and $|\vec{a}| = 5$, then find the value
of $|\vec{b}|$. All India 2014

Given, $|\vec{a} + \vec{b}| = 13$, and $|\vec{a}| = 5$

Now,

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b}$$

$$= |\vec{a}|^2 + 0 + 0 + |\vec{b}|^2$$

$$[\because \vec{a} \cdot \vec{a} = |\vec{a}|^2, \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = 0 \text{ as } \vec{a} \perp \vec{b}]$$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$$

$$\Rightarrow (13)^2 = (5)^2 + |\vec{b}|^2$$

$$\Rightarrow 169 = 25 + |\vec{b}|^2 \Rightarrow 169 - 25 = |\vec{b}|^2$$

$$\Rightarrow 144 = |\vec{b}|^2 \Rightarrow |\vec{b}| = 12 \quad (1)$$

2. If \vec{a} and \vec{b} are two unit vectors such that

$\vec{a} + \vec{b}$ is also a unit vector, then find the

angle between \vec{a} and \vec{b} .

Delhi 2014

Given, $|\vec{a}| = 1, |\vec{b}| = 1$ and $|\vec{a} + \vec{b}| = 1$

$$\text{Now, } |\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$$

$$= \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}$$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2$$

$$[\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \text{ and } \vec{x} \cdot \vec{x} = |\vec{x}|^2]$$

$$\Rightarrow 1 = 1 + 2\vec{a} \cdot \vec{b} + 1$$

$$\Rightarrow 2\vec{a} \cdot \vec{b} = -1$$

$$\Rightarrow |\vec{a}| |\vec{b}| \cos \theta = -\frac{1}{2} \quad [\because \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta]$$

$$\Rightarrow \cos \theta = -\frac{1}{2} \quad [\because |\vec{a}| = |\vec{b}| = 1]$$

$$\Rightarrow \cos \theta = \cos \frac{2\pi}{3} \Rightarrow \theta = \frac{2\pi}{3}$$

Hence, angle between \vec{a} and \vec{b} is $\frac{2\pi}{3}$. (1)

3. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$

on the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$.

Delhi 2014



The projection of vector \vec{a} on vector \vec{b} is given by $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$.

$$\text{Let } \vec{a} = \hat{i} + 3\hat{j} + 7\hat{k} \text{ and } \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

The projection of vector \vec{a} on the vector \vec{b} is given by

$$\frac{1}{|\vec{b}|} (\vec{a} \cdot \vec{b}) = \frac{1 \times 2 - 3 \times 3 + 7 \times 6}{\sqrt{(2)^2 + (-3)^2 + (6)^2}}$$

$$= \frac{2 - 9 + 42}{\sqrt{4 + 9 + 36}} = \frac{35}{\sqrt{49}} = \frac{35}{7} = 5 \quad (1)$$

4. Write the projection of vector $\hat{i} + \hat{j} + \hat{k}$ along the vector \hat{j} .
Foreign 2014

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j}$

The projection of \vec{a} on \vec{b} is given by

$$\frac{1}{|\vec{b}|} (\vec{a} \cdot \vec{b}) = \frac{1 \times 0 + 1 \times 1 + 1 \times 0}{\sqrt{1^2}} = 1$$

Hence, the projection of vector $\hat{i} + \hat{j} + \hat{k}$ along the vector \hat{j} is 1. (1)

5. If vectors \vec{a} and \vec{b} are such that
 $|\vec{a}| = 3$, $|\vec{b}| = 2/3$ and $\vec{a} \times \vec{b}$ is a unit vector,
 then write the angle between \vec{a} and \vec{b} .
 Delhi 2014

Given, $|\vec{a}| = 3$ and $|\vec{b}| = 2/3$

Let θ be the angle between \vec{a} and \vec{b} .

Also, given $|\vec{a} \times \vec{b}| = 1$ (1/2)

$$\Rightarrow |\vec{a}| |\vec{b}| \sin \theta = 1 \Rightarrow 3 \times \frac{2}{3} \sin \theta = 1$$

$$\Rightarrow 2 \sin \theta = 1$$

$$\Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} \quad (1/2)$$

6. Find $\vec{a} \cdot (\vec{b} \times \vec{c})$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$,
 $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$.

All India 2014

Given, $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

$$\begin{aligned} \text{Now, } \vec{a} \cdot (\vec{b} \times \vec{c}) &= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} \\ &= 2(4 - 1) - 1(-2 - 3) + 3(-1 - 6) \\ &= 2 \times 3 - 1 \times (-5) + 3 \times (-7) \\ &= 6 + 5 - 21 = 11 - 21 = -10 \quad (1) \end{aligned}$$

7. If \vec{a} and \vec{b} are unit vectors, then find the angle between \vec{a} and \vec{b} , given that $(\sqrt{3}\vec{a} - \vec{b})$ is a unit vector. Delhi 2014C

Given, \vec{a} and \vec{b} are two unit vectors, then

$$|\vec{a}| = |\vec{b}| = 1$$

Also, $(\sqrt{3}\vec{a} - \vec{b})$ is a unit vector.

$$\therefore |\sqrt{3}\vec{a} - \vec{b}| = 1 \Rightarrow |\sqrt{3}\vec{a} - \vec{b}|^2 = 1^2$$

$$\Rightarrow (\sqrt{3} \vec{a} - \vec{b}) \cdot (\sqrt{3} \vec{a} - \vec{b}) = 1 \quad [\because |\vec{a}|^2 = \vec{a} \cdot \vec{a}]$$

$$\Rightarrow 3(\vec{a} \cdot \vec{a}) - \sqrt{3}(\vec{a} \cdot \vec{b}) - \sqrt{3}(\vec{b} \cdot \vec{a}) + \vec{b} \cdot \vec{b} = 1$$

$$\Rightarrow 3|\vec{a}|^2 - \sqrt{3}|\vec{a}||\vec{b}|\cos\theta$$

$$- \sqrt{3}|\vec{b}||\vec{a}|\cos\theta + |\vec{b}|^2 = 1 \quad (1/2)$$

[let θ be the angle between \vec{a} and \vec{b}]

$$\Rightarrow 3 - \sqrt{3}\cos\theta - \sqrt{3}\cos\theta + 1 = 1$$

$$\Rightarrow 3 = 2\sqrt{3}\cos\theta$$

$$\Rightarrow \cos\theta = \frac{3}{2\sqrt{3}} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$$

Hence, required angle between \vec{a} and \vec{b} is $\frac{\pi}{6}$.

- 8.** If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, find the angle between \vec{a} and \vec{b} . All India 2014C

Let θ be the angle between \vec{a} and \vec{b} .

Given, $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$

We know that, $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$$\therefore |\vec{a}||\vec{b}|\sin\theta = 12$$

$$\Rightarrow \sin\theta = \frac{12}{|\vec{a}||\vec{b}|} \Rightarrow \sin\theta = \frac{12}{8 \times 3}$$

$$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

Hence, the required angle between \vec{a} and \vec{b} is $\frac{\pi}{6}$. (1)

9. Write the projection of the vector

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k} \text{ on the vector } \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}.$$

Delhi 2014C

Given, $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$

Then, projection of \vec{a} on \vec{b} is given by

$$\begin{aligned} \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} &= \left[\frac{(2\hat{i} - \hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{(1)^2 + (2)^2 + (2)^2}} \right] \\ &= \frac{2 \times 1 + (-1) \times (2) + 1 \times 2}{\sqrt{1 + 4 + 4}} \\ &= \frac{2 - 2 + 2}{\sqrt{9}} = \frac{2}{\sqrt{9}} = \frac{2}{3} \end{aligned}$$

Hence, the projection of vector $(2\hat{i} - \hat{j} + \hat{k})$ on $(\hat{i} + 2\hat{j} + 2\hat{k})$ is $\frac{2}{3}$. (1)

10. Write the value of λ , so that the vectors

$$\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{j} + 3\hat{k} \text{ are}$$

perpendicular to each other. Delhi 2013C, 2008

Given, vectors are $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$

and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

Since, vectors are perpendicular.

$$\therefore \vec{a} \cdot \vec{b} = 0 \quad (1/2)$$

$$\Rightarrow (2\hat{i} + \lambda\hat{j} + \hat{k}) \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow 2 - 2\lambda + 3 = 0$$

$$\therefore \lambda = 5/2 \quad (1/2)$$

11. Write the projection of the vector $7\hat{i} + \hat{j} - 4\hat{k}$

on the vector $2\hat{i} + 6\hat{j} + 3\hat{k}$.

Delhi 2013C

Let $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$$\begin{aligned}\therefore \text{Projection of } \vec{a} \text{ on } \vec{b} &= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \\ &= \frac{(7\hat{i} + \hat{j} - 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})}{|2\hat{i} + 6\hat{j} + 3\hat{k}|} \\ &= \frac{14 + 6 - 12}{\sqrt{(2)^2 + (6)^2 + (3)^2}} \\ &= \frac{8}{\sqrt{4 + 36 + 9}} = \frac{8}{\sqrt{49}} = \frac{8}{7} \quad (1)\end{aligned}$$

12. If \vec{a} and \vec{b} are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a} + \vec{b}$ is perpendicular to vector \vec{b} . HOTS; Delhi 2013

To prove, $(2\vec{a} + \vec{b}) \perp \vec{b}$

Given, $|\vec{a} + \vec{b}| = |\vec{a}|$

On squaring both sides, we get

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2$$

$$\Rightarrow |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{a}|^2$$

$$\Rightarrow 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = 0 \quad [\because |\vec{x}|^2 = \vec{x} \cdot \vec{x} = x^2]$$

$$\begin{aligned}\Rightarrow (2\vec{a} + \vec{b}) \cdot \vec{b} &= 0 \Rightarrow (2\vec{a} + \vec{b}) \perp \vec{b} \\ &[\because \text{If } \vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0] \quad (1)\end{aligned}$$

Hence proved.

13. Find $|\vec{x}|$, if for a unit vector \hat{a} ,

$$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15. \quad \text{HOTS; All India 2013}$$

Given, \hat{a} is a unit vector. Then, $|\hat{a}| = 1$

Now, we have $(\vec{x} - \hat{a}) \cdot (\vec{x} + \hat{a}) = 15$

$$\Rightarrow \vec{x} \cdot \vec{x} - \hat{a} \cdot \vec{x} + \vec{x} \cdot \hat{a} - \hat{a} \cdot \hat{a} = 15$$

$$\Rightarrow \vec{x} \cdot \vec{x} - \hat{a} \cdot \vec{x} + \hat{a} \cdot \vec{x} - \hat{a} \cdot \hat{a} = 15$$

[\because scalar product is commutative]

$$\Rightarrow |\vec{x}|^2 - |\hat{a}|^2 = 15 \quad [\because \vec{z} \cdot \vec{z} = |\vec{z}|^2]$$

$$\Rightarrow |\vec{x}|^2 - 1 = 15 \Rightarrow |\vec{x}|^2 = 16$$

$$\therefore |\vec{x}| = 4 \quad (1)$$

- 14.** Find λ , when projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

HOTS; Delhi 2012

Given, $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ and
projection of \vec{a} on $\vec{b} = 4$

$$\Rightarrow \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 4$$

$$\left[\because \text{projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right]$$

$$\Rightarrow \frac{(\lambda\hat{i} + \hat{j} + 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = 4$$

$$\Rightarrow \frac{2\lambda + 6 + 12}{\sqrt{49}} = 4 \Rightarrow 2\lambda + 18 = 28$$

$$\therefore \lambda = 5 \quad (1)$$

- 15.** Write the value of $(\hat{k} \times \hat{j}) \cdot \hat{i} + \hat{j} \cdot \hat{k}$.

HOTS; All India 2012



Use the results $\hat{j} \times \hat{k} = \hat{i}$,

$$\hat{j} \cdot \hat{k} = 0 \quad \text{and} \quad \hat{i} \cdot \hat{i} = 1$$

We have, $(\hat{k} \times \hat{j}) \cdot \hat{i} + \hat{j} \cdot \hat{k} = (-\hat{i}) \cdot \hat{i} + \hat{j} \cdot \hat{k}$

$$[\because \hat{j} \times \hat{k} = \hat{i} \Rightarrow \hat{k} \times \hat{j} = -\hat{i} \text{ and } \hat{j} \cdot \hat{k} = 0]$$

$$= -\hat{i}^2 + 0 = -1 \quad [\because \hat{i}^2 = 1] \quad (1)$$

16. If $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded about the vector \vec{b} ? **Foreign 2011**

$$\text{Given, } \vec{a} \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0 \quad \dots(i)$$

$$\text{and} \quad \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow |\vec{a}| |\vec{b}| \cos \theta = 0 \quad \dots(ii)$$

From Eqs. (i) and (ii), it may be concluded that \vec{b} is either zero or non-zero perpendicular vector. **(1)**

17. Write the projection of vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$. **All India 2011**

Let given vector are $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$.

Projection of \vec{a} on \vec{b}

$$\begin{aligned} &= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{(\hat{i} - \hat{j}) \cdot (\hat{i} + \hat{j})}{\sqrt{(1)^2 + (1)^2}} \\ &= \frac{1-1}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0 \quad \left[\begin{array}{l} \because \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = 1 \\ \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0 \end{array} \right] \quad (1) \end{aligned}$$

18. Write the angle between vectors \vec{a} and \vec{b} with magnitudes $\sqrt{3}$ and 2 respectively, having

$$\vec{a} \cdot \vec{b} = \sqrt{6}.$$

All India 2011



Let θ be the angle between \vec{a} and \vec{b} , then use the following formula

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

Given, $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$

Now, angle between \vec{a} and \vec{b} is given by

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{\sqrt{6}}{\sqrt{3} \times 2} = \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{3} \times 2} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}} = \cos \frac{\pi}{4}$$

$$\therefore \theta = \frac{\pi}{4} \quad \left[\because \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \right] \text{ (1)}$$

19. For what value of λ are the vectors $\hat{i} + 2\lambda\hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} - 3\hat{k}$ perpendicular?

All India 2011C



If two vectors \vec{a} and \vec{b} are perpendicular, then $\vec{a} \cdot \vec{b} = 0$

Given vectors are $(\hat{i} + 2\lambda\hat{j} + \hat{k})$

and $(2\hat{i} + \hat{j} - 3\hat{k})$.

Also given, the vectors are perpendicular, so their dot product is zero.

$$\therefore (\hat{i} + 2\lambda\hat{j} + \hat{k}) \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 0$$

$$\Rightarrow 2 + 2\lambda - 3 = 0$$

$$\Rightarrow 2\lambda - 1 = 0$$

$$\Rightarrow 2\lambda = 1 \text{ or } \lambda = \frac{1}{2} \quad \text{(1)}$$

Hence, required value of λ is $1/2$.

💡 If two vectors \vec{a} and \vec{b} are perpendicular, then
 $\vec{a} \cdot \vec{b} = 0$

Given vectors are $(\hat{i} + 2\lambda\hat{j} + \hat{k})$

and $(2\hat{i} + \hat{j} - 3\hat{k})$.

Also given, the vectors are perpendicular, so their dot product is zero.

$$\therefore (\hat{i} + 2\lambda\hat{j} + \hat{k}) \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 0$$

$$\Rightarrow 2 + 2\lambda - 3 = 0$$

$$\Rightarrow 2\lambda - 1 = 0$$

$$\Rightarrow 2\lambda = 1 \text{ or } \lambda = \frac{1}{2} \quad (1)$$

Hence, required value of λ is $1/2$.

20. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and angle between \vec{a} and \vec{b} is 60° , then find $\vec{a} \cdot \vec{b}$. Delhi 2011C

We know that, $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos \theta$

On putting $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\theta = 60^\circ$, we get

$$\vec{a} \cdot \vec{b} = \sqrt{3} \times 2 \cos 60^\circ$$

$$\Rightarrow \vec{a} \cdot \vec{b} = \frac{1}{2} \times 2\sqrt{3} = \sqrt{3} \quad \left[\because \cos 60^\circ = \frac{1}{2} \right]$$

$$\therefore \vec{a} \cdot \vec{b} = \sqrt{3} \quad (1)$$

21. Find the value of λ , if the vectors $2\hat{i} + \lambda\hat{j} + 3\hat{k}$ and $3\hat{i} + 2\hat{j} - 4\hat{k}$ are perpendicular to each other. All India 2010C

Do same as Que. 19. [Ans. $\lambda = 3$]

22. If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 3$, then find the projection of \vec{b} on \vec{a} . All India 2010C

Given, $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 3$

$$\therefore \text{Projection of } \vec{b} \text{ on } \vec{a} = \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|}$$

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \quad [\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}]$$

$$= \frac{3}{2} \quad [\because \vec{a} \cdot \vec{b} = 3 \text{ and } |\vec{a}| = 2] \quad (1)$$

23. Vectors \vec{a} and \vec{b} are such that

$$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2/3 \text{ and } \vec{a} \times \vec{b} \text{ is a unit}$$

vector. Write the angle between \vec{a} and \vec{b} .

All India 2010

Do same as Que. 5. [Ans. $\frac{\pi}{3}$]

24. If \vec{a} and \vec{b} are two vectors, such that

$$|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|, \text{ then find the angle}$$

between \vec{a} and \vec{b} .

HOTS; All India 2010



Use the following formulae:

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

and $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \cdot \hat{n}$

where, θ is the angle between \vec{a} and \vec{b} .

Given, $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$

$$\Rightarrow |\vec{a}| |\vec{b}| \cos \theta = |\vec{a}| |\vec{b}| \sin \theta \quad [\because |\hat{n}| = 1]$$

$$\Rightarrow \cos \theta = \sin \theta$$

On dividing both sides by $\cos \theta$, we get

$$\tan \theta = 1$$

$$\Rightarrow \tan \theta = \tan \frac{\pi}{4} \quad \left[\because 1 = \tan \frac{\pi}{4} \right]$$

$$\therefore \theta = \frac{\pi}{4}$$

So, angle between \vec{a} and \vec{b} is $\frac{\pi}{4}$. (1)

25. Find λ , if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

All India 2010



If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Given, $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 14 \\ 1 & -\lambda & 7 \end{vmatrix} = \vec{0}$$

$$\Rightarrow \hat{i}(42 + 14\lambda) - \hat{j}(14 - 14) + \hat{k}(-2\lambda - 6) = \vec{0}$$

$$\Rightarrow \hat{i}(42 + 14\lambda) + \hat{k}(-2\lambda - 6) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$[\because \vec{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}]$$

On comparing coefficients of \hat{i} and \hat{k} from both sides, we get

$$42 + 14\lambda = 0$$

$$\Rightarrow \lambda = -3$$

$$\text{and } -2\lambda - 6 = 0$$

$$\Rightarrow \lambda = -3 \quad (1)$$

Hence, required value of λ is -3 .

26. Find $\vec{a} \cdot \vec{b}$, if $\vec{a} = -\hat{i} + \hat{j} - 2\hat{k}$ and

$$\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}. \quad \text{All India 2009C}$$

Given, $\vec{a} = -\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

$$\text{Then, } \vec{a} \cdot \vec{b} = (-\hat{i} + \hat{j} - 2\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k})$$

$$= -2 + 3 + 2 = 3 \quad (1)$$

27. Find $\vec{a} \cdot \vec{b}$, if $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 3\hat{k}$.

Delhi 2009C

Do same as Que. 26. [Ans. 9]

28. Find the value of P , if

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + P\hat{k}) = \vec{0}. \quad \text{All India 2009}$$

Do same as Que. 25. [Ans. $\frac{27}{2}$]

29. If \hat{P} is a unit vector and $(\vec{x} - \hat{P}) \cdot (\vec{x} + \hat{P}) = 80$,

$$\text{then find } |\vec{x}|. \quad \text{HOTS; All India 2009}$$

Do same as Que. 13. [Ans. 9]

30. Find the angle between \vec{a} and \vec{b} with magnitudes 1 and 2 respectively, when

$$|\vec{a} \times \vec{b}| = \sqrt{3}. \quad \text{Delhi 2009}$$

Given, $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = \sqrt{3}$

$$\Rightarrow |\vec{a}| |\vec{b}| \sin \theta = \sqrt{3}$$

$$[\because \vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \cdot \hat{n} \text{ and } |\hat{n}| = 1]$$

$$\Rightarrow 1 \times 2 \times \sin \theta = \sqrt{3} \quad [\because |\vec{a}| = 1 \text{ and } |\vec{b}| = 2]$$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$$

Hence, angle between \vec{a} and \vec{b} is $\frac{\pi}{3}$. (1)

31. Write the value of P , for which

$\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + P\hat{j} + 3\hat{k}$ are parallel vectors.

Delhi 2009

Given vectors are $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + P\hat{j} + 3\hat{k}$.

Also, \vec{a} and \vec{b} are parallel vectors.

$$\text{So, } \vec{a} \times \vec{b} = 0$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 9 \\ 1 & P & 3 \end{vmatrix} = \vec{0}$$

$$\Rightarrow \hat{i} (6 - 9P) - \hat{j} (9 - 9) + \hat{k} (3P - 2) = \vec{0}$$

$$\Rightarrow \hat{i} (6 - 9P) + \hat{k} (3P - 2) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

On comparing the coefficients of \hat{i} or \hat{k} from both sides, we get

$$6 - 9P = 0 \Rightarrow P = \frac{2}{3} \quad (1)$$

Alternate Method



If the two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are parallel to each other, then use the following relation.

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Given, $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and

$\vec{b} = \hat{i} + P\hat{j} + 3\hat{k}$ are parallel vectors.

$$\text{Then, } \frac{3}{1} = \frac{2}{P} = \frac{9}{3} \Rightarrow P = \frac{2}{3} \quad (1)$$

32. Find the projection of \vec{a} on \vec{b} , if $\vec{a} \cdot \vec{b} = 8$ and

$$\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}.$$

Delhi 2009

Do same as Que. 9.

$$\left[\text{Ans. } \frac{8}{7} \right]$$

33. Find value of the following:

$$\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j}).$$

HOTS; All India 2008C

We have, $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$

$$= \hat{i} \cdot \hat{i} + \hat{j} \cdot (-\hat{j}) + \hat{k} \cdot \hat{k}$$

$$\left[\begin{array}{l} \because \hat{i} \times \hat{j} = \hat{k}; \quad \hat{j} \times \hat{k} = \hat{i} \\ \hat{k} \times \hat{i} = \hat{j} \Rightarrow \hat{i} \times \hat{k} = -\hat{j} \end{array} \right]$$

$$= \hat{i}^2 - \hat{j}^2 + \hat{k}^2$$

$$= 1 - 1 + 1 = 1$$

(1)

34. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and

$$\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}.$$

Delhi 2008C

Given, $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\begin{aligned}\therefore \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix} \\ &= \hat{i}(-14 + 14) - \hat{j}(2 - 21) + \hat{k}(-2 + 21) \\ &= 19\hat{j} + 19\hat{k}\end{aligned}$$

$$\begin{aligned}\text{Now, } |\vec{a} \times \vec{b}| &= \sqrt{(19)^2 + (19)^2} \\ &= \sqrt{2(19)^2} = 19\sqrt{2} \quad (1)\end{aligned}$$

35. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 3$, then find the angle between \vec{a} and \vec{b} . All India 2008

Do same as Que. 18. $\left[\text{Ans. } \frac{\pi}{6} \right]$

36. Find angle between vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. Delhi 2008

Given, vectors are

$$\vec{a} = \hat{i} - \hat{j} + \hat{k} \text{ and } \vec{b} = \hat{i} + \hat{j} - \hat{k}.$$

$$\begin{aligned}\text{Then, } \vec{a} \cdot \vec{b} &= (\hat{i} - \hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) \\ &= 1 - 1 - 1 = -1\end{aligned}$$

$$|\vec{a}| = \sqrt{(1)^2 + (-1)^2 + (1)^2} = \sqrt{3}$$

$$\text{and } |\vec{b}| = \sqrt{(1)^2 + (1)^2 + (-1)^2} = \sqrt{3}$$

We know that, angle between \vec{a} and \vec{b} is given by

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\therefore \cos \theta = \frac{-1}{\sqrt{3} \times \sqrt{3}} = -\frac{1}{3} \Rightarrow \theta = \cos^{-1} \left(-\frac{1}{3} \right)$$

which is the required angle between \vec{a} and \vec{b} .
(1)

4 Marks Questions

37. Prove that, for any three vectors \vec{a} , \vec{b} and \vec{c} ,

$$[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2 [\vec{a} \quad \vec{b} \quad \vec{c}] \text{ Delhi 2014}$$



If use the property that in a scalar triple product, if any two vectors are equal, then value of scalar triple product will be zero and $[\vec{a} \quad \vec{b} \quad \vec{c}] = [\vec{b} \quad \vec{c} \quad \vec{a}]$.

$$\text{We have, LHS} = [\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}]$$

$$= (\vec{a} + \vec{b}) \cdot [(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})]$$

$$= (\vec{a} + \vec{b}) \cdot (\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{c} + \vec{c} \times \vec{a})$$

$$= (\vec{a} + \vec{b}) \cdot (\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a})$$

$$[\because \vec{c} \times \vec{c} = 0] \quad (2)$$

$$= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a})$$

$$+ \vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{a}) + \vec{b} \cdot (\vec{c} \times \vec{a})$$

$$= [\vec{a} \quad \vec{b} \quad \vec{c}] + [\vec{a} \quad \vec{b} \quad \vec{a}] + [\vec{a} \quad \vec{c} \quad \vec{a}]$$

$$+ [\vec{b} \quad \vec{b} \quad \vec{c}] + [\vec{b} \quad \vec{b} \quad \vec{a}] + [\vec{b} \quad \vec{c} \quad \vec{a}]$$

$$= 2 [\vec{a} \quad \vec{b} \quad \vec{c}] = \text{RHS}$$

Hence proved. (2)

38. Vectors \vec{a} , \vec{b} and \vec{c} are such that

$$\vec{a} + \vec{b} + \vec{c} = \vec{0} \text{ and } |\vec{a}| = 3, |\vec{b}| = 5 \text{ and}$$

$$|\vec{c}| = 7. \text{ Find the angle between } \vec{a} \text{ and } \vec{b}.$$

All India 2008; Delhi 2014, 2008



Firstly, write the given expression $\vec{a} + \vec{b} + \vec{c} = 0$ as $\vec{a} + \vec{b} = -\vec{c}$ and then square both sides and simplify to get the angle between \vec{a} and \vec{b} .

$$\text{Given, } |\vec{a}| = 3, |\vec{b}| = 5 \text{ and } |\vec{c}| = 7$$

$$\text{Also, } \vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{a} + \vec{b} = -\vec{c}$$

$$\Rightarrow (\vec{a} + \vec{b})^2 = (-\vec{c})^2 \text{ [squaring on both sides]}$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (-\vec{c}) \cdot (-\vec{c})$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} = \vec{c} \cdot \vec{c}$$

$$\Rightarrow |\vec{a}|^2 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{c}|^2$$

$$[\because \vec{x} \cdot \vec{x} = |\vec{x}|^2 \text{ and } \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}] \text{ (1)}$$

$$\Rightarrow |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{c}|^2$$

$$\Rightarrow |\vec{a}|^2 + 2|\vec{a}||\vec{b}|\cos\theta + |\vec{b}|^2 = |\vec{c}|^2 \quad \dots(i) \quad (1)$$

$$[\because \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta]$$

$$\Rightarrow (3)^2 + 2 \times 3 \times 5 \cos\theta + (5)^2 = (7)^2$$

$$[\because |\vec{a}| = 3, |\vec{b}| = 5 \text{ and } |\vec{c}| = 7]$$

$$\Rightarrow 9 + 30 \cos\theta + 25 = 49 \quad (1)$$

$$\Rightarrow 30 \cos\theta = 49 - 34$$

$$\Rightarrow \cos\theta = \frac{15}{30} = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \cos\frac{\pi}{3} \quad \left[\because \frac{1}{2} = \cos\frac{\pi}{3} \right]$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

Hence, the angle between \vec{a} and \vec{b} is $\frac{\pi}{3}$. (1)

- 39.** Show that the four points A, B, C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}, -\hat{j} - \hat{k}, 3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively are coplanar. All India 2014

Given, points are $A = 4\hat{i} + 5\hat{j} + \hat{k}$, $B = -\hat{j} - \hat{k}$,
 $C = 3\hat{i} + 9\hat{j} + 4\hat{k}$ and $D = 4(-\hat{i} + \hat{j} + \hat{k})$.

We know that, the four points A , B , C , and D
 will be coplanar, if the three vectors \vec{AB} , \vec{AC}
 and \vec{AD} are coplanar, i.e. if

$$[\vec{AB} \ \vec{AC} \ \vec{AD}] = 0$$

$$\begin{aligned}\therefore \vec{AB} &= -\hat{j} - \hat{k} - (4\hat{i} + 5\hat{j} + \hat{k}) \\ &= -4\hat{i} - 6\hat{j} - 2\hat{k}\end{aligned}\quad (1)$$

$$\begin{aligned}\vec{AC} &= (3\hat{i} + 9\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k}) \\ &= -\hat{i} + 4\hat{j} + 3\hat{k}\end{aligned}$$

$$\begin{aligned}\text{and } \vec{AD} &= 4(-\hat{i} + \hat{j} + \hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k}) \\ &= -8\hat{i} - \hat{j} + 3\hat{k}\end{aligned}\quad (1)$$

$$\text{Now, } [\vec{AB} \ \vec{AC} \ \vec{AD}] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}\quad (1)$$

$$\begin{aligned}&= -4(12 + 3) + 6(-3 + 24) - 2(1 + 32) \\ &= -60 + 126 - 66 = -126 + 126 = 0\end{aligned}$$

Hence, points A , B , C and D are coplanar. (1)

40. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$
 with a unit vector along the sum of vectors
 $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal
 to one. Find the value of λ and hence, find
 the unit vector along $\vec{b} + \vec{c}$. All India 2014

Given, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and
 $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$.

$$\begin{aligned}\text{Now, } \vec{b} + \vec{c} &= 2\hat{i} + 4\hat{j} - 5\hat{k} + \lambda\hat{i} + 2\hat{j} + 3\hat{k} \\ &= (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}\end{aligned}$$

$$\begin{aligned}
 \therefore |\vec{b} + \vec{c}| &= \sqrt{(2 + \lambda)^2 + (6)^2 + (-2)^2} \\
 &= \sqrt{4 + \lambda^2 + 4\lambda + 36 + 4} \\
 &= \sqrt{\lambda^2 + 4\lambda + 44} \quad (1)
 \end{aligned}$$

The unit vector along $\vec{b} + \vec{c}$

$$\begin{aligned}
 &= \frac{\vec{b} + \vec{c}}{|\vec{b} + \vec{c}|} \\
 &= \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} \quad \dots(i)
 \end{aligned}$$

Given, scalar product of $(\hat{i} + \hat{j} + \hat{k})$ with this unit vector is 1. (1)

$$\therefore (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{\vec{b} + \vec{c}}{|\vec{b} + \vec{c}|} = 1$$

$$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \frac{1(2 + \lambda) + 1(6) + 1(-2)}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \frac{(2 + \lambda) + 6 - 2}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \lambda + 6 = \sqrt{\lambda^2 + 4\lambda + 44}$$

$$\Rightarrow (\lambda + 6)^2 = \lambda^2 + 4\lambda + 44$$

[squaring on both sides]

$$\Rightarrow \lambda^2 + 36 + 12\lambda = \lambda^2 + 4\lambda + 44$$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1$$

Hence, the value of λ is 1.

On substituting the value of λ in Eq. (i), we get

$$\begin{aligned}\text{Unit vector along } \vec{b} + \vec{c} \\&= \frac{(2+1)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(1)^2 + 4(1) + 44}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{1+4+44}} \\&= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{49}} = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k} \quad (1)\end{aligned}$$

- 41.** Find the vector \vec{p} which is perpendicular to both $\vec{\alpha} = 4\hat{i} + 5\hat{j} - \hat{k}$ and $\vec{\beta} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{p} \cdot \vec{q} = 21$, where $\vec{q} = 3\hat{i} + \hat{j} - \hat{k}$. All India 2014C

Given $\alpha = 4\hat{i} + 5\hat{j} - \hat{k}$, $\beta = \hat{i} - 4\hat{j} + 5\hat{k}$

$$q = 3\hat{i} + \hat{j} - \hat{k}$$

Also, vector \vec{p} is perpendicular to α and β .

Then, $\vec{p} = \lambda (\vec{\alpha} \times \vec{\beta})$

$$\begin{aligned} \text{Now, } \vec{\alpha} \times \vec{\beta} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & -1 \\ 1 & -4 & 5 \end{vmatrix} \\ &= \hat{i} (25 - 4) - \hat{j} (20 + 1) + \hat{k} (-16 - 5) \\ &= \hat{i} (21) - \hat{j} (21) + \hat{k} (-21) \\ &= 21\hat{i} - 21\hat{j} - 21\hat{k} \end{aligned}$$

So, $\vec{p} = 21\lambda\hat{i} - 21\lambda\hat{j} - 21\lambda\hat{k}$... (i)

Also, given that $\vec{p} \cdot \vec{q} = 21$

$$\therefore (21\lambda\hat{i} - 21\lambda\hat{j} - 21\lambda\hat{k}) \cdot (3\hat{i} + \hat{j} - \hat{k}) = 21$$

$$\Rightarrow 63\lambda - 21\lambda + 21\lambda = 21$$

$$\Rightarrow 63\lambda = 21 \Rightarrow \lambda = 1/3 \quad (1)$$

On putting $\lambda = \frac{1}{3}$ in Eq. (i), we get

$$\vec{p} = 21 \times \frac{1}{3} \hat{i} - 21 \times \frac{1}{3} \hat{j} - 21 \times \frac{1}{3} \hat{k}$$

$$\Rightarrow \vec{p} = 7\hat{i} - 7\hat{j} - 7\hat{k}$$

which is the required vector. (1)

42. Find a unit vector perpendicular to both of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$

where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

Foreign 2014

Given, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

Let the required unit vector be

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k},$$

then $\sqrt{x^2 + y^2 + z^2} = 1$

$$\Rightarrow x^2 + y^2 + z^2 = 1 \quad \dots(i)$$

$$\begin{aligned}\text{Now, } \vec{a} + \vec{b} &= (\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + 2\hat{j} + 3\hat{k}) \\ &= 2\hat{i} + 3\hat{j} + 4\hat{k}\end{aligned}$$

$$\text{and } \vec{a} - \vec{b} = (\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = -\hat{j} - 2\hat{k} \quad (1)$$

Since, \vec{r} is perpendicular to $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$,

$$\therefore \vec{r} \cdot (\vec{a} + \vec{b}) = 0 \quad \text{and} \quad \vec{r} \cdot (\vec{a} - \vec{b}) = 0$$

$$\text{i.e. } (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 0$$

$$\Rightarrow 2x + 3y + 4z = 0 \quad \dots(ii)$$

$$\text{and } (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (-\hat{j} - 2\hat{k}) = 0$$

$$\Rightarrow -y - 2z = 0$$

$$\Rightarrow y = -2z \quad \dots(iii)$$

On putting the value of y in Eq. (ii), we get

$$2x + 3(-2z) + 4z = 0$$

$$\Rightarrow x = z \quad (1)$$

On substituting the value of x and y in Eq. (i), we get

$$z^2 + 4z^2 + z^2 = 1 \Rightarrow z = \pm \frac{1}{\sqrt{6}} \text{ and}$$

$$\text{then, } x = \pm \frac{1}{\sqrt{6}} \quad \text{and} \quad y = \mp \frac{2}{\sqrt{6}} \quad (1)$$

Hence, the required vectors are

$$\frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$$

$$\text{and } \frac{-1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}. \quad (1)$$

- 43.** Find the unit vector perpendicular to the plane ABC where the position vectors of A , B and C are $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{k}$ respectively. All India 2014C



A unit vector perpendicular to plane ABC is

$$\frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|}$$

Let O be the origin of reference.

Then, given $\vec{OA} = 2\hat{i} - \hat{j} + \hat{k}$,

$$\vec{OB} = \hat{i} + \hat{j} + 2\hat{k} \text{ and } \vec{OC} = 2\hat{i} + 3\hat{k}$$

$$\begin{aligned} \therefore \vec{AB} &= \vec{OB} - \vec{OA} = \hat{i} + \hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - \hat{k} \\ &= -\hat{i} + 2\hat{j} + \hat{k} \quad (1) \end{aligned}$$

$$\begin{aligned} \text{and } \vec{AC} &= \vec{OC} - \vec{OA} = 2\hat{i} + 3\hat{k} - 2\hat{i} + \hat{j} - \hat{k} \\ &= \hat{j} + 2\hat{k} \end{aligned}$$

$$\begin{aligned} \text{Now, } \vec{AB} \times \vec{AC} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} \\ &= \hat{i}(4 - 1) - \hat{j}(-2 - 0) + \hat{k}(-1 - 0) \\ &= 3\hat{i} + 2\hat{j} - \hat{k} \quad (1) \end{aligned}$$

$$\begin{aligned} \text{Then, } |\vec{AB} \times \vec{AC}| &= \sqrt{(3)^2 + (2)^2 + (-1)^2} \\ &= \sqrt{9 + 4 + 1} = \sqrt{14} \quad (1) \end{aligned}$$

Unit vector perpendicular to the plane ABC

$$\begin{aligned} &= \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} = \frac{3\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{14}} \\ &= \frac{3}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} - \frac{1}{\sqrt{14}}\hat{k} \quad (1) \end{aligned}$$

- 44.** Show that the vectors \vec{a} , \vec{b} and \vec{c} are coplanar, if and only if $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are coplanar. Foreign 2014

Consider,

$$\begin{aligned} & [(\vec{a} + \vec{b})(\vec{b} + \vec{c})(\vec{c} + \vec{a})] \\ &= (\vec{a} + \vec{b}) \cdot \{(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})\} \\ &= (\vec{a} + \vec{b}) \cdot (\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{c} + \vec{c} \times \vec{a}) \\ &= (\vec{a} + \vec{b}) \cdot (\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a}) \\ & \qquad \qquad \qquad \because \vec{c} \times \vec{c} = \vec{0} \quad (2) \end{aligned}$$

$$\begin{aligned} &= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a}) \\ & \qquad + \vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{a}) + \vec{b} \cdot (\vec{c} \times \vec{a}) \\ &= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{c} \times \vec{a}) \end{aligned}$$

$$[\because [\vec{a} \ \vec{b} \ \vec{a}] = [\vec{b} \ \vec{b} \ \vec{a}] = [\vec{a} \ \vec{c} \ \vec{a}] = 0]$$

$$= 2 [\vec{a} \ \vec{b} \ \vec{c}]$$

Now, we can see that

$$[(\vec{a} + \vec{b})(\vec{b} + \vec{c})(\vec{c} + \vec{a})] = 2 [\vec{a} \ \vec{b} \ \vec{c}]$$

Hence, the vectors \vec{a} , \vec{b} , \vec{c} are coplanar, if and only if $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are coplanar (2)

- 45.** If \vec{a} , \vec{b} and \vec{c} are three mutually perpendicular vectors of the same magnitude, then prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors \vec{a} , \vec{b} and \vec{c} .

HOTS; Delhi 2013C, 2011



If three vectors \vec{a} , \vec{b} and \vec{c} are mutually perpendicular to each other. then

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ and if all three vectors \vec{a} , \vec{b} and \vec{c} are equally inclined with the vector $(\vec{a} + \vec{b} + \vec{c})$, that means each vector \vec{a} , \vec{b} and \vec{c} makes equal angle with $(\vec{a} + \vec{b} + \vec{c})$ by using formula $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$.

Given, $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$ [say]

and $\vec{a} \cdot \vec{b} = 0, \vec{b} \cdot \vec{c} = 0$ and $\vec{c} \cdot \vec{a} = 0$ (1/2)

$$\begin{aligned}
 \text{Now, } |\vec{a} + \vec{b} + \vec{c}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 \\
 &\quad + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) \\
 &= \lambda^2 + \lambda^2 + \lambda^2 + 2(0 + 0 + 0) = 3\lambda^2
 \end{aligned}$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \pm \sqrt{3} \lambda \quad (1)$$

Suppose $(\vec{a} + \vec{b} + \vec{c})$ is inclined at angles θ_1, θ_2 and θ_3 respectively with vectors \vec{a}, \vec{b} and \vec{c} , then

$$(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{a} = |\vec{a} + \vec{b} + \vec{c}| \cdot |\vec{a}| \cos \theta_1$$

$$[\because \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta]$$

$$\Rightarrow |\vec{a}|^2 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = \pm \sqrt{3} \lambda \times \lambda \cos \theta_1$$

$$\Rightarrow \lambda^2 + 0 + 0 = \pm \sqrt{3} \lambda^2 \cos \theta_1$$

$$\therefore \cos \theta_1 = \pm \frac{1}{\sqrt{3}}$$

$$(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{b} = |\vec{a} + \vec{b} + \vec{c}| |\vec{b}| \cos \theta_2 \quad (1)$$

$$\Rightarrow \vec{a} \cdot \vec{b} + |\vec{b}|^2 + \vec{b} \cdot \vec{c} = \pm \sqrt{3} \lambda \cdot \lambda \cos \theta_2$$

$$\Rightarrow 0 + \lambda^2 + 0 = \pm \sqrt{3} \lambda^2 \cos \theta_2$$

$$\Rightarrow \quad \vec{a} + \vec{b} + \vec{c} = \pm \sqrt{3} \vec{a} \cos \theta_2$$

$$\Rightarrow \quad \cos \theta_2 = \pm \frac{1}{\sqrt{3}}$$

$$\text{Similarly, } (\vec{a} + \vec{b} + \vec{c}) \cdot \vec{c} = |\vec{a} + \vec{b} + \vec{c}| |\vec{c}| \cos \theta_3$$


$$\Rightarrow \quad \cos \theta_1 = \pm \frac{1}{\sqrt{3}} \quad (1)$$

$$\text{Thus, } \cos \theta_1 = \cos \theta_2 = \cos \theta_3 = \pm \frac{1}{\sqrt{3}}$$

Hence, it is proved that $(\vec{a} + \vec{b} + \vec{c})$ is equally inclined with the vectors \vec{a} , \vec{b} and \vec{c} . (1/2)

46. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a vector \vec{c} , such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

Delhi 2013, 2008

 If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are two vectors, then

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\text{and } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\text{Given, } \vec{a} = \hat{i} + \hat{j} + \hat{k} \text{ and } \vec{b} = \hat{j} - \hat{k}$$

$$\text{and } \vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\therefore \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$$

$$= \hat{i}(z - y) - \hat{j}(z - x) + \hat{k}(y - x) \quad (1)$$

$$\text{Now, } \vec{a} \times \vec{c} = \vec{b} \quad [\text{given}]$$

$$\Rightarrow \hat{i}(z - y) + \hat{j}(x - z) + \hat{k}(y - x)$$

$$= 0\hat{i} + 1\hat{j} + (-1)\hat{k} \quad [\because \vec{b} = \hat{j} - \hat{k}]$$

On comparing the coefficients from both sides, we get

$$z - y = 0, x - z = 1, y - x = -1$$

$$\Rightarrow y = z \text{ and } x - y = 1 \quad \dots(i)$$

Also given, $\vec{a} \cdot \vec{c} = 3$

$$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k}) = 3$$

$$\Rightarrow x + y + z = 3 \quad (1)$$

$$\Rightarrow x + 2y = 3 [\because y = z] \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$3y = 2$$

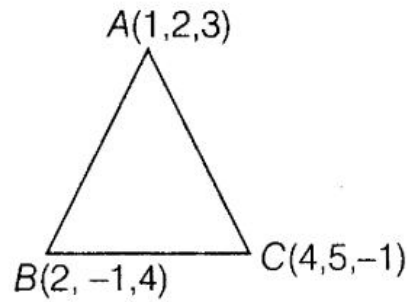
$$\Rightarrow y = \frac{2}{3} = z \quad [\because y = z]$$

$$\text{From Eq. (i)} \quad x = 1 + y = 1 + \frac{2}{3} = \frac{5}{3} \quad (1)$$

$$\text{Hence, } \vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k} \quad (1)$$

- 47.** Using vectors, find the area of the $\triangle ABC$, whose vertices are $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$. All India 2013

Let the position vectors of the vertices A, B and C of ΔABC be (1)



$$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \overrightarrow{OB} = 2\hat{i} - \hat{j} + 4\hat{k}$$

and $\overrightarrow{OC} = 4\hat{i} + 5\hat{j} - \hat{k}$, respectively.

Then, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$$= (2\hat{i} - \hat{j} + 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= \hat{i} - 3\hat{j} + \hat{k}$$

and $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$$= (4\hat{i} + 5\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= (3\hat{i} + 3\hat{j} - 4\hat{k}) \quad (1)$$

Then, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$

$$= \hat{i}(12 - 3) - \hat{j}(-4 - 3) + \hat{k}(3 + 9)$$

$$= 9\hat{i} + 7\hat{j} + 12\hat{k} \quad (1)$$


$$\therefore |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(9)^2 + (7)^2 + (12)^2}$$

$$= \sqrt{81 + 49 + 144} = \sqrt{274}$$

Hence, area of $\Delta ABC = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$

$$= \frac{1}{2} \sqrt{274} \text{ sq units} \quad (1)$$

- 48.** If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ , so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors. **All India 2013**

 Use the result that if \vec{a} and \vec{b} are perpendicular, then their dot product should be zero and simplify it.

Given, $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

Then, $\vec{a} + \vec{b} = (\hat{i} - \hat{j} + 7\hat{k}) + (5\hat{i} - \hat{j} + \lambda\hat{k})$
 $= 6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}$ **(1)**

and $\vec{a} - \vec{b} = (\hat{i} - \hat{j} + 7\hat{k}) - (5\hat{i} - \hat{j} + \lambda\hat{k})$
 $= -4\hat{i} + (7 - \lambda)\hat{k}$ **(1)**

Since, $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular vectors, then $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$

$\Rightarrow [6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}] \cdot [-4\hat{i} + (7 - \lambda)\hat{k}] = 0$ **(1)**

$\Rightarrow -24 + (7 + \lambda)(7 - \lambda) = 0$

$\Rightarrow 49 - \lambda^2 = 24 \Rightarrow \lambda^2 = 25$

$\therefore \lambda = \pm 5$ **(1)**

- 49.** If $\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$, then find the value of λ , so that $\vec{p} + \vec{q}$ and $\vec{p} - \vec{q}$ are perpendicular vectors. **All India 2013**

Do same as Que. 48. [**Ans.** $\lambda = \pm 1$]

- 50.** If \vec{a}, \vec{b} and \vec{c} are three vectors, such that

$|\vec{a}| = 5, |\vec{b}| = 12, |\vec{c}| = 13$ and

$\vec{a} + \vec{b} + \vec{c} = 0$, then find the value of

$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

Delhi 2012



Use the following formula:

$$(\vec{a} + \vec{b} + \vec{c})^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

Given, $|\vec{a}| = 5, |\vec{b}| = 12, |\vec{c}| = 13$

and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

On multiplying both sides by $(\vec{a} + \vec{b} + \vec{c})$, we get

$$(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = \vec{0} \cdot (\vec{a} + \vec{b} + \vec{c}) \quad (1)$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c} = 0$$

$$\Rightarrow |\vec{a}|^2 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{a} + |\vec{b}|^2 + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} + |\vec{c}|^2 = 0$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0 \quad (1\frac{1}{2})$$

$$[\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}, \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{b}, \vec{c} \cdot \vec{a} = \vec{a} \cdot \vec{c}]$$

$$\text{and } \vec{x} \cdot \vec{x} = |\vec{x}|^2]$$

$$\Rightarrow (5)^2 + (12)^2 + (13)^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$[\because |\vec{a}| = 5, |\vec{b}| = 12 \text{ and } |\vec{c}| = 13]$$

$$\Rightarrow 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -338 \quad (1\frac{1}{2})$$

$$\text{Hence, } \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -169$$

- 51.** Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector \vec{p} , which is perpendicular to both \vec{a} and \vec{b} and $\vec{p} \cdot \vec{c} = 18$.

HOTS; All India 2012, 2010

Given, vectors are $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$,

$$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$$

and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

We have to find a vector \vec{p} , such that

$$\vec{p} \cdot \vec{a} = 0 \quad \dots(i)$$

$$\text{and} \quad \vec{p} \cdot \vec{b} = 0 \quad \dots(ii)$$

[$\because \vec{p}$ is perpendicular to both \vec{a} and \vec{b} , given]

$$\text{and} \quad \vec{p} \cdot \vec{c} = 18 \quad \dots(iii)(1)$$

So, let $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$

From Eqs. (i), (ii) and (iii), we get

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 4\hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow x + 4y + 2z = 0 \quad \dots(iv)$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} - 2\hat{j} + 7\hat{k}) = 0$$

$$\Rightarrow 3x - 2y + 7z = 0 \quad \dots(v)$$

$$\text{and } (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} - \hat{j} + 4\hat{k}) = 0$$

$$\Rightarrow 2x - y + 4z = 18 \quad \dots(vi)$$

On multiplying Eq. (iv) by 3 and subtracting it from Eq. (v), we get

$$-14y + z = 0 \quad \dots(vii)$$

Now, multiplying Eq. (iv) by 2 and subtracting it from Eq. (vi), we get

$$-9y = 18 \Rightarrow y = -2 \quad (1)$$

On putting $y = -2$ in Eq. (vii), we get

$$-14(-2) + z = 0$$

$$\Rightarrow 28 + z = 0$$

$$\Rightarrow z = -28$$

On putting $y = -2$ and $z = -28$ in Eq. (iv), we get

$$x + 4(-2) + 2(-28) = 0$$

$$\Rightarrow x - 8 - 56 = 0$$

$$\Rightarrow x = 64 \quad (1\frac{1}{2})$$


Hence, the required vector

$$\vec{p} = x\hat{i} + y\hat{j} + z\hat{k} \text{ is}$$

$$\vec{p} = 64\hat{i} - 2\hat{j} - 28\hat{k} \quad (1/2)$$

52. Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}. \text{ Delhi 2011}$$

 Use the concept, if \vec{a} and \vec{b} are two vectors, then a unit vector perpendicular to both of them is
$$= \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}.$$

Given, $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Then, $\vec{a} + \vec{b} = (3\hat{i} + 2\hat{j} + 2\hat{k}) + (\hat{i} + 2\hat{j} - 2\hat{k})$
 $= 4\hat{i} + 4\hat{j}$

and $\vec{a} - \vec{b} = (3\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - 2\hat{k})$
 $= 2\hat{i} + 4\hat{k} \quad (1)$

Let $\vec{a} + \vec{b} = \vec{c}$ and $\vec{a} - \vec{b} = \vec{d}$, so that we have
 $\vec{c} = 4\hat{i} + 4\hat{j}$ and $\vec{d} = 2\hat{i} + 4\hat{k}.$

Now, $\vec{c} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \end{vmatrix}$

$$\begin{aligned}
& \begin{vmatrix} 2 & 0 & 4 \end{vmatrix} \\
& = \hat{i}(16 - 0) - \hat{j}(16 - 0) + \hat{k}(0 - 8) \\
\Rightarrow \quad \vec{c} \times \vec{d} &= 16\hat{i} - 16\hat{j} - 8\hat{k} \quad \dots(i) \quad (1) \\
\therefore |\vec{c} \times \vec{d}| &= \sqrt{(16)^2 + (-16)^2 + (-8)^2} \\
&= \sqrt{256 + 256 + 64} \\
&= \sqrt{576} = 24 \quad \dots(ii) \quad (1)
\end{aligned}$$

On putting the values from Eq. (i) and (ii), we get

$$\begin{aligned}
\text{Required vector} &= \frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{24} \\
&= \frac{8(2\hat{i} - 2\hat{j} - \hat{k})}{24} \\
&= \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k} \quad (1)
\end{aligned}$$

53. If \vec{a} and \vec{b} are two vectors, such that $|\vec{a}| = 2$,
 $|\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 1$, then find

Given, $|\vec{a}| = 2$, $|\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 1$

Now, $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$

$$= 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{b} \cdot \vec{a} - 35\vec{b} \cdot \vec{b}$$

$$= 6|\vec{a}|^2 + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35|\vec{b}|^2 \quad (1)$$

$$[\because \vec{x} \cdot \vec{x} = |\vec{x}|^2 \text{ and } \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}]$$

$$= 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$$

$$= 6(2)^2 + 11(1) - 35(1)^2 = 0 \quad (1)$$

$$[\because |\vec{a}| = 2 \text{ and } |\vec{b}| = 1]$$

$$\text{Hence, } (3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}) = 0 \quad (1)$$

- 54.** If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda \vec{b}$ is perpendicular to \vec{c} , then find the value of λ .

Foreign 2011; All India 2009C

Given, $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$,

$$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k} \text{ and } \vec{c} = 3\hat{i} + \hat{j}.$$

Also, $\vec{a} + \lambda \vec{b}$ is perpendicular to \vec{c} .

$$\therefore (\vec{a} + \lambda \vec{b}) \cdot \vec{c} = 0 \quad \dots(i)(1)$$

$$[\because \text{when } \vec{a} \perp \vec{b}, \text{ then } \vec{a} \cdot \vec{b} = 0]$$

$$\begin{aligned} \text{Now, } \vec{a} + \lambda \vec{b} &= (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k}) \\ \Rightarrow \vec{a} + \lambda \vec{b} &= \hat{i}(2 - \lambda) + \hat{j}(2 + 2\lambda) + \hat{k}(3 + \lambda) \quad (1) \end{aligned}$$

Then, from Eq. (i), we get

$$[\hat{i}(2 - \lambda) + \hat{j}(2 + 2\lambda) + \hat{k}(3 + \lambda)] \cdot [3\hat{i} + \hat{j}] = 0 \quad (1)$$

$$\Rightarrow 3(2 - \lambda) + 1(2 + 2\lambda) = 0$$

$$\Rightarrow 8 - \lambda = 0$$

$$\therefore \lambda = 8 \quad (1)$$

- 55.** Using vectors, find the area of triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5).

All India 2011

Do some as Que. 47. [Ans. $\frac{1}{2}\sqrt{61}$ sq units]

- 56.** If \vec{a} , \vec{b} and \vec{c} are three vectors, such that

$|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 5$ and each one of these is perpendicular to the sum of other

two, then find $|\vec{a} + \vec{b} + \vec{c}|$. All India 2011C, 2010C

$$\text{Given, } |\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5 \quad \dots(i)$$

Also, given that each of the vectors \vec{a} , \vec{b} and \vec{c} is perpendicular to sum of the other two vectors, i.e.

$$\vec{a} \perp (\vec{b} + \vec{c})$$

$$\Rightarrow \vec{a} \cdot (\vec{b} + \vec{c}) = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0 \quad \dots(ii)$$

$$\vec{b} \perp (\vec{c} + \vec{a})$$

$$\Rightarrow \vec{b} \cdot (\vec{c} + \vec{a}) = 0$$

$$\Rightarrow \vec{b} \cdot \vec{c} + \vec{b} \cdot \vec{a} = 0 \quad \dots(iii)$$

$$\text{and } \vec{c} \perp (\vec{a} + \vec{b}) \Rightarrow \vec{c} \cdot (\vec{a} + \vec{b}) = 0$$

$$\Rightarrow \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} = 0 \quad \dots(iv)$$

[\because when two vectors are perpendicular, then their dot product is zero] **(1)**

Now, adding Eqs. (ii), (iii) and (iv), we get

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0 \quad \dots(v)$$

$$[\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}, \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{b} \text{ and } \vec{c} \cdot \vec{a} = \vec{a} \cdot \vec{c}]$$

Now, consider

$$\begin{aligned} (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) &= \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \\ &\quad + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c} \end{aligned}$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) \quad (1)$$

$$[\because \vec{a} \cdot \vec{a} = |\vec{a}|^2]$$

On putting the values from Eqs. (i) and (v) we get

$$\begin{aligned} |\vec{a} + \vec{b} + \vec{c}|^2 &= (3)^2 + (4)^2 + (5)^2 + 2(0) \quad (1) \\ &= 9 + 16 + 25 = 50 \end{aligned}$$

$$\begin{aligned} \text{Hence, } |\vec{a} + \vec{b} + \vec{c}| &= \sqrt{50} \\ &= \sqrt{25 \times 2} = 5\sqrt{2} \quad (1) \end{aligned}$$

- 57.** Using vectors, find the area of triangle with vertices A (2, 3, 5), B (3, 5, 8) and C (2, 7, 8).
Delhi 2010C

Do same as Que. 47. [Ans. $\frac{1}{2}\sqrt{61}$ sq units]

- 58.** The scalar product of vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ . All India 2009, 2008C

Do same as Que. 40. [Ans. $\lambda = 1$]

- 59.** If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.
HOTS; Delhi 2009



If two vectors are parallel, then their cross-product will be a zero vector.

Given, $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$... (i)

and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$... (ii) (1)

On subtracting Eq. (ii) from Eq. (i), we get

$$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) = (\vec{c} \times \vec{d}) - (\vec{b} \times \vec{d})$$

$$\Rightarrow (\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) + (\vec{b} \times \vec{d}) - (\vec{c} \times \vec{d}) = \vec{0}$$

$$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) + (\vec{b} - \vec{c}) \times \vec{d} = \vec{0}$$

$$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = \vec{0}$$

$$[\because \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}] (1)$$

$$\Rightarrow (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{0}$$

$$[\because \vec{a} \neq \vec{d} \text{ and } \vec{b} \neq \vec{c}] (1/2)$$

Thus, we have that cross-product of vectors $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ as a zero vector, so $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$. (1½)

60. Three vectors \vec{a} , \vec{b} and \vec{c} satisfy the

condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Find the value of

$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $|\vec{a}| = 1$, $|\vec{b}| = 4$ and

$$|\vec{c}| = 2.$$

All India 2008C

Do same as Que. 50.

$$\left[\text{Ans.} -\frac{21}{2} \right]$$

61. Find a vector of magnitude 5 units,

perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and

$(\vec{a} - \vec{b})$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and

$$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}.$$

Delhi 2008C

Given, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\vec{a} = \hat{i} + \hat{j} + \hat{k} \quad \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\begin{aligned}\therefore a + b &= (i + j + k) + (i + 2j + 3k) \\ &= 2\hat{i} + 3\hat{j} + 4\hat{k} \quad (1)\end{aligned}$$

$$\begin{aligned}\text{and } \vec{a} - \vec{b} &= \hat{i} + \hat{j} + \hat{k} - (\hat{i} + 2\hat{j} + 3\hat{k}) \\ &= -\hat{j} - 2\hat{k}\end{aligned}$$

$$\text{Let } \vec{a} + \vec{b} = \vec{c} \text{ and } \vec{a} - \vec{b} = \vec{d}$$

$$\text{Then we get, } \vec{c} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\text{and } \vec{d} = -\hat{j} - 2\hat{k}$$

We know that, unit vector which is perpendicular to both \vec{c} and \vec{d} is given by

$$\hat{n} = \frac{\vec{c} \times \vec{d}}{|\vec{c} \times \vec{d}|}$$

$$\begin{aligned}\therefore \vec{c} \times \vec{d} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} \\ &= \hat{i}(-6 + 4) - \hat{j}(-4 - 0) + \hat{k}(-2 - 0) \quad (1) \\ &= -2\hat{i} + 4\hat{j} - 2\hat{k}\end{aligned}$$

$$\begin{aligned}\text{and } |\vec{c} \times \vec{d}| &= \sqrt{(-2)^2 + (4)^2 + (-2)^2} \\ &= \sqrt{4 + 16 + 4} \\ &= \sqrt{24} = 2\sqrt{6}\end{aligned}$$

$$\begin{aligned}\therefore \hat{n} &= \frac{-2\hat{i} + 4\hat{j} - 2\hat{k}}{2\sqrt{6}} \\ &= \frac{-\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}} \quad (1)\end{aligned}$$

Hence, required vector of magnitude 5 units

$$= 5 \left(\frac{-\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}} \right)$$

$$= -\frac{5}{\sqrt{6}}\hat{i} + \frac{10}{\sqrt{6}}\hat{j} - \frac{5}{\sqrt{6}}\hat{k} \quad (1)$$