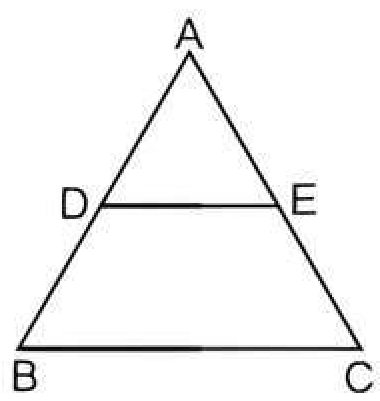
6 Triangles

Fastrack Revision

- **Similar Figures:** Two figures having the same shapes (and not necessarily the same size) are called similar figures.
- **Similar Triangles:** Two triangles are said to be similar if:
  1. their corresponding angles are equal.
  2. their corresponding sides are in the same ratio (or proportional).

  Note: Suppose ΔABC is similar to ΔPQR, we write as $\Delta ABC \sim \Delta PQR$. But we do not write as $\Delta ABC \sim \Delta QRP$ or $\Delta BAC \sim \Delta PQR$.
- **Equiangular Triangles:** If corresponding angles of two triangles are equal, then they are equiangular triangles. The ratio of any two corresponding sides of each pair in two equiangular triangles is always the same.
- **Basic Proportionality Theorem—BPT (Thales' Theorem):** In a triangle, a line drawn parallel to one side, to intersect the other two sides at distinct points, divides the two sides in the same ratio.

  In Δ ABC, DE || BC, then $\frac{AD}{DB} = \frac{AE}{EC}$,

  $\frac{AD}{AB} = \frac{AE}{AC}$ and $\frac{AB}{DB} = \frac{AC}{EC}$
- **Converse of Basic Proportionality Theorem:** If a line divides any two sides of a ΔABC in the same ratio, *i.e.*, $\frac{AD}{DB} = \frac{AE}{EC}$, then the line must be parallel to the third side, *i.e.*, DE || BC.
- **Criterion for Similarity of Triangles:** There are three criteria for similarity of triangles :
  1. **AAA Similarity:** In two triangles, if three angles of one triangle are respectively equal to the three angles of the other triangle, then the two triangles are similar.

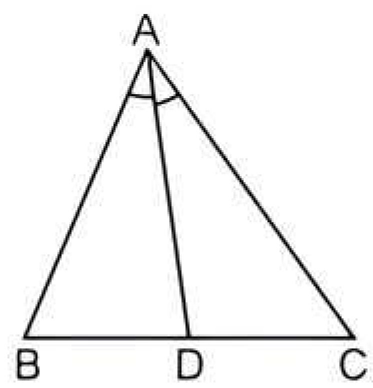
     If two of their angles are equal, then the third angle must also be equal, because sum of angles of a triangle always make 180°. So, AA could also be called similarity.
  2. **SSS Similarity:** In two triangles, if the corresponding sides are proportional, then they are similar.

     Or

     In two triangles, if sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.
  3. **SAS Similarity:** In two triangles, if one pair of corresponding sides are proportional and the included angles are also equal, then the two triangles are similar.

Knowledge BOOSTER

1. *All congruent triangles are similar but the similar triangles need not be congruent.*
2. ***Mid-point Theorem:*** *The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.*
3. ***Angle Bisector Theorem:*** *The internal bisector of an angle of a triangle divides the opposite side in two segments that are proportional to the other two sides of the triangle.*

   In ΔABC, $\frac{AB}{AC} = \frac{BD}{DC}$
4. *If two triangles are similar, then their corresponding sides, medians and altitudes are proportional.*

Practice Exercise

Multiple Choice Questions

Q 1. Two polygons have same number of sides are similar, if:

a. their corresponding sides are proportional

b. their corresponding angles are equal

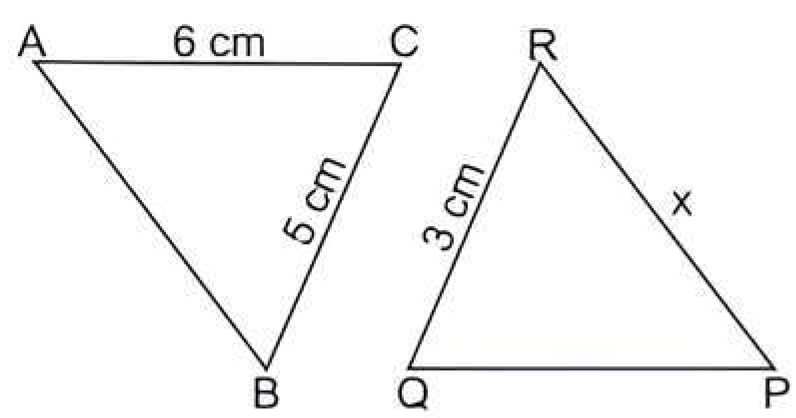
c. Both a. and b.

d. None of the above

Q 2. If $\Delta ABC \sim \Delta PQR$ with $\angle A = 32°$ and $\angle R = 65°$, then the measures of $\angle$ B is: *[CBSE 2023]*

a. 32° b. 65° c. 83° d. 97°

Q 3.



In the given figure, $\Delta ABC \sim \Delta QPR$. If AC = 6 cm, BC = 5 cm, QR = 3 cm and PR = x, then the value of x is: *[CBSE 2023]*

a. 3.6 cm b. 2.5 cm c. 10 cm d. 3.2 cm

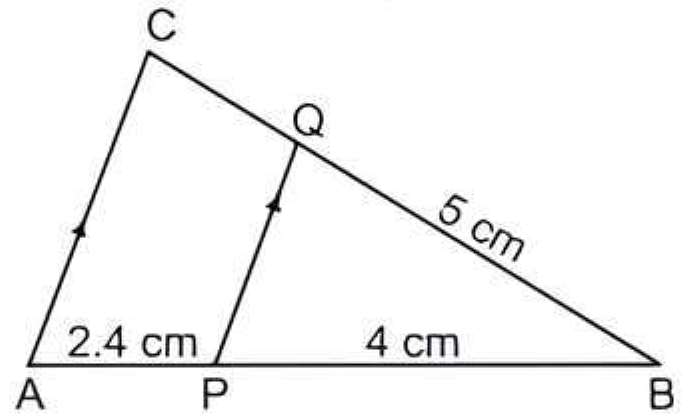
Q 4. In a $\triangle ABC$, it is given that $AB = 6$ cm, $AC = 8$ cm and AD is the bisector of $\angle A$. Then, BD : DC =

a. $3:4$ b. $9:16$ c. $4:3$ d. $\sqrt{3}:2$

Q 5. $\triangle ABC \sim \triangle PQR$. If AM and PN are altitudes of $\triangle ABC$ and $\triangle PQR$ respectively and $AB^2 : PQ^2 = 4:9$, then AM : PN = [CBSE SQP 2021 Term-I]

a. $16:81$ b. $4:9$ c. $3:2$ d. $2:3$

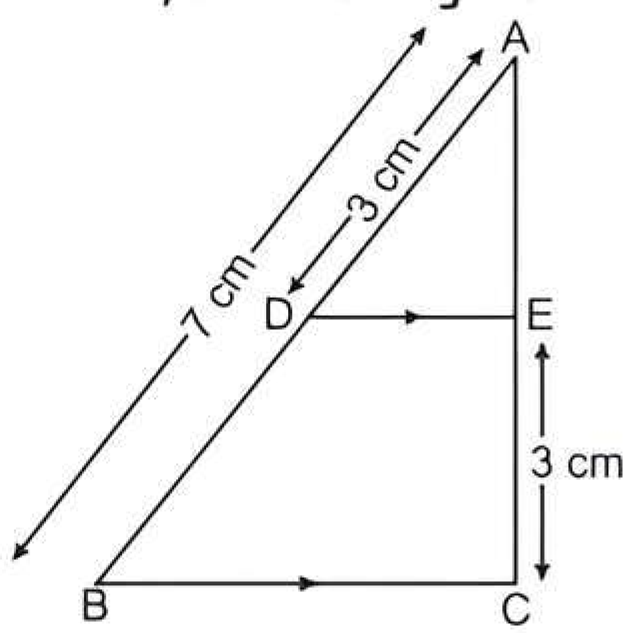
Q 6. In the given figure, PQ || AC. If $BP = 4$ cm, $AP = 2.4$ cm and $BQ = 5$ cm, then length of BC is: [CBSE 2023]



a. 8 cm b. 3 cm

c. 0.3 cm d. $\frac{25}{3}$ cm

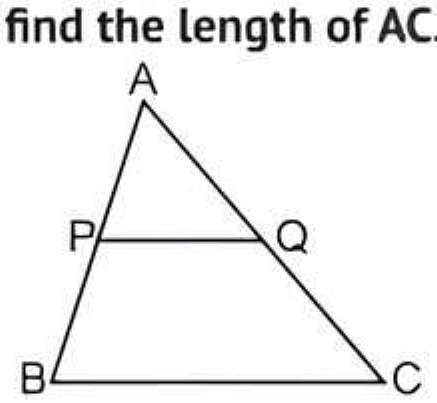
Q 7. In the given figure, DE || BC. If $AD = 3$ cm, $AB = 7$ cm and $EC = 3$ cm, then the length of AE is: [CBSE 2023]



a. 2 cm b. 2.25 cm

c. 3. 5 cm d. 4 cm

Q 8. In $\triangle ABC$, PQ || BC. If $PB = 6$ cm, $AP = 4$ cm, $AQ = 8$ cm, find the length of AC. [CBSE 2023]



a. 12 cm b. 20 cm

c. 6 cm d. 14 cm

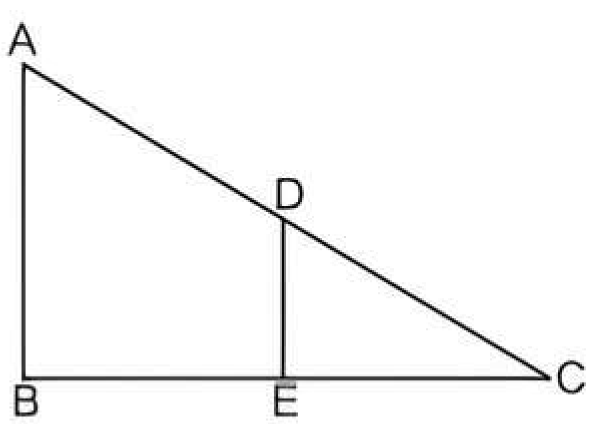
Q 9. In $\triangle ABC$, DE || AB. If $AB = a$, $DE = x$, $BE = b$ and $EC = c$. Express x in terms of a, b and c. [CBSE SQP 2023-24]

a. $\frac{ac}{b}$

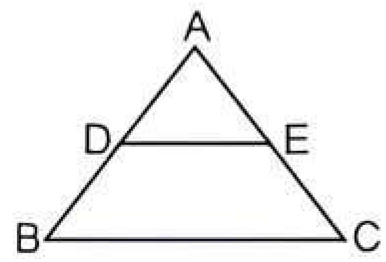
b. $\frac{ac}{b+c}$

c. $\frac{ab}{c}$

d. $\frac{ab}{b+c}$



Q 10. In the given figure, if DE || BC, $AD = 3$ cm, $BD = 4$ cm and $BC = 14$ cm, then DE equals: [CBSE SQP 2021 Term-I]



a. 7 cm b. 6 cm c. 4 cm d. 3 cm

Q 11. In $\triangle ABC$ and $\triangle DEF$, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = \frac{1}{2}DE$. Then, the two triangles are: [CBSE 2021 Term-I]

a. congruent but not similar

b. similar but not congruent

c. neither congruent nor similar

d. congruent as well as similar

Q 12. It is given that $\triangle ABC \sim \triangle DFE$, $\angle A = 30°$, $\angle C = 50°$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm. Then, which of the following is true? [NCERT EXEMPLAR]

a. $DE = 12$ cm, $\angle F = 50°$ b. $DE = 12$ cm, $\angle F = 100°$

c. $EF = 12$ cm, $\angle D = 100°$ d. $EF = 12$ cm, $\angle D = 30°$

Q 13. If the corresponding medians of two similar triangles are in the ratio $5:7$, then the ratio of their corresponding sides is: [CBSE 2015]

a. $25:49$ b. $5:7$

c. $7:5$ d. $49:25$

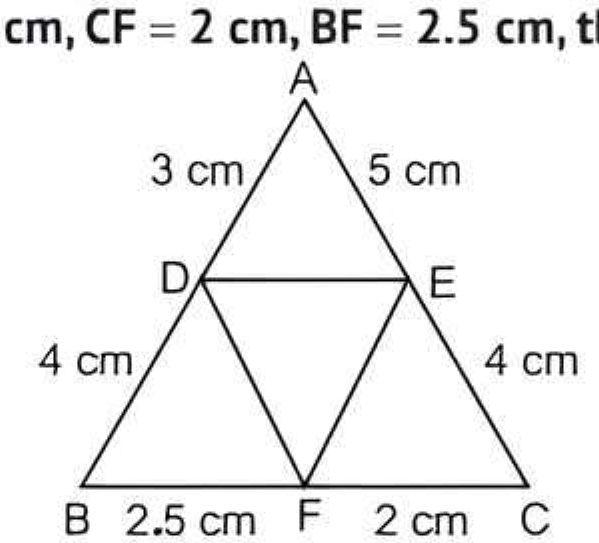
Q 14. The parallel sides of a trapezium are 3 cm and 9 cm. The non-parallel sides are 4 cm and 6 cm. A line parallel to the base divides the trapezium into two trapeziums of equal perimeters. The ratio in which each of the non-parallel sides is divided, is:

a. $4:5$ b. $3:2$ c. $4:1$ d. $3:1$

Q 15. ABCD is a trapezium with AD || BC and $AD = 4$ cm. If the diagonals AC and BD intersect each other at O such that $AO/OC = DO/OB = 1/2$, then BC = [CBSE SQP 2022-23]

a. 6 cm b. 7 cm c. 8 cm d. 9 cm

Q 16. In the given figure, $AD = 3$ cm, $AE = 5$ cm, $BD = 4$ cm, $CE = 4$ cm, $CF = 2$ cm, $BF = 2.5$ cm, then:



a. DE || BC b. DF || AC

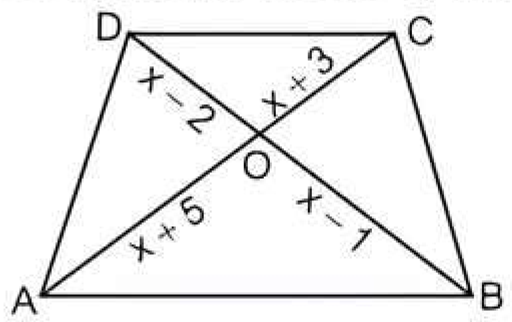
c. EF || AB d. None of these

Q 17. It is given that, $\triangle ABC \sim \triangle EDF$, such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm, then the sum of the remaining sides of the triangles is:

a. 23.05 cm b. 16.8 cm

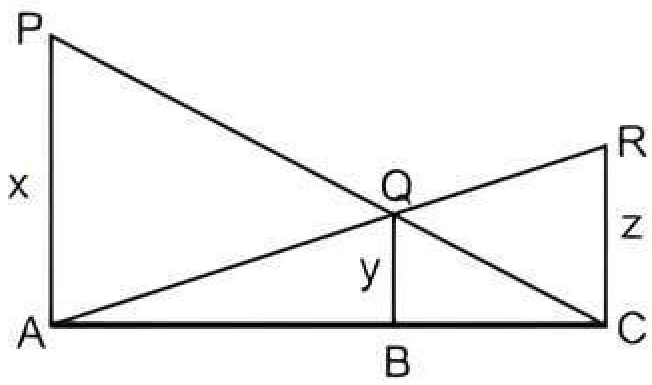
c. 6.25 cm d. 24 cm

Q 18. **In the given figure, if AB || DC, find the value of x.**



a. 5 b. 7 c. 6 d. 4

Q 19. **In the given figure, PA, QB and RC are each perpendicular to AC. If $x = 8$ cm and $z = 6$ cm, then y is equal to:** *[CBSE 2021 Term-I]*



a. $\frac{56}{7}$ cm b. $\frac{7}{56}$ cm

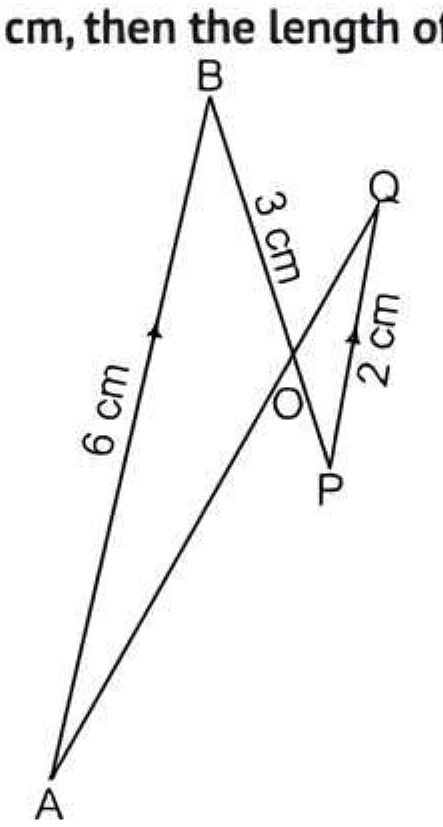
c. $\frac{25}{7}$ cm d. $\frac{24}{7}$ cm

Q 20. **If $\triangle PQR \sim \triangle ABC$; PQ = 6 cm, AB = 8 cm and the perimeter of $\triangle ABC$ is 36 cm, then the perimeter of $\triangle PQR$ is:** *[CBSE 2023]*

a. 20.25 cm b. 27 cm

c. 48 cm d. 64 cm

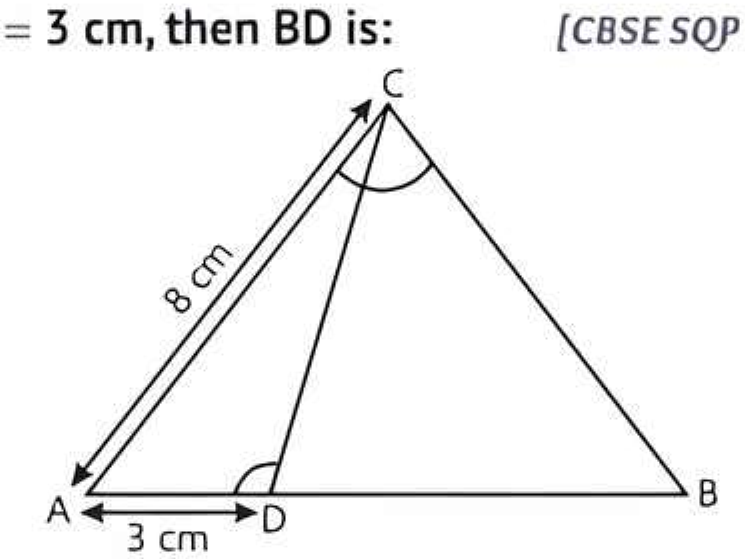
Q 21. **In the given figure, AB || PQ. If AB = 6 cm, PQ = 2 cm and OB = 3 cm, then the length of OP is:** *[CBSE 2023]*



a. 9 cm b. 3 cm

c. 4 cm d. 1 cm

Q 22. **In the given figure, if $\angle ACB = \angle CDA$, AC = 8 cm, AD = 3 cm, then BD is:** *[CBSE SQP 2021 Term-I]*



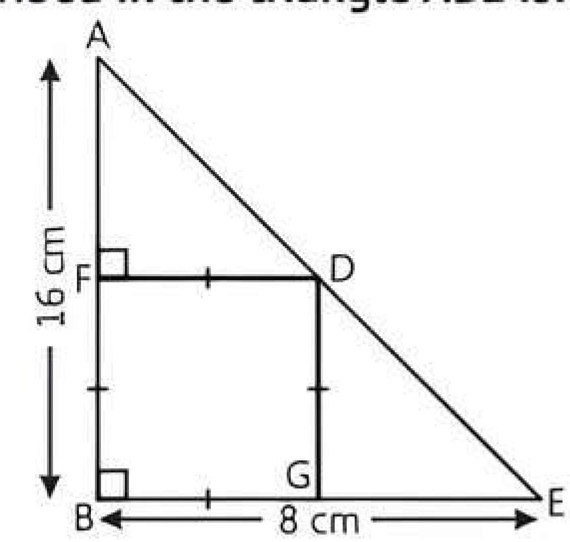
a. 22/3 cm b. 26/3 cm

c. 55/3 cm d. 64/3 cm

Q 23. **A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time, a tower casts the shadow 40 m long on the ground. Then, the height of the tower is:**

a. 65 m b. 60 m c. 70 m d. 72 m

Q 24. **Sides AB and BE of a right triangle, right angled at B are of lengths 16 cm and 8 cm respectively. The length of the side of largest square FDGB that can be inscribed in the triangle ABE is:**



a. 32/3 cm b. 16/3 cm

c. 8/3 cm d. 4/3 cm

Assertion & Reason Type Questions

Directions (Q.Nos. 25-29): *In the following questions given below, there are two statement marked as Assertion (A) and Reason (R). Read the statements and choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 25. **Assertion (A): All regular polygons of the same number of sides such as equilateral triangles, squares etc, are similar.**

Reason (R): Two polygons of the same number of sides are said to be similar, if their corresponding angles are equal and lengths of corresponding sides are proportional.

Q 26. **Assertion (A): In a $\triangle ABC$, if DE || BC and intersects AB at D and AC at E, then $\frac{AB}{AD} = \frac{AC}{AE}$.**

Reason (R): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then these sides are divided in the same ratio.

Q 27. **Assertion (A): If the bisector of an angle of a triangle bisects the opposite side, then the triangle is isosceles.**

Reason (R): The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

Q 28. Assertion (A): In a $\triangle ABC$, D and E are points on sides AB and AC respectively such that $BD = CE$. If $\angle B = \angle C$, then DE is not parallel to BC.

Reason (R): If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Q 29. Assertion (A): In $\triangle ABC$, $DE \parallel BC$, such that $AD = (7x-4)$ cm, $AE = (5x-2)$ cm, $DB = (3x+4)$ cm and $EC = 3x$ cm then x is equal to 5.

Reason (R): If a line is drawn parallel to one side of triangle to intersect the other two sides at a distinct point, then the other two sides are divided in the same ratio.

Fill in the Blanks Type Questions

Q 30. Two triangles are similar, if their corresponding sides are .... [NCERT EXERCISE; CBSE 2020]

Q 31. In $\triangle ABC$ and $\triangle DEF$, if $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 2DE$. Then, the two triangles are ....

Q 32. If in two triangles DEF and PQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then $\frac{DE}{QR} = \frac{DF}{PQ} =$ ....

Q 33. The line segment joining the mid-points of any two sides of a triangle is .... to the third side.

Q 34. All congruent triangles are similar but the similar triangles need not to be ....

True/False Type Questions

Q 35. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. [NCERT EXEMPLAR]

Q 36. If AD and PM are medians of $\triangle ABC$ and $\triangle PQR$ respectively, where $\triangle ABC \sim \triangle PQR$, then $\frac{AB}{PQ} = \frac{AD}{PM}$. [NCERT EXERCISE]

Q 37. In two triangles, if one pair of corresponding sides are proportional and the included angles are also equal, then two triangles are similar.

Solutions

1. (c) Two polygons of the same number of sides are similar if their corresponding angles are equal and their corresponding sides are in the same ratio *i.e.* proportional.

2. (c) Given $\triangle ABC \sim \triangle PQR$

$\Rightarrow \quad \angle A = \angle P, \angle B = \angle Q$ and $\angle C = \angle R$

$\therefore \quad \angle A = \angle P = 32^\circ$ and $\angle R = \angle C = 65^\circ$

Now, in $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \quad 32^\circ + \angle B + 65^\circ = 180^\circ$$

$$\Rightarrow \quad \angle B = 180^\circ - 97^\circ = 83^\circ$$

3. (b) Given, $\triangle ABC \sim \triangle QPR$

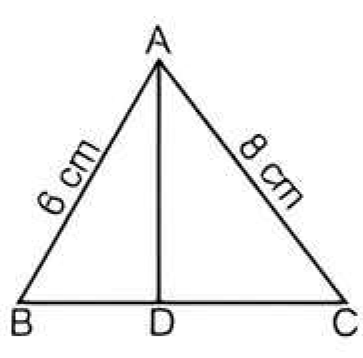
$$\therefore \quad \frac{AB}{QP} = \frac{BC}{PR} = \frac{CA}{RQ}$$

$$\Rightarrow \quad \frac{BC}{PR} = \frac{CA}{RQ} \quad \Rightarrow \quad \frac{5}{x} = \frac{6}{3}$$

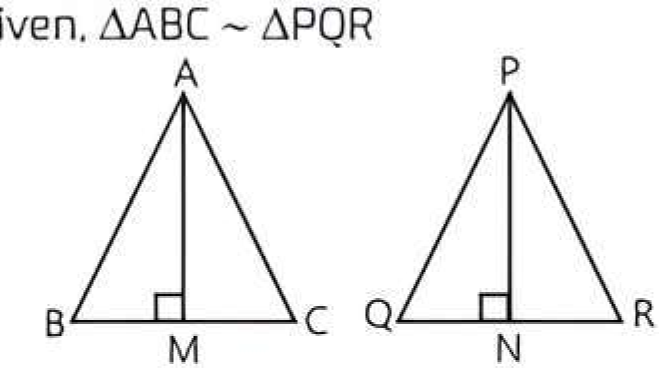
$$\Rightarrow \quad x = \frac{5 \times 3}{6} = \frac{5}{2} = 2.5 \text{ cm}$$

4. (a) We know that, the bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle.

$\therefore \quad BD : DC = AB : AC = 6 : 8 = 3 : 4$



5. (d) Given, $\triangle ABC \sim \triangle PQR$



TiP

If two triangles are similar, then their corresponding sides and altitudes are in same proportion.

AM and PN are altitudes of $\triangle ABC$ and $\triangle PQR$ respectively.

Also. $$\frac{AB^2}{PQ^2} = \frac{4}{9} \quad \Rightarrow \quad \frac{AB}{PQ} = \frac{2}{3} \quad \text{...(1)}$$

As we know that,

Ratio of altitudes = Ratio of sides for similar triangles.

So, $$\frac{AM}{PN} = \frac{AB}{PQ} = \frac{2}{3}$$

6. (a) In $\triangle ABC$, $PQ \parallel AC$

$$\therefore \quad \frac{AP}{BP} = \frac{CQ}{BQ} \quad \text{[By Thales theorem]}$$

$$\Rightarrow \quad \frac{2.4}{4} = \frac{CQ}{5}$$

$$\Rightarrow \quad CQ = \frac{2.4 \times 5}{4} = 3 \text{ cm}$$

$\therefore$ Length of $BC = BQ + CQ = 5 + 3 = 8$ cm

7. (b) In $\triangle ABC$, $BC \parallel DE$

$$\therefore \quad \frac{AD}{BD} = \frac{AE}{CE} \quad \text{[By Thales theorem]}$$

$$\Rightarrow \quad \frac{AD}{BD} + 1 = \frac{AE}{CE} + 1$$

$$\Rightarrow \quad \frac{AD + BD}{BD} = \frac{AE + CE}{CE}$$

$$\Rightarrow \quad \frac{AB}{AB-AD}=\frac{AE+CE}{CE}$$

$$\Rightarrow \quad \frac{7}{7-3}=\frac{AE+3}{3}$$

$$\Rightarrow \quad \frac{7}{4}=\frac{AE}{3}+1$$

$$\Rightarrow \quad \frac{AE}{3}=\frac{7}{4}-1=\frac{3}{4}$$

$$\Rightarrow \quad AE=\frac{9}{4}=2.25 \text{ cm}$$

8. (b) Given that.
In $\triangle ABC$, $PQ \parallel BC$
$PB = 6$ cm, $AP = 4$ cm, $AQ = 8$ cm

$$\therefore \quad \frac{AP}{AB}=\frac{AQ}{AC}$$

$$\Rightarrow \quad \frac{AP}{AP+PB}=\frac{AQ}{AC} \qquad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{4}{4+6}=\frac{8}{AC} \Rightarrow \frac{4}{10}=\frac{8}{AC}$$

$$\Rightarrow \quad AC=\frac{8\times 10}{4}=20\text{ cm}$$

9. (b) Given $AB = a$, $DE = x$
$BE = b$ and $EC = c$
In $\triangle ABC$, $DE \parallel AB$
By basic proportionality theorem,

$$\Rightarrow \quad \frac{DE}{ABC}=\frac{EC}{BC}$$

$$\Rightarrow \quad \frac{DE}{AB}=\frac{EC}{BE+EC}$$

$$\Rightarrow \quad \frac{x}{a}=\frac{c}{b+c} \Rightarrow x=\frac{ac}{b+c}$$

10. (b) Given, $AD = 3$ cm, $BD = 4$ cm, $BC = 14$ cm
and $DE \parallel BC$

$$\therefore \quad \triangle ADE \sim \triangle ABC \qquad \text{(By AA similarity)}$$

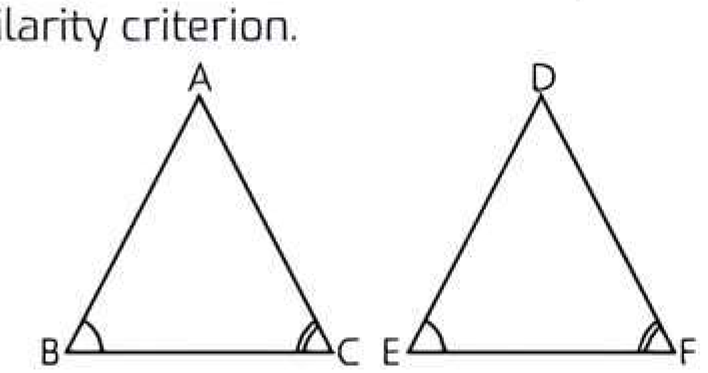
$$\Rightarrow \quad \frac{AD}{AB}=\frac{DE}{BC} \Rightarrow \frac{AD}{AD+BD}=\frac{DE}{BC}$$

$$\Rightarrow \quad \frac{3}{3+4}=\frac{DE}{14} \Rightarrow DE=\frac{3\times 14}{7}=6\text{ cm}$$

11. (b) In $\triangle ABC$ and $\triangle DEF$,

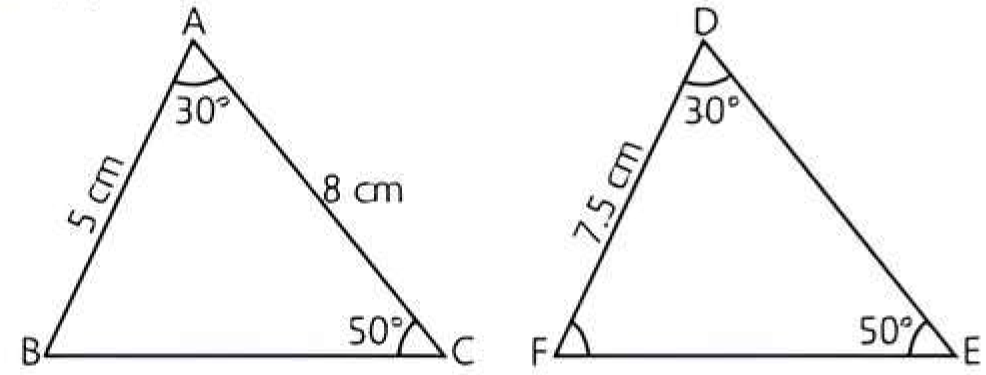
$$\angle B=\angle E, \angle F=\angle C \text{ and } AB=\frac{1}{2}DE.$$

We know that, if in two triangles corresponding two angles are same, then they are similar by AA similarity criterion.



Since, $AB \neq DE$
Therefore, $\triangle ABC$ and $\triangle DEF$ are not congruent.

12. (b) Given, $\triangle ABC \sim \triangle DFE$



Then $\quad \angle A = \angle D = 30°$,
$\angle C = \angle E = 50°$

$$\therefore \quad \angle B=\angle F=180°-(30°+50°)=100°$$

Also, $\quad AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm

$$\therefore \quad \frac{AB}{DF}=\frac{AC}{DE} \Rightarrow \frac{5}{7.5}=\frac{8}{DE}$$

$$\Rightarrow \quad DE=\frac{8\times 7.5}{5}=12\text{ cm}$$

Hence, $DE = 12$ cm, $\angle F = 100°$

13. (b) Let ABC and PQR be two similar triangles with medians AD and PS respectively.

$$\text{Then,} \quad \frac{AD}{PS}=\frac{5}{7} \qquad \text{(Given)}$$

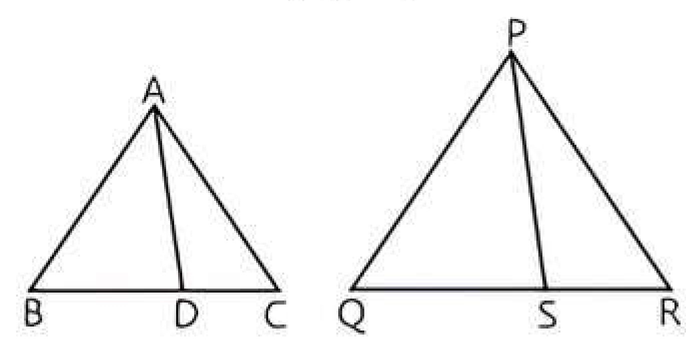
TR!CK

The ratios of the medians of two similar triangles is equal to the ratio of their corresponding sides.

$$\because \quad \triangle ABC \sim \triangle PQR$$

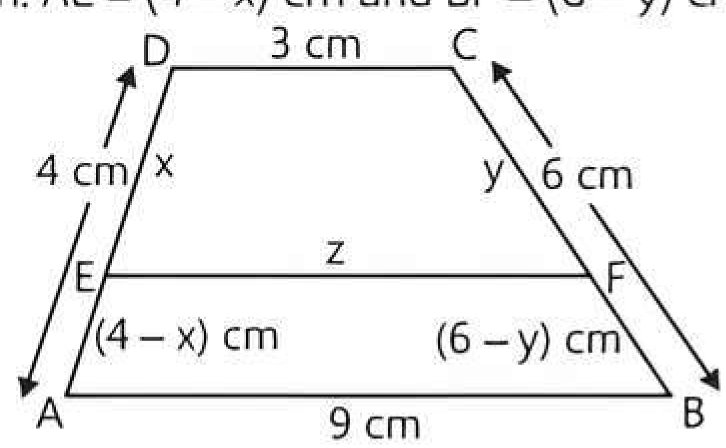
$$\frac{AB}{PQ}=\frac{AD}{PS}$$

$$\Rightarrow \quad \frac{AB}{PQ}=\frac{5}{7}$$



Hence, the ratio of corresponding sides is 5 : 7.

14. (c) Given, $AD = 4$ cm and $BC = 6$ cm.
Let $DE = x$ cm, $CF = y$ cm and $EF = z$ cm
Then, $AE = (4 - x)$ cm and $BF = (6 - y)$ cm



According to the given condition,
Perimeter of DCFE = Perimeter of ABFE

$$\Rightarrow \quad x+3+y+z=9+6-y+z+4-x$$

$$\Rightarrow \quad x+3+y=19-x-y$$

$$\Rightarrow \quad 2x+2y=16$$

$$\Rightarrow \quad x+y=8$$

$$\Rightarrow \quad y=8-x$$

We know that line parallel to the base divides the two non-parallel lines in the same ratio.

$$\therefore \quad \frac{DE}{AE} = \frac{CF}{BF}$$

$$\Rightarrow \quad \frac{x}{4-x} = \frac{y}{6-y}$$

$$\Rightarrow \quad \frac{x}{4-x} = \frac{8-x}{6-8+x}$$

$$\Rightarrow \quad x(x-2) = (8-x)(4-x)$$

$$\Rightarrow \quad x^2 - 2x = 32 - 12x + x^2$$

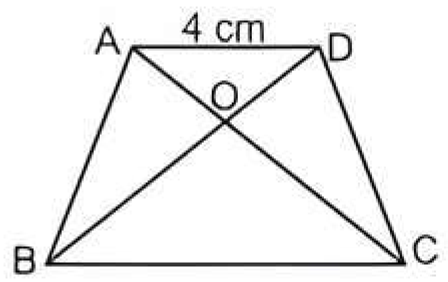
$$\Rightarrow \quad 10x = 32 \text{ cm}$$

$$\Rightarrow \quad x = 3.2 \text{ cm}$$

$$\therefore \quad \text{Ratio} = \frac{DE}{AE} = \frac{3.2}{4-3.2} = \frac{3.2}{0.8} = \frac{4}{1} = 4:1$$

15. (c) Given, $AD = 4$ cm

and $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$ ...(1)



In ΔAOD and ΔCOB

$\angle ADO = \angle CBO$

($\because AD \parallel BC$, so alternate angles are equal)

$\angle AOD = \angle COB$ (Vertically opposite angles)

and $\angle OAD = \angle OCB$

(Alternate angles are equal)

$\therefore \Delta AOD \sim \Delta COB$ (By AAA similarity rule)

$$\Rightarrow \quad \frac{AO}{OC} = \frac{OD}{OB} = \frac{AD}{BC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{4}{BC} \quad \text{[From eq. (1)]}$$

$$\Rightarrow \quad BC = 8 \text{ cm}$$

16. (c) Given, $AD = 3$ cm, $AE = 5$ cm, $BD = 4$ cm,

$CE = 4$ cm, $CF = 2$ cm, $BF = 2.5$ cm

Here $\frac{CF}{FB} = \frac{2}{2.5}$

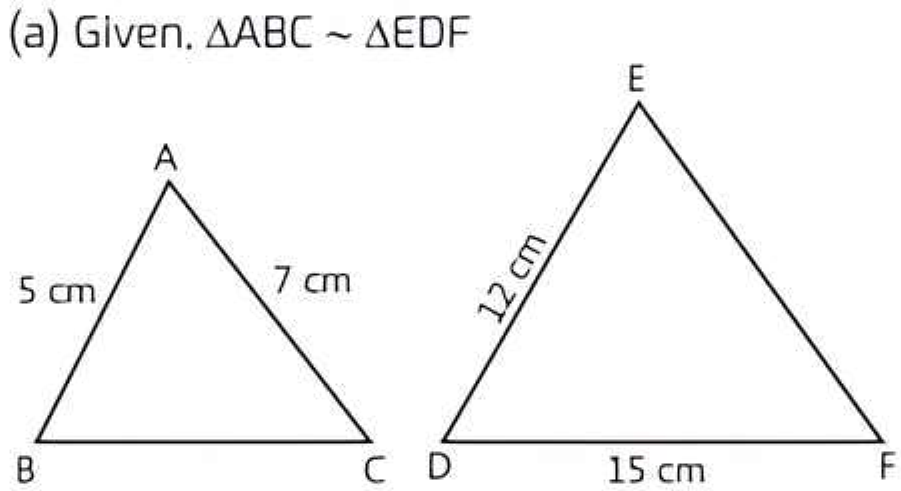
$= \frac{20}{25} = \frac{4}{5}$

and $\frac{CE}{AE} = \frac{4}{5}$

$$\therefore \quad \frac{CF}{FB} = \frac{CE}{AE}$$

$\Rightarrow$ EF || AB (By converse of Thales theorem)

17. (a) Given, $\Delta ABC \sim \Delta EDF$



Since, $\Delta ABC \sim \Delta EDF$

$$\therefore \quad \frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$$

$$\Rightarrow \quad \frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

On taking first and second ratios, we get

$$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5} = 16.8 \text{ cm}$$

On taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12} = 6.25 \text{ cm}$$

Now, sum of the remaining sides of triangle.

$= EF + BC = 16.8 + 6.25 = 23.05$ cm

18. (b) Given, AB || DC

$\therefore \angle ODC = \angle OBA$ (Alternate interior angles)

and $\angle OCD = \angle OAB$ (Alternate interior angles)

$\therefore \Delta DOC \sim \Delta BOA$ (By AA similarity criterion)

Thus, $\frac{OD}{OB} = \frac{OC}{OA}$

$$\Rightarrow \quad \frac{x-2}{x-1} = \frac{x+3}{x+5}$$

$$\Rightarrow \quad (x-2)(x+5) = (x+3)(x-1)$$

$$\Rightarrow \quad x^2 + 3x - 10 = x^2 + 2x - 3$$

$$\Rightarrow \quad x = 7$$

19. (d) ΔPAC and ΔQBC is similar.

$$\therefore \quad \frac{AP}{BQ} = \frac{AC}{BC} \Rightarrow \frac{x}{y} = \frac{AC}{BC}$$

or $\frac{y}{x} = \frac{BC}{AC}$ ...(1)

Also, ΔRAC and ΔQAB is similar.

$$\therefore \quad \frac{CR}{BQ} = \frac{AC}{AB} \Rightarrow \frac{z}{y} = \frac{AC}{AB}$$

or $\frac{y}{z} = \frac{AB}{AC}$ ...(2)

Adding eqs. (1) and (2), we get

$$\frac{y}{x} + \frac{y}{z} = \frac{BC}{AC} + \frac{AB}{AC}$$

$$\Rightarrow \quad y\left(\frac{1}{x} + \frac{1}{z}\right) = \frac{BC + AB}{AC}$$

$$\Rightarrow \quad y\left(\frac{1}{x} + \frac{1}{z}\right) = \frac{AC}{AC}$$

$$\Rightarrow \quad y\left(\frac{1}{8} + \frac{1}{6}\right) = 1 \Rightarrow y\left(\frac{6+8}{48}\right) = 1$$

$$\Rightarrow \quad y = \frac{48}{14} = \frac{24}{7} \text{ cm}$$

20. (b) Given, $\Delta PQR \sim \Delta ABC$

$$\therefore \quad \frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC} = \frac{PQ + QR + PR}{AB + BC + AC}$$

$$\Rightarrow \quad \frac{PQ}{AB} = \frac{\text{perimeter of } \Delta PQR}{\text{perimeter of } \Delta ABC} \quad \text{... (1)}$$

Given PQ = 6 cm, AB = 8 cm and perimeter of $\Delta ABC = 36$ cm

From eq. (1), we get

$$\frac{6}{8} = \frac{\text{Perimeter of } \Delta PQR}{36}$$

$$\text{Perimeter of } \Delta PQR = \frac{6 \times 36}{8} = 27 \text{ cm}$$

21. (d) In the given figure, AB || PQ

In ΔPQR and ΔBOA,

$\angle OPQ = \angle OBA$ (Alternate interior angles)

$\angle OQP = \angle OAB$ (Alternative interior angles)

and $\angle POQ = \angle BOA$ (Vertically opposite angles)

$\because$ $\Delta POQ \sim \Delta BOA$

$\Rightarrow$ $\frac{PO}{BO} = \frac{OQ}{OA} = \frac{PQ}{BA}$

Given AB = 6 cm , PQ = 2 cm and OB = 3 cm

$\Rightarrow$ $\frac{PO}{BO} = \frac{PQ}{BA} \Rightarrow \frac{OP}{3} = \frac{2}{6}$

$\Rightarrow$ $OP = \frac{2 \times 3}{6} = 1 \text{ cm}$

22. (c) Given, AC = 8 cm, AD = 3 cm

In ΔACD and ΔABC,

$\angle CDA = \angle ACB$ (Given)

$\angle CAD = \angle CAB$ (Common)

$\therefore$ $\Delta ACD \sim \Delta ABC$ (By AA similarity)

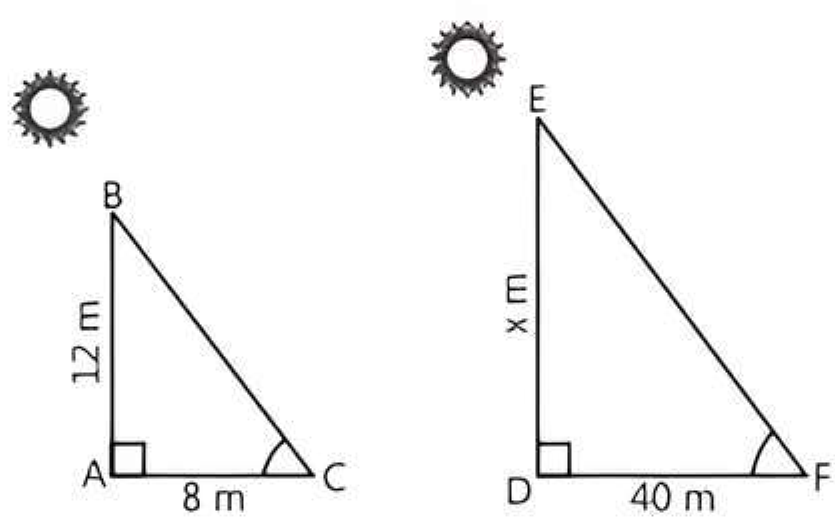
$\Rightarrow$ $\frac{AC}{AB} = \frac{AD}{AC}$ (By CPCT)

$\Rightarrow$ $\frac{8}{AB} = \frac{3}{8}$

$\Rightarrow$ $AB = \frac{64}{3} \text{cm}$

So, $BD = AB - AD = \frac{64}{3} - 3 = \frac{55}{3} \text{cm}$

23. (b) Let AB be the vertical stick and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Join BC and EF. Let DE = x m



We have, AB = 12 m, AC = 8 m and DF = 40 m

In ΔABC and ΔDEF, we have

$\angle A = \angle D = 90°$

$\angle C = \angle F$ (Angular elevation of the sun)

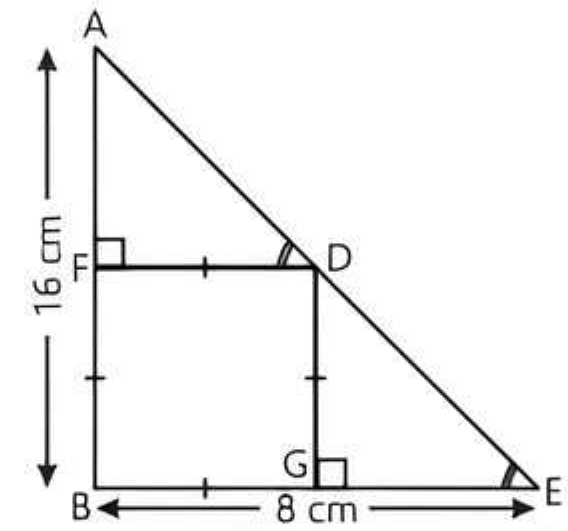
$\therefore$ $\Delta ABC \sim \Delta DEF$ (By AA similarity criterion)

$\Rightarrow$ $\frac{AB}{DE} = \frac{AC}{DF} \Rightarrow \frac{12}{x} = \frac{8}{40}$

$\Rightarrow$ $\frac{12}{x} = \frac{1}{5} \Rightarrow x = 60 \text{ m}$

24 (b) Given, ΔABE is a right-angled triangle with

AB = 16 cm and BE = 8 cm



Let each side of square FDGB be x cm.

In ΔAFD and ΔDGE,

$\angle AFD = \angle DGE$ (Each 90°)

$\angle ADF = \angle DEG$ (From figure)

$\therefore$ $\Delta AFD \sim \Delta DGE$ (By AA similarity)

$\Rightarrow$ $\frac{AF}{DG} = \frac{FD}{GE}$

$\Rightarrow$ $\frac{AB - FB}{DG} = \frac{FD}{BE - BG}$ (From figure)

$\Rightarrow$ $\frac{16 - x}{x} = \frac{x}{8 - x}$

$\Rightarrow$ $128 - 24x + x^2 = x^2$

$\Rightarrow$ $24x = 128$

$\Rightarrow$ $x = \frac{128}{24} = \frac{16}{3} \text{cm}$

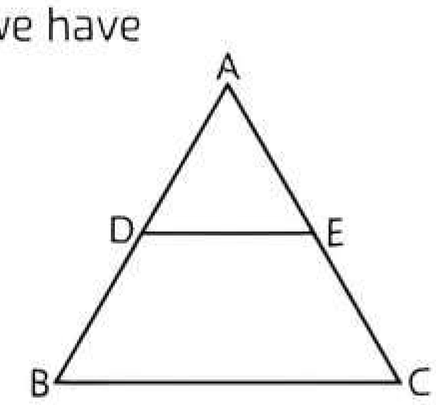
25. (a) **Assertion (A) :** Two polygons of the same number of sides are similar, if their corresponding angles are equal and corresponding sides are proportional.

$\because$ In equilateral triangles or squares, each angle is equal and sides are also proportional, therefore all regular polygons are similar.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

26. (a) **Assertion (A):** In ΔABC, DE || BC, by using Thale's theorem, we have



$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

$\Rightarrow$ $1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$

$\Rightarrow$ $\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$

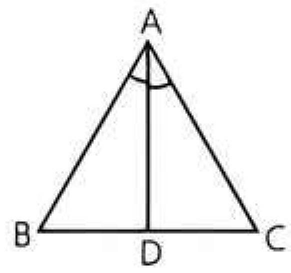
$\Rightarrow$ $\frac{AB}{AD} = \frac{AC}{AE}$

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

27. (a) **Assertion (A):** In ΔABC, AD is the bisector of $\angle A$.



$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$

$\Rightarrow \quad \frac{AB}{AC} = 1$

($\because$ D is the mid-point of BC, $\therefore$ BD = DC)

$\Rightarrow \quad AB = AC$

Hence, ΔABC is an isosceles.

So, Assertion (A) is true.

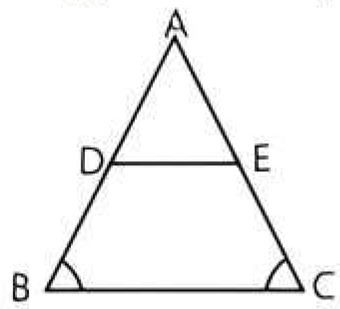
Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

28. (d) **Assertion (A):** In ΔABC, we have $\angle B = \angle C$

$\Rightarrow \quad AC = AB$

($\because$ Sides opposite to equal angles are equal)



$\Rightarrow \quad AE + EC = AD + DB$

$\Rightarrow \quad AE + CE = AD + DB$

$\Rightarrow \quad AE + CE = AD + CE \qquad$ [BD = CE (Given)]

$\Rightarrow \quad AE = AD$

Thus, we have

$AD = AE$

and $\quad BD = CE$

$\therefore \quad \frac{AD}{BD} = \frac{AE}{CE} \quad \Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \quad DE \parallel BC$

So, Assertion (A) is false.

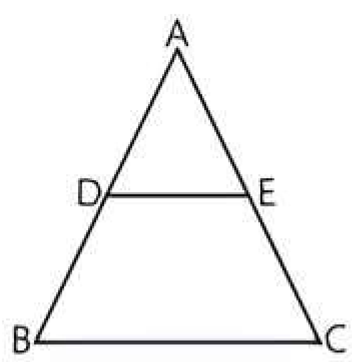
Reason (R): It is true statement.

Hence, Assertion (A) is false but Reason (R) is true.

29. (d) **Assertion (A):** We have,

$\frac{AD}{DB} = \frac{AE}{EC} \qquad [\because DE \parallel BC]$

$\Rightarrow \quad \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$



$\Rightarrow \quad 21x^2 - 12x = 15x^2 + 20x - 6x - 8$

$\Rightarrow \quad 6x^2 - 26x + 8 = 0$

$\Rightarrow \quad 3x^2 - 13x + 4 = 0$

TR!CK

Product of extreme terms $= 3 \times 4 = 12$

$\because \quad 12 = 6 \times 2 = 3 \times 4 = 12 \times 1$

Here, we will take 12 and 1 as a factors of 12. So, middle term

$-13 = -12 - 1$

$\Rightarrow \quad 3x^2 - 12x - x + 4 = 0$

$\Rightarrow \quad 3x(x-4) - 1(x-4) = 0$

$\Rightarrow \quad (x-4)(3x-1) = 0$

$\Rightarrow \quad x = 4, \frac{1}{3}$

So, Assertion (A) is false.

Reason (R): It is true statement.

Hence, Assertion (A) is false but Reason (R) is true.

30. Proportional

31. In ΔABC and ΔDEF,

$\angle B = \angle E \qquad$ (Given)

$\angle C = \angle F \qquad$ (Given)

$\therefore \quad \Delta ABC \sim \Delta DEF \qquad$ (By AA similarity)

32. In ΔDEF and ΔQRP,

$\angle D = \angle Q \qquad$ (Given)

$\angle E = \angle R \qquad$ (Given)

$\therefore \quad \Delta DEF \sim \Delta QRP \qquad$ (By AA similarity)

$\Rightarrow \quad \frac{DE}{QR} = \frac{DF}{QP} = \frac{EF}{RP}$

33. Parallel

34. Congruent

35. True

36. True

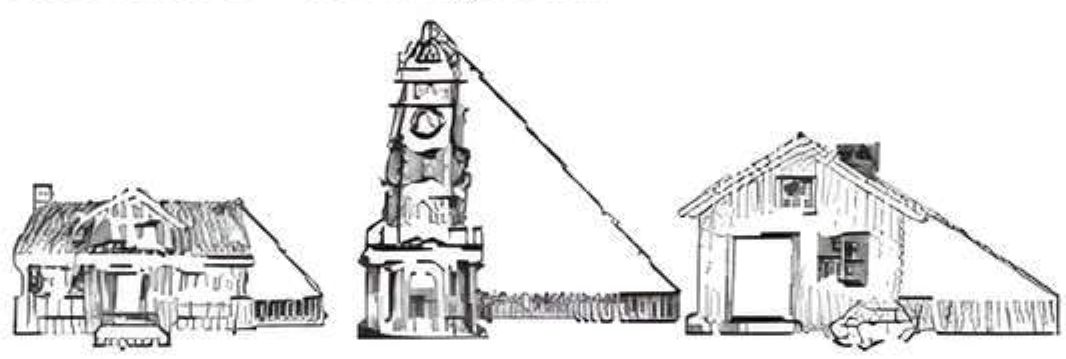
37. True

Case Study Based Questions

Case Study 1

Digvijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Digvijay's house is 20 m when Digvijay's house casts a shadow 10 m long on the ground.

At the same time, the tower casts a shadow 50 m long on the ground and the house of Anshul casts 20 m shadow on the ground.



Based on the above information, solve the following questions:

Q 1. The height of the tower is:

a. 10 m b. 20 m° c. 50 m d. 100 m

Q 2. When Digvijay's house casts a shadow of 18 cm, the length of the shadow of the tower is:

a. 18 m b. 20 m c. 90 m d. 100 m

Q 3. The height of Anshul's house is:

a. 20 m b. 40 m c. 50 m d. 100 m

Q 4. When the tower casts a shadow of 40 m, same time the length of the shadow of Anshul's house is:

a. 16 m b. 40 m
c. 100 m d. None of these

Q 5. Which of the following similarity criterion does not exist?

a. AA b. SAS c. SSS d. RHS

Solutions

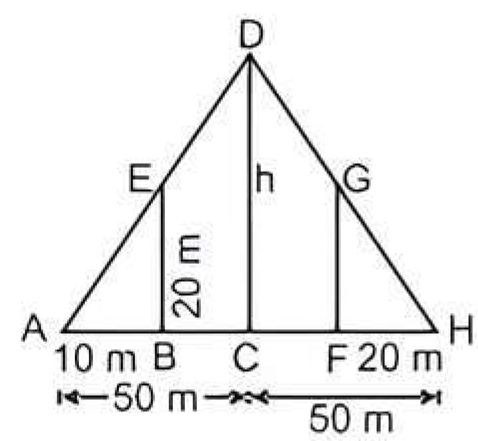
1. Let CD = h m be the height of the tower.
Let BE = 20 m be the height of Digvijay house and GF be the height of Anshul's house.

$$\Delta ACD \sim \Delta ABE$$

$$\frac{AC}{AB} = \frac{CD}{EB}$$

$$\Rightarrow \quad \frac{50}{10} = \frac{h}{20}$$

$$\Rightarrow \quad h = 100 \text{ m}$$



So, option (d) is correct.

2. Given AB = 18 m, let AC = x
In similar ΔABE and ΔACD

$$\frac{AB}{AC} = \frac{BE}{CD} \Rightarrow \frac{18}{x} = \frac{20}{100}$$

$$\Rightarrow \quad x = \frac{18 \times 100}{20} = 18 \times 5 = 90 \text{ m}$$

So, option (c) is correct.

3. Let height of Anshul's house be GF = h_1
Since, $\Delta HFG \sim \Delta HCD$

$$\therefore \quad \frac{HF}{HC} = \frac{FG}{CD} \Rightarrow \frac{20}{50} = \frac{h_1}{100}$$

$$h_1 = \frac{20 \times 100}{50} = 40 \text{ m}$$

So, option (b) is correct.

4. Given, HC = 40 cm
Let length of the shadow of Anshul's house be HF = l m.
Since, $\Delta HFG \sim \Delta HCD$

$$\therefore \quad \frac{HF}{HC} = \frac{FG}{CD}$$

$$\Rightarrow \quad \frac{l}{40} = \frac{40}{100}$$

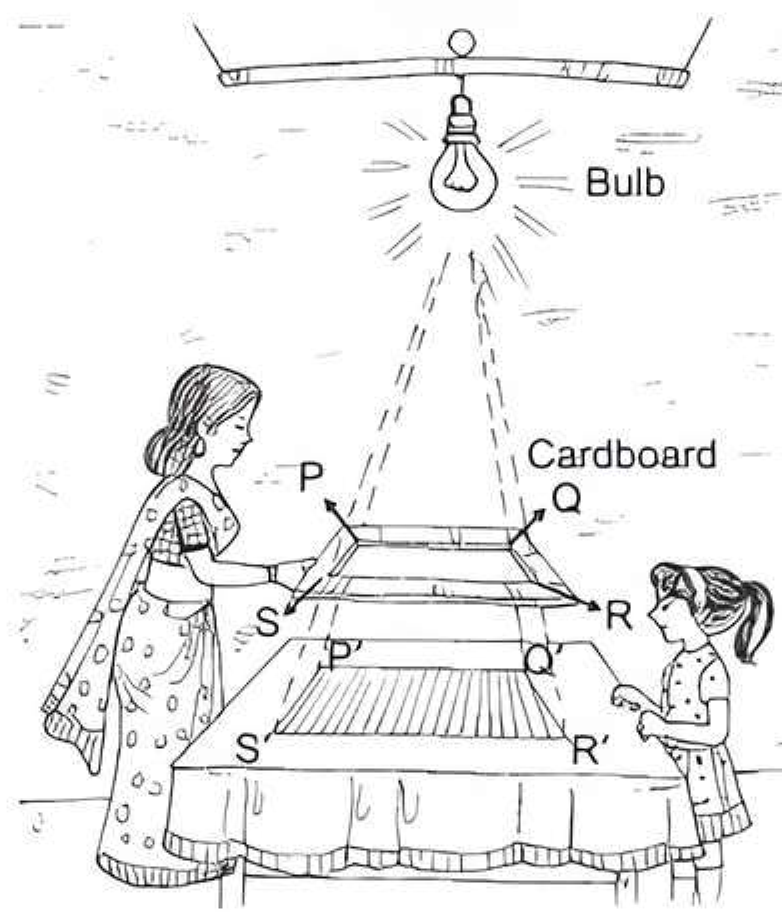
$$\Rightarrow \quad l = \frac{40 \times 40}{100} = 16 \text{ m}$$

So, option (a) is correct.

5. RHS similarity
Criterion does not exist.
So, option (d) is correct.

Case Study 2

Gaurav placed a light bulb at a point O on the ceiling and directly below it placed a table. He cuts a polygon, say a quadrilateral PQRS, from a plane cardboard and place this cardboard parallel to the ground between the lighted bulb and the table. Then a shadow of PQRS is cast on the table as P′Q′R′S′. Quadrilateral P′Q′R′S′ is an enlargement of the quadrilateral PQRS with scale factor 1 : 3. Given that PQ = 2.5 cm, QR = 3.5 cm. RS = 3.4 cm and PS = 3.1 cm; $\angle P = 115^\circ$, $\angle Q = 95^\circ$, $\angle R = 65^\circ$ and $\angle S = 85^\circ$.



Based on the above information, solve the following questions:

Q 1. The length of R′S′ is:

a. 3.4 cm b. 10.2 cm c. 6.8 cm d. 9.5 cm

Q 2. The ratio of sides P′Q′ and Q′R′ is:

a. 5 : 7 b. 7 : 5 c. 7 : 2 d. 2 : 7

Q 3. The measurement of $\angle Q'$ is:

a. 115° b. 95° c. 65° d. 85°

Q 4. The sum of the lengths Q′R′ and P′S′ is:

a. 12.3 cm b. 6.7 cm c. 19.8 cm d. 9 cm

Q 5. The sum of angles of quadrilateral P′Q′R′S′ is:

a. 180° b. 270° c. 300° d. 360°

Solutions

1. Given, scale factor is 1 : 3.

$$\because \quad R'S' = 3RS$$

$$\therefore \quad R'S' = 3 \times 3.4 = 10.2 \text{ cm}$$

So, option (b) is correct.

2. Since, $P'Q' = 3\,PQ = 3 \times 2.5 = 7.5$ cm
and $Q'R' = 3\,QR = 3 \times 3.5 = 10.5$ cm

$$\therefore \quad \frac{P'Q'}{Q'R'} = \frac{7.5}{10.5} = \frac{5}{7} \text{ or } 5:7$$

So, option (a) is correct.

3. Quadrilateral P′Q′R′S′ is similar to PQRS

$$\therefore \quad \angle Q' = \angle Q = 95^\circ$$

So, option (b) is correct.

4. $Q'R' = 3\,QR = 3 \times 3.5 = 10.5$ cm
and $P'S' = 3\,PS = 3 \times 3.1 = 9.3$ cm
$\therefore \quad Q'R' + P'S' = 10.5 + 9.3 = 19.8$ cm
So, option (c) is correct.

5. Since, PQRS ~ P′Q′R′S′

$\therefore \quad \angle P' = P = 115°$

$\angle Q' = \angle Q = 95°$

$\angle R' = \angle R = 65°$

and $\angle S' = \angle S = 85°$

$\therefore \quad \angle P' + \angle Q' + \angle R' + \angle S' = 115° + 95° + 65° + 85°$

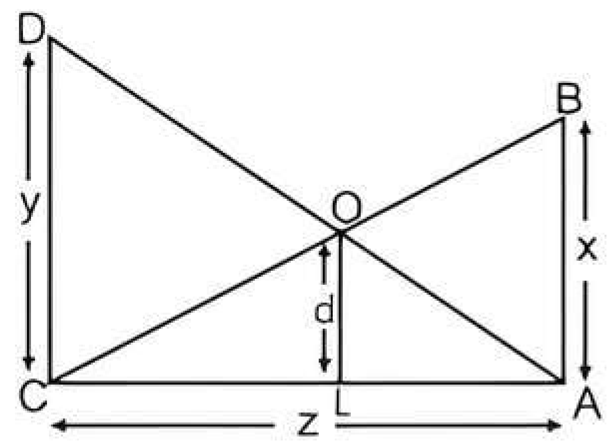
$= 360°$

i.e., the sum of angles of quadrilateral P′Q′R′S′ is 360°.

So, option (d) is correct.

Case Study 3

Anika is studying in class X. She observe two poles DC and BA. The heights of these poles are *x* m and *y* m respectively as shown in figure:



These poles are *z* m apart and O is the point of intersection of the lines joining the top of each pole to the foot of opposite pole and the distance between point O and L is *d*. Few questions came to his mind while observing the poles.

Based on the above information, solve the following questions:

Q 1. Which similarity criteria is applicable in ΔCAB and ΔCLO?

Q 2. If CL = *a*, then find *a* in terms of *x*, *y* and *d*.

Q 3. If AL = *b*, then find *b* in terms of *x*, *y* and *d*.

Solutions

1. In ΔCAB and ΔCLO, we have

$\angle CAB = \angle CLO = 90°$

$\angle C = \angle C$ (common)

$\therefore$ By AA similarity criterion,

$\Delta CAB \sim \Delta CLO$

2. $\because \quad \Delta CAB \sim \Delta CLO$

TiP

Corresponding sides of similar triangles are proportional.

$$\therefore \quad \frac{CA}{CL} = \frac{AB}{LO} \Rightarrow \frac{z}{a} = \frac{x}{d} \Rightarrow a = \frac{zd}{x}$$

3. In ΔALO and ΔACD,

We have

$\angle ALO = \angle ACD = 90°$

$\angle A = \angle A$ (common)

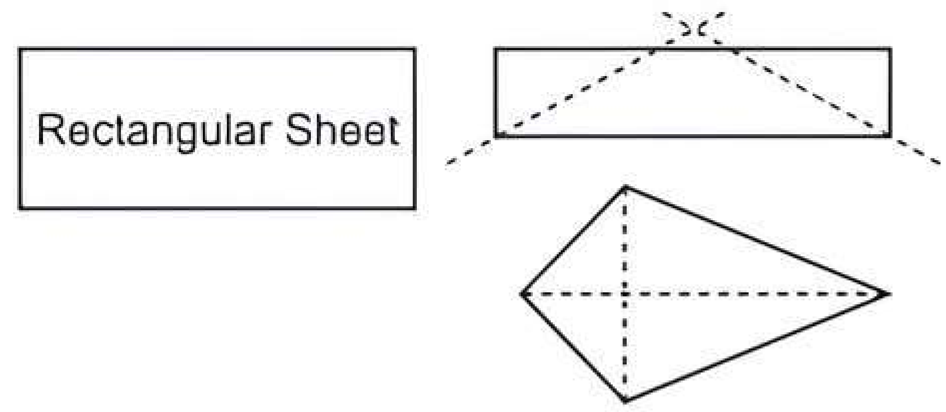
$\therefore$ By AA similarity criterion,

$\Delta A\angle O \sim \Delta ACD$

$$\therefore \frac{AL}{AC} = \frac{OL}{DC} \Rightarrow \frac{b}{z} = \frac{d}{y} \Rightarrow b = \frac{zd}{y}$$

Case Study 4

Before Basant Panchami, Samarth is trying to make kites at home. So, he take a rectangular sheet and fold it horizontally, then vertically and fold it transversally. After cutting transversally, he gets a kite shaped figure as shown below:



Based on the above information, solve the following questions:

Q 1. What is the angle between diagonals of a rectangle?

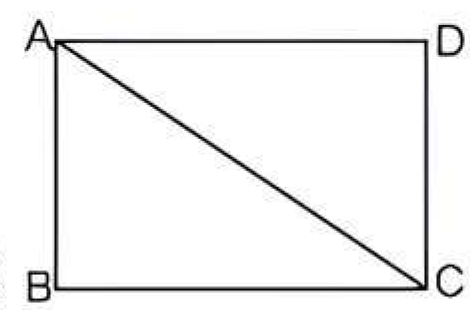
Q 2. Prove that two triangles divided by a diagonal in rectangle are similar as well as congruent.

Q 3. By which similarity criterion the triangles formed by longest diagonal in a kite are similar?

Solutions

1. Diagonals of a rectangle can bisect each other at any angle.

2. In ΔABC and ΔCDA



$AB = CD$

$\angle B = \angle D$

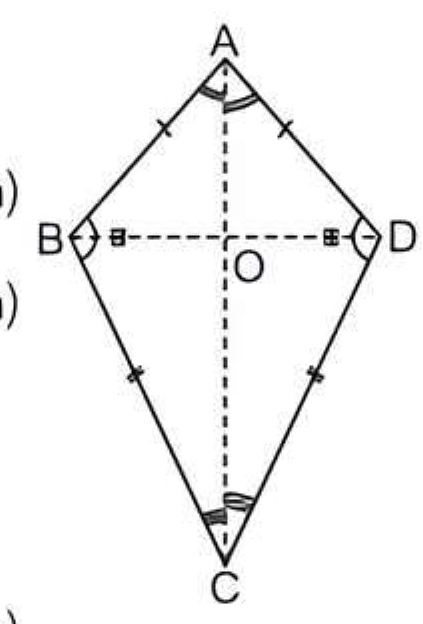
$BC = DA$

$\Delta ABC \cong \Delta CDA$

(By SAS)

When two triangles are congruent, then they are similar also.

3. In ΔABC and ΔADC,



$AB = AD$

$BC = DC$

$AC = AC$ (common)

$\therefore \quad \Delta ABC \sim \Delta ADC$

(by SSS criterion)

In ΔABC and ΔADC,

$AB = AD$

$\angle ABC = \angle ADC$

$BC = DC$

$\therefore \quad \Delta ABC \sim \Delta ADC$

(by SAS criterion)

In ΔAOB and ΔAOD,

$AB = AD$

$OA = OA$ (common)

$BO = DO$

(diagonal AC bisect the other diagonal BD)

$\therefore \quad \Delta AOB \sim \Delta AOD$ (by SSS similarity)

$\Rightarrow \quad \angle BAO = \angle DAO$

In ΔBOC and ΔDOC,

$BC = DC$

$OC = OC$ (common)

$BO = OD$

[Diagonal AC bisect the other diagonal BD]

$\therefore \quad \Delta BOC \sim \Delta DOC$ [by SSS similarity]

$\Rightarrow \quad \angle BCO = \angle DCO$

In ΔABC and ΔADC,

$\angle B = \angle D$

$\angle BAC = \angle DAC$

($\because$ $\angle BAO = \angle BAC$, $\angle DAO = \angle DAC$, proved above)

$\angle BCA = \angle DCA$

($\because$ $\angle BCO = \angle BCA$, $\angle DCO = \angle DCA$, proved above)

$\therefore$ $\triangle ABC \sim \triangle ADC$ (by AAA similarity)

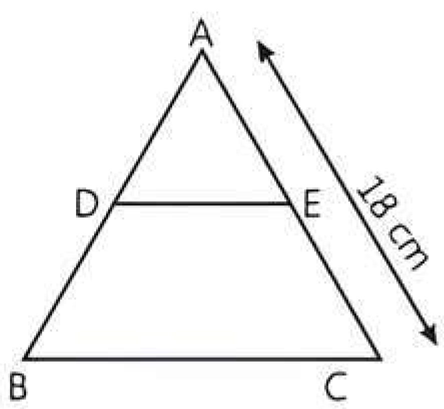
So, required similarity criterions are SSS, SAS and AAA.

Very Short Answer Type Questions

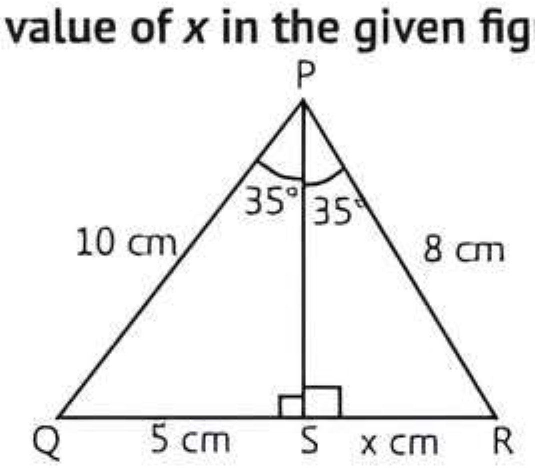
Q 1. In $\triangle PQR$, S and T are points on the sides PQ and PR respectively, such that ST || QR. If PS = 4 cm, PQ = 9 cm and PR = 4.5 cm, then find PT. *[CBSE 2015, 16, 17]*

Q 2. In two triangles ABC and DEF, if $\angle A = \angle E$ and $\angle B = \angle F$. Then, prove that $\frac{AB}{AC} = \frac{EF}{ED}$.

Q 3. In the given figure, DE is parallel to BC. If $\frac{AD}{DB} = \frac{2}{3}$ and AC = 18 cm, then find AE.

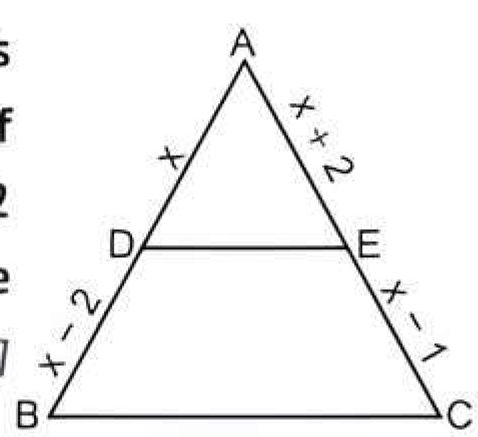


Q 4. Find the value of x in the given figure.

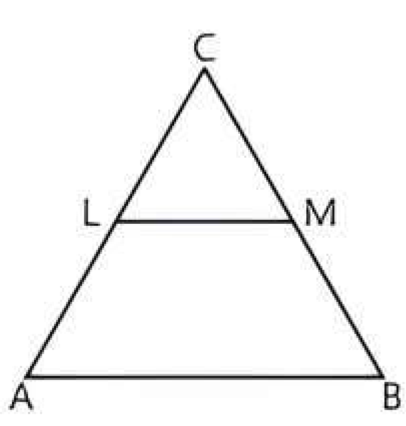


Q 5. If the corresponding medians of two similar triangles are in the ratio 5 : 7, then find the ratio of their corresponding sides. *[CBSE 2015]*

Q 6. In the given figure, ABC is a triangle which DE || BC. If AD= x, DB = x– 2, AE = x + 2 and EC = x – 1, then find the value of x. *[CBSE 2023]*

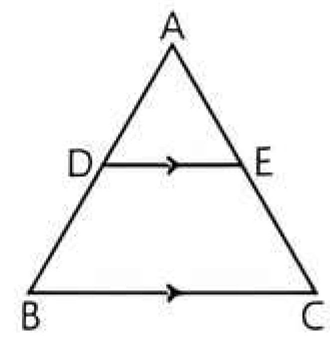


Q 7. In the given figure, LM || AB. If AL = x – 3, AC = 2x, BM = x – 2 and BC = 2x + 3, find the value of x.

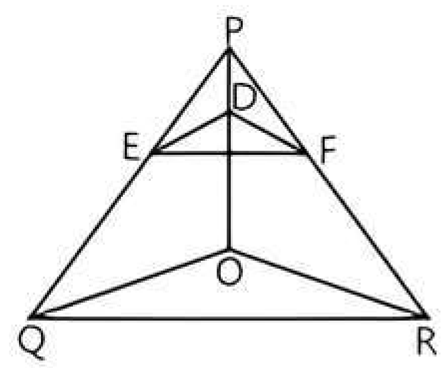


Short Answer Type-I Questions

Q 1. In the given figure, a $\triangle ABC$, DE || BC, so that AD = (4x – 3) cm, AE = (8x – 7) cm, BD = (3x – 1) cm and CE = (5x – 3) cm. Find the value of x. *[CBSE 2015]*

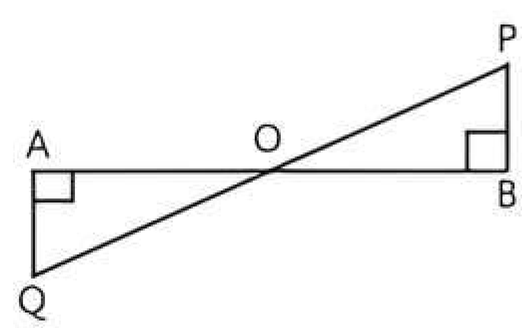


Q 2. In the following figure, DE || OQ and DF || OR, show that EF || QR. *[NCERT EXERCISE]*

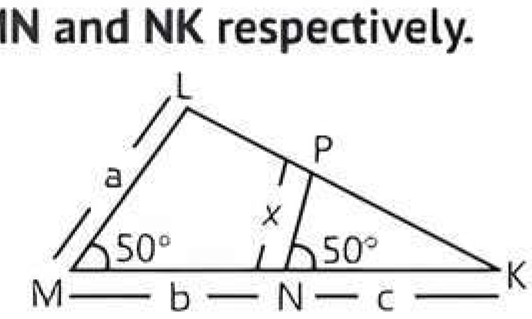


Q 3. X and Y are points on the sides AB and AC respectively of a triangle ABC, such that $\frac{AX}{AB} = \frac{1}{4}$, AY = 2 cm and YC = 6 cm. Find whether XY || BC or not. *[CBSE 2015]*

Q 4. In the given figure, QA $\perp$ AB and PB $\perp$ AB. If AO = 20 cm, BO = 12 cm, PB = 18 cm, find AQ. *[CBSE 2017]*



Q 5. In the given figure, $\angle M = \angle N = 50°$. Express x in terms of a, b and c where a, b and c are the lengths of LM, MN and NK respectively.

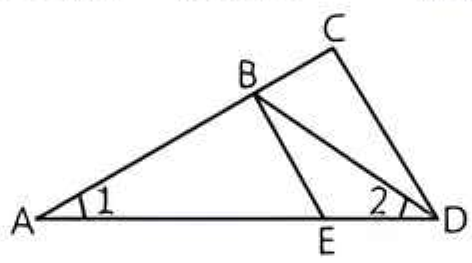


Q 6. A vertical stick which is 15 cm long casts a 12 cm long shadow on ground. At the same time, a vertical tower casts a 50 m long shadow on the ground. Find the height of the tower. *[CBSE 2016]*

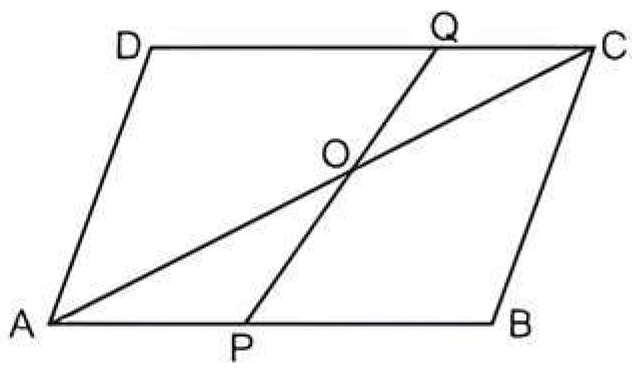
Q 7. A girl of height 100 cm is walking away from the base of a lamp post at a speed of 1.9 m/s. If the lamp is 5 m above the ground, find the length of her shadow after 4s. *[CBSE 2015]*

Q 8. ABCD is a trapezium such that BC||AD and AB = 4 cm. If the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{OB}{OD} = \frac{1}{2}$, then find CD.

Q 9. In the given figure below, $\frac{AD}{AE} = \frac{AC}{BD}$ and $\angle 1 = \angle 2$. Show that $\triangle BAE \sim \triangle CAD$. *[CBSE SQP 2022-23]*

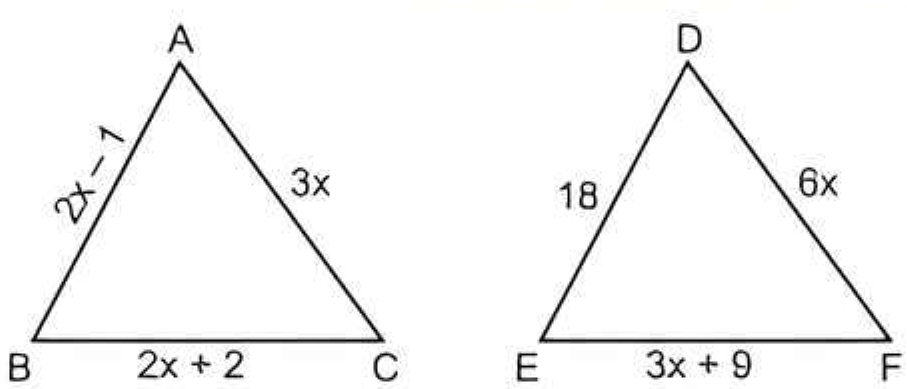


Q 10. ABCD is a parallelogram. Point P divide AB in the ratio 2 : 3 and point Q divides DC in the ratio 4 : 1. Prove that OC is half of OA. *[CBSE SQP 2023-24]*

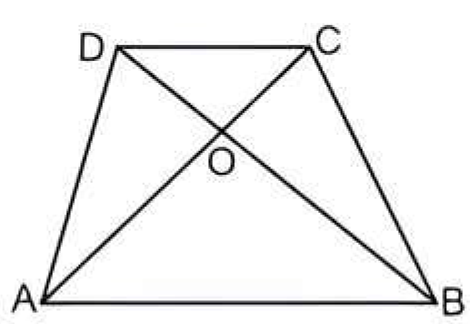


Short Answer Type-II Questions

Q 1. In the given figure, if $\triangle ABC \sim \triangle DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle. *[NCERT EXEMPLAR; CBSE 2020]*

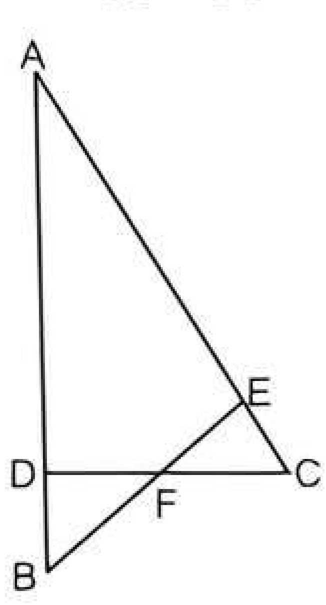


Q 2. Diagonals AC and BD of trapezium ABCD with AB || DC intersect each other at point O. Show that $\frac{OA}{OC} = \frac{OB}{OD}$. *[CBSE 2023, NCERT EXERCISE]*

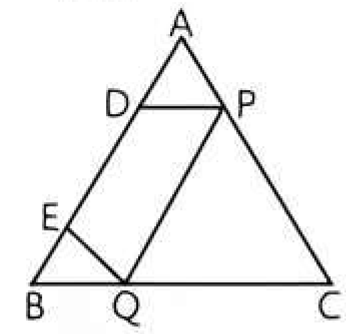


Q 3. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium. *[NCERT EXERCISE]*

Q 4. In the given figure $\angle CEF = \angle CFE$. F is the mid-point of DC. Prove that $\frac{AB}{BD} = \frac{AE}{FD}$ *[CBSE SQP 2023-24]*



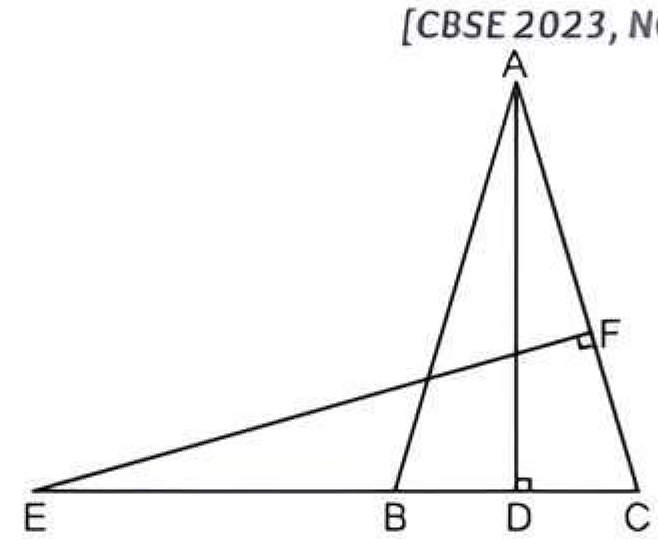
Q 5. In the given figure, D and E are two points lying on side AB, such that AD = BE. If DP || BC and EQ || AC, then prove that PQ || AB.



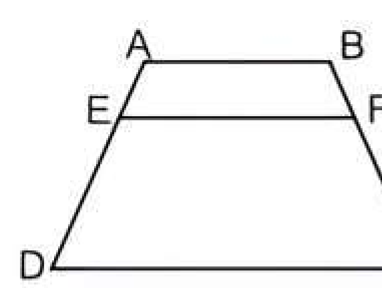
Q 6. If AD and PM are medians of triangles ABC and PQR, respectively, where $\triangle ABC \sim \triangle PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$. *[NCERT EXERCISE; CBSE 2017]*

Q 7. If two poles 5 m and 15 m high are 100 m apart, then find the height of the point of intersection of the line joining the top of each pole to the foot of the opposite pole. *[CBSE 2015]*

Q 8. In the given figure, E is a point on the side CB produced of an isosceles triangle ABC with AB= AC. If $AD \perp BC$ and $EF \perp AC$, then prove that $\triangle ABD \sim \triangle ECF$. *[CBSE 2023, NCERT EXERCISE]*



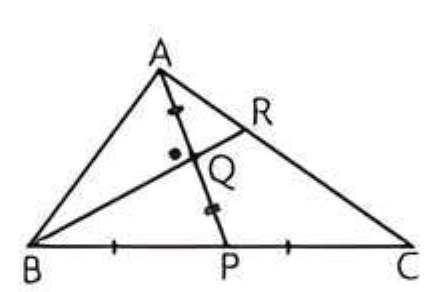
Q 9. In the given figure, EF || DC || AB. Prove that $\frac{AE}{ED} = \frac{BF}{FC}$.



Q 10. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD (produced) in E. Prove that EL = 2BL. *[CBSE 2023]*

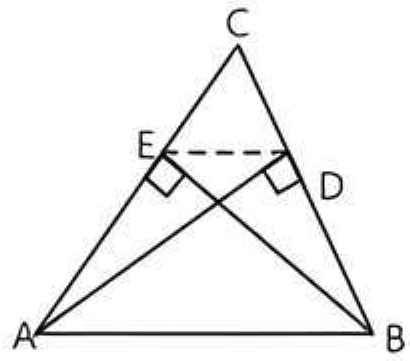
Long Answer Type Questions

Q 1. In the given figure of $\triangle ABC$, P is the mid-point of BC and Q is the mid-point of AP. If extended BQ meets AC in R, prove that $AR = \frac{1}{3}CA$. *[CBSE 2016]*



Q 2. Sides AB and AC and median AD of a $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$. *[NCERT EXERCISE; CBSE 2023, 17]*

Q 3. In the given figure, AD and BE are respectively perpendiculars to BC and AC. Show that:



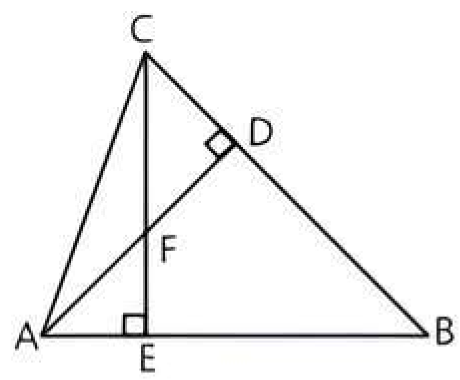
(i) $\Delta ADC \sim \Delta BEC$ (ii) $CA \times CE = CB \times CD$

(iii) $\Delta ABC \sim \Delta DEC$ (iv) $CD \times AB = CA \times DE$

Q 4. ABCD is a parallelogram, P is a point on side BC and DP when produced meets AB produced at L. Prove that:

(i) $\frac{DP}{PL} = \frac{DC}{BL}$ (ii) $\frac{DL}{DP} = \frac{AL}{DC}$

Q 5. In the given figure, AD and CE are two altitudes of ΔABC. Prove that:



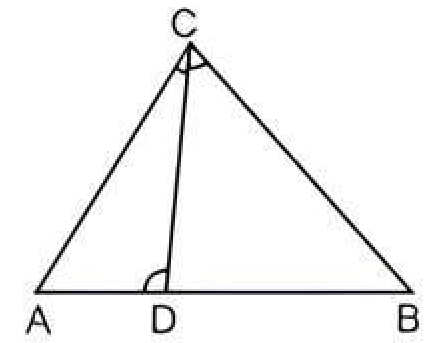
(i) $\Delta AEF \sim \Delta CDF$

(ii) $\Delta ABD \sim \Delta CBE$

(iii) $\Delta AEF \sim \Delta ADB$

(iv) $\Delta FDC \sim \Delta BEC$ *[NCERT EXERCISE]*

Q 6. In the given figure, $\angle ADC = \angle BCA$; prove that $\Delta ACB \sim \Delta ADC$. Hence find BD if AC = 8 cm and AD = 3 cm. *[CBSE SQP 2023-24, NCERT EXEMPLAR]*



Q 7. In a ΔPQR, N is a point on PR, such that $QN \perp PR$. If $PN \times NR = QN^2$, prove that $\angle PQR = 90°$.

[CBSE SQP 2023-24, NCERT EXEMPLAR]

Q 8. Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio. *[CBSE 2023]*

Using the above theorem, prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non-parallel sides in the same ratio.

[CBSE SQP 2022-23]

Or

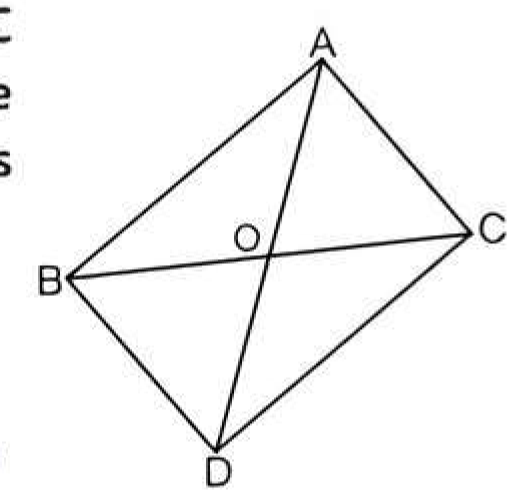
State and prove Basic Proportionality Theorem.

[CBSE SQP 2023-24]

Q 9. In the given figure, ΔABC and ΔDBC are on the same base BC. If AD intersects BC at O, prove that

$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.

[NCERT EXEMPLAR; CBSE 2023]



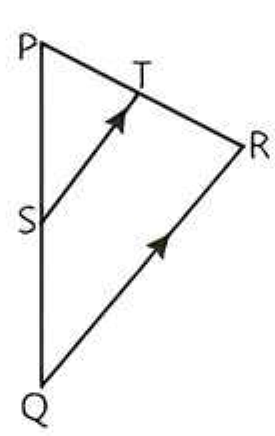
Solutions

Very Short Answer Type Questions

1. Given, PS = 4 cm, PQ = 9 cm and PR = 4.5 cm

> **TiP**
>
> *If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.*

Since, ST || QR, then by BPT, we have



$$\frac{PS}{PQ} = \frac{PT}{PR}$$

$$\Rightarrow \quad \frac{4}{9} = \frac{PT}{4.5}$$

or $$PT = \frac{4 \times 4.5}{9} = 2\text{ cm}$$

Hence, PT = 2 cm.

2. In ΔABC and ΔEFD,

$\angle A = \angle E$ and $\angle B = \angle F$ (Given)

$\therefore \Delta ABC \sim \Delta EFD$ (By AA similarity criterion)

$$\therefore \quad \frac{AB}{EF} = \frac{AC}{ED} \Rightarrow \frac{AB}{AC} = \frac{EF}{ED}$$ **Hence proved.**

3. In ΔABC, DE || BC.

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$ (By Thales theorem)

$$\Rightarrow \quad \frac{2}{3} = \frac{AE}{EC} \quad \Rightarrow \quad \frac{3}{2} = \frac{EC}{AE}$$

Adding 1 on both the sides, we get

$$\frac{3}{2} + 1 = \frac{EC}{AE} + 1 \quad \Rightarrow \quad \frac{3+2}{2} = \frac{EC + AE}{AE}$$

$$\Rightarrow \quad \frac{5}{2} \ \frac{AC}{AE} \quad \Rightarrow \quad \frac{5}{2} = \frac{18}{AE}$$

$$\Rightarrow \quad 5AE = 36$$

$$\Rightarrow \quad AE = \frac{36}{5} \Rightarrow AE = 7.2\text{cm}$$

4. In ΔPSQ and ΔPSR,

$\angle QSP = \angle RSP = 90°$

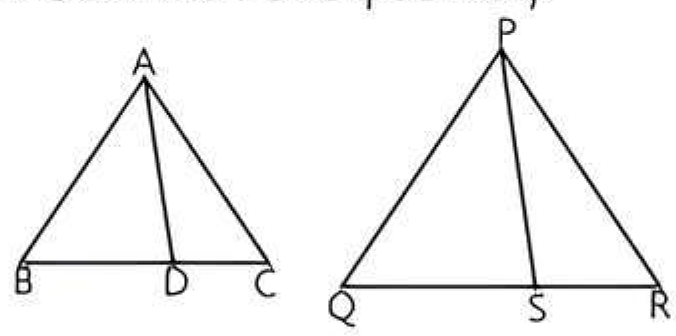
and $\angle QPS = \angle RPS = 35°$

$$\Delta PSQ \sim \Delta PSR \quad \text{(By AA similarity)}$$

$$\therefore \quad \frac{PQ}{PR} = \frac{SQ}{SR}$$

$$\Rightarrow \quad \frac{10}{8} = \frac{5}{x} \Rightarrow x = 4\text{cm}$$

5. Let ABC and PQR are two similar triangles with medians AD and PS respectively.



Then, $\frac{AD}{PS} = \frac{5}{7}$ (Given)

TR!CK

The ratio of the medians of two similar triangles is equal to the ratio of their corresponding sides.

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PS} = \frac{5}{7} \quad (\because \Delta ABC \sim \Delta PQR)$$

Hence, the ratio of corresponding sides is 5 : 7.

6. In ΔABC, DE || BC, so by BPT.

$$\frac{AD}{BD} = \frac{AE}{CE}$$

$$\Rightarrow \quad \frac{x}{x-2} = \frac{x+2}{x-1}$$

$$\Rightarrow \quad x(x-1) = (x-2)(x+2)$$

$$x^2 - x = x^2 - 4$$

$$\Rightarrow \quad -x = -4 \Rightarrow x = 4$$

7.

TiP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

In ΔABC, LM || AB

$$\therefore \quad \frac{AL}{LC} = \frac{BM}{MC} \quad \text{(By Thales theorem)}$$

$$\Rightarrow \quad \frac{AL}{AC - AL} = \frac{BM}{BC - BM}$$

$$\Rightarrow \quad \frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)} \Rightarrow \frac{x-3}{x+3} = \frac{x-2}{x+5}$$

$$\Rightarrow \quad (x-3)(x+5) = (x-2)(x+3)$$

$$\Rightarrow \quad x^2 + 2x - 15 = x^2 + x - 6$$

$$\Rightarrow \quad x = 9$$

Short Answer Type-I Questions

1. In Δ ABC, DE || BC, so by BPT.

$$\frac{AD}{BD} = \frac{AE}{CE} \quad \Rightarrow \quad \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$\Rightarrow \quad (4x-3)(5x-3) = (8x-7)(3x-1)$$

$$\Rightarrow \quad 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$$

$$\Rightarrow \quad 24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$$

$$\Rightarrow \quad 4x^2 - 2x - 2 = 0$$

$$\Rightarrow \quad 2(2x^2 - x - 1) = 0$$

TR!CK

$\because \quad 2 = 2 \times 1$

$\therefore$ *Here, we have taken 2 and 1 as a factors of 2.*

So, middle term, −1 = 1 −2.

$$\Rightarrow \quad 2x^2 - 2x + x - 1 = 0$$

$$\Rightarrow \quad 2x(x-1) + 1(x-1) = 0$$

$$\Rightarrow \quad (x-1)(2x+1) = 0$$

$$\Rightarrow \quad x - 1 = 0 \text{ and } 2x + 1 = 0$$

$$\Rightarrow \quad x = 1 \text{ and } x = -\frac{1}{2}$$

When $x = -\frac{1}{2}$, then AD, BD, AE and CE all are negative.

$$\therefore \quad x \neq -\frac{1}{2}$$

Hence, the value of x is 1.

COMMON ERR(!)R

Sometimes students take both value of x as a answer but it is wrong. Students should cross check the value of x.

2.

TiP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

In Δ POQ, DE || OQ

$$\therefore \quad \frac{PE}{EQ} = \frac{PD}{DO} \quad \text{(By BPT) ...(1)}$$

In Δ POR, DF || OR

$$\therefore \quad \frac{PF}{FR} = \frac{PD}{DO} \quad \text{(By BPT) ...(2)}$$

From eqs. (1) and (2), we get

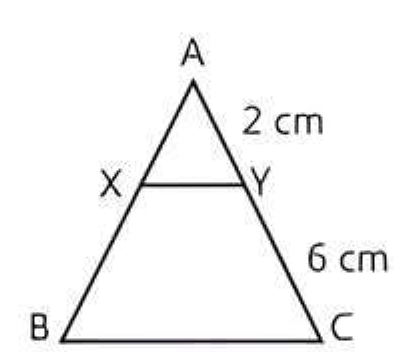
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

$\therefore$ EF || QR (Converse of BPT) **Hence proved.**

3. Given, $\frac{AX}{AB} = \frac{1}{4}$



Let $AX = k$, $AB = 4k$

$\therefore XB = AB - AX = 4k - k = 3k$

Now, $\frac{AX}{XB} = \frac{k}{3k} = \frac{1}{3}$

and $\frac{AY}{YC} = \frac{2}{6} = \frac{1}{3}$

TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

$\therefore \quad \frac{AX}{XB} = \frac{AY}{YC}$

Hence, XY || BC (By converse of BPT)

4. In ΔOAQ and ΔOBP,

$\angle OAQ = \angle OBP$ (Each 90°)

$\angle AOQ = \angle BOP$ (Vertically opposite angles)

$\therefore \quad \Delta OAQ \sim \Delta OBP$ (By AA similarity)

$\Rightarrow \quad \frac{AO}{BO} = \frac{AQ}{PB}$ (Corresponding sides are proportional)

$\Rightarrow \quad \frac{20}{12} = \frac{AQ}{18}$

$\Rightarrow \quad AQ = \frac{18 \times 20}{12} = 30$ cm

5. Given, $LM = a$, $PN = x$, $MN = b$ and $NK = c$

In Δ PNK and Δ LMK,

$\angle PNK = \angle LMK$ (Each 50°)

$\angle PKN = \angle LKM$ (Common angle)

$\therefore \quad \Delta PNK \sim \Delta LMK$ (By AA similarity)

So, $\frac{NK}{MK} = \frac{PN}{LM}$ (Corresponding sides are proportional)

$\Rightarrow \quad \frac{NK}{MN + NK} = \frac{PN}{LM}$

$\Rightarrow \quad \frac{c}{b+c} = \frac{x}{a}$ or $x = \frac{ac}{b+c}$

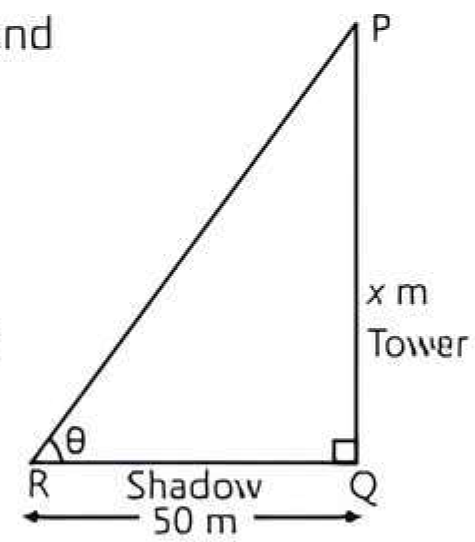
6. Let AB be the vertical stick and BC be its shadow.

Given, $AB = 15$ cm $= 0.15$ m

and $BC = 12$ cm $= 0.12$ m

Let PQ be the vertical tower and QR be its shadow.

In Δ ABC and Δ PQR,



TiP

Two triangles are similar, if their corresponding sides are in proportional.

$\angle ABC = \angle PQR$ (Each 90°)

$\angle ACB = \angle PRQ$

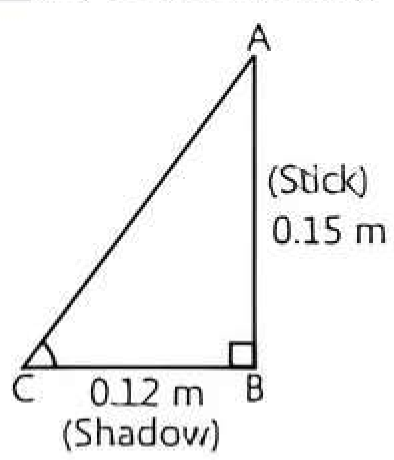
(Angular elevation of the sun at the same time)

$\therefore \Delta ABC \sim \Delta PQR$ (By AA similarity)

So, $\frac{AB}{PQ} = \frac{BC}{QR}$

(Corresponding sides are proportional)

$\Rightarrow \quad \frac{0.15}{x} = \frac{0.12}{50}$

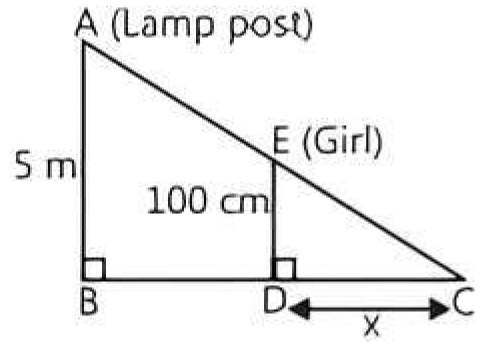


$\Rightarrow \quad x = \frac{0.15 \times 50}{0.12}$

or $\quad x = 62.5$ m

Hence, the height of the tower is 62.5 m.

7. Let AB be the lamp post and ED be the position of girl after 4s.



Given, height of the girl $ED = 100$ cm

and height of the lamp post $AB = 5$ m $= 500$ cm

Distance of the girl from lamp post after 4s

$= 1.9 \times 4 = 7.6$ m $= 760$ cm

($\because$ Distance = Speed × Time)

i.e., $BD = 760$ cm

Let $DC = x$ cm

TiP

Students should know about AAA criteria for similarity of triangles.

In Δ CDE and Δ CBA,

$\angle DCE = \angle BCA$ (Common angle)

$\angle CDE = \angle CBA$ (Each 90°)

$\therefore \quad \Delta CDE \sim \Delta CBA$ (By AA similarity)

So, $\frac{CD}{CB} = \frac{DE}{BA} \quad \Rightarrow \quad \frac{x}{x+760} = \frac{100}{500}$

$\Rightarrow \quad 5x = x + 760 \quad \Rightarrow \quad 4x = 760$

$\Rightarrow \quad x = 190$ cm

Hence, the length of her shadow after 4 s is 190 cm.

8.

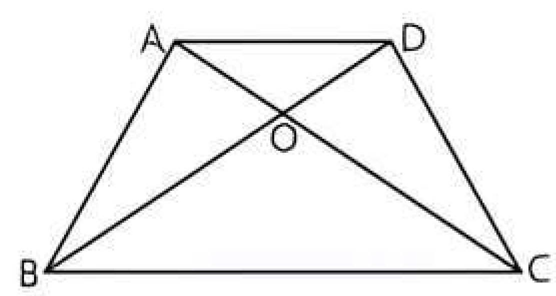
TiP

When two non-parallel rays intersect at a point, the angles formed between these rays at point of intersection in opposite directions are called vertically opposite angles (VOA).

In ΔAOB and ΔCOD, we have

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\frac{AO}{OC} = \frac{OB}{OD} = \frac{1}{2}$ (Given)



$\therefore \quad \Delta AOB \sim \Delta COD$ (By SAS similarity criterion)

$\therefore \quad \frac{AB}{CD} = \frac{AO}{OC} \Rightarrow \frac{4}{CD} = \frac{1}{2} \Rightarrow CD = 8$ cm

9. In ΔABD,

$\angle 1 = \angle 2$ (Given)

$\therefore \quad BD = AB$ (Sides opposite to equal angles are equal) ...(1)

Given,

$$\frac{AD}{AE} = \frac{AC}{BD}$$

Using eq. (1),

$$\frac{AD}{AE} = \frac{AC}{AB} \qquad ...(2)$$

In ΔBAE and ΔCAD,

$$\frac{AC}{AB} = \frac{AD}{AE} \qquad \text{(From eq. (2))}$$

$$\angle A = \angle A \qquad \text{(Common)}$$

$$\Delta BAE \sim \Delta CAD$$

(By SAS similarity criterion)

Hence proved.

10. Given ABCD is a parallelogram.

Let $AB = DC = a$

Now point P divides AB in the ratio 2 : 3

$$\therefore \quad AP = \frac{2}{2+3} \cdot a = \frac{2a}{5} \text{ and } BP = \frac{3}{2+5} \cdot a = \frac{3a}{7}$$

Again, point Q divides DC in the ratio 4 : 1.

$$\therefore \quad DQ = \frac{4}{4+1} \cdot a = \frac{4a}{5} \text{ and } CQ = \frac{1}{4+1} \cdot a = \frac{a}{5}$$

In ΔAPO and ΔCQO

$\angle AOP = \angle COQ$ (vertically opposite angles)

$\angle OAP = \angle OCQ$

(In parallelogram, AB||CD and AC is transverse)

$\therefore \quad \Delta APO \sim \Delta CQO$

(By AA similarity criterion)

$$\Rightarrow \quad \frac{AP}{CQ} = \frac{PO}{QO} = \frac{AO}{CO}$$

$$\Rightarrow \quad \frac{2a/5}{a/5} = \frac{AO}{CO} \quad \Rightarrow \quad \frac{AO}{CO} = \frac{2a}{a} = \frac{2}{1}$$

$$\Rightarrow \quad OC = \frac{1}{2} OA$$

Hence proved.

Short Answer Type-II Questions

1. Since, $\Delta ABC \sim \Delta DEF$, so ratio of their corresponding sides is equal.

So,
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\Rightarrow \quad \frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$$

$$\Rightarrow \quad \frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{1}{2}$$

Taking first and third parts, we get

$$\frac{2x-1}{18} = \frac{1}{2}$$

$$\Rightarrow \quad 2x = 9 + 1$$

$$\Rightarrow \quad x = \frac{9+1}{2} = \frac{10}{2} = 5$$

$\therefore$ Sides of ΔABC,

$$AB = (2x-1) = 2 \times 5 - 1 = 10 - 1 = 9 \text{ cm},$$

$$BC = 2x + 2 = 2 \times 5 + 2 = 10 + 2 = 12 \text{ cm}$$

and $AC = 3x = 3 \times 5 = 15$ cm

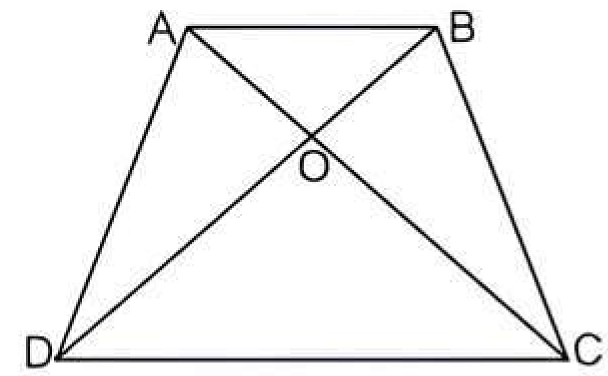
$\therefore$ Sides of ΔDEF,

$$DE = 18 \text{ cm},$$

$$EF = 3 \times 5 + 9 = 15 + 9 = 24 \text{ cm}$$

and $DF = 6x = 6 \times 5 = 30$ cm

2. Given: ABCD is a trapezium in which AB||CD and its diagonals AC and BD intersect at point O.



To Prove: $\frac{OA}{OC} = \frac{OB}{OD}$.

Proof : $\because$ AB||CD and AC is a transversal.

$\therefore \quad \angle OAB = \angle OCD$ (alternate angles)

and $\quad \angle AOB = \angle COD$

(vertically opposite angles)

Now, In ΔAOB and ΔOCD,

$$\angle AOB = \angle COD \text{ and } \angle OAB = \angle OCD$$

From AA similarity,

$$\Delta AOB \sim \Delta OCD$$

$$\frac{OA}{OC} = \frac{OB}{OD} \quad \text{[From the proportionality of side]}$$

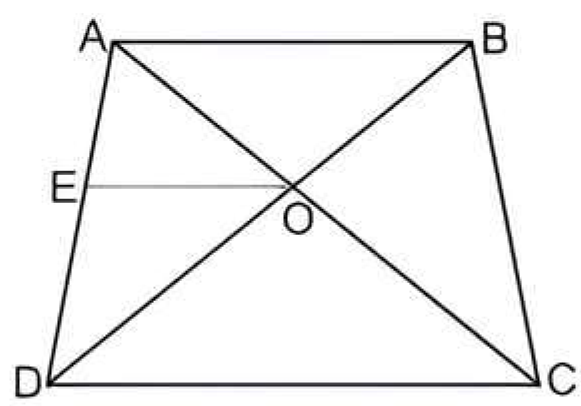
Hence proved.

3. Given : In a quadrilateral ABCD, $\frac{AO}{BO} = \frac{CO}{DO}$

To Prove : ABCD is a trapezium.

Construction : Let us draw a quadrilateral ABCD.

Draw a line OE || AB.



Proof : In Δ ABD, OE || AB

> **TiP**
>
> *If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.*

By using basic proportionality theorem, we get

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad ...(1)$$

But, it is given that

$$\frac{AO}{BO} = \frac{CO}{DO}$$

$$\Rightarrow \quad \frac{AO}{OC} = \frac{BO}{OD} \qquad ...(2)$$

From eqs. (1) and (2), we get

$$\frac{AE}{ED} = \frac{AO}{OC}$$

TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

$\Rightarrow$ EO || DC (By the converse of basic proportionality theorem)

$\Rightarrow$ AB || OE || DC

$\Rightarrow$ AB || DC

$\therefore$ ABCD is a trapezium. **Hence proved.**

4. Given: In ΔCAD, BE intersects CD at F, F is the mid-point of CD and $\angle CFE = \angle CEF$.

To Prove: $\frac{AB}{BD} = \frac{AE}{FD}$

Construction: Draw a line DG parallel to BE.

Proof: Since, F is the mid-point of DC, so CF = FD and

$\angle CEF = \angle CFE$ (Given)

$\therefore$ CF = CE

[$\because$ Sides opposite to equal angles are equal]

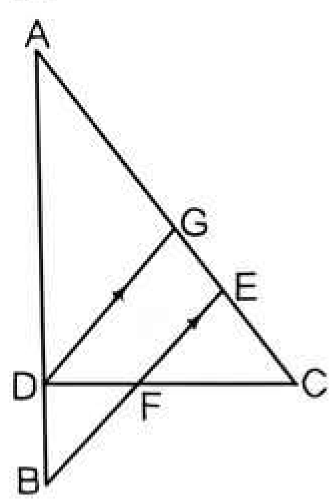
Now, in ΔCDG, FE || DG

$\Rightarrow \frac{CE}{GE} = \frac{CF}{DF}$ [By BPT]

$\Rightarrow \frac{CE}{GE} = \frac{CE}{DF}$ [$\therefore$ CF = CE]

GE = DF ... (1)

In ΔABE, DG || BE



$\Rightarrow \frac{AB}{DB} = \frac{AE}{GE}$ (By BPT)

$\Rightarrow \frac{AB}{DB} = \frac{AE}{DF}$ [From eq. (1)]

or $\frac{AB}{BD} = \frac{AE}{FD}$ **Hence proved.**

5. Given : AD = BE, DP || BC and EQ || AC

To Prove : PQ || AB

TiP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Proof : In Δ ABC, by BPT we have

$\frac{AD}{DB} = \frac{AP}{PC}$ ($\because$ DP || BC) ...(1)

Again, in Δ ABC, by BPT we have

$\frac{BE}{EA} = \frac{BQ}{QC}$ ($\because$ EQ || AC)

or $\frac{AD}{DB} = \frac{BQ}{QC}$ ...(2)

($\because$ AD = BE and EA = ED + DA = ED + BE = DB)

TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

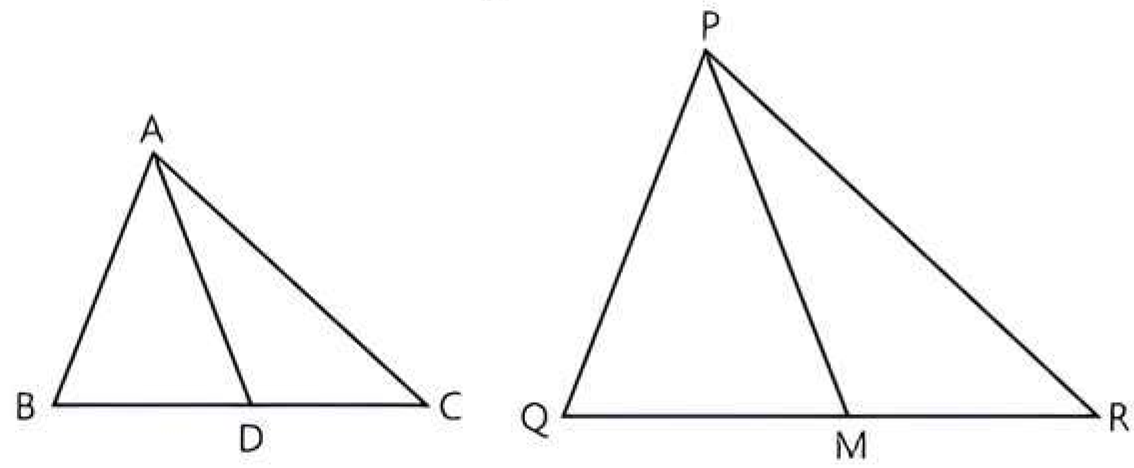
From eqs. (1) and (2), we get

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

In ΔABC, P and Q divide the sides CA and CB respectively in the same ratio.

$\therefore$ PQ || AB **Hence proved.**

6. Given: $\Delta ABC \sim \Delta PQR$



To Prove : $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof : Since, $\Delta ABC \sim \Delta PQR$

$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ ...(1)

(Corresponding sides are proportional)

Also, $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ ...(2)

(Corresponding angles are equal)

Since, AD and PM are medians, they will divide their opposite sides.

$\therefore BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$ ...(3)

From eqs. (1) and (3), we get

$\frac{AB}{PQ} = \frac{BD}{QM}$ ...(4)

Now, in Δ ABD and Δ PQM,

$\angle B = \angle Q$ [Using eq. (2)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Using eq. (4)]

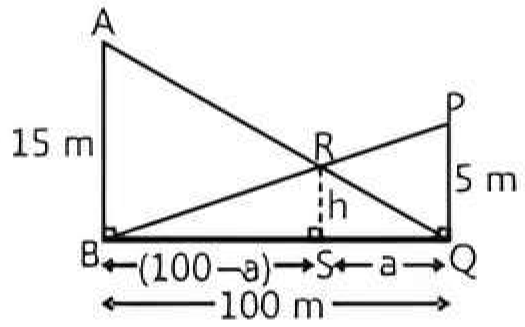
$\Delta ABD \sim \Delta PQM$ (By SAS similarity)

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

(Corresponding sides are proportional)

Hence proved.

7. Let the required height RS = h m and SQ = a



$\therefore \quad BS = 100 - a \qquad (\because BQ = 100 \text{ m})$

$AB \perp BQ$ and $RS \perp BQ$

$\Rightarrow \quad RS \parallel AB$

In Δ ABQ and Δ RSQ,

$\angle AQB = \angle RQS$ (Common angle)

$\angle QBA = \angle QSR$ (Corresponding angles)

$\Delta ABQ \sim \Delta RSQ$ (By AA similarity)

So, $\quad \dfrac{AB}{RS} = \dfrac{BQ}{SQ}$

(Corresponding sides are proportional)

$\Rightarrow \quad \dfrac{15}{h} = \dfrac{100}{a} \Rightarrow a = \dfrac{20h}{3}$ ...(1)

Now, in Δ RSB and Δ PQB,

$\angle RBS = \angle PBQ$ (Common angle)

$\angle RSB = \angle PQB$ (Corresponding angles)

$\Delta RSB \sim \Delta PQB$ (By AA similarity)

So, $\quad \dfrac{RS}{PQ} = \dfrac{BS}{BQ}$ (Corresponding sides are proportional)

$\Rightarrow \quad \dfrac{h}{5} = \dfrac{100 - a}{100} \Rightarrow 20h = 100 - a$

$\Rightarrow \quad 20h = 100 - \dfrac{20h}{3}$ [From eq. (1)]

$\Rightarrow \quad 20h + \dfrac{20h}{3} = 100$

$\Rightarrow \quad \dfrac{80h}{3} = 100$

$\Rightarrow \quad h = \dfrac{300}{80} = \dfrac{15}{4}$

Hence, the required height is $\dfrac{15}{4}$ m.

8. It is given that ABC is an isosceles triangle.

$\therefore \quad AB = AC$

$\angle ABD = \angle ECF$

[$\therefore$ Angles opposite to equal sides are equal]

In ΔABD and ΔECF

$\angle ADB = \angle EFC$ (Each 90°)

$\angle ABD = \angle ECF$ (Proved above)

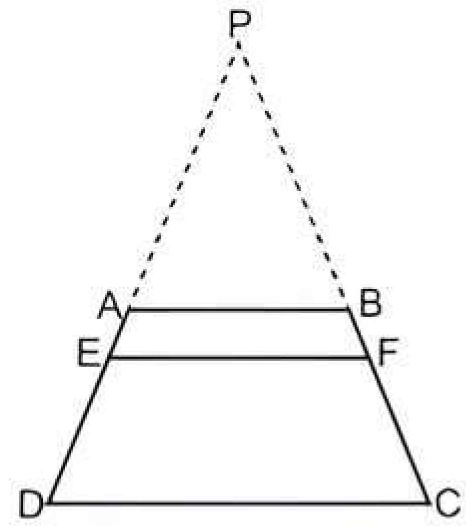
$\therefore \quad \Delta ABD \sim \Delta ECF$ (By AA similarity) **Proved.**

9. Given: In the given figure, EF || DC || AB.

To Prove: $\dfrac{AE}{ED} = \dfrac{BF}{FC}$.

Construction: Produce DA and CB to meet at P (say).

Proof: In ΔPEF, we have



AB || EF

$$\frac{PA}{AE} = \frac{PB}{BF} \qquad \text{(By Thales theorem)}$$

$$\Rightarrow \quad \frac{PA}{AE} + 1 = \frac{PB}{BF} + 1$$

$$\Rightarrow \quad \frac{PA + AE}{AE} = \frac{PB + BF}{BF}$$

$$\Rightarrow \quad \frac{PE}{AE} = \frac{PF}{BF} \qquad ...(1)$$

In ΔPDC, we have

EF || DC

$$\frac{PE}{ED} = \frac{PF}{FC} \qquad ...(2)$$

(By Basic Proportionality Theorem)

On dividing eq. (1) by eq. (2), we get

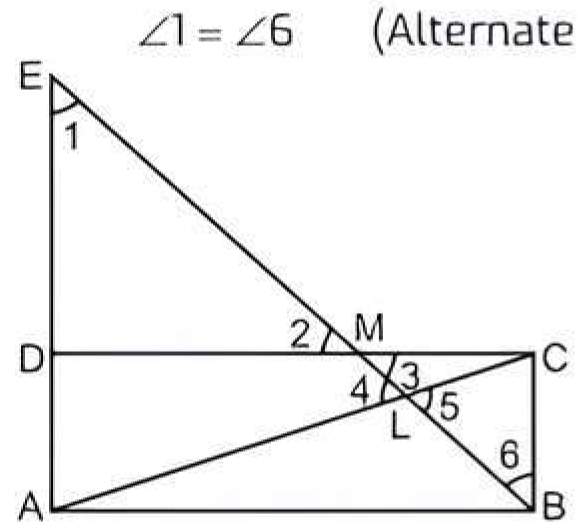
$$\frac{\frac{PE}{AE}}{\frac{PE}{ED}} = \frac{\frac{PF}{BF}}{\frac{PF}{FC}}$$

$$\Rightarrow \quad \frac{ED}{AE} = \frac{FC}{BF}$$

or $\quad \dfrac{AE}{ED} = \dfrac{BF}{FC}$ **Hence proved.**

10. In ΔDEM and ΔCBM

$\angle 1 = \angle 6$ (Alternate interior angles)



$\angle 2 = \angle 3$ (Vertically opposite angle)

DM = MC (M is the mid-point of CD)

$\Delta DEM \cong \Delta CBM$

(By AAS congruence criterion)

So, $\quad$ DE = BC [By CPCT]

Also, $\quad$ AD = BC

(Opposite sides of a parallelogram)

$\Rightarrow \quad AE = AD + DE = 2BC$

Now, $\quad \angle 1 = \angle 6$ and $\angle 4 = \angle 5$

$\Delta ELA \sim \Delta BLC$ (By AA similarity)

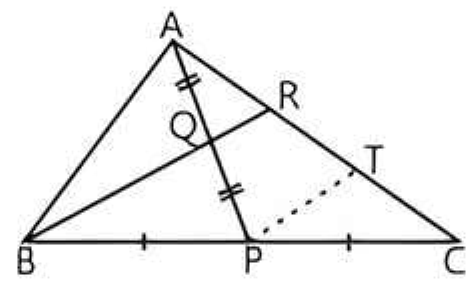
$\Rightarrow \quad \frac{EL}{BL} = \frac{EA}{BC}$

$\Rightarrow \quad \frac{EL}{BL} = \frac{2BC}{BC} = \frac{2}{1} \Rightarrow EL = 2BL$ **Hence proved.**

Long Answer Type Questions

1. **Given:** In $\triangle ABC$, P is the mid-point of BC and Q is the mid-point of AP.

To Prove: $RA = \frac{1}{3}CA$



Construction: Draw PT || BR.

Proof: In $\triangle$ CBR, PT || BR

$$\frac{CT}{TR} = \frac{CP}{PB} \quad \text{(By BPT)}$$

TiP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

$\Rightarrow \quad \frac{CT}{TR} = 1$ ($\because$ P is mid-point of BC *i.e.*, PB = CP)

$\Rightarrow \quad CT = TR$ ...(1)

In $\triangle$ APT, QR || PT

$$\frac{AQ}{QP} = \frac{AR}{RT} \quad \text{(By BPT)}$$

$\Rightarrow \quad 1 = \frac{AR}{RT}$ ($\because$ Q is mid-point of AP *i.e.*, AQ = QP)

$\Rightarrow \quad AR = RT$ ...(2)

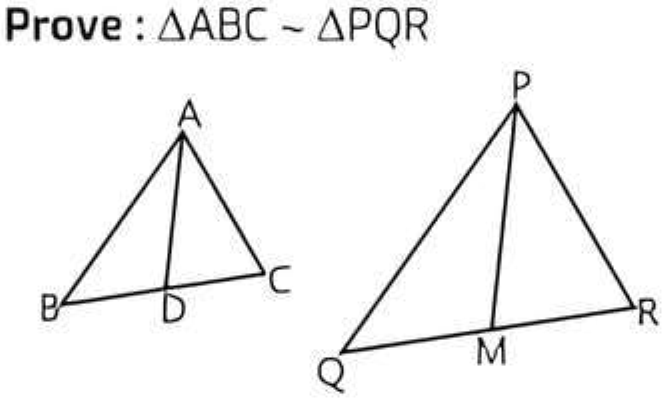
From eqs. (1) and (2), we get

$$AR = RT = CT$$

$$AR = \frac{1}{3}AC$$ **Hence proved.**

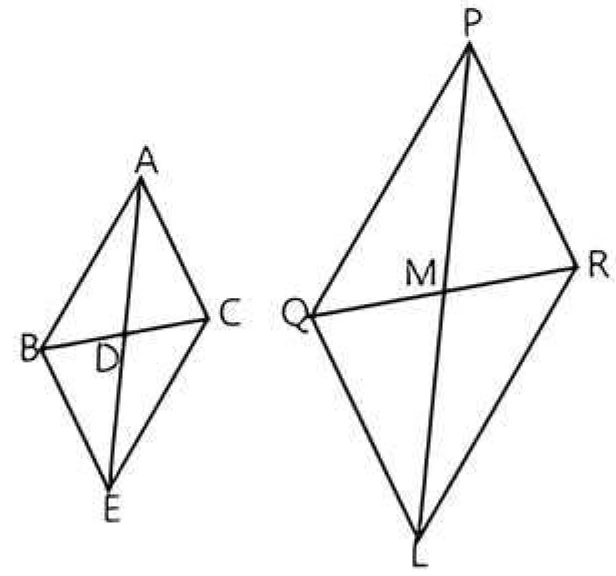
2. **Given :** $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

To Prove : $\triangle ABC \sim \triangle PQR$



Construction : Extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML.

Then join B to E, C to E, Q to L and R to L.



Proof : We know that, medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

and PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

$\therefore$ AC = BE and AB = CE

(Opposite sides of a parallelogram are equal)

Similarly, quadrilateral PQLR is a parallelogram.

$\therefore$ PR = QL and PQ = RL

$\because \quad \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ (Given)

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$ (Proved above)

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$

TiP

If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (SSS).

$\therefore \quad \triangle ABE \sim \triangle PQL$ (By SSS similarity)

$\Rightarrow \quad \angle BAE = \angle QPL$ ...(1)

(Corresponding angles of similar triangles)

Similarly, $\triangle AEC \sim \triangle PLR \Rightarrow \angle CAE = \angle RPL$ ...(2)

Adding eqs. (1) and (2), we get

$$\angle BAE + \angle CAE = \angle QPL + \angle RPL$$

$\Rightarrow \quad \angle CAB = \angle RPQ$ ...(3)

TiP

If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the triangles are similar (SAS).

In $\triangle$ ABC and $\triangle$ PQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

$\angle CAB = \angle RPQ$ [Using eq. (3)]

$\therefore \quad \triangle ABC \sim \triangle PQR$ (By SAS similarity)

Hence proved.

3. (i) In $\triangle ADC$ and $\triangle BEC$, we have

$\angle ADC = \angle BEC = 90°$ (Given)

$\angle ACD = \angle BCE$ (Common)

So, by AA criterion of similarity,

$$\triangle ADC \sim \triangle BEC$$

(ii) We have,

$\triangle ADC \sim \triangle BEC$ (As proved above)

$\Rightarrow \quad \frac{AC}{BC}=\frac{DC}{EC}$ ...(1)

$\Rightarrow \quad CA \times CE = CB \times CD$

(iii) In ΔABC and ΔDEC, we have

$\frac{AC}{BC}=\frac{DC}{EC}$ [From eq. (1)]

$\Rightarrow \quad \frac{AC}{DC}=\frac{BC}{EC}$

Also, $\angle ACB = \angle DCE$ (Common)

$\therefore \quad \Delta ABC \sim \Delta DEC$

(iv) We have,

$\Delta ABC \sim \Delta DEC$ (As proved above)

$\Rightarrow \quad \frac{AB}{DE}=\frac{AC}{DC}$

$\Rightarrow \quad AB \times DC = AC \times DE$

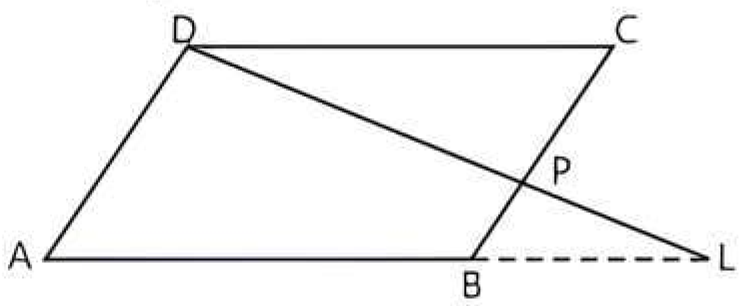
$\Rightarrow \quad CD \times AB = CA \times DE$ **Hence proved.**

4. Given: A parallelogram ABCD in which P is a point on side BC such that DP produced meets AB produced at L.

To Prove: (i) $\frac{DP}{PL}=\frac{DC}{BL}$ (ii) $\frac{DL}{DP}=\frac{AL}{DC}$

Proof: (i) In ΔALD, we have

BP || AD



$\therefore \quad \frac{LB}{BA}=\frac{LP}{PD}$

$\Rightarrow \quad \frac{BL}{AB}=\frac{PL}{DP}$

$\Rightarrow \quad \frac{BL}{DC}=\frac{PL}{DP}$ ($\because$ AB = DC)

$\Rightarrow \quad \frac{DP}{PL}=\frac{DC}{BL}$ (Taking reciprocals of both sides)

Hence proved.

(ii) From part (i), we have

$\frac{DP}{PL}=\frac{DC}{BL}$

$\Rightarrow \quad \frac{PL}{DP}=\frac{BL}{DC}$ (Taking reciprocals of both sides)

$\Rightarrow \quad \frac{PL}{DP}=\frac{BL}{AB}$ ($\because$ DC = AB)

$\Rightarrow \quad \frac{PL}{DP}+1=\frac{BL}{AB}+1$

$\Rightarrow \quad \frac{DP+PL}{DP}=\frac{BL+AB}{AB}$

$\Rightarrow \quad \frac{DL}{DP}=\frac{AL}{AB}$

$\Rightarrow \quad \frac{DL}{DP}=\frac{AL}{DC}$ ($\because$ AB = DC)

Hence proved.

5. (i) In ΔAEF and ΔCDF, we have

$\angle AEF = \angle CDF = 90°$ ($\because$ CE $\perp$ AB and AD $\perp$ BC)

$\angle AFE = \angle CFD$ (Vertically opposite angles)

$\therefore \quad \Delta AEF \sim \Delta CDF$ (By AA similarity)

(ii) In ΔABD and ΔCBE, we have

$\angle ABD = \angle CBE = \angle B$ (Common angle)

$\angle ADB = \angle CEB = 90°$

($\because$ AD $\perp$ BC and CE $\perp$ AB)

$\therefore \quad \Delta ABD \sim \Delta CBE$ (By AA similarity)

(iii) In ΔAEF and ΔADB, we have

$\angle AEF = \angle ADB = 90°$

($\because$ AD $\perp$ BC and CE $\perp$ AB)

$\angle FAE = \angle DAB$ (Common angle)

$\therefore \quad \Delta AEF \sim \Delta ADB$ (by AA similarity)

(iv) In ΔFDC and ΔBEC, we have

$\angle FDC = \angle BEC = 90°$

($\because$ AD $\perp$ BC and CE $\perp$ AB)

$\therefore \quad \angle FCD = \angle ECB$ (Common angle)

$\therefore \quad \Delta FDC \sim \Delta BEC$ (By AA similarity)

Hence proved.

6. Given $\angle BCA = \angle ADC$

In ΔACB and ΔADC,

$\angle BCA = \angle ADC$ (Given)

$\angle CAB = \angle DAC$ (Common angle)

$\therefore \quad \Delta ACB \sim \Delta ADC$ [By AA similarity]

Hence proved.

Also given, AC = 8 cm and AD = 3 cm.

We know that,

Sides of similar triangle are in same proportion.

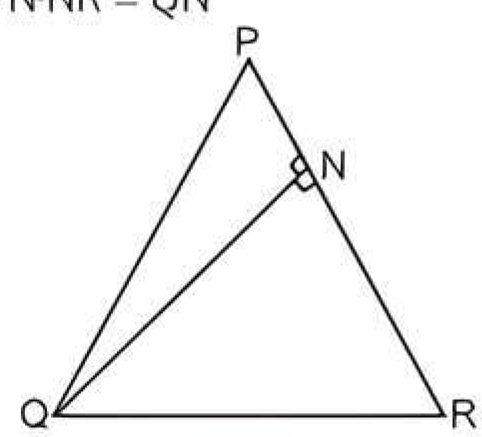
$\therefore \quad \frac{AC}{AD}=\frac{AB}{AC} \Rightarrow AC^2 = AB \times AD$

$AB = \frac{AC^2}{AD}=\frac{(8)^2}{3}=\frac{64}{3}$

So, $BD = AB - AD$

$=\frac{64}{3}-3=\frac{64-9}{3}=\frac{55}{3}$

7. Given: ΔPQR, *N* is a point on PR, such that QN $\perp$ PR and $PN \cdot NR = QN^2$



To Prove: $\angle PQR = 90°$

Proof: We have $PN \cdot NR = QN^2$

$\Rightarrow \quad PN \cdot NR = QN \cdot QN$

$\Rightarrow \quad \frac{PN}{QN}=\frac{QN}{NR}$ ...(1)

In ΔQNP and ΔRNQ, $\frac{PN}{QN}=\frac{QN}{NR}$

and $PNQ = \angle RNQ$ (Each equal to 90°)

$\therefore \quad \Delta QNP \sim \Delta RNQ$ (By SAS similarity criterion)

Then ΔQNP and ΔRNQ are equiangular.

i.e. $\angle PQN = \angle QRN$

$\angle RQN = \angle QPN$

On adding both sides, we get

$\angle PQN + \angle RQN = \angle QRN + \angle QPN$

$\Rightarrow \quad \angle PQR = \angle QRN + \angle QPN$ ...(2)

We know that sum of angles of a triangle = 180°

In ΔPQR, $\angle PQR + \angle QPR + \angle QRP = 180°$

$\Rightarrow \quad \angle PQR + \angle QPN + \angle QRN = 180°$

$[\because \angle QPR = \angle QPN$ and $\angle QRP = \angle QRN]$
[using Eq. (2)]

$\Rightarrow \quad \angle PQR + \angle PQR = 180°$

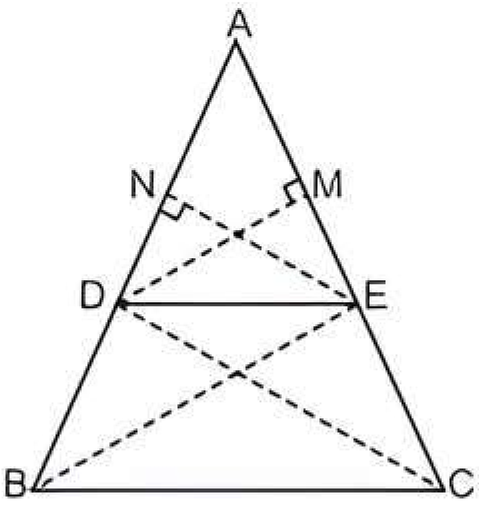
$\Rightarrow \quad 2\angle PQR = 180°$

$\Rightarrow \quad \angle PQR = \frac{180°}{2} = 90°$

$\angle PQR = 90°$ **Hence proved.**

8. Given: In ΔABC, $DE \parallel BC$

To Prove: $\frac{AD}{BD} = \frac{AE}{EC}$



Construction: Join BE and CD.

Draw $DM \perp AC$ and $EN \perp AB$

Proof: Here.

$$ar(\Delta ADE) = \frac{1}{2} \times AD \times EN$$

$(\because$ Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height})$

and $ar(\Delta BDE) = \frac{1}{2} \times DB \times EN$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad ...(1)$$

Also, $ar(\Delta ADE) = \frac{1}{2} \times AE \times DM$

and $ar(\Delta DEC) = \frac{1}{2} \times EC \times DM$

$$\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$$

Since, ΔBDE and ΔDEC are on the same base DE and between the same parallel lines BC and DE.

$\therefore \quad ar(\Delta BDE) = ar(\Delta CED)$

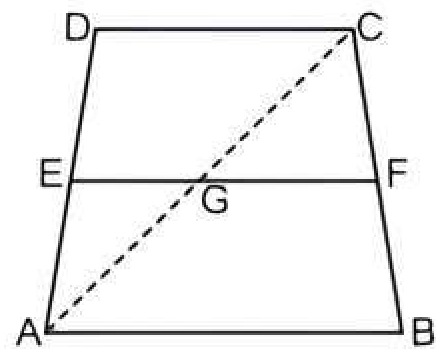
$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AE}{EC} \quad ...(2)$

From eqs. (1) and (2),

$\frac{AD}{DB} = \frac{AE}{EC}$ **Hence proved.**

Let ABCD be a trapezium with $DC \parallel AB$ and EF be a line parallel to AB.

To Prove: $\frac{DE}{EA} = \frac{CF}{FB}$



Construction: Join AC, meeting EF in G.

Proof: Given, $AB \parallel DC$ and $EF \parallel AB$

So, $EF \parallel DC$ (Lines parallel to the same line are parallel to each other)

In ΔABC, $GF \parallel AB$ (As $EF \parallel DC$)

$\frac{CG}{GA} = \frac{CF}{FB}$ (By BPT) ...(1)

In ΔADC, $EG \parallel DC$ (As $EF \parallel DC$)

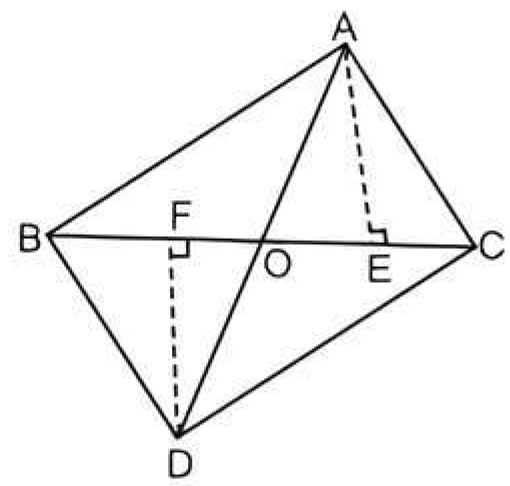
$\frac{DE}{EA} = \frac{CG}{GA}$ (By BPT) ...(2)

From eqs. (1) and (2),

$\frac{DE}{EA} = \frac{CF}{FB}$ **Hence proved.**

9. Given: ΔABC and ΔDBC are two triangles on same base BC. AC intersects BD at point O.

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$



Construction: Draw the perpendicular AE from vertex A to BC and DF from vertex D to BC.

Proof: Since perpendiculars AD and DF are drawn from vertices A and D respectively to BC, therefore ΔAEO and ΔDFO are right angled.

In right angled ΔAEO and ΔDFO,

$\angle AEO = \angle DFO$ (each 90°)

$\angle AOE = \angle DOF$ [vertically opposite angles]

$\Delta AEO \sim \Delta DFO$ (by AA similarity)

$\frac{AE}{DF} = \frac{AO}{DO}$ ...(1)

Now, area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BC \times AE$

and area of $\Delta DBC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BC \times DF$

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DBC} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF} = \frac{AE}{DF}$$

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AE}{DF} \quad ..... (2)$

From eqs. (1) and (2), we get

$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$ **Hence proved.**

Chapter Test

Multiple Choice Questions

Q 1. If in $\triangle ABC$, AB = 6 cm and DE || BC such that $AE = \frac{1}{4}AC$, then the length of AD is:

a. 2 cm b. 12 cm
c. 1.5 cm d. 4 cm

Q 2. In $\triangle PQR$ and $\triangle MNS$, $\frac{PQ}{NS} = \frac{QR}{MS} = \frac{PR}{MN}$, then symbolically we write as:

a. $\triangle QRP \sim \triangle SMN$ b. $\triangle PQR \sim \triangle SMP$
c. $\triangle PQR \sim \triangle MNS$ d. None of these

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): ABC is a triangle in which AB = AC and D is a point on AC such that $BC^2 = AC \times CD$. Then $\triangle ABC \sim \triangle BDC$ by SAS similarity criterion.

Reason (R): If two angles of one triangle are respectively equal to the two angles of another triangle, then the two triangles are similar. This is known as SAS similarity criterion.

Q 4. Assertion (A): In a $\triangle ABC$, D and E are points on sides AB and AC respectively, such that BD = CE. If $\angle B = \angle C$, then DE is not parallel to BC.

Reason (R): If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Fill in the Blanks

Q 5. Two polygons of the same number of sides are similar, if their corresponding angles are .... and their corresponding sides are, in the .... ratio.

Q 6. All equilateral triangles are .... (similar/not similar).

True/False

Q 7. Two figures having the same shapes is said to be similar figures.

Q 8. In two triangles, if one pair of the corresponding sides are proportional and the included angles are also equal, then two triangles are not similar.

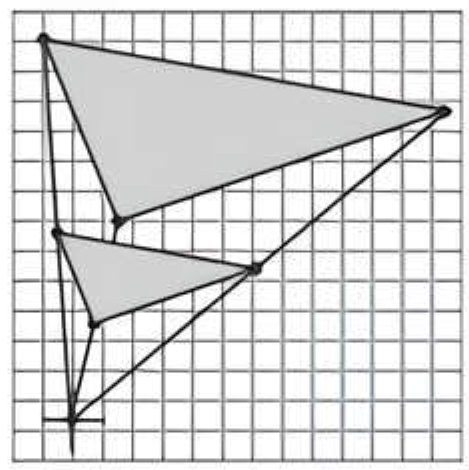
Case Study Based Question

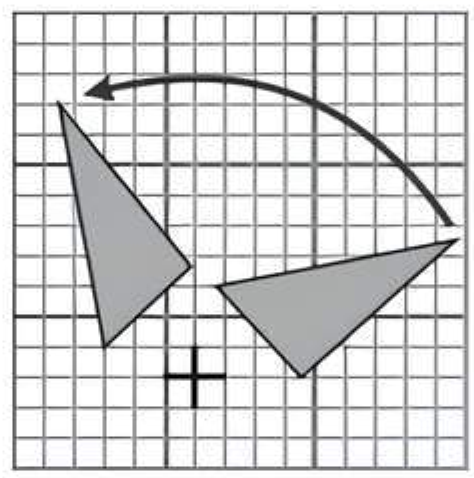
Q 9. Scale Factor: A scale drawing of an object is of the same shape as the object but of a different dimension.

The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio.

Similar Figures: The ratio of two corresponding sides in similar figures is called the scale factor.

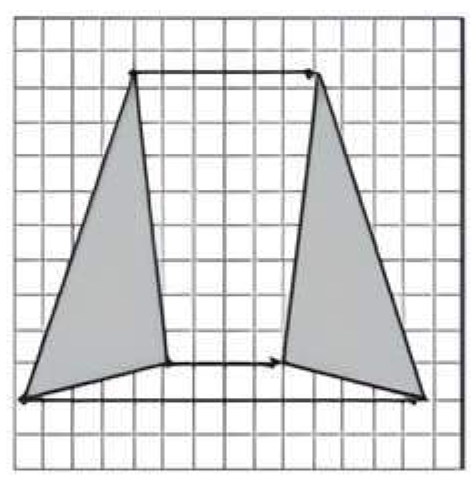
If one shape can become another using resizing then the shapes are similar.

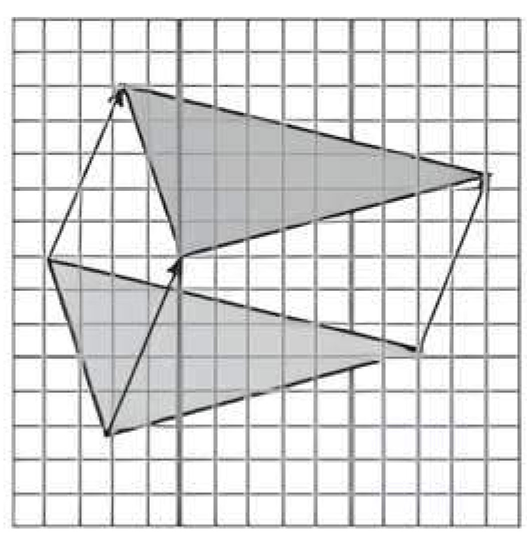




Rotation or Turn

Reflection or Flip





Translation or Slide

Hence, two shapes are similar when one can become the other after a resize, flip, slide or turn.

Based on the above information, solve the following questions:

(i) A model of an aeroplane is made to a scale of 1 : 400. Find the length (in cm) of the model, if the length of the aeroplane is 40 m.

(ii) Find the length (in m) of the aeroplane if length of its model is 16 cm.

(iii) A $\triangle ABC$ has been enlarged by scale factor $m = 2.5$ to the $\triangle A'B'C'$. Find the length of $A'B'$, if AB is 6 cm.

Or

Find the length of $C'A'$, if $CA = 4$ cm.

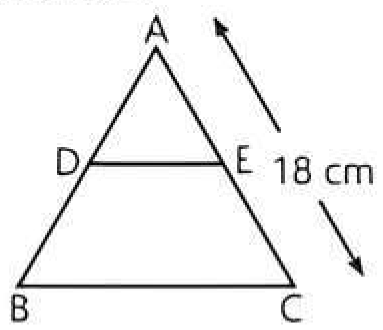
Very Short Answer Type Questions

Q 10. If the corresponding altitudes of two similar triangles are in the ratio 3 : 5, then find the ratio of their corresponding sides.

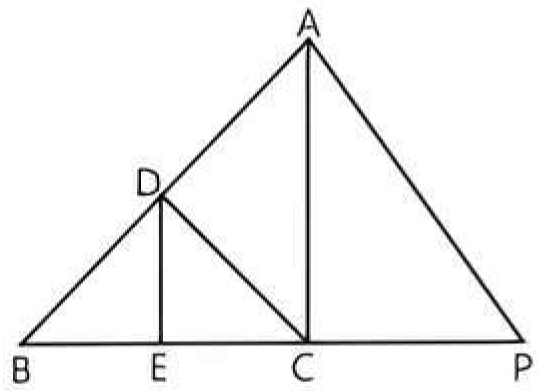
Q 11. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$ and $\angle B = \angle D$, then these triangles will be similar by which criteria?

Short Answer Type-I Questions

Q 12. In the given figure, DE || BC. If $\frac{AD}{DB} = \frac{2}{3}$ and $AC = 18$ cm, find AE.

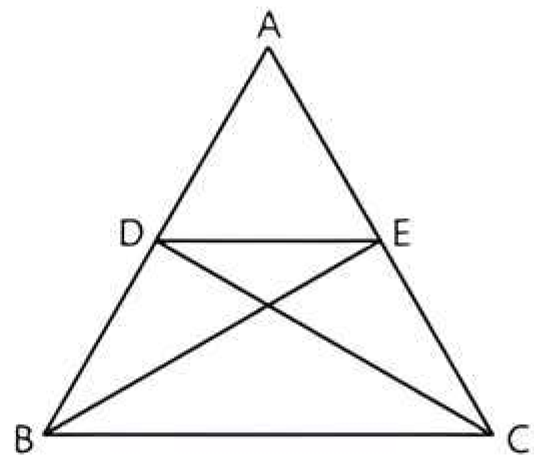


Q 13. In the given figure, DE || AC and DC || AP. Prove that $\frac{BC}{CP} = \frac{BE}{EC}$.

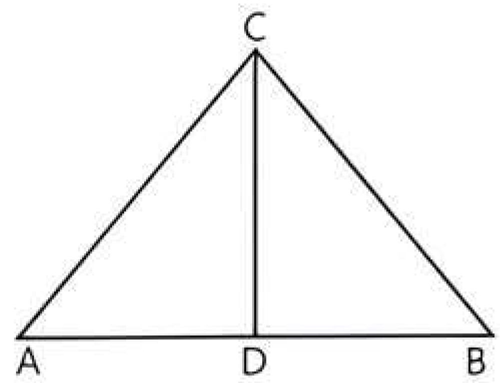


Short Answer Type-II Questions

Q 14. In the given figure, if $\triangle ABE \cong \triangle ACD$, prove that $\triangle ADE \sim \triangle ABC$.



Q 15. In the given figure, $\angle ACB = 90°$ and $CD \perp AB$. Prove that $\frac{CB^2}{CA^2} = \frac{BD}{AD}$.



Long Answer Type Question

Q 16. Through the vertex D of a parallelogram ABCD, a line is drawn to intersect the sides BA and BC produced at E and F respectively. Prove that

$$\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}.$$