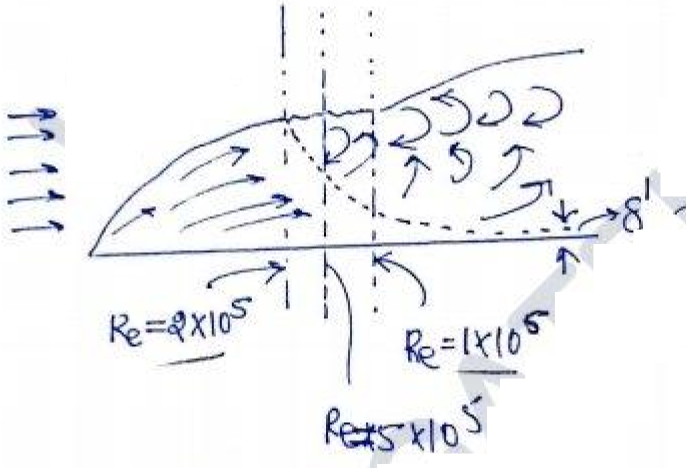
Viscous Pipe flow / laminar Pipe flow.

Features of laminar flow:

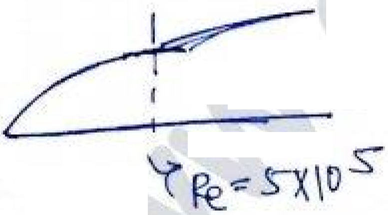
① Follow newtons low of viscosity.
② Negligible mixing b/w layer.
③ Head loss due to friction is proportional to first order of velocity
④ No-slip at the surface i.e. velocity profile approaches at the surface

Critical Reynolds Number:- The reynolds number below which flow is fully developed laminar and after which fully developed turbulant is known as critical reynolds number.



laminar sublayer

$$\boxed{\delta' = \frac{11.6\,\nu}{\sqrt{\frac{\tau_w}{\rho}}}}$$



Pipe flow

$Re \le 2000$ laminar
↳ (lower critical reynold number)

$Re \ge 4000$ turbulant

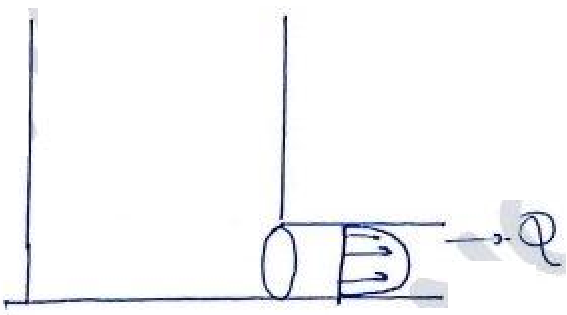
$Re = 2300$ Velocity corresponding to critical reynold no. is Critical Velocity
↳ Critical Reynold Number

Note:

Navier-stoke's eqn is fundamental eqn for laminar flow.

Applications of Navier stoke's equations:-

1. Viscous flow through circular pipe (Hagen-Poiseuille's flow)
2. Viscous flow b/w parallel plats (stationary) (Plane-Poiseuille's flow)
3. Viscous flow b/w parallel plates (Relative motion) (couette flow)



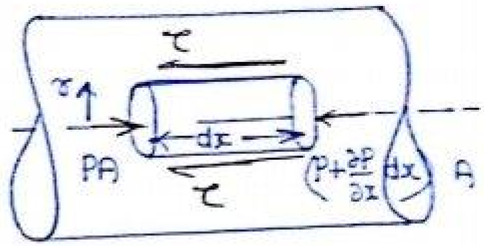
$\tau \rightarrow u \rightarrow u_m \rightarrow h_f$?

$u \rightarrow Q = ?$

Case-1 Hagen - Poiseuille's Flow

Shear stress distribution in circular pipe:-

Assumptions ① steady flow.
② uniform flow
③ laminar or turbulant flow



Navier-Stoke's eqn

$$F_p + F_v + \cancel{F_g} = m\left(\cancel{V\frac{dv}{ds}} + \cancel{\frac{dv}{dt}}\right)$$

0   0   0

uniform   steady

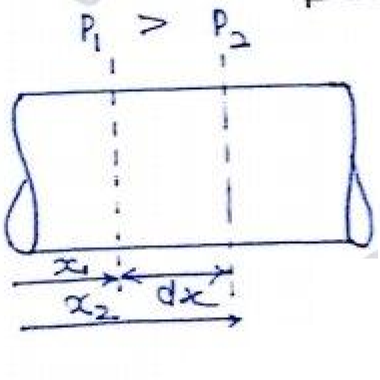
$$PA - \left(P + \frac{\partial P}{\partial x}dx\right)A - \tau(2\pi r\, dx) = 0$$

$$-\frac{\partial P}{\partial x}dx\,(\pi r^2) = \tau(2\pi r\, dx)$$

$$\star\star \quad \boxed{\tau = \left(-\frac{\partial P}{\partial x}\right)\frac{r}{2}}$$

laminar and turb

*



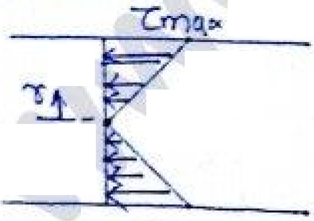
$P_1 > P_2$ $x_2 > x_1$

$dP = P_2 - P_1$ $dx = x_2 - x_1$

$dP < 0$ $dx > 0$

$(-dP) > 0$ $\left(\frac{-dP}{dx}\right) > 0$

* shear stress variation



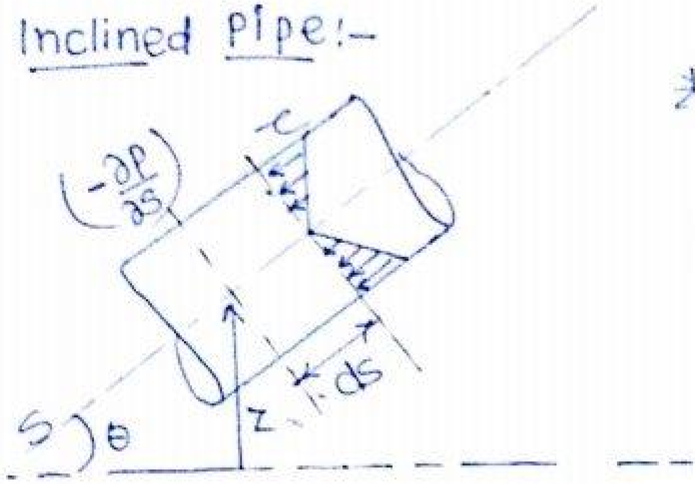
$\left(\frac{-\partial P}{\partial x}\right) = \text{Const}$

$\tau \propto r$

$\tau_{max} = \tau_w$

$$\tau_{max} = \left(\frac{-\partial P}{\partial x}\right)\frac{R}{2}$$

* Inclined pipe :-

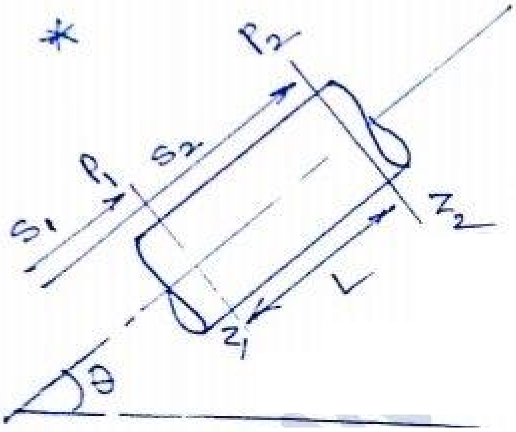


*

$$\boxed{\tau = -\frac{\partial(p+\rho g z)}{\partial s}\frac{r}{2}}$$

in inclined replace
$\begin{cases} p \to p+\rho g z \\ x \to s \end{cases}$

*



$$\tau = -\left[\frac{(P_2+\rho g z_2)-(P_1+\rho g z_1)}{S_2-S_1}\right]\frac{r}{2}$$

$$\tau = \left[\frac{(P_1+\rho g z_1)-(P_2+\rho g z_2)}{S_2-S_1}\right]\frac{r}{2}$$

H.Z. pipe $z_1 = z_2$

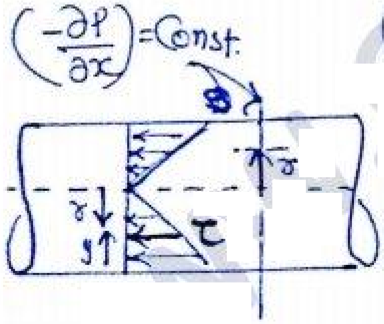
$S_2 - S_1 = x_2 - x_1$

Velocity distribution in circular pipe :→

Assumption :- ① Steady flow

② Uniform flow

③ laminar flow.



$$\tau = \left(\frac{-\partial p}{\partial x}\right)\frac{r}{2} \quad -(i)$$

$$\tau = \mu\left(\frac{du}{dy}\right) \quad -(ii)$$

} at $\underline{\underline{y}}$

(i) & (ii)

$$\left(\frac{-\partial p}{\partial x}\right)\frac{r}{2} = \mu\left(\frac{du}{dy}\right)$$

$$dU = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right) r\,dy$$

$r + y = R$

$dr + dy = 0$

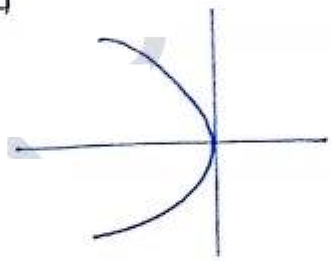
$dy = -dr$

$\left(-\frac{\partial P}{\partial x}\right)$ = Const for a section

$$\int dU = -\frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\int r\,dr$$

$$\boxed{U = -\frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) r^2 + C}$$

$$U \propto (-r^2)$$



at $r = R$, $U = 0$

$$C = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) R^2$$

$$\boxed{U = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)(R^2 - r^2)}$$ Actual velocity

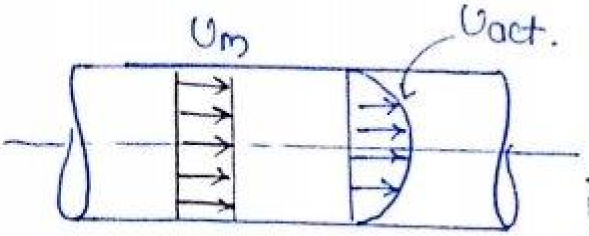
at $r = 0$, $U = U_{max}$.

$$U_{max} = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) R^2$$

$$U = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) R^2 \left[1 - \frac{r^2}{R^2}\right]$$

$$\boxed{U = U_{max}\left[1 - \frac{r^2}{R^2}\right]}$$ Actual Velocity

Mean Velocity / Avg. Velocity:-



$$Q_{avg} = Q_{act.}$$

$$* \boxed{U_m = \frac{2\int_0^R u r\,dr}{R^2}}$$

laminar

→ turb.

$$U_m = \frac{2\int_0^R \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)(R^2-r^2)\,r\,dr}{R^2}$$

$$U_m = \frac{\frac{2}{4\mu}\left(\frac{-\partial p}{\partial x}\right)}{R^2}\left(\frac{R^2 r^2}{2} - \frac{r^4}{4}\right)\Bigg|_0^R$$

$$U_m = \frac{\frac{2}{4\mu}\left(\frac{-\partial p}{\partial x}\right)}{R^2}\frac{R^4}{4}$$

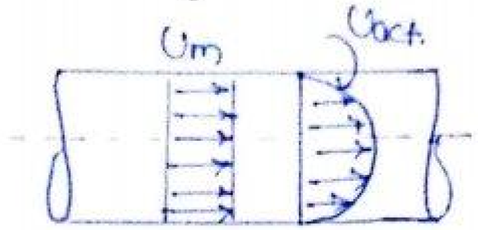
Average velocity

$$* \boxed{U_m = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2}$$

$$U_{max} = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)R^2$$

$$* \boxed{U_{mean} \text{ or } U_{avg.} = \frac{U_{max}}{2}}$$

Discharge equation for laminar flow:-



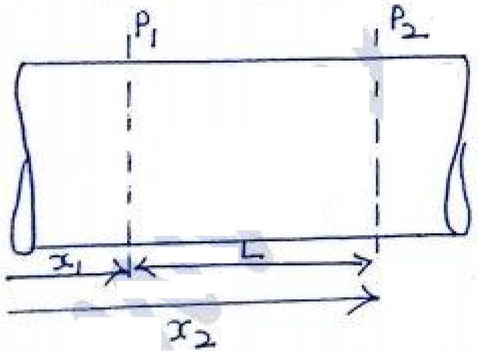
$$Q_{act} = U_m \pi R^2$$

$$Q_{act} = \left(\frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^2\right)\pi R^2$$

$R = D/2$

$$\boxed{Q_{act} = \frac{\pi}{128\mu}\left(\frac{-\partial p}{\partial x}\right)D^4}$$ Hagen-Posiulles' equation

Heat loss equation for laminar eq^n:-

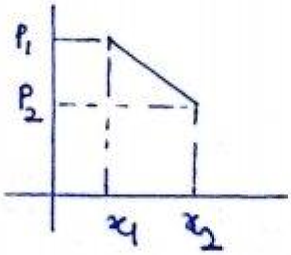


$$U_{mean} = V$$

$$V = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^2$$

$$\frac{8\mu V}{R^2} = -\left[\frac{P_2 - P_1}{x_2 - x_1}\right]$$

$$P_1 - P_2 = \frac{8\mu V L}{R^2}$$



$R = \frac{D}{2}$

$$\boxed{P_1 - P_2 = \frac{32\mu V L}{D^2}}$$ Horizontal $Z_1 = Z_2$

$$\frac{P_1}{\omega} - \frac{P_2}{\omega} = \frac{32 \mu V L}{\omega D^2}$$

$$\boxed{h_f = \frac{32 \mu V L}{\rho g D^2}}$$

inclined pipe

$$\boxed{\left(\frac{P_1}{\omega} + z_1\right) - \left(\frac{P_2}{\omega} + z_2\right) = \frac{32 \mu V L}{\rho g D^2}} \rightarrow \text{laminar}$$

$$\left(\frac{P_1}{\omega} + z_1\right) - \left(\frac{P_2}{\omega} + z_2\right) = h_f$$ ← head loss

piezometric

Darcy's eq

$$h_f \rightarrow h_f = \frac{4 f' L V^2}{2 g D_h}$$

$$h_f \rightarrow h_f = \frac{f L V^2}{2 g D_h}$$

laminar & turb.

$\boxed{D_h = D}$ pipe

f' = friction const.

f = friction factor.

for laminar flow

$$\frac{f L V^2}{2 g d} = \frac{32 \mu V L}{\rho g d^2}$$

$$\boxed{f = \frac{64}{\left(\frac{\rho V d}{\mu}\right)} = \frac{64}{Re}}$$

Friction factor $\boxed{f = \frac{64}{Re}}$ laminar flow

$$4f' = \frac{64}{Re} \qquad \boxed{f' = \frac{16}{Re}}$$

$$\boxed{h_f = \frac{32\mu VL}{\rho g d^2}} \qquad h_f \propto V$$

laminar flow

$$f' = \frac{16}{Re} \qquad \boxed{f' = \frac{\tau_w}{\frac{1}{2}\rho V^2}}$$

$$\frac{\tau_w}{\frac{1}{2}\rho V^2} = \frac{16}{\frac{\rho V D}{\mu}}$$

wall shear stress. $\boxed{\tau_w = \frac{8\mu V}{D}}$ ← mean velocity.

↳ Const. for fully developed laminar flow.

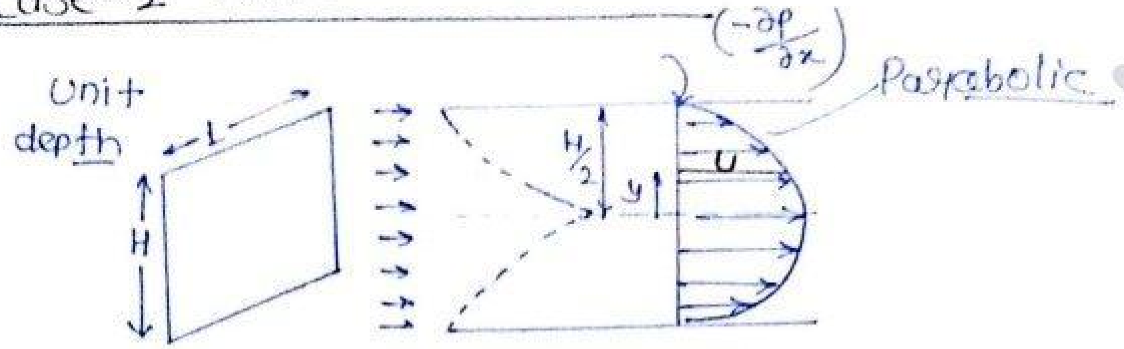
* for laminar flow

$$f_{min} = \frac{64}{Re_{max}}$$

$$f_{min} = \frac{64}{2000}$$

$$f_{min} = 0.032$$

Case - 2 Plane - Poiseuille's :-



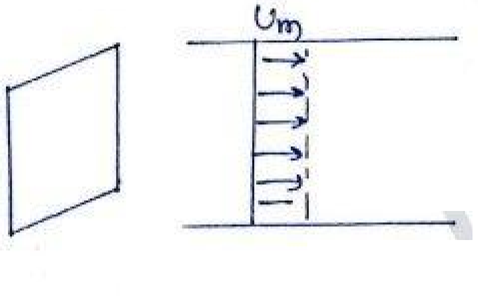
* $$\boxed{U = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left(\left(\frac{H}{2}\right)^2 - y^2\right)}$$

$y = 0, \quad U = U_{max}$

* $$\boxed{U_{max} = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left(\frac{H}{2}\right)^2}$$

Mean Velocity *
Avg. Velocity $\boxed{U_m = \frac{2}{3} U_{max}}$

* $$\boxed{U_{avg} = U_{mean} = \frac{1}{3\mu}\left(-\frac{\partial p}{\partial x}\right)\left(\frac{H}{2}\right)^2}$$



$Q = A \cdot U_{mean}$

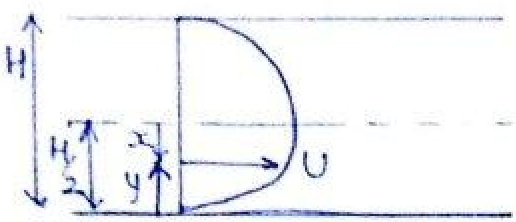
$$\boxed{Q = \frac{1}{3\mu}\left(\frac{-\partial p}{\partial x}\right)\left(\frac{H}{2}\right)^2 \cdot H}$$

Discharge per unit depth

* $$\boxed{h_f = \frac{12 \mu V L}{\rho g H^2}}$$ head loss

* $$\boxed{\tau_w = \frac{6 \mu V}{H}}$$ wall stress

*



$$U = \frac{1}{2\mu}\left(\frac{-\partial p}{\partial x}\right)\left(\left(\frac{H}{2}\right)^2 - x^2\right)$$

$$x = \left(\frac{H}{2} - y\right)$$

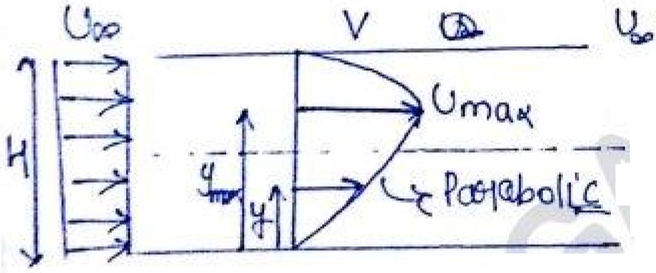
$$U = \frac{1}{2\mu}\left(\frac{-\partial p}{\partial x}\right)\left[\left(\frac{H}{2}\right)^2 - \left(\frac{H}{2}\right)^2 - y^2 + 2\times\frac{H}{2}\times y\right]$$

$$\boxed{U = \frac{1}{2\mu}\left(\frac{-\partial p}{\partial x}\right)\left(Hy - y^2\right)}$$ y taken from wall

$$\boxed{\tau = \mu \frac{dU}{dy}}\Bigg|_{y=0}$$ Gate 2016

Case-3

Coutte Flow :- one flat stationary another is moving.



$$\boxed{U = \frac{V}{H}y + \frac{1}{2\mu}\left(\frac{-\partial p}{\partial x}\right)\left(Hy - y^2\right)}$$

$$\frac{du}{dy} = 0 \text{ at } y = y_{max}$$

$$\boxed{\tau_w = \mu \frac{du}{dy}\Big|_{y=0}}$$

Q. 7, 5, 3,

Q.3 Pg 43 W.B.

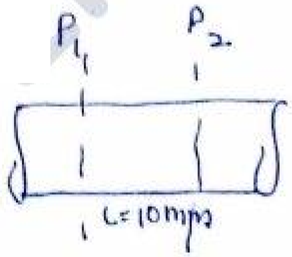
$$U = -\frac{h^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)\left(1-4\left(\frac{y}{h}\right)^2\right)$$

$$U = +\frac{1}{8\mu}\left(\frac{h}{2}\right)\left(-\frac{\partial p}{\partial x}\right)\left(1-\frac{y^2}{(h/2)^2}\right)$$

$$U = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)\left(\left(h/2\right)^2-y^2\right)$$

Q.9

$$\left(-\frac{\partial p}{\partial x}\right) = \frac{50\ KPa}{10}$$



$$P_1 - P_2 = 50\ KPa$$

$$\tau = -\left(\frac{\partial p}{\partial x}\right)\frac{r}{2} = \frac{+50\ KPa}{10} \times \frac{0.05}{2}$$

$$\tau = 0.125\ \underline{KPa}$$

Ques 15

$\rho = 800\ Kg/m^3$

$\mu = 0.8 Kg/m\cdot s$

$D = 0.1 m$

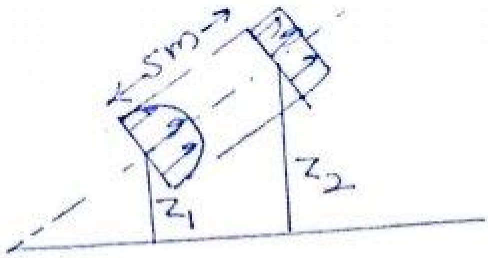
$$= (U_m) \cdot A$$

$$= -\frac{1}{8\mu}\left(-\frac{\partial(p+\rho g z)}{\partial x}\right) \cdot \pi R^2 \cdot R^2$$

$$= \frac{1}{8\mu}\left\{\frac{P_1+\rho g z_1 - P_2 - \rho g z_2}{x_2-x_1}\right\}\pi R^2 R^2$$

$$Q = \frac{1}{8\times 0.8}\left\{\frac{435\times10^3 + 800\times 9.81\times\frac{5}{\sqrt{2}} - 200\times10^3}{5}\right\}\times \pi\left(\frac{0.1}{2}\right)^2 (0.1)^2$$

$Q = 0.127\ m^3/s$



$z_2 - z_1 = 5\cos 45^\circ$

$P_1 = 435$ KPa

$P_2 = 200$ KPa

$$Q_{act} = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^4$$

$$Q_{act} = \frac{\pi}{8\mu}\left(\frac{(P_1 + \rho_o g z_1) - (P_2 + \rho_o g z_2)}{S_2 - S_1}\right) R^4$$

$$= \frac{\pi}{8\mu}\left(\frac{(P_1 - P_2) - \rho_o g (z_2 - z_1)}{S_2 - S_1}\right) R^4$$

$$= \frac{\pi}{8 \times 0.8}\left(\frac{(435-200)\times 10^3 - 800 \times 9.81 \times \left(\frac{5}{\sqrt{2}}\right)}{5}\right)\left(\frac{0.1}{2}\right)^4$$

$$Q_{act} = 0.127 \ m^3/s$$

or B.E. at 1 & 2

$$\frac{P_1}{\omega} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{P_2}{\omega} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f$$

$V_1 = V_2 = V_{mean}$

$$h_f = \frac{P_1 - P_2}{\omega_o} + (z_1 - z_2) = \frac{32\mu V L}{\omega_o D^2}$$

$V = ?$ $\quad Q = \frac{\pi}{4} d^2 \times V$

Que Find the distance from the centre in laminar pipe flow where the actual velocity is equal to the average velocity.

$$U_{mean} = \frac{U_{max}}{2}$$

$$\frac{1}{2} = 1 - \frac{r^2}{R^2}$$

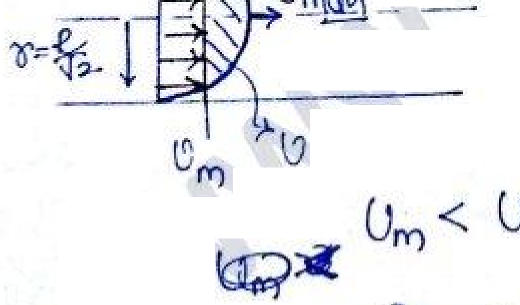
$$\frac{r^2}{R^2} = \frac{1}{2} \rightarrow r = \frac{R}{\sqrt{2}}$$

$$U = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)$$

$$U_{max} = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

$$U_{mean} = \frac{U_{max}}{2}$$

$$U = \frac{U_{max}}{2} \rightarrow \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)R^2\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{2}\frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)R^2$$



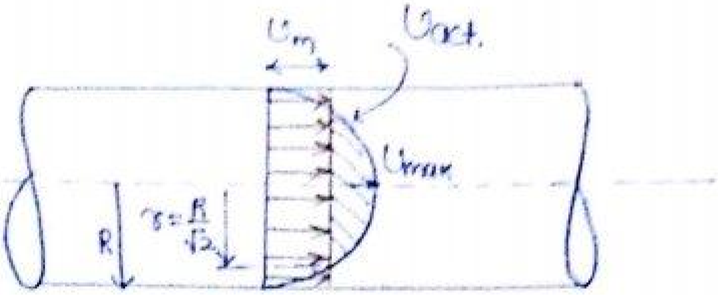
$$1 - \frac{r^2}{R^2} = \frac{1}{2}$$

$$\boxed{r = 0.707\,R}$$

$U_m < U_{act.}$

$Q_{avg} = Q_{act.}$

*



$U_m < U_{act.}$ $\qquad U_m = \frac{U_{max}}{2}$

$Q_{avg} = Q_{act.}$

Momentum $\dot{P}_{avg} = \dot{m}_{avg} V_{avg}.$ $\qquad \dot{P}_{act} = \dot{m}_{act} V_{act}.$

kinetic energy $\rightarrow \dot{KE} = \frac{1}{2} \dot{m}_{avg} V_{avg}^2$ $\qquad \dot{KE} = \frac{1}{2} \dot{m}_{act} V_{act}^2$

$\dot{P}_{avg} < \dot{P}_{act}$

$\Rightarrow \quad \beta \dot{P}_{avg} = \dot{P}_{act}$ $\qquad \beta$ - momentum Correction factor

$$\boxed{\beta = \frac{\dot{P}_{act}}{\dot{P}_{avg}}} > 1 \qquad \boxed{\alpha = \frac{\dot{KE}_{act.}}{\dot{KE}_{avg}}} > 1$$

$\dot{P} = \dot{m} V$

$= (\rho A V) V$

$\dot{P} = \rho A V^2$

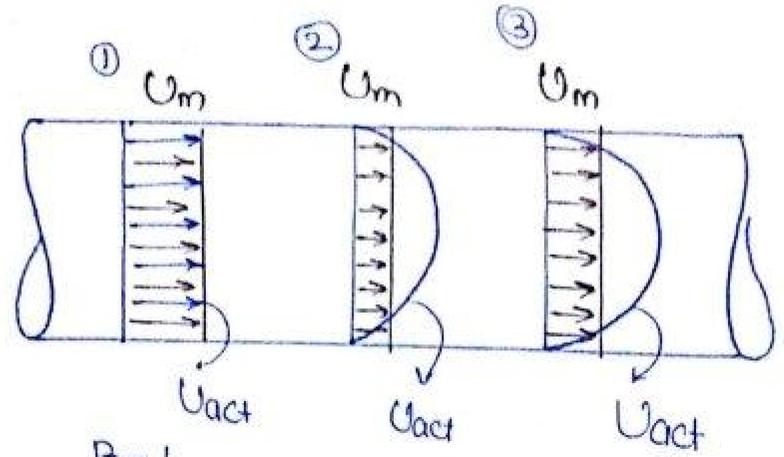
$KE = \frac{1}{2} \dot{m} V^2$

$KE = \frac{1}{2} \rho A V^3$

$$\beta = \frac{\int_A \rho \, dA \, U^2}{\rho A U_m^2} \qquad \alpha = \frac{\frac{1}{2}\int_A \rho \, dA \, U^3}{\frac{1}{2} \rho A U_m^3}$$

$dA = 2\pi r dr$ → elemental area

$$\boxed{\beta = \frac{\int_A U^2 dA}{A U_m^2}} \qquad \boxed{\alpha = \frac{\int_A U^3 dA}{A U_m^3}} \qquad \boxed{\alpha > \beta > 1}$$

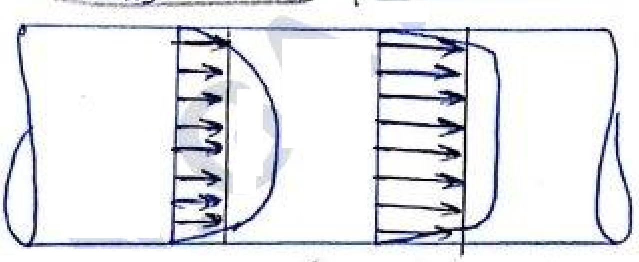


$\beta = 1$

$\beta_1 < \beta_2 < \beta_3$ ⟹ (more deviation more momentum correction factor)

laminar & turbulant

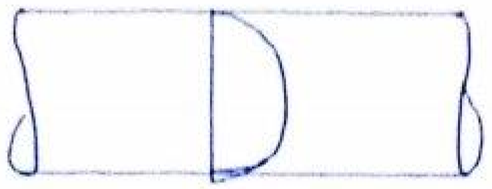
$\boxed{U_m = 0.5\, U_{max}}$ $\boxed{U_m = 0.82\, U_{max}}$ Imp



$\boxed{\beta_L > \beta_T}$

$$\frac{U}{U_{max}} = \left(1 - \frac{r^2}{R^2}\right) \qquad \frac{U}{U_{max}} = \left(1 - \frac{r}{R}\right)^{1/7}$$

* In fluid flow the actual velocities are more then avg. velocities & so the momentum transfer rate & K.E. transfer rate at any section in actual is more than calculated by average velocities so some correction factors are used for actual momentum transfer & K.E. transfer are β & α respectively.



laminar $\frac{U}{U_{max}} = \left(1 - \frac{r^2}{R^2}\right)$

$$\beta = \frac{\int_0^R U^2 (2\pi r \, dr)}{\pi R^2 U_m^2}$$

$$\beta = \frac{2\int_0^R \left(U_{max}\left(1 - \frac{r^2}{R^2}\right)\right)^2 \cdot r\,dr}{R^2 \cdot \left(\frac{U_{max}}{2}\right)^2}$$

$$\beta = \frac{8\int_0^R \left(1 - \frac{r^2}{R^2}\right)^2 \cdot r\,dr}{R^2}$$

$\beta = 1.33$

$$\beta = \frac{8}{R^2}\int_0^R \left(r + \frac{r^5}{R^4} - \frac{2r^3}{R^2}\right) dr$$

Imp

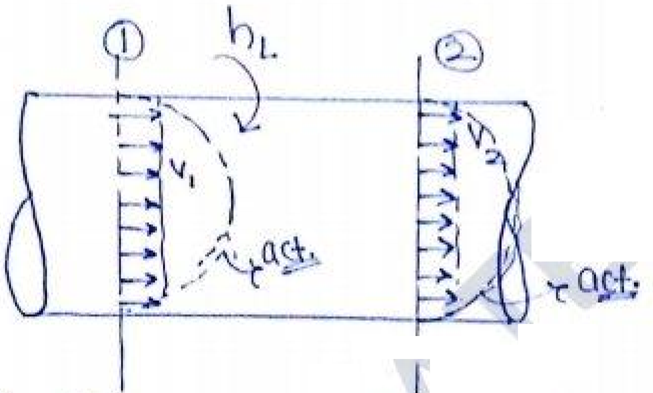
β — laminar $= 1.33$

β — turb. $= 1.03$

α — laminar $= 2$

α — turb $= 1.33$

Note



$$\alpha = \frac{(KE)_{act.}}{(KE)_{avg.}}$$

$$\alpha V_{avg} = V_{act.}$$

$$\frac{P_1}{\omega} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{P_2}{\omega} + \frac{\alpha_2 V_2^2}{2g} + z_1 + h_f$$

$$\boxed{\frac{P_1}{\omega} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{P_2}{\omega} + \frac{\alpha_2 V_2^2}{2g} + h_f}$$

Q. 21

1.g 44

WB