EXERCISE 11.1

In each of following exercises 1 to 5, find equation of circle wi

1. Centre (0,2) and radius 2.

Sol. By Central form of equation of circle.

Required equation of circle is

$$(x-0)^2+(y-2)^2=(2)^2$$

or $x^2+y^2-4y+4=4$ or $x^2+y^2-4y=0$

2. Centre (−2,3) and radius 4.

Sol. Required eqn of circle is $[x-(-2)]^2+[y-3]^2=(4)^2$

or. $x^2+4x+4+y^2-6y+9=16$ or $x^2+y^2+4x-6y-3=0$

3. Centre $\left(\frac{1}{2},\frac{1}{4}\right)$ and radius $\frac{1}{12}$

Sol. Required eqn. is $\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2$

or $x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}=\frac{1}{144}$

or $144x^2-144x+36+144y^2-72y+9=1$

or $144x^2+144y^2-144x-72y+44=0$

or $36x^2+36y^2-36x-18y+11=0$

4. Centre (1,1) and radius $\sqrt{2}$

Sol. Required Eqn is $(x-1)^2+(y-1)^2=(\sqrt{2})^2$

or $x^2-2x+1+y^2-2y+1=2$

or $x^2+y^2-2x-2y=0$

5. Centre (−a,−b) and radius $\sqrt{a^2-b^2}$

Sol. Eqn. of circle is $(x+a)^2+(y+b)^2=\left(\sqrt{a^2-b^2}\right)^2$

or $x^2+2ax+a^2+y^2+2by+b^2=a^2-b^2$

or $x^2+y^2+2ax+2by+2b^2=0$

In each of following exercises from 6 to 9, find the centre and radius of circle.

6. $(x+5)^2+(y-3)^2=36$

Sol. Given eqn is $(x+5)^2+(y-3)^2=36$

or $[x-(-5)]^2+(y-3)^2=(6)^2$

Which is of Central form i.e. $(x-h)^2+(y-k)^2=(r)^2$ with $h=-5$, $k=3$, $r=6$

$\therefore$ Centre is $(-5,3)$ and radius is 6.

Q No 7. $x^2+y^2-4x-8y-45=0$

Sol: Given eqn is $x^2+y^2-4x-8y-45=0$

or $x^2+y^2+2(-2)x+2(-4)y+(-45)=0$

Comparing with General form of eqn. of Circle

$$x^2+y^2+2gx+2fy+c=0$$

$g=-2$, $f=-4$, $c=-45$

$\therefore$ Centre is $(-g,-f)$ ie $(2,4)$

and radius $=\sqrt{g^2+f^2-c}=\sqrt{(-2)^2+(-4)^2-(-45)}$

$=\sqrt{4+16+45}=\sqrt{65}$

Alternatively

$$x^2+y^2-4x-8y-45=0$$

or $(x^2-4x)+(y^2-8y)=45$

or $(x^2-4x+4)+(y^2-8y+16)=45+4+16$

or $(x-2)^2+(y-4)^2=(\sqrt{65})^2$ [Central form]

$\therefore$ Centre is $(2,4)$ and $r=\sqrt{65}$.

Q No 8. $x^2+y^2-8x+10y-12=0$

Sol Given eqn. is $x^2+y^2-8x+10y-12=0$

or $(x^2-8x)+(y^2+10y)=12$

or $(x^2-8x+16)+(y^2+10y+25)=12+16+25$

or $(x-4)^2+(y+5)^2=53$

or $(x-4)^2+(y-(-5))^2=(\sqrt{53})^2$

which is of Central form $(x-h)^2+(y-k)^2=r^2$

$\therefore$ Centre is $(+4,-5)$ and radius $=\sqrt{53}$

Q No. 9 $2x^2+2y^2-x=0$

Sol. Given eqn. is $2x^2+2y^2-x=0$

or $x^2+y^2-\frac{1}{2}x=0$ or $\left(x^2-\frac{1}{2}x\right)+y^2=0$

or $\left(x^2-\frac{1}{2}x+\frac{1}{16}\right)+y^2=\frac{1}{16}$

or $\left(x-\frac{1}{4}\right)^2+(y-0)^2=\left(\frac{1}{4}\right)^2$

which is of Central form $(x-h)^2+(y-k)^2=r^2$

$\therefore$ Centre is $\left(\frac{1}{4},0\right)$ and radius $=\frac{1}{4}$.

Q No 10. Find the equation of Circle passing through the points $(4,1)$ and $(6,5)$ and whose centre is on line $4x+y=16$.

Sol: Let the equation of the circle be

$$x^2+y^2+2gx+2fy+c=0 \quad \cdots (1)$$

Since (1) passes through $(4,1)$

$\therefore$ $16+1+8g+2f+c=0$

ie $8g+2f+c=-17 \quad \cdots (2)$

Also it passes through $(6,5)$

$\therefore$ $36+25+12g+10f+c=0$

or $12g+10f+c=-61 \quad \cdots (3)$

Subtracting (2) from (3), we get

$4g+8f=-44$ or $g+2f+11=0 \quad \cdots (4)$

Now Since Centre $(-g,-f)$ lies on $4x+y=16$

$\therefore$ $-4g-f=16$ or $4g+f+16=0 \quad \cdots (5)$

Solving (4) and (5) for g and f.

$$\frac{g}{32-11}=\frac{f}{44-16}=\frac{1}{1-8}$$

$$\therefore \frac{g}{21} = \frac{f}{28} = \frac{1}{-7}$$

or $g = -3$ and $f = -4$.

$\therefore$ from (2) $-24 - 8 + c = -17 \Rightarrow c = 15$

Putting values of g, f, c in (1) we got

$$x^2 + y^2 - 6x - 8y + 15 = 0$$

Which is required equation of circle.

QNo 11 Find the equation of circle passing through the points $(2,3)$ and $(-1,1)$ and whose centre is on the line $x - 3y - 11 = 0$

Sol: Let the equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots (1)$$

$\because$ It passes through $(2,3)$

$$\therefore 4 + 9 + 4g + 6f + c = 0$$

ie $$4g + 6f + c = -13 \quad \ldots (2)$$

Also it passes through $(-1,1)$

$$\therefore 1 + 1 - 2g + 2f + c = 0$$

$$\therefore -2g + 2f + c = -2 \quad \ldots (3)$$

Subtracting (3) from (2), we get

$$6g + 4f = -11 \quad \text{or} \quad 6g + 4f + 11 = 0 \quad \ldots (4)$$

$\because$ Centre $(-g,-f)$ lies on $x - 3y - 11 = 0$

$$\therefore -g + 3f - 11 = 0 \quad \ldots (5)$$

or $$g - 3f + 11 = 0 \quad \ldots (5)$$

Solving (4) and (5), we get

$$\frac{g}{44+33} = \frac{f}{11-66} = \frac{1}{-18-4}$$

$$\therefore \frac{g}{77} = \frac{f}{-55} = \frac{1}{-22}$$

$$\therefore g = -\frac{77}{22} = -\frac{7}{2} \quad \text{and} \quad f = \frac{-55}{-22} = \frac{5}{2}$$

Putting values of g, f in (3) we get

$7+5+c = -2 \Rightarrow c = -14.$

Putting values of g, f, c in (1) we get

$x^2+y^2-7x+5y-14=0$ which is reqd eqn of circle.

Q No. 12 Find the equation of circle with radius 5 whose centre lies on x-axis and passes through the pt. $(2,3)$.

Sol: Since the centre of circle lies on x-axis

$\therefore$ Let centre be $(h, 0)$.

Radius $= 5$

$\therefore$ Equation of circle (using central form)

$(x-h)^2+(y-0)^2 = (5)^2$

i.e. $(x-h)^2+y^2 = 25$ ---(1)

$\because$ It passes through the point $(2,3)$

$\therefore$ $(2-h)^2+(3)^2 = 25$

i.e. $(2-h)^2 = 25-9 = 16$ i.e. $2-h = \pm 4$

$\Rightarrow h = -2, 6$

When $h=-2$, (1) becomes $(x+2)^2+y^2 = 25$

i.e. $x^2+y^2+4x-21=0$

When $h=6$, (1) becomes $(x-6)^2+y^2 = 25$

i.e. $x^2+y^2-12x+11=0$

Which are required eqns of circles.

Q No. 13 Find the equation of circle passing through $(0,0)$ and making intercepts a and b on the coordinate axes.

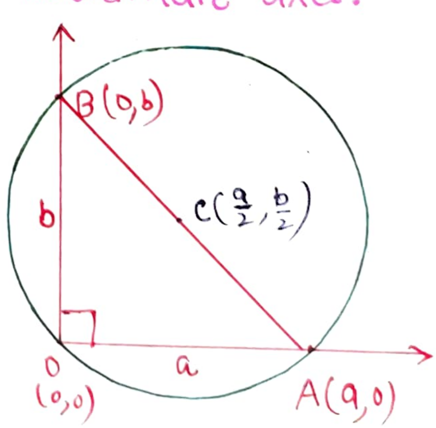
Sol: Let the circle meet x-axis in A and y-axis in B so that

$OA = a$, $OB = b$ where O is origin

$\therefore$ A is $(a,0)$, B is $(0,b)$

Now since $\angle AOB = 90^\circ$ $\therefore$ AB is diameter.

$\therefore$ Centre will be mid pt $C\left(\frac{a}{2}, \frac{b}{2}\right)$



and sadius $= \sqrt{\left(\frac{a}{2}-0\right)^2+\left(\frac{b}{2}-0\right)^2} = \sqrt{\frac{a^2}{4}+\frac{b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2}$

$\therefore$ Equation of Circle will be.

$$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b}{2}\right)^2 = \left(\frac{\sqrt{a^2+b^2}}{2}\right)^2$$

ie $\quad x^2+\frac{a^2}{4}-ax+y^2+\frac{b^2}{4}-by = \frac{a^2+b^2}{4}$

ie $\quad x^2+y^2-ax-by=0$

Alternatively Using Diameter form.

Equation of Circle with $(x_1,y_1),(x_2,y_2)$ as end points of diameter, is given by

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$\therefore \quad (x-a)(x-0)+(y-0)(y-b)=0$

$\therefore \quad x^2-ax+y^2-bx=0 \quad$ or $\quad x^2+y^2-ax-bx=0.$

Q.No. 14. Find the equation of Circle with centse $(2,2)$ and passing through point $(4,5)$

Sol: Let $C(2,2)$ be the centse of circle passing thsough the point $P(4,5)$

$\therefore$ Radius $= |CP| = \sqrt{(4-2)^2+(5-2)^2}$

$= \sqrt{4+9} = \sqrt{13}.$

$\therefore$ Equation of circle is

$$(x-2)^2+(y-2)^2=(\sqrt{13})^2$$

or $\quad x^2-4x+4+y^2-4y+4=13$

or $\quad x^2+y^2-4x-4y-5=0$

Q No 15: Does the point $(-2.5, 3.5)$ lie inside, outside or on circle $x^2+y^2=25$?

Sol: The equation of circle is

$$x^2+y^2=25$$

$\therefore$ Its centre is $C(0,0)$ and radius $= 5$

Let $P(-2.5, 3.5)$ be given point.

$$CP = \sqrt{(-2.5-0)^2+(3.5-0)^2}$$

$$= \sqrt{6.25+12.25}$$

$$= \sqrt{18.5}$$

$$\approx 4.301 < 5$$

$\therefore$ Distance CP is less than the radius of circle.

$\therefore$ P lies inside the circle

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