CBSE Test Paper 05

CH-04 Principle of Mathematical Induction

1. The smallest +ve integer n , for which $n! < \left(\frac{n+1}{2}\right)^n$ holds is
   a. 4
   b. 1
   c. 2
   d. 3
2. The product of three consecutive natural numbers is divisible by
   a. 3
   b. 11
   c. 6
   d. 8
3. For all $n \in N, 7^{2n} - 48n - 1$ is divisible by :
   a. 1234
   b. 2304
   c. 25
   d. 26
4. 3+13+29+51+79+...........to n terms is
   a. $2n^2 + 7n$
   b. $n^3 + 2n^2$
   c. none of these

d. $n^2 + 5n^3$

5. The sum of n terms of the series $1 + (1 + a) + \left(1 + a + a^2\right) + \left(1 + a + a^2 + a^3\right) + \ldots\ldots\ldots$, is

   a. none of these

   b. $-\frac{n}{1-a} + \frac{a(1-a^n)}{(1-a)^2}$

   c. $\frac{n}{1-a} + \frac{a(1+a^n)}{(1-a)^2}$

   d. $\frac{n}{1-a} - \frac{a(1-a^n)}{(1-a)^2}$

6. Fill in the blanks:

   The step 'if the statement is true for n = k, then it is also true for n = k + 1' is sometimes referred as the ________ step.

7. State true or false:

   If P(n) : $49^n + 16^n + k$ is divisible by 64 for n $\in$ N is true, then the least negative integral value of k is ________.

8. Using the principle of mathematical induction, prove that $(2^{3n} - 1)$ is divisible by 7 for all n $\in$ N.

9. Prove by using Mathematical Induction that $3\times6 + 6\times9 + 9\times12 + \ldots\ldots + (3n)\times(3n + 3) = 3n(n + 1)(n + 2)$, for all n$\in$N.

10. Show by the Principle of Mathematical Induction that the sum S_n of the n terms of the series $1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 \ldots$ is given by

$$S_n = \begin{cases} \frac{n(n+1)^2}{2}, & \text{if } n \text{ is even} \\ \frac{n^2(n+1)}{2}, & \text{if } n \text{ is odd} \end{cases}$$

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Solution

1. (c) 2

   Explanation: when n = 2 , L H S : n ! = 2 ! = 2x1 = 2 RHS:

   But if n = 1 , the inequation does not hold good.

2. (c) 6

   Explanation: By replacing n by 1 we get 6.
3. (b) 2304

   Explanation: When n = 1 the value is 0. When n = 2 the value is 2304..... Hence by the principle of mathematical induction the expression is divisible by 2304.
4. (b) $n^3 + 2n^2$

   Explanation: When n = 1 we have 3 when n = 2 we have LHS : 3 + 13 = 16 RHS : 8 + 8 = 16 .....By PMI this is the sum of the n terms of the series.

5. (d) $\frac{n}{1-a} - \frac{a(1-a^n)}{(1-a)^2}$

   Explanation: Replace n = 1 we get 1.replace n = 2 we get LHS ; a $+2$ RHS : a $+2$

6. inductive

7. -1

8. Let P(n) be the statement given by

   P(n) : 2^{3n} - 1 is divisible by 7

   P(1) : $2^{3\times 1}$ - 1 is divisible by 7.

   Clearly, $2^{3\times 1}$ -1 = 8 -1 = 7, which is divisible by 7.
   So, P(1) is true
   Let P(m) be true. Then,

   2^{3m} - 1 is divisible by 7.

   2^{3m} - 1 = 7λ, for some $\lambda \in$ N ...(i)

We shall now show that P(m + 1) is true. For this we have to show that $2^{3(m+1)}$ - 1 is divisible by 7.

Now,

$2^{3(m+1)} - 1 = 2^{3m} \times 2^3 - 1 = (7\lambda + 1)2^3 - 1 = 56\lambda + 8 - 1 = 7(8\lambda + 1)$, which is divisible by 7 [Using (i)]

$\therefore$ P(m + 1) is true.

Thus, P(m) is true $\Rightarrow$ P(m + 1) is true

Hence, by the principle of mathematical induction, P(n) is true for all n $\in$ N i.e. $2^{3n} - 1$ is divisible by 7 for all n $\in$ N.

9. **Step I** Let P(n) be the given statement, then

P(n) =

$3 \times 6 + 6 \times 9 + 9 \times 12 + \ldots\ldots\ldots + (3n) \times (3n + 3) = 3n(n + 1)(n + 2)$,for all n $\in$N.

Step II For n = 1, we have

LHS $= 3 \times 1 \times (3 \times 1 + 3) = 3(6)$

$= 18$

RHS $= 3.1(1 + 1)(1 + 2)$

$=3(2)(3)$

$= 18 = LHS$

Thus, P(1) is true.

Step III

For n = k, assume that P (k) is true,

i.e., P(k): $3 \times 6 + 6 \times 9 + 9 \times 12 + \ldots + (3k) \times (3k + 3) = 3k(k + 1)(k + 2)$..........(i)

Step IV For n = k + 1, we have to show that P (k + 1) is true, whenever P (k) is true, i.e.,

$P(k + 1) : 3 \times 6 + 6 \times 9 + 9 \times 12 + \ldots + 3k(3k + 3) + 3(k + 1)[3(k + 1) + 3]$

$= 3(k + 1)(k + 1 + 1)(k + 1 + 2)$

Now, consider LHS = $3 \times 6 + 6 \times 9 + 9 \times 12 + \ldots + 3k(3k + 3) + 3(k + 1)[3(k + 1) + 3]$

$= 3k(k + 1)(k + 2) + 3(k + 1)[3(k + 1) + 3]$ [from Eq. (i)]

$= 3(k + 1)[k(k + 2) + 3(k + 1) + 3]$

$= 3(k + 1)[k^2 + 2k + 3k + 3 + 3]$

$= 3(k + 1)[k^2 + 5k + 6]$

$= 3(k+1)(k+3)(k+2)$

$= 3(k+1)(k+1+1)(k+1+2) =$RHS

So, P (k + 1) is true, whenever P (k) is true.

Hence, by Principle of Mathematical Induction, P(n) is true for all n$\in$N.

10. **Step I** Let P(n) be given statement, then

P(n) : S_n = $\begin{cases} \frac{n(n+1)^2}{2}, \text{ if } n \text{ is even} \\ \frac{n^2(n+1)}{2}, \text{ if } n \text{ is odd} \end{cases}$

Also, note that any term T_n of the series is given by

T_n = $\begin{cases} n^2, \text{ if } n \text{ is odd} \\ 2n^2, \text{ if } n \text{ is even} \end{cases}$

Step II For $n = 1$, we have

$P(1) : S_1 = 1^2 = 1 = \frac{1 \cdot 2}{2} = \frac{1^2 \cdot (1+1)}{2}$ [$\because$ 1 is odd]

Thus, $P(1)$ is true.

Step III For n = k, assume that, P(k) is true for some natural number k.

If k is odd, then $P(k) : 1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + \ldots + k^2$

= $\frac{k^2(k+1)}{2}$ ...(i)

If k is even, then P(k) = $P(k) : 1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + \ldots + 2k^2$ =

$\frac{(k)(k+1)^2}{2}$ ....(ii)

Step IV For n = k + 1, we have to show that P (k + 1) is true whenever P(k) is true.

Now, if k is odd, then k + 1 is even and in that case we have to show

$P(k+1) : 1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + \ldots + k^2 + 2 \times (k+1)^2$

= $\frac{(k+1)(k+1+1)^2}{2}$ ...(iii)

If k is even, then (k + 1) odd and in that case we have to show

$P(k+1) : 1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + \ldots + 2k^2 + (k+1)^2$

= $\frac{(k+1)^2(k+1+1)}{2}$ ...(iv)

Case I When k is odd, then k + 1 is even and we have,

LHS $= 1^2 + 2 \times 2^2 + \ldots + k^2 + 2 \times (k+1)^2$

= $\frac{k^2(k+1)}{2} + 2 \times (k+1)^2$ [from Eq. (i)]

= $\frac{k+1}{2} [k^2 + 4(k+1)]$

= $\frac{k+1}{2} [k^2 + 4k + 4] = \frac{k+1}{2} (k+2)^2$

$= (k+1)\ \frac{[(k+1)+1]^2}{2}$ = RHS

Thus, $P(k+1)$ is true, whenever $P(k)$ is true for the case, when k is odd.

Case II When k is even, then k + 1 is odd and we have

LHS $= 1^2\ + 2\ \times\ 2^2\ + \ldots + 2.k^2\ + (k+1)^2$

= $\frac{k(k+1)^2}{2}\ + (k+1)^2$ [from Eq. (ii)]

= $\frac{(k+1)^2(k+2)}{2} = \frac{(k+1)^2[(k+1)+1]}{2}$

= RHS

Thus, P(k + 1) is true, whenever P(k) is true for the case when k is even. So, P(k + 1) is true, whenever P (k) is true for any natural number k.

Hence, by the Principle of Mathematical Induction, P(n) is true for all natural numbers.