3

Integer-Answer-Type Questions

*In this chapter the answer to each question is a **single-digit integer**, ranging from 0 to 9 (both inclusive). For each question, darken the bubble corresponding to the correct integer in the ORS.*

3.1 General Physics

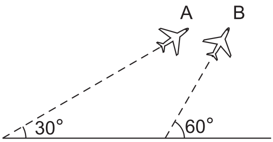
1. During Searle's experiment, the zero of a vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale divisions. The 20th division of the vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of the vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8.0×10^{-7} m^2. The least count of the vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young modulus of the wire is

2. To find the distance d over which a signal can be seen clearly in foggy conditions, a railway engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

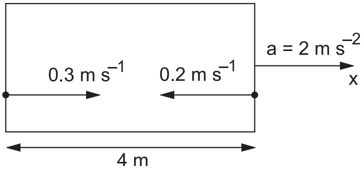
3. The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2$ s^{-1}. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, find the percentage error in the value of $E(t)$ at $t = 5$ s.

4. A train is moving along a straight line with a constant acceleration a. A boy standing inside the train throws a ball forward with a speed of 10 m s^{-1}, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m s^{-2}, is

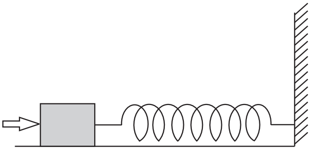
5. Aeroplanes A and B are flying with constant velocities in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in the figure. The speed of A is $100\sqrt{3}$ m s^{-1}. At time $t = 0$ s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B then t_0 in seconds is



6. A rocket is moving in a gravity-free space with a constant acceleration of 2 m s^{-2} along the $+x$ direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in the $+x$ direction with a speed of 0.3 m s^{-1} relative to the rocket. At the same time, another ball is thrown in the $-x$ direction with a speed of 0.2 m s^{-1} from the right end relative to the rocket. The time in seconds when the two balls hit each other is

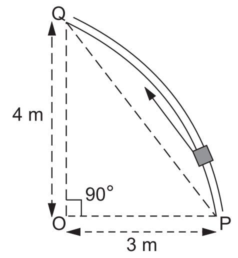


7. A block of mass 0.18 kg is attached to a spring of force constant 2 N m^{-1}. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m s^{-1} is $v = N/10$. Then N is



8. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power of 0.5 W to the particle. If the initial speed (in m s^{-1}) of the particle is zero, the speed (in m s^{-1}) after 5 s is

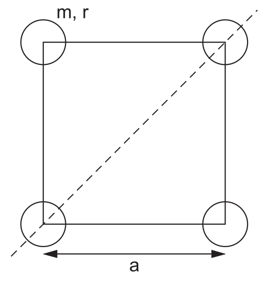
9. Consider an elliptical-shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is ($n \times 10$) joules. The value of n is (take $g = 10$ m s^{-2})



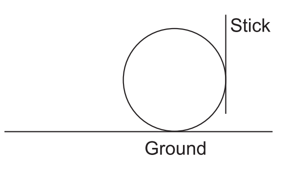
10. A block is moving on an inclined plane making an angle 45° with the horizontal. The coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\mu$ then N is

11. A bob of mass m, suspended by a string of length l_1, is given a minimum speed required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m, suspended by a string of length l_2, which is initially at rest. Both the strings are massless and inextensible. If the second bob, after the collision acquires a minimum speed required to complete a full circle in the vertical plane, the ratio l_1/l_2 is

12. Four solid spheres, each of diameter $\sqrt{5}$ cm and mass 0.5 kg, are placed with their centres at the corners of a square of side 4 cm. If the moment of inertia of the system about a diagonal of the square is $N \times 10^{-4}$ kg m^2 then N is



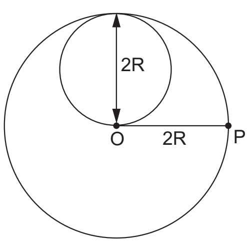
13. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m s^{-2}. The coefficient of friction between the ground and the ring is large enough so that rolling always occurs. The coefficient of friction between the stick and the ring is $P/10$. The value of P is



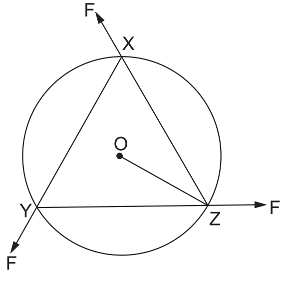
14. A binary star consists of two stars A (mass 2.2 M_s) and B (mass 11 M_s), where M_s is the mass of the sun. They are separated by a distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is

15. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is

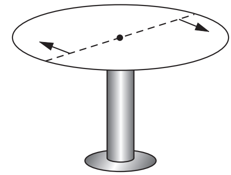
16. A lamina is made by removing a small disc of diameter $2R$, as shown in the figure. The moment of inertia of this lamina about the axes passing through O and P are I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is



17. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude $F = 0.5$ N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc is

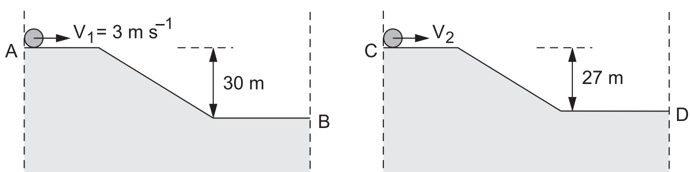


18. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy guns, each carrying a steel ball of mass 0.05 kg, are attached to the platform at a



distance of 0.25 m from the centre on its either side along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 m s^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is

19. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure), starting at A and C with linear speeds v_1 and v_2 respectively and always remaining in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m s^{-1} then v_2 in m s^{-1} is ($g = 10$ m s^{-2})

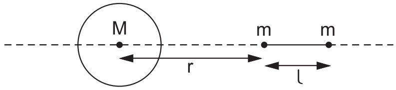


20. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$ respectively, where k is a constant. The moment of inertia of the individual spheres about the axes passing through their centres are I_A and I_B respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, find the value of n.

21. The gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is 2/3 times that of the earth. If the escape velocity on the surface of the earth is taken to be 11 km s^{-1}, the escape velocity on the surface of the planet in km s^{-1} will be

22. A bullet is fired vertically upwards with a velocity v from the surface of a spherical planet. When the bullet reaches its maximum height, its acceleration due to the planet's gravity is 1/4th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$ then N is (ignore the energy loss due to atmosphere)

23. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M as shown in the figure. The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3l$ from M, the tension in the rod is zero for $m = K\left(\frac{M}{288}\right)$. Find the value of K.

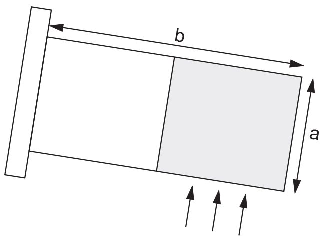


24. A 0.1-kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young modulus of the material of the wire is $n \times 10^9$ N m^{-2}, the value of n is

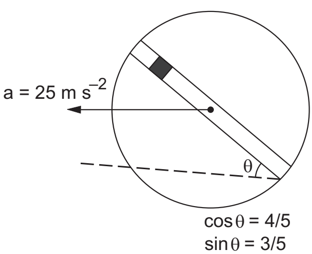
25. Consider two solid spheres P and Q each of density 8 g cm^{-3} and diameter 1 cm and 0.5 cm respectively. The sphere P is dropped into a liquid of density 0.8 g cm^{-3} and viscosity $\eta = 3$ poiseuilles. The sphere Q is dropped into a liquid of density 1.6 g cm^{-3} and viscosity $\eta = 2$ poiseuilles. Find the ratio of the terminal velocities of P and Q.

26. A steel wire of length L at 40 °C is suspended from the ceiling and then a mass m is hung from its free end. The wire is cooled down from 40 °C to 30 °C to regain its original length L. The coefficient of linear thermal expansion of the steel is 10^{-5} °C^{-1}, Young modulus of steel is 10^{11} N m^{-2} and the radius of the wire is 1 mm. Assume that $L >>$ diameter of the wire. Then, the value of m in kilogram is nearly

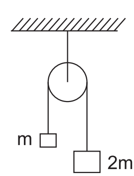
27. A rectangular plate of mass M and dimensions ($a \times b$) is held in a horizontal position by striking n small balls each of mass m per unit area per unit time. These are striking in the shaded half-region of the plate. The balls are colliding elastically with a velocity v. Find v.
(Given: $n = 100$, $M = 3$ kg, $m = 0.01$ kg, $b = 2$ m, $a = 1$ m, $g = 10$ m s^{-2})



28. A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown. The coefficient of friction between the block and all the surfaces of the groove in contact is $\mu = 0.4$. The disc has an acceleration of 25 m s^{-2}. Find the acceleration of the block with respect to the disc.

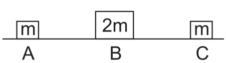


29. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10$ m s^{-2}, find the work done (in joules) by the string on the block of 0.36 kg during the first second after the system is released from rest.



30. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at 8 N m^{-2}. The radii of bubbles A and B are 2 cm and 4 cm respectively. The surface tension of the soap solution to form bubbles is 0.04 N m^{-1}. Find the ratio n_B/n_A, where n_A and n_B are the numbers of moles of air in bubbles A and B respectively. (Neglect the effect of gravity.)

31. Three blocks A, B and C are kept along a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m respectively. The block A moves towards B with a speed of 9 m s^{-1} and makes an elastic collision with it. Thereafter, B makes a completely inelastic collision with C. All motions occur along the same straight line. Find the final speed of block C in m s^{-1}.



32. A cylindrical vessel of height 500 mm has an orifice at its bottom. The orifice is initially closed and water is filled in it up to a height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out through the orifice and the water level in the vessel becomes steady with the height of water column being 200 mm. Find the fall in height (in mm) of the water level due to the opening of the orifice.

(Take atmospheric pressure = 1.0×10^5 N m^{-2}, density of water = 1000 kg m^{-3} and g = 10 m s^{-2}. Neglect any effect of surface tension.)

33. A drop of liquid of radius $R = 10^{-2}$ m having the surface tension $S = \frac{0.1}{4\pi}$ N m^{-1} divides itself into K identical drops. In this process, the total change in the surface energy ΔU is 10^{-3} J. If $K = 10^{\alpha}$ then the value of α is

3.2 Heat and Thermodynamics

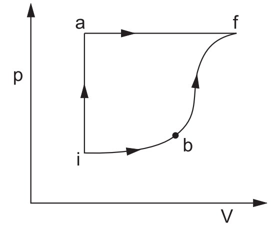
1. A piece of ice (specific heat capacity = 2100 J kg^{-1} °C^{-1} and latent heat = 3.36×10^5 J kg^{-1}) of mass m g is at –5 °C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally, when the ice–water mixture is in equilibrium, it is found that 1 g of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

2. In an insulated vessel, 0.05 kg of steam at 373 K and 0.45 kg of ice at 253 K are mixed. What would be the final temperature of the mixture in degree Celsius?

3. A metal rod AB of length $10x$ has its one end A in ice at 0 °C and the other end B in water at 100 °C. If a point P on the rod is maintained at 400 °C then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal g^{-1} and the latent heat of melting of ice is 80 cal g^{-1}. If the point P is at a distance of λx from the ice end A, find the value of λ. (Neglect any heat loss to the surroundings.)

4. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is

5. A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the paths iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio $\dfrac{Q_{bf}}{Q_{ib}}$ is



6. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2 respectively. The maximum intensity in the emission spectrum of A is at 500 nm and that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

7. Two spherical stars A and B emit black-body radiations. The radius of A is 400 times that of B, and A emits 10^4 times the power emitted by B. The ratio of their wavelengths λ_A to λ_B (i.e., λ_A/λ_B) at which the peaks occur in their respective radiation curves is

8. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C?

3.3 Sound Waves

1. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

2. Four harmonic waves of equal frequencies and equal intensities (I_0) have phase angles 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is

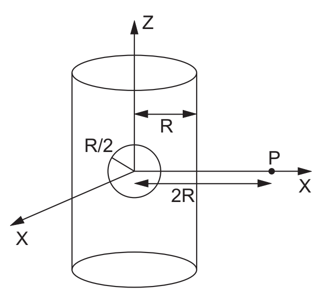
3. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much lower than the speed of sound which is 330 m s^{-1}.

4. A 20-cm-long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.

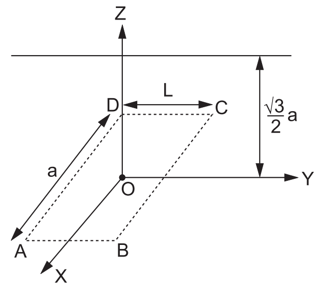
5. A stationary source emits sound of frequency f_0 = 492 Hz. The sound is reflected by a large car approaching the source with a speed of 2 m s^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 m s^{-1} and the car reflects the sound at the frequency it has received.)

3.4 Electrostatics

1. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16K\varepsilon_0}$. The value of K is



2. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the yz-plane at $z = \frac{\sqrt{3}}{2}a$, as shown in the figure. If the magnitude of the flux of electric field through the rectangular surface ABCD lying in the xy-plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space) then the value of n is



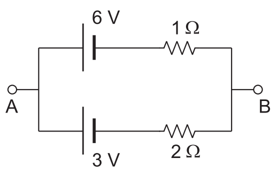
3. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is S. The system of charges and planar film are in equilibrium, and $a = K\left[\frac{q^2}{S}\right]^{1/N}$, where K is a constant. Then N is

4. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = Kr^n$, where K and n are constants and r is the distance of a point from its centre. If the electric field at the distance $r = R/2$ is 1/8 times that at $r = R$, find the value of n.

3.5 Current Electricity and Magnetism

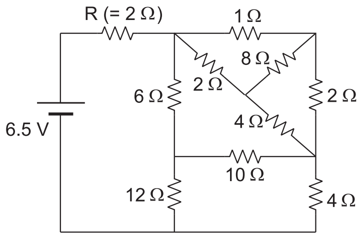
1. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25J_2$ then the value of R in Ω is

2. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volt is



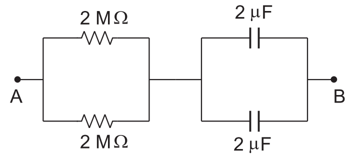
3. A galvanometer gives a full-scale deflection with 0.006-A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0–30 V. If connected to a $\frac{2n}{249}$-Ω resistance, it becomes an ammeter of range 0–1.5 A. The value of n is

4. In the given circuit, the current through the resistor R (= 2 Ω) is I amperes. The value of I is

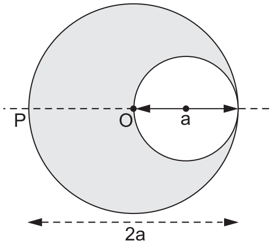


5. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V?

(Take: ln 5 = 1.6, ln 3 = 1.1.)

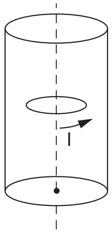


6. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$ then the value of N is

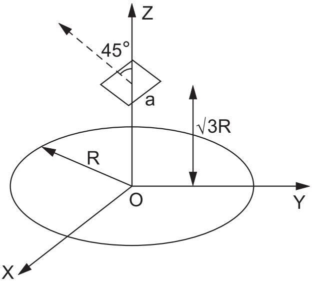


7. Two parallel wires in the plane of a paper are at a distance X_0 apart. A point charge is moving with a speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents of magnitude I in the two wires have directions opposite to each other, the curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, the value of $\frac{R_1}{R_2}$ is

8. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire loop of resistance 0.005 Ω and radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos 300\, t$, where I_0 is a constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin 300\, t$ then N is



9. A circular wire loop of radius R is placed in the xy-plane centred at the origin O. A square loop of side a ($a << R$) having two turns is placed with its centre at $\sqrt{3}R$ along the axis of the circular wire loop, as shown in the figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\dfrac{\mu_0 a^2}{2^{p/2} R}$ then the value of p is



10. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5-V battery. The circuit is switched on at time $t = 0$. Find the ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery.

11. A series RC combination is connected to an AC voltage of angular frequency $\omega = 500$ rad s^{-1}. If the impedance of the RC circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

12. A steady current I goes through a wire loop PQR having the shape of a right-angled triangle with PQ = $3x$, PR = $4x$ and QR = $5x$. If the magnitude of the magnetic field at P due to this loop is $K(\mu_0 I/48\pi x)$, find the value of K.

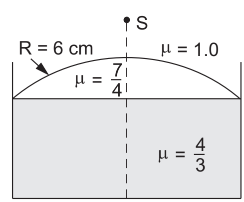
3.6 Ray Optics and Wave Optics

1. The image of an object that is approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from 25/3 m to 50/7 m in 30 seconds. What is the speed of the object in km per hour?

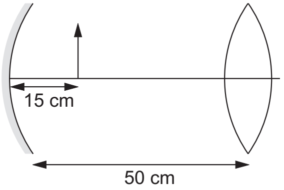
2. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

3. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is

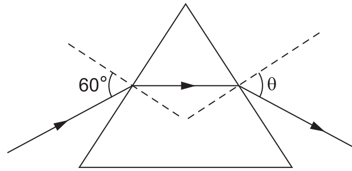
4. The water (refractive index 4/3) kept in a tank is 18 cm deep. A layer of oil of refractive index 7/4 lies on this water making a convex surface of radius of curvature R = 6 cm as shown in the figure. Consider the oil to act as a thin lens. An object S is placed 24 cm above the water surface. The location of its image is at X cm above the bottom of the tank. Then X is



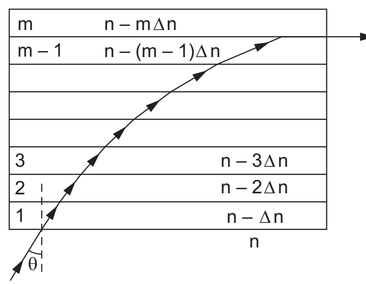
5. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



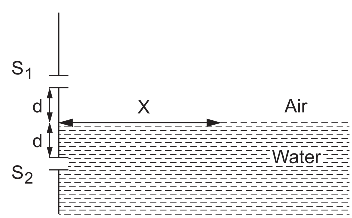
6. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal. For $n = \sqrt{3}$, the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is



7. A monochromatic light is travelling in a medium of refractive index $n = 1.6$. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of the different glass layers are monotonically decreasing as $n_s = n - m\Delta n$, where n_m is the refractive index of the mth slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m - 1)$th and mth slabs from the right side of the stack. What is the value of m?



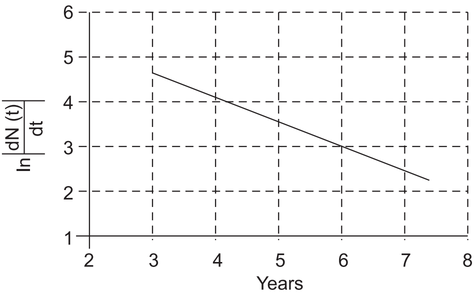
8. A Young's double-slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of the maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, when λ is the wavelength of light in air (refractive index = 1), $2d$ is the separation between the slits and m is an integer. The value of p is



3.7 Modern Physics

1. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

2. To determine the half-life of a radioactive element, a student plots a graph of $\ln\left|\dfrac{dN(t)}{dt}\right|$ versus t. Here $\dfrac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is



3. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in a free space. It is under continuous illumination of a 200-nm-wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^n$ (where $1 < A < 10$). The value of n is

4. The work function of silver and sodium are 4.6 eV and 2.3 eV respectively. The ratio of the slope of the stopping potential versus frequency plot for silver to that of sodium is

5. A proton is fired from very far away towards a nucleus with charge $Q = 120\,e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is

(proton-mass $= \dfrac{5}{3} \times 10^{-27}$ kg, $h/e = 4.2 \times 10^{-15}$ J s C^{-1})

6. Consider a hydrogen atom with its electron in the nth orbital. An electromagnetic radiation of wavelength 90 nm is used to ionise the atom. If the kinetic energy of the ejected electron is 10.4 eV then the value of n is ($hc = 1242$ eV nm)

7. An electron in an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is

8. A hydrogen atom in its ground state is irradiated by a light of wavelength 970 Å. Taking $hc = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as –13.6 eV, find the number of lines present in the emission spectrum.

9. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_{i} to another with quantum number n_{f}. V_{i} and V_{f} are respectively the initial and final potential energies of the electron. If $\dfrac{V_{\mathrm{i}}}{V_{\mathrm{f}}} = 6.25$ then the smallest possible n_{f} is

10. A freshly prepared sample of radioisotope of half-life 1386 s has activity 10^3 disintegration per second. Given that $\ln 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after the preparation of the sample is

11. A nuclear power plant supplying electrical power to a village uses a radioactive material of half-life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is met by 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years then the value of n is

12. For a radioactive material, its activity A and the rate of change of its activity R are defined as $A = -\dfrac{dN}{dt}$ and $R = -\dfrac{dA}{dt}$, where $N(t)$ is the number of active nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rates of change of activities at $t = 2\tau$ are R_{P} and R_{Q} respectively. If $R_{\mathrm{P}}/R_{\mathrm{Q}} = \dfrac{n}{e}$ then the value of n is

13. The isotope ${}^{12}_{5}B$ having a mass 12.014 u undergoes β-decay to ${}^{12}_{6}C$, ${}^{12}_{6}C$ has an excited state of the nucleus ${}^{12}_{6}C^*$ at 4.041 MeV above the ground state. If ${}^{12}_{5}B$ decays to ${}^{12}_{6}C^*$, find the maximum kinetic energy of the β-particle in units of MeV. (Take 1 u = 931.5 MeV c^{-2}, where c is the speed of light in vacuum.)

14. ${}^{131}I$ is an isotope of iodine that β decays to an isotope of xenon with a half-life of 8 days. A small amount of a serum labelled with ${}^{131}I$ is injected into the blood of a person. The activity of the amount of ${}^{131}I$ injected was 2.4×10^5 becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood serum in less than half an hour. After 11.5 hours, 2.5 mL of blood is drawn from the person's body and it gives an activity of 11.5 Bq. The total volume of blood in the person's body in litres, is approximately

15. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second and whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

Answers

3.1 General Physics

1. 4	**2.** 3	**3.** 4	**4.** 5	**5.** 5	**6.** 2
7. 4	**8.** 5	**9.** 5	**10.** 5	**11.** 5	**12.** 9
13. 4	**14.** 6	**15.** 8	**16.** 3	**17.** 2	**18.** 4
19. 7	**20.** 6	**21.** 2	**22.** 2	**23.** 7	**24.** 4
25. 3	**26.** 3	**27.** 10	**28.** 10	**29.** 8	**30.** 6
31. 4	**32.** 6	**33.** 6			

3.2 Heat and Thermodynamics

1. 8	**2.** 0	**3.** 9	**4.** 4	**5.** 2	**6.** 9
7. 2	**8.** 9				

3.3 Sound Waves

1. 5	**2.** 3	**3.** 7	**4.** 5	**5.** 6

3.4 Electrostatics

1. 6	**2.** 6	**3.** 3	**4.** 2

3.5 Current Electricity and Magnetism

1. 4	**2.** 5	**3.** 5	**4.** 1	**5.** 2	**6.** 5
7. 3	**8.** 6	**9.** 7	**10.** 8	**11.** 4	**12.** 7

3.6 Ray Optics and Wave Optics

1. 3	**2.** 6	**3.** 6	**4.** 2	**5.** 7	**6.** 2
7. 8	**8.** 3				

3.7 Modern Physics

1. 3	**2.** 8	**3.** 7	**4.** 1	**5.** 7	**6.** 2
7. 2	**8.** 6	**9.** 5	**10.** 4	**11.** 3	**12.** 2
13. 9	**14.** 5	**15.** 1			

Hints and Solutions

3.1 General Physics

1. $Y = \frac{FL}{Al}.$

Here measurement is for l only,

$\therefore \frac{\Delta Y}{Y} = \frac{\Delta l}{l}.$

From observation, l_1 = MS + 20(LC), and $l_2 = MS$ + 45(LC).

Change in length = $l_2 - l_1$ = 25 × LC, and the maximum permissible error in measurement of elongation is one LC.

$$\therefore \frac{\Delta Y}{Y} \times 100\% = \frac{1\text{ LC}}{25\,(\text{LC})} \times 100\% = 4\%.$$

2. Distance $d \propto (\text{density})^a \left(\frac{\text{power}}{\text{area}}\right)^b (\text{frequency})^c$

$$\Rightarrow \quad M^0LT^0 = (ML^{-3})^a\,(MT^{-3})^b\,(\text{T}^{-1})^c$$
$$= M^{a+b}\,L^{-3a}\,T^{-3b-c}.$$

Thus, $a + b = 0, -3a = 1, -3b - c = 0$

or $\quad a = -\frac{1}{3}, b = \frac{1}{3}, c = -1.$

$\because \quad b = \frac{1}{n} = \frac{1}{3} \Rightarrow n = 3.$

3. $E(t) = A^2 e^{-\alpha t}.$

Given: $\alpha = 0.2\text{ s}^{-1}, \frac{dA}{A} = 1.25\%, \frac{dt}{t} = 1.50\%.$

$$\log E = 2\log A - \alpha t$$
$$\Rightarrow \quad \frac{dE}{E} = \pm 2\frac{dA}{A} \pm \alpha\frac{dt}{t}\cdot t$$
$$= \pm 2(1.25) \pm 0.2\,(1.5)(5)$$
$$= \pm 2.50 \pm 1.5 = \pm 4\%.$$

4. $0 = 10\frac{\sqrt{3}}{2}t - \frac{1}{2}(10)t^2. \quad \therefore t = \sqrt{3}\text{ s}.$

$\therefore 1.15 = 5\sqrt{3} - \frac{3}{2}a \Rightarrow a \approx 5\text{ m s}^{-2}.$

5. The relative velocity of B with respect to A is perpendicular to the line of motion of A as shown in Figure (ii). From this vector triangle,

$V_B \cos 30° = V_A = 100\sqrt{3}$ m s^{-1}.

$\Rightarrow \quad V_B = 200$ m s^{-1}.

$\therefore$ time $t_0 = \dfrac{\text{relative distance}}{\text{relative velocity}}$

$$= \frac{500}{V_B \sin 30°}$$

$$= \frac{500 \text{ m s}^{-1}}{200 \text{ m} \times \frac{1}{2}}$$

$= 5$ seconds.

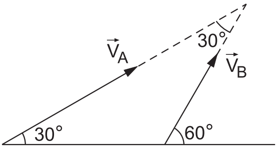


Fig. (i)

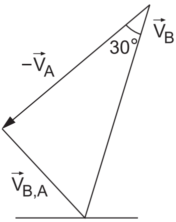


Fig. (ii)

6. Consider the motions of the balls A and B relative to the rocket. Maximum distance of the ball A from the left wall

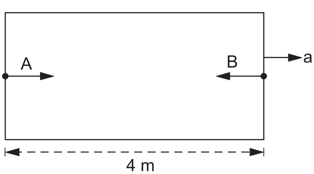
$$= \frac{u^2}{2a} = \frac{(0.3)^2}{2 \times 2} \approx 0.02 \text{ m}.$$

The ball B will collide with the ball A when it is very close to the left wall. Thus, for B, $S = ut + \frac{1}{2}at^2$

$\Rightarrow 4 = (0.2)t + \frac{1}{2} \cdot 2t^2$.

Solving, $t \approx 1.9$ s

or nearest integer = 2 s.



7. From the work–energy principle for the given system,

$$-\frac{1}{2}kx^2 - \mu mgx = 0 - \frac{1}{2}mv^2$$

$\Rightarrow \quad v = \dfrac{4}{10}$, so $N = 4$.

8. Power $= \dfrac{dW}{dt} = 0.5$ W

$\Rightarrow$ work done $= \displaystyle\int_0^{5\text{ s}} (0.5 \text{ W})\,dt = \frac{1}{2} \times 5 \text{ J} = 2.5 \text{ J}.$

From the work–energy theorem,

$W = \text{KE}_f - \text{KE}_i$

$\Rightarrow \dfrac{5}{2} = \dfrac{1}{2}\left(\dfrac{2}{10}\right)v^2$, hence $v = 5$ m s^{-1}.

9. From the work–energy principle,

$W_{\text{gravity}} + W_{\text{force}} = \Delta\text{KE} = (\text{KE})_f - (\text{KE})_i$

$\Rightarrow \quad -mgh + Fd = (\text{KE})_f$

$\Rightarrow \quad -1 \times 10 \times 4 + 18(5) = (\text{KE})_f = 50 = n \times 10.$

10. $mg(\sin\theta + \mu\cos\theta) = 3mg(\sin\theta - \mu\cos\theta)$.

$\therefore \mu = 0.5$, so $N = 10\mu = 5$.

11. The initial speed of the first bob (suspended by an ideal string of length l_1) is $\sqrt{5gl_1}$ and its speed at the highest point will be $\sqrt{gl_1}$.

When this bob collides elastically with another identical bob, their speeds get interchanged, so the speed acquired by the second bob (to complete a full vertical circle), is

$$\sqrt{5gl_2} = \sqrt{gl_1} \Rightarrow \frac{l_1}{l_2} = 5.$$

12. $I = 2\left[\frac{2}{5}mR^2\right] + 2\left[\frac{2}{5}mR^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right]$.

Substituting, $I = 9 \times 10^{-4}$ kg m^2 = $N \times 10^{-4}$ kg m^2; $\therefore N = 9$.

13. For translation, $N - f_s = ma$,

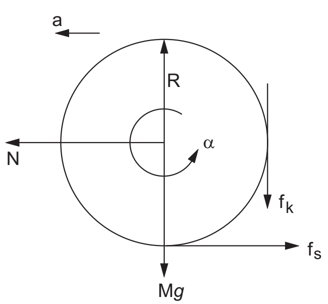
for rotation $(f_s - f_k)R = I\alpha$ and $a = R\alpha$.

Solving, $f_k = 0.8$ N; $f_s = 1.4$ N

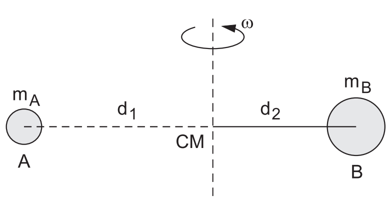
$$\Rightarrow \mu = \frac{P}{10} = \frac{f_k}{2N}$$

$$\Rightarrow \frac{P}{10} = \frac{0.8}{2}$$

$$\Rightarrow P = 4.$$



14. Distance of the centre of the mass of the system from A (mass $2.2M_s$) is $\frac{m_B d}{m_A + m_B} = \left(\frac{11}{2.2+11}\right)d = \frac{5d}{6}$ and that of B from the CM is $\left(d - \frac{5}{6}d\right) = \frac{d}{6}$. The system rotates about the axis through the CM of the system, so $I_A = m_A d_1^2$ and $I_B = m_B d_2^2$.



$\therefore$ angular momentum of A, $L_A = I_A\omega$ and that for B, $L_B = I_B\omega$

$$\Rightarrow \frac{\text{total angular momentum of the system}}{\text{angular momentum of star B}} = \frac{I_A\omega + I_B\omega}{I_B\omega} = \frac{I_A}{I_B} + 1$$

$$= \frac{(2.2M_s)\left(\frac{5}{6}d\right)^2}{(11M_s)\left(\frac{d}{6}\right)^2} + 1 = 6.$$

15. Mass of the disc = 50 kg = M.

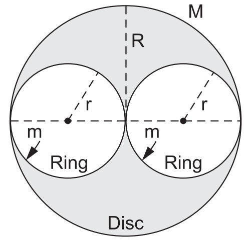
Mass of each ring = 6.25 kg = m.

$\therefore M = 8m$.

Radius of the disc = 0.4 m = R.

Radius of each ring = 0.2 m = r.

$\therefore R = 2r$.



Conserving angular momentum about the vertical axis of the disc,

$$\frac{1}{2}MR^2\cdot\omega = \left[\frac{1}{2}MR^2 + 2(2mr^2)\right]\omega'$$

$$\Rightarrow \quad \frac{1}{2}MR^2\cdot 10 \text{ rad s}^{-1} = \left[\frac{1}{2}MR^2 + 4\times\frac{M}{8}\times\frac{R^2}{4}\right]\omega'$$

$\therefore \quad \omega' = 8 \text{ rad s}^{-1}$.

16. If σ = mass per unit area then total mass $M = 4\sigma\pi R^2$ and the mass of cut-out disc $m = \sigma\pi R^2$.

$\therefore \quad M = 4m$.

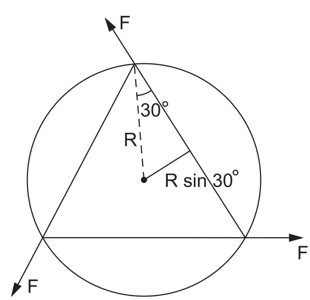
Now, $I_O = \frac{1}{2}M\cdot 4R^2 - \left(\frac{1}{2}mR^2 + mR^2\right) = \frac{13}{2}mR^2$, and

$$I_P = \frac{1}{2}M\cdot 4R^2 + M\cdot 4R^2 - \left(\frac{1}{2}mR^2 + m\cdot 5R^2\right) = \frac{37}{2}mR^2.$$

$$\Rightarrow \quad \frac{I_P}{I_O} = \frac{37}{2}\times\frac{2}{13} \approx 3.$$

17. Torque $\tau = 3F(R\sin 30°)$

$= \frac{3}{2}FR$.



Now, $\omega = \omega_0 + \alpha t$

$= 0 + \frac{\tau}{I}t$.

$$\omega = \frac{\frac{3}{2}FR}{M\frac{R^2}{2}}(1\text{ s})$$

$$= \frac{3\left(\frac{1}{2}\text{N}\right)(1\text{ s})}{(1.5\text{ kg})(0.5\text{ m})} = 2 \text{ rad s}^{-1}.$$

18. Since the net torque about the centre of rotation is zero, the angular momentum about the centre of the disc is conserved.

$\therefore I\omega + 2mv\left(\frac{r}{2}\right) = 0$.

In magnitude, $\frac{1}{2}Mr^2\cdot\omega = mvr$

$$\Rightarrow \frac{1}{2}(0.45)\left(\frac{1}{2}\right)^2 \omega = (0.05)(9)\left(\frac{1}{2}\right)$$

$$\Rightarrow \omega = 4 \text{ rad s}^{-1}.$$

19. The kinetic energy of a pure rolling disc moving with v_{CM} is

$$\frac{1}{2} m v_{CM}^2 \left(1 + \frac{K^2}{R^2}\right) = \frac{3}{4} m v_{CM}^2.$$

Conserving total mechanical energy for both:

$$\frac{3}{4} m(3)^2 + mg(30) = \frac{3}{4} m v_2^2 + mg27$$

$$\Rightarrow v_2 = 7 \text{ ms}^{-1}.$$

20. For the sphere A:

$$I_A = \int \frac{2}{3}\rho(r) 4\pi r^2 dr \cdot r^2$$

$$= \frac{2}{3}\frac{k}{R}\int_0^R r 4\pi r^4 dr = A\int r^5 dr = \frac{AR^6}{6}.$$

For the sphere B:

$$I_B = \int \frac{2}{3} k \frac{r^5}{R^5} 4\pi r^2 dr \cdot r^2 = \frac{A}{R^4}\int_0^R r^9 dr$$

$$= \frac{AR^{10}}{10R^4} = \frac{AR^6}{10}.$$

$$\therefore \frac{I_B}{I_A} = \frac{6}{10} = \frac{n}{10}, n = 6.$$

21. Escape velocity, $v = \sqrt{2gR}$.

But acceleration due to gravity, $g = \frac{GM}{R^2} = G \cdot \frac{4}{3}\pi R\rho$.

$$\therefore \quad R = \frac{3g}{4\pi G\rho}$$

$$\text{or} \quad v = \sqrt{\frac{2g \cdot 3g}{4\pi G\rho}} = K\sqrt{\frac{g^2}{\rho}} \Rightarrow v \propto \sqrt{\frac{g^2}{\rho}}.$$

$$\therefore \frac{v_{\text{planet}}}{v_{\text{earth}}} = \sqrt{\left(\frac{g_p}{g_e}\right)^2 \left(\frac{\rho_e}{\rho_p}\right)} = \sqrt{\left(\frac{\sqrt{6}}{11}\right)^2 \cdot \left(\frac{3}{2}\right)} = \frac{3}{11}.$$

$$\therefore v_{\text{planet}} = \frac{3}{11} \times 11 \text{ km s}^{-1} = 3 \text{ km s}^{-1}.$$

22. At the height h, $g' = \dfrac{g}{4}$, so $\dfrac{gR^2}{(R+h)^2} = \dfrac{g}{4} \Rightarrow R = h$.

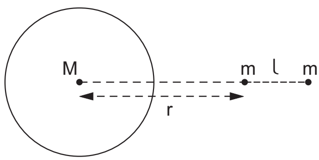
Conserving mechanical energy,

$$\frac{-GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{(R+h)} = -\frac{GMm}{2R}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2R} \quad \Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{2GM}{2R}}$$

$$\Rightarrow v = \frac{1}{\sqrt{2}}v_{\text{esc}} \text{ or } v_{\text{esc}} = v\sqrt{2} = v\sqrt{N}. \quad \therefore N = 2.$$

23. For m closer to M at $r = 3l$,



$$\frac{GMm}{9l^2} - \frac{Gm^2}{l^2} = ma. \qquad \text{...(i)}$$

For the other mass (m)

$$\frac{GMm}{16l^2} + \frac{Gm^2}{l^2} = ma. \qquad \text{...(ii)}$$

Equating (i) and (ii), $m = \dfrac{7M}{288} \Rightarrow K\left(\dfrac{M}{288}\right) = 7\left(\dfrac{M}{288}\right)$

$$\Rightarrow K = 7.$$

24. For the wire–block system in equilibrium,

Young modulus, $Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{mg/A}{\Delta l/L} = \dfrac{mg}{A}\dfrac{L}{\Delta l}$.

$\therefore$ weight $mg = \dfrac{YA}{L}\Delta l = k\Delta l$,

where $\dfrac{YA}{L} = k$ = force constant.

The force equation for the block is,

$$F = -kx$$

$\Rightarrow \quad a = -\dfrac{k}{m}x \Rightarrow$ angular frequency, $\omega = \sqrt{\dfrac{k}{m}}$

or $\quad \omega = \sqrt{\dfrac{YA}{Lm}} = \sqrt{\dfrac{(n)(10^9\,\text{N m}^{-2})(4.9\times 10^{-7}\,\text{m}^2)}{(1\,\text{m})(0.1\,\text{kg})}} = (70\,\text{rad s}^{-1})\sqrt{n}$.

But $\omega = 140\,\text{rad s}^{-1}$

$\Rightarrow \quad n = 4.$

25. Terminal velocity $v_{\text{T}} = \dfrac{2}{9}\dfrac{r^2}{\eta}(\rho - \sigma)g$,

where r = radius of the sphere,

ρ = density of the solid sphere,
σ = density of the viscous liquid.

$$\frac{v_P}{v_Q} = \frac{(8-0.8)\times\left(\frac{1}{2}\right)^2\times 2}{(8-1.6)\times\left(\frac{1}{4}\right)^2\times 3} = 3.$$

26. Increase in the length of the wire due to weight mg, $\Delta L = \frac{mgL}{YA}$.

Decrease in length due to cooling $= |\Delta L| = L\alpha\Delta T$.

$$\therefore \quad \frac{mgL}{YA} = L\alpha\Delta T$$

or $$m = \frac{YAL\alpha\Delta T}{gL} = \frac{(10^{11}\text{ N m}^{-2})(\pi 10^{-6}\text{ m}^2)(10^{-5}\text{ °C}^{-1})(10\text{ °C})}{(10\text{ m s}^{-2})}$$

$$= 3.1\text{ kg} \approx 3\text{ kg}.$$

27. The force exerted on the plate by the balls on the shaded area is

$$F = n(2mv)\left(\frac{ab}{2}\right) = mnv\cdot ab.$$

Torque of this force on the hinged end,

$$\tau_1 = F \times \text{distance of the CM of the shaded part from the left end}$$

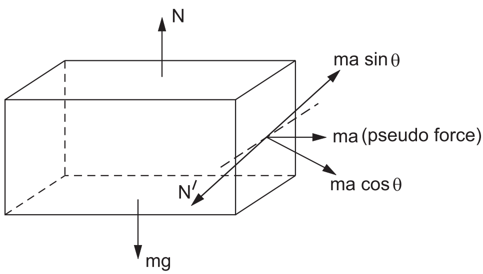
$$= mnvab \times \frac{3b}{4}.$$

This is balanced by the torque due to weight Mg

$$\Rightarrow \quad mnvab\cdot\frac{3}{4}b = Mg\cdot\frac{b}{2}.$$

$$\therefore \quad v = \frac{4Mg}{6mnab} = 10\text{ m s}^{-1}.$$

28. Consider the forces shown in the free-body diagram.



$N = mg$, $N' = ma\sin\theta$ and $ma\cos\theta - \mu(N + N') = ma_{\text{block}}$, where a_{block} is the acceleration of the block relative to the disc.

$$\Rightarrow \; m \cdot a \cdot \frac{2}{5} - \frac{2}{5}\left(mg + ma \times \frac{3}{5}\right) = ma_{\text{block}}.$$

Simplify to get $a_{\text{block}} = 10 \text{ m s}^{-2}$.

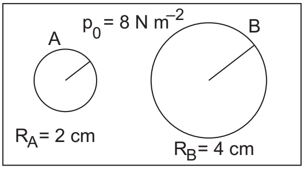
29. $2mg - T = 2ma$, $T - mg = ma$.

$\therefore$ acceleration of blocks, $a = \dfrac{g}{3}$ and tension in the string, $T = \dfrac{4mg}{3}$.

Work done by the string,

$$W = T \cdot S = T \cdot \frac{1}{2}at^2 = 8 \text{ J}.$$

30. The pressures inside soap bubbles A and B are



$$p_A = p_0 + \frac{4S}{R_A} = 16 \text{ N m}^{-2},$$

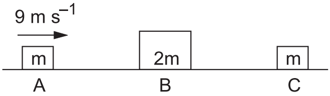
$$p_B = p_0 + \frac{4S}{R_B} = 12 \text{ N m}^{-2}.$$

$$p_A V_A = n_A RT.$$

$$p_B V_B = n_B RT.$$

$$\therefore \; \frac{n_A}{n_B} = \frac{p_A}{p_B} \times \left(\frac{R_A}{R_B}\right)^3 = \frac{1}{6} \Rightarrow \frac{n_B}{n_A} = 6.$$

31. After the first (elastic) collision between A and B,



$$m(9 \text{ m s}^{-1}) = mv'_A + 2mv'_B$$

$$\Rightarrow \quad v'_A + 2v'_B = 9 \text{ m s}^{-1}. \qquad \text{...(i)}$$

Coefficient of restitution, $e = 1$ (elastic collision).

Velocity of separation = e × velocity of approach.

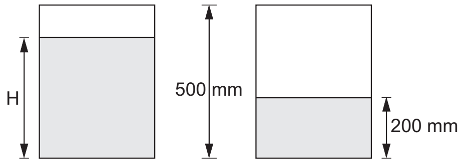
$$\therefore \quad v'_B - v'_A = 1 \times v_A. \qquad \text{...(ii)}$$

Solving (i) and (ii), $v'_B = 6 \text{ m s}^{-1}$.

After the second collision

$$2mv'_B = (2m + m)\, v_C \quad \Rightarrow \quad v_C = 4 \text{ m s}^{-1}.$$

32.



$p = p_0 - \rho g H = 98 \times 10^3 \text{ N m}^{-2}$.

$$p_0 V_0 = pV$$

$$\Rightarrow \quad 10^5\,[A(500 - H)] = 98 \times 10^3\,[A(500 - 200)]$$

$$\Rightarrow \quad H = 206 \text{ mm.}$$

Fall in water level = (206 – 200) mm = 6 mm.

33. Surface energy of the liquid drop, $U = 4\pi R^2 S$.

If r = radius of each droplet then

$$\frac{4}{3}\pi R^3 = K \cdot \frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{K^{\frac{1}{3}}}.$$

Increase in the surface energy

$$\Delta U = S(4\pi R^2)\left(K^{\frac{1}{3}} - 1\right)$$

$$\Rightarrow K^{\frac{1}{3}} = \frac{\Delta U}{4\pi R^2 \cdot S} + 1 = 101 \approx 10^2.$$

$\therefore K \approx 10^6 = 10^{\alpha}.$ $\quad \therefore \alpha = 6.$

3.2 Heat and Thermodynamics

1. Heat required to raise the temperature of the ice block,

$$Q_1 = mc\Delta\theta = m(2100 \text{ J kg}^{-1}\,°\text{C}^{-1})(5\,°\text{C})$$

Heat required to melt 1 g of ice $= mL = \left(\frac{1}{1000}\text{ kg}\right)(3.36 \times 10^5 \text{ J kg}^{-1})$.

$$\therefore \quad Q = Q_1 + Q,$$

or

$$420 \text{ J} = \left(\frac{m}{1000}\text{ kg}\right)(2100 \text{ J kg}^{-1}\,°\text{C}^{-1})(5\,°\text{C}) + 3.36 \times 100 \text{ J}$$

Simplifying, we get

$$m = \frac{(420 - 336)\,10}{105}\text{ g} = 8 \text{ g}.$$

2. The heat lost by the water during its condensation from steam (at 100 °C) to water (at 0 °C) is

$$\begin{aligned} Q_1 &= mL + mc_w\,\Delta\theta \\ &= (0.05 \text{ kg})(540 \text{ cal g}^{-1}) + (0.05 \text{ kg})(1 \text{ cal g}^{-1}\text{ K}^{-1})(100 \text{ K}) \\ &= 27 \text{ kcal} + 5 \text{ kcal} = 32 \text{ kcal}. \end{aligned}$$

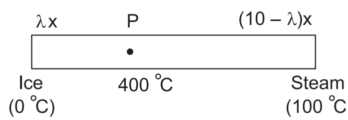
The heat required by the ice to reach 0 °C and then to melt (at 0 °C) is

$$\begin{aligned} Q_2 &= m's'\Delta\theta + m'L' \\ &= (0.45 \text{ kg})(0.5 \text{ cal g}^{-1}\text{ K}^{-1})(20 \text{ K}) + (0.45 \text{ kg})(80 \text{ cal g}^{-1}) \\ &= 4.5 \text{ kcal} + 36 \text{ kcal} = 40.5 \text{ kcal}. \end{aligned}$$

Since $Q_1 < Q_2$, the steam will not provide the required amount of heat to melt the entire ice. The temperature of the mixture remains at 0 °C.

3. Steady thermal current, $\dfrac{\Delta Q}{\Delta t} = kA\left(\dfrac{\Delta T}{\Delta x}\right)$

$$\Rightarrow \quad \frac{kA\,(400-0)\ ^\circ\mathrm{C}}{\lambda x \cdot L_{\text{ice}}} = \frac{kA\,(400-100)\ ^\circ\mathrm{C}}{(10-\lambda)\,x \cdot L_{\text{water}}}$$



Substituting the values, $\lambda = 9$.

4. For a diatomic gas undergoing adiabatic process,

$$TV^{\gamma-1} = \text{constant, where } \gamma = \frac{C_p}{C_V} = \frac{7}{5}.$$

$$\therefore \quad T_i V^{2/5} = \alpha T_i \left(\frac{V}{32}\right)^{2/5}$$

$$\Rightarrow \quad \alpha = (32)^{2/5} = (2^5)^{2/5} = 2^2 = 4.$$

5. $U_b = 200$ J, $U_i = 200$ J

For the process iaf:

Process	W	ΔU	Q
ia		0	
af		200 J	
Net	300 J	200 J	500 J

$$\Rightarrow U_f = 400 \text{ J}.$$

For the process ibf:

Process	W	ΔU	Q
ib	100 J	50 J	150 J
bf	200 J	100 J	300 J
Net	300 J	150 J	450 J

$$\Rightarrow \frac{Q_{\text{bf}}}{Q_{\text{ib}}} = \frac{300\text{ J}}{150\text{ J}} = 2.$$

6. $\lambda_m T = $ constant.

Rate of radiated energy, $Q = (4\pi R^2)(\sigma T^4)$.

$$\therefore \quad \frac{Q_1}{Q_2} = \left(\frac{R_1}{R_2}\right)^2\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{R_1}{R_2}\right)^2\left(\frac{\lambda_2}{\lambda_1}\right)^4 = \left(\frac{6\text{ cm}}{18\text{ cm}}\right)^2\left(\frac{1500\text{ nm}}{500\text{ nm}}\right)^4$$

$$= \left(\frac{1}{3}\right)^2 (3)^4 = 9.$$

7. According to Wien's displacement law

$$\lambda_A T_A = \lambda_B T_B.$$

Ratio of power radiated

$$\frac{E_A}{E_B} = \frac{\sigma T_A^4 A_A}{\sigma T_B^4 A_B}$$

$$\Rightarrow 10^4 = \left(\frac{T_A}{T_B}\right)^4\left(\frac{r_A}{r_B}\right)^2 = \left(\frac{\lambda_B}{\lambda_A}\right)^4\left(\frac{r_A}{r_B}\right)^2 = \left(\frac{\lambda_B}{\lambda_A}\right)^4\left(\frac{400}{1}\right)^2$$

$$\Rightarrow \frac{\lambda_A}{\lambda_B} = 2.$$

8. Given: $\log_2\left(\frac{P_1}{P_0}\right) = 1 \qquad \Rightarrow P_0 = \frac{P_1}{2}.$

According to Stefan's law,

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4 = \frac{(2767+273)^4}{(487+273)^4} = 4^4.$$

$$\frac{P_2}{P_1} = \frac{P_2}{2P_0} = 4^4 \Rightarrow \frac{P_2}{P_0} = 2\times 4^4 = 4^{9/2}.$$

$$\therefore \log_2\left(\frac{P_2}{P_0}\right) = \log_2(4)^{9/2} = \log_2 2^9 = 9.$$

3.3 Sound Waves

1. Phase difference $= \frac{\pi}{2}\cdot$ Hence resultant amplitude

$$A = \sqrt{a_1^2+a_2^2} = \sqrt{4^2+3^2} = 5.$$

2. The first and the fourth waves have phase difference π, so they interfere destructively leading to zero intensity.

The second and the third waves differ in phase by $\left(\frac{2\pi}{3}-\frac{\pi}{3}\right) = \frac{\pi}{3}\cdot$ So the net intensity,

$$I_{net} = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos\frac{\pi}{3} = 3I_0.$$

3. Apparent frequency of the sound wave received by the car (observer) approaching the source is

$$f' = \frac{v+v_0}{v}f_0,$$

where v_0 = velocity of the observer,

v = velocity of sound wave, and

f_0 = true frequency.

Frequency of the reflected wave,

$$f'' = \left(\frac{v}{v - v_0}\right) f' = \left(\frac{v + v_0}{v - v_0}\right) f_0.$$

$\therefore$ difference of the frequencies reflected from the two cars,

$$\Delta f = (1.2\%) f_0 = \left[\frac{v + v_1}{v - v_1} - \frac{v + v_2}{v - v_2}\right] f_0$$

$$\Rightarrow \qquad \frac{1.2}{100} = \left(1 + \frac{v_1}{v}\right)\left(1 - \frac{v_1}{v}\right)^{-1} - \left(1 + \frac{v_2}{v}\right)\left(1 - \frac{v_2}{v}\right)^{-1}$$

$$\approx \frac{2}{v}(v_1 - v_2) \qquad \text{(from binomial theorem).}$$

$\therefore$ difference in speeds of cars,

$$\Delta v = v_1 - v_2 = \frac{12}{1000}\,\frac{330\ \text{m s}^{-1}}{2} = 1.98\ \text{m s}^{-1} \approx 2\ \text{m s}^{-1}$$

$$= 2 \times \frac{18}{5}\ \text{km h}^{-1} = 7.2\ \text{km h}^{-1} \approx 7\ \text{km h}^{-1}.$$

4. $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{(0.5\ \text{N})\,(20 \times 10^{-2}\ \text{m})}{(1 \times 10^{-3}\ \text{kg})}} = 10\ \text{m s}^{-1}.$

Wavelength, $\lambda = \dfrac{v}{f} = \dfrac{10\ \text{m s}^{-1}}{100\ \text{s}^{-1}} = 0.1\ \text{m} = 10\ \text{cm}.$

Separation between two successive nodes

$$= \frac{\lambda}{2} = 5\ \text{cm}.$$

5. Frequency received by the approaching car (as observer)

$$f_1 = \left(\frac{v + v_0}{v}\right) f.$$

Frequency of the reflected wave as received by the source

$$f_2 = \left(\frac{v}{v - v_0}\right) f_1 = \left(\frac{v + v_0}{v - v_0}\right) f.$$

$\therefore$ beat frequency $= f_2 - f = \left(\dfrac{v + v_0}{v - v_0} - 1\right) f = \left(\dfrac{332}{328} - 1\right) 492\ \text{s}^{-1} = 6\ \text{s}^{-1}.$

3.4 Electrostatics

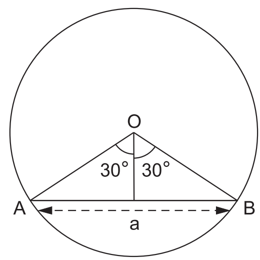
1. Due to cylinder: $E_1 = \dfrac{\rho R^2}{\varepsilon_0 \cdot 4R}$.

Due to sphere: $E_2 = \dfrac{1}{4\pi\varepsilon_0} \dfrac{\frac{4}{3}\pi \frac{R^3}{8}\rho}{(2R)^2}$.

Net field, $E_1 - E_2 = \dfrac{\rho R}{4\varepsilon_0} - \dfrac{\rho R}{4\varepsilon_0 \times 24} = \dfrac{23\,\rho R}{4 \times 24\varepsilon_0} = \dfrac{23\rho R}{16K\varepsilon_0}$

$\Rightarrow \quad K = 6.$

2. Flux from the total cylindrical surface of radius OA ($= a/2$) is equally distributed to the hexagonal shape formed by six given rectangular surface ABCD. Hence the required flux



$\phi = \dfrac{Q_{\text{inside}}}{6\varepsilon_0} = \dfrac{\lambda L}{6\varepsilon_0} = \dfrac{\lambda L}{n\varepsilon_0} \Rightarrow n = 6.$

3. $F_{\text{electrical}} \propto \dfrac{q^2}{a^2} \propto Sa.$

$\therefore\ a = K\left(\dfrac{q^2}{S}\right)^{1/3} \qquad \Rightarrow N = 3.$

4. Total charge enclosed, $Q = \displaystyle\int_0^R (4\pi r^2 dr)\, Kr^n = 4\pi K \frac{R^{n+3}}{n+3}$.

Charge contained from $r = 0$ to $\dfrac{R}{2}$ is

$Q' = \displaystyle\int_0^{R/2} 4\pi r^2 dr \cdot Kr^n = 4\pi K \frac{(R/2)^{n+3}}{n+3}$.

$E(R) = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q}{R^2}$ and $E\left(\dfrac{R}{2}\right) = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q'}{(R/2)^2}$.

Given $\dfrac{1}{8} E(R) = E\left(\dfrac{R}{2}\right)$

$\Rightarrow\ 2^{n+3} = 2^5$

$\Rightarrow\ n = 2.$

3.5 Current Electricity and Magnetism

1. Rate of heat production, $J = I^2R = \left(\frac{\mathcal{E}}{R_{\text{eq}}}\right)^2 R.$

When in series, $J_1 = \left(\frac{2\mathcal{E}}{R+2r}\right)^2 R = \frac{4\mathcal{E}^2}{(R+2)^2}R \qquad [\because r = 1\,\Omega].$

When in parallel, $J_2 = \left(\frac{2\mathcal{E}}{2R+r}\right)^2 R = \frac{4\mathcal{E}^2}{(2R+1)^2}R.$

Given $J_1 = 2.25J_2$, so $\frac{1}{(R+2)^2} = \frac{2.25}{(2R+1)^2}$

$\Rightarrow \frac{2R+1}{R+2} = 1.5 \qquad \Rightarrow R = 4\,\Omega.$

2. The given circuit is a closed loop, so the current through the loop,

$$I = \frac{(6-3)\,\text{V}}{(2+1)\,\Omega} = 1\text{ A}.$$

From the loop rule, $V_A - 6 + 1 - V_B = 0 \Rightarrow V_A - V_B = 5$ V.

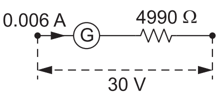
<u>Alternative method:</u> Equivalent emf of the given combination is

$$\mathcal{E} = \frac{(e_1r_2 + e_2r_1)}{(r_1+r_2)} = \frac{(6\times 2 + 3\times 1)}{(2+1)} = \frac{15}{3} = 5\text{ V}.$$

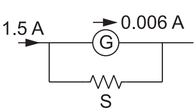
3. Let G be the resistance of the galvanometer coil.

When used as a voltmeter:

$$0.006 = \frac{30}{4990+G} \quad \Rightarrow G = 10\,\Omega.$$



When used as an ammeter:

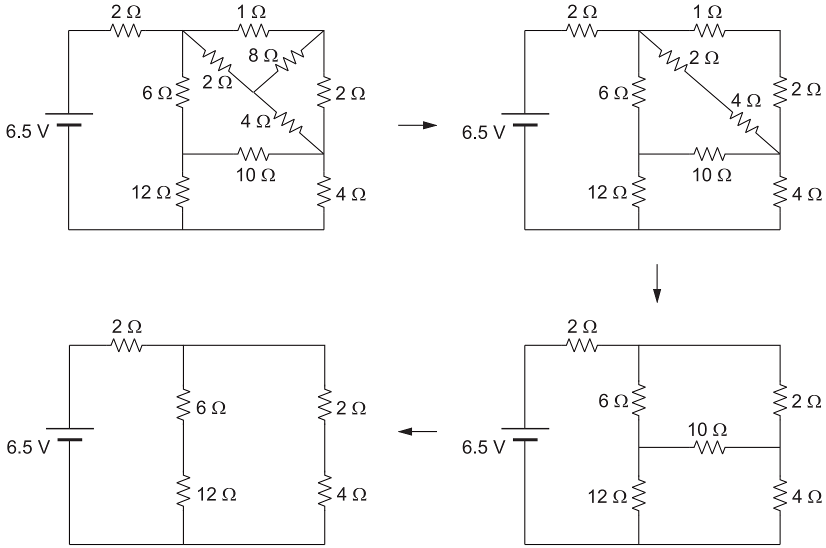


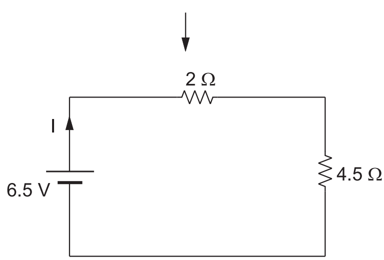
$$I_g = \frac{S}{G+S}I$$

$$\Rightarrow \frac{6}{1000} = \frac{S}{10+S}\times\frac{3}{2}.$$

$$\therefore \quad S = \frac{10}{249}\,\Omega = \frac{2n}{249}\,\Omega \quad \Rightarrow n = 5.$$

4. The equivalent circuit for the given network can be redrawn in steps as given below:





$\therefore I = 1$ A.

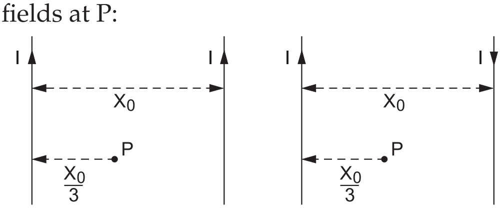
5. Time constant $\tau = RC = (10^6\ \Omega)\ (4\ \mu\text{F}) = 4$ s.

$4\text{ V} = (10\text{ V})\ [1 - e^{-t/\tau}] \Rightarrow e^{-t/4} = 0.6 = 3/5.$

$t/4 = \ln 5 - \ln 3 = 0.5 \Rightarrow t = 2$ s.

6. Net field at P $= \dfrac{\mu_0 Ja}{2} - \dfrac{\mu_0 Ja}{12} = \dfrac{5}{12}\mu_0 Ja = \dfrac{N}{12}\mu_0 Ja. \quad \therefore N = 5.$

7. Magnetic fields at P:



Case I Case II

In case I. $$B_1 = \frac{\mu_0 I}{2\pi\left(\frac{X_0}{3}\right)} - \frac{\mu_0 I}{2\pi\left(\frac{2X_0}{3}\right)} = \frac{3\mu_0 I}{4\pi X_0}.$$

In case II. $$B_2 = \frac{\mu_0 I}{2\pi\left(\frac{X_0}{3}\right)} + \frac{\mu_0 I}{2\pi\left(\frac{2X_0}{3}\right)} = \frac{9\mu_0 I}{4\pi X_0}.$$

$$\therefore \quad \frac{B_2}{B_1} = 3.$$

If R_1 and R_2 be the corresponding radii,

$$\frac{R_1}{R_2} = \frac{mv/qB_1}{mv/qB_2} = \frac{B_2}{B_1} = 3.$$

8. Magnetic field in the cylindrical tube,

$$B = \mu_0 ni = \mu_0 \frac{I}{L}.$$

The emf induced in the wire loop,

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -\frac{d}{dt}\left(\mu_0 \frac{I}{L}\cdot \pi r^2\right) = -\frac{\mu_0 \pi r^2}{L}\frac{dI}{dt}.$$

Magnetic moment $= (I_{loop})(\pi r^2) = \frac{\mathcal{E}}{R}\pi r^2.$

$$\therefore \; -\frac{\mu_0 \pi r^2}{RL}\frac{d}{dt}(I_0 \cos 300t) \times \pi r^2 = N\mu_0 I_0 \sin 300t.$$

Solving, $N = 6$.

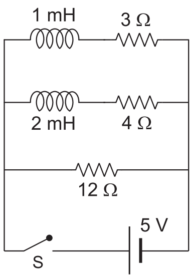
9. $$B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} = \frac{\mu_0 I R^2}{2(4R^2)^{3/2}} \qquad [\because x = \sqrt{3}R].$$

Magnetic flux, $\phi = NBA \cos 45° = \frac{\mu_0 I a^2}{8\sqrt{2}R}.$

Now, mutual inductance,

$$M = \frac{\phi}{I} = \frac{\mu_0 a^2}{8\sqrt{2}R} = \frac{\mu_0 a^2}{2^{7/2}R} = \frac{\mu_0 a^2}{2^{p/2}R} \Rightarrow p = 7.$$

10. At $t = 0$, an inductor offers large resistance and at $t = \infty$, inductors behave as conductors (zero resistance).



At $t = \infty$, $I_{max} = \frac{5\text{ V}}{R_{eq}} = \frac{5}{3/2}\text{ A} = \frac{10}{3}\text{ A}.$

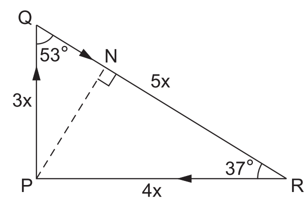
At $t = 0$, $I_{min} = \frac{5\text{ V}}{R_{eq}} = \frac{5\text{ V}}{12\,\Omega} = \frac{5}{12}\text{ A}.$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{10/3}{5/12} = 8.$$

11. $R\sqrt{1.25} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

$\Rightarrow$ time constant of the RC circuit = 4 ms.

12. $PN = 4x \cdot \sin 37° = \frac{12x}{5}.$



$\therefore\ B = \frac{\mu_0}{4\pi} I\left[\frac{\cos 53° + \cos 37°}{(12x/5)}\right]$

$= 7\left(\frac{\mu_0 I}{48\pi x}\right)$

$= K\left(\frac{\mu_0 I}{48\pi x}\right). \Rightarrow K = 7.$

3.6 Ray Optics and Wave Optics

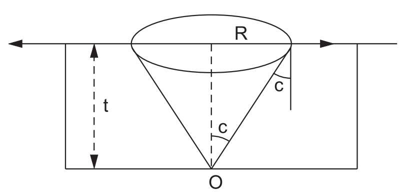
1. $f = 10$ m; for $v = \frac{50}{7}$ m, $u = -25$ m

for $v = \frac{25}{3}$ m, $u = -50$ m, $t = 30$ s

speed $= \left(\frac{25\ \text{m}}{30\ \text{s}}\right) = 3\ \text{km h}^{-1}$

2. $\sin C = \frac{1}{n} = \frac{3}{5} = \frac{R}{(R^2 + t^2)^{\frac{1}{2}}}$

$t = 8$ cm, $R = 6$ cm.



3. From lens formula for real image, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, so magnification $m = \frac{f}{u - f}$

$\Rightarrow \frac{m_{25}}{m_{50}} = \frac{50 - 20}{25 - 20} = 6 \qquad [\because f = 20\ \text{cm}].$

4. For refraction through the spherical surface

$$\frac{7}{4v_1} - \frac{1}{-24} = \frac{\frac{7}{4} - 1}{6}, \text{ so } v_1 = 21 \text{ cm.}$$

Finally, $\frac{\frac{4}{3}}{v_2} - \frac{\frac{7}{4}}{21} = 0$, so $v_2 = 16$ cm.

$\therefore\ x = 18 - 16 = 2$ cm.

5. From the mirror formula, $\dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u}$, $u = 15$ cm, $f = 10$ cm.

So, $v = 30$ cm (right). Now u for the lens = 20 cm = $2f_{\text{lens}}$. From the lens formula, $v_{\text{lens}} = 20$ cm (right)

$\Rightarrow$ magnification by the lens $|M_L| = 1$ (in air). When kept in liquid $\left(\mu' = \dfrac{7}{6}\right)$, the focal length of the lens is changed from $f = 10$ cm to f'.

$$\frac{\frac{1}{f}}{\frac{1}{f'}} = \frac{(\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{\left(\frac{\mu_g}{\mu'} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = \frac{\left(\frac{3}{2} - 1\right)}{\left(\frac{3}{2}\cdot\frac{6}{7} - 1\right)} \Rightarrow f' = \frac{35}{2} \text{ cm.}$$

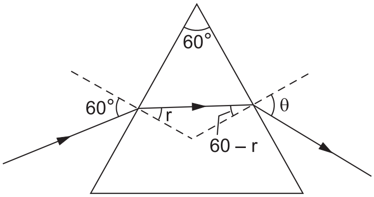
Change of medium does not affect f for mirror and hence magnification as well.

So, for lens, $u = 20$ cm and $\dfrac{1}{v'} - \dfrac{1}{-30} = \dfrac{2}{35} \Rightarrow v' = 140$ cm.

$\therefore M_L' = \dfrac{v'}{u'} = \dfrac{140}{20} = 7$. Now, for the system, $\dfrac{M'}{M} = \dfrac{M_L'}{M_L} = 7$.

6. $\sin 60° = n \sin r$. ...(i)

$\sin\theta = n\sin(60° - r)$. ...(ii)



Differentiating (ii) w.r.t. n,

$$\cos\theta\frac{d\theta}{dn} = -n\cos(60° - r)\frac{dr}{dn} + \sin(60° - r). \quad \text{...(iii)}$$

Differentiating (i)

$$n\cos r\cdot\frac{dr}{dn} + \sin r = 0.$$

From (iii), $\cos\theta\dfrac{d\theta}{dn} = -n\cos(60° - r)\left(\dfrac{-\tan r}{n}\right) + \sin(60° - r)$

or $\dfrac{d\theta}{dn} = \dfrac{1}{\cos\theta}[\cos(60° - r)\tan r + \sin(60° - r)]$

$\Rightarrow$ $\dfrac{d\theta}{dn} = \dfrac{1}{\cos 60°}[\cos 30° \tan 30° + \sin 30°]$

$= 2\left[\dfrac{1}{2} + \dfrac{1}{2}\right] = 2$, so $m = 2$.

7. For the internal reflection,

$$n \sin\theta = (n - m\Delta n) \sin 90°$$

$$\Rightarrow 1.6 \sin 30° = (1.6 - m \times 0.1)$$

$$\Rightarrow \quad 0.8 = 1.6 - \frac{m}{10} \quad \Rightarrow m = 8.$$

8. For maxima at p,

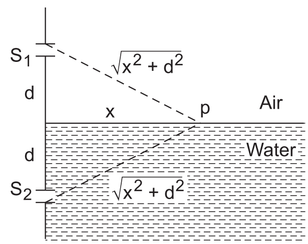
$$\Delta x = m\lambda, \text{ where } m = \text{integer}.$$

$$\mu\sqrt{x^2+d^2} - \sqrt{x^2+d^2} = m\lambda$$

$$\Rightarrow \quad \left(\frac{4}{3}-1\right)\sqrt{d^2+x^2} = m\lambda$$

$$\Rightarrow \quad \frac{d^2+x^2}{9} = m^2\lambda^2.$$

$\therefore \ x^2 = 9m^2\lambda^2 - d^2.$



Comparing with the given expression,

$$x^2 = p^2m^2\lambda^2 - d^2, \text{ we get } p = 3.$$

3.7 Modern Physics

1. $\text{KE} = \dfrac{p^2}{2m} = qV$, or momentum $p = \sqrt{2mqV}$.

de Broglie wavelength $\lambda = \dfrac{h}{p}$.

$$\therefore \ \frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{2m_\alpha q_\alpha V}}{\sqrt{2m_p q_p V}} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{4\times 2} = \sqrt{8} \approx 3.$$

2. Slope of line $= -\dfrac{1}{2}$. $\quad \therefore \lambda = \dfrac{1}{2}\text{yr}^{-1}, \ t_{1/2} = \dfrac{0.693}{\lambda} = 1.386$ yr.

4.16 yr $= 3t_{1/2}$. $\qquad \therefore p = 2^3 = 8.$

3. If V_s = stopping potential then

$$eV_s = \frac{hc}{\lambda} - \phi_0 = \frac{1240 \text{ eVnm}}{200 \text{ nm}} - 4.7 \text{ eV} = 1.5 \text{ eV}.$$

$$\therefore \ V_s = 1.5 \text{ V} = K\frac{Q}{r} = K\frac{ne}{r}.$$

$$\therefore \quad n = \frac{(1.5)(1\times 10^{-2})}{(9\times 10^9)(1.6\times 10^{-19})} = 1.05 \times 10^7.$$

4. From Einstein's photoelectric equation,

$$h\nu = \phi_0 + eV_s \quad \text{or} \quad V_s = \frac{h}{e}\nu - \frac{\phi_0}{e}.$$

The plot of V_s vs ν is a straight line with a constant slope $\frac{h}{e}$ for all samples.

Thus, the ratio of slopes = 1.

5. $$\frac{p^2}{2m} = \frac{KZe^2}{r} \Rightarrow p = \sqrt{\frac{KZe^2 \cdot 2m}{r}}$$

$$\Rightarrow \lambda = \frac{h}{p} = \frac{h/e}{\sqrt{2K\frac{Zm}{r}}} = \frac{4.2 \times 10^{-15}}{\sqrt{\frac{2 \times 9 \times 10^9 \times 120 \times \frac{5}{3} \times 10^{-27}}{10 \times 10^{-15}}}} = 7 \text{ fm.}$$

6. Energy of absorbed photon = ionisation energy + KE of the electron

$$\Rightarrow \frac{hc}{\lambda} = \frac{13.6 \text{ eV}}{n^2} + 10.4 \text{ eV}$$

$$\Rightarrow \frac{1242 \text{ eV nm}}{90 \text{ nm}} - 10.4 \text{ eV} = \frac{13.6}{n^2} \text{ eV}$$

$$\Rightarrow n^2 = 4 \quad \Rightarrow n = 2.$$

7. $$L = \frac{nh}{2\pi} = \frac{3h}{2\pi} \Rightarrow n = 3.$$

$$\lambda = \frac{h}{p} = \frac{hr}{mvr} = \frac{hr}{L} = \frac{hr}{3h/2\pi} = \frac{2\pi r}{3}.$$

But, $r = a_0 \frac{n^2}{Z} = \frac{a_0 9}{3} = 3a_0.$

$$\Rightarrow \quad \lambda = 2\pi a_0 = p\pi a_0.$$

$$\therefore \quad p = 2.$$

8. Energy of the incident photon,

$$E = \frac{hc}{\lambda} = \frac{1.237 \times 10^{-6} \text{ eV m}}{970 \times 10^{-10} \text{ m}} = 12.75 \text{ eV}.$$

Final energy,

$$E_n = (-13.6 + 12.75) \text{ eV} = -0.85 \text{ eV} = -\frac{13.6}{4^2} \text{eV}\cdot$$

This corresponds to the state $n = 4$ of H-atom.

The number of spectral lines present in the emission spectrum is ${}^4C_2 = 6$.

9. Potential energy $V \propto \frac{1}{n^2}\cdot$

$$\Rightarrow \frac{V_i}{V_f} = \frac{n_f^2}{n_i^2} \Rightarrow 6.25 = \left(\frac{n_f}{n_i}\right)^2 \Rightarrow \left(\frac{5}{2}\right)^2 = \left(\frac{n_f}{n_i}\right)^2 \cdot \; \therefore \frac{n_f}{n_i} = \frac{5}{2}.$$

$\therefore$ minimum value of n_f is 5.

10. The required fraction $f = \dfrac{N_0 - N}{N_0} = 1 - e^{-\lambda t}$

$$\approx 1 - (1 - \lambda t) = \lambda t = \frac{\ln 2}{T} \times t = 0.04.$$

$\therefore$ decay $\approx$ 4%.

11. If activity at $t = 0$ and at t be A_0 and A respectively then

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

$$\Rightarrow \frac{12.5}{100} = \left(\frac{1}{2}\right)^{\frac{t}{T}} \Rightarrow \left(\frac{1}{8}\right) = \left(\frac{1}{2}\right)^{\frac{t}{T}} \Rightarrow \left(\frac{1}{2}\right)^{3} = \left(\frac{1}{2}\right)^{\frac{t}{T}}.$$

$\therefore \quad t = 3T = nT \quad \Rightarrow n = 3.$

12. For P, $\lambda_P = \dfrac{1}{\tau}$; for Q, $\lambda_Q = \dfrac{1}{2\tau}$

$$\Rightarrow \quad A_P = A_0 e^{-t/\tau};\ A_Q = A_0\, e^{-t/2\tau}$$

$$\Rightarrow \quad R_P = -\frac{dA_P}{dt} = \frac{A_0}{\tau}\, e^{-t/\tau};$$

$$R_Q = -\frac{dA_Q}{dt} = \frac{A_0}{2\tau} e^{-t/2\tau}.$$

$$\therefore \quad \frac{R_P}{R_Q} = 2\,e^{-t/2\tau} = 2e^{-1} \quad (\because t = 2\tau)$$

$$= \frac{2}{e} = \frac{n}{e} \quad \Rightarrow n = 2.$$

13. The nuclear process can be represented as

${}^{12}_{5}\text{B} \longrightarrow {}^{12}_{6}\text{C} + \beta^- + \gamma$, and the corresponding value of

$$Q = \left[m({}^{12}_{5}\text{B}) - m({}^{12}_{6}\text{C}^*)\right]c^2$$

$$= \left[m({}^{12}_{5}\text{B}) - \left(m{}^{12}_{6}\text{C} + \Delta m\right)\right]c^2$$

$$= \left[m({}^{12}_{5}\text{B}) - m({}^{12}_{6}\text{C})\right]c^2 - \Delta m \cdot c^2$$

$$= [12.014\text{ u} - 12u]\,c^2 - 4.041\text{ MeV}$$

$$= (0.014)\ 931.5\,\frac{\text{MeV}}{c^2}\cdot c^2 - 4.041\text{ MeV}$$

$$= 13.041\text{ MeV} - 4.04\text{ MeV} = 9\text{ MeV}.$$

Hence β^- particle has KE of 9 MeV.

14. Final activity, $A_\mathrm{f} = \dfrac{V_\mathrm{body}}{V} \times A_0 \times e^{-\lambda t}$

$$\Rightarrow \; V_\mathrm{body} = \frac{V}{A_\mathrm{f}} \times A_0 \times e^{-\frac{\ln 2 \times t}{8\text{ days}}}$$

$$= V \times \frac{A_0}{A_\mathrm{f}} \times e^{-\frac{\ln 2 \times t}{192}} \simeq 4.998 \text{ litres} = 5 \text{ litres.}$$

15. Activity $\left|\dfrac{dN}{dt}\right| = \lambda N.$

$$\therefore \; N = \frac{\left(\dfrac{dN}{dt}\right)}{\lambda} = 10^{10} \times 10^{9} = 10^{19} \text{ atoms.}$$

Mass of the sample = $(10^{-25}\text{ kg})(10^{19}) = 10^{-6}\text{ kg} = 1\text{ mg}$.

❑