SECTION - I
Straight Objective Type

1. $(x-a)^2$ is a factor of x^3+px+q

$$x^3-2cx+a^2 \overline{\left) \begin{array}{l} \not{x^3}+0+px+q \\ \not{x^3}\mp 2ax^2 \pm a^2x \end{array} \right(} x+2a$$

$2ax^2+\left(p-a^2\right)x+q$

$2ax^2 \mp 2ax^2 \pm 2a^3$

0

$\left(p-a^2+4a^2\right)x=0$

$p-3a^2=0$

$p=3a^2$

$q-2a^3=0$

$q=2a^3$

Hence (c) is the correct option.

2. The homogeneous function of the second degree

$=ax^2+bxy+cy^2$

$2x-y$ is a factor of $ax^2+bxy+cy^2$

$\Rightarrow$ $(2x-y)(ax+by)=Ax^2+Bxy+Cy^2$

$x=y=1$

$(2\times1-1)(a+b)=Ax^2+Bxy+Cy^2$

$a+b=A+B+C$

Hence (a) is the correct option.

3. $15-x-6x^2=-\left(6x^2+x-15\right)$

$=-\left(6x^2-9x+10x-15\right)$

$=-(3x(2x-3)+5(2x-3))$
$=-(3x+5)(2x-3)$
$=(3x+5)(3-2x)$
Hence (c) is the correct option.

4. Let $f(x)=7x^3+6x^3-x+1$

$f(2)=7(2)^3+6(2)^2-2+1$
$=56+24-2+1$
$=81-2=79$
Hence the remainder is 79.
If $f(x)$ is divided by n-a then the remainder is $f(a)$
Hence (a) is the correct option.

5. If $\mathrm{f}(\mathrm{x})=\mathrm{x}^2$ $\qquad$ $\mathrm{g}(\mathrm{x})=\mathrm{x}^3$

then the value of $\dfrac{f(b)-f(a)}{g(b)-g(a)}$

$=\dfrac{b^2-a^2}{b^3-a^3}=\dfrac{(b-a)(b+a)}{(b-a)(b^2+ab+a^2)}$

$=\dfrac{a+b}{a^2+ab+b^2}$
Hence (b) is the correct option.

6. Let $f(x)=5x^3-(p+4)x^2-px-(p+y)$
$x-4$ is a factor of $f(x)$
$\Rightarrow$ $f(4)=0$
$\Rightarrow$ $f(4)=5(4^3)-(p+4)4^2$
$-p(4)-(p+4)=0$
$320-16p-4p-p-4=0$
$320-21p-68=0$

$252-21p=0$

$\Rightarrow \quad 21p=252$

$p=\dfrac{252}{21}=12$

Hence (b) is the correct option.

7. $x=\dfrac{1}{x}=7$

$\Rightarrow \quad \left(x-\dfrac{1}{x}\right)^3=7^3$

$\Rightarrow \quad x^3-\dfrac{1}{x^3}-3.x.\dfrac{1}{x}\left(x-\dfrac{1}{x}\right)=343$

$x^3-\dfrac{1}{x^3}-3.(7)=343$

$x^3-\dfrac{1}{x^3}-343+21=364$

Hence (c) is the correct option.

8. If x^2-3x+2 is a factor of the expression x^4-ax^2+b

$x^2+3x+2 \left| \begin{array}{l} \not{x^4}+0+ax^2+0+b \\ \not{x^4}+\underline{3x^3}+\underline{2x^2} \end{array} \right.$

$-\not{3x^5}-2x^2+0$

$\mp\not{3x^5}\mp 9x^2\mp 6x$

$\left[(a-2)+9\right]x^2=0$

$(a+7)x^2=0$

$a=-7$

Hence (a) is the correct option.

9. If $a+b+c=6$

$bc+ca+ab=11$

$abc = 6$

$(1-a)(1-b)(1-c)$

$= 1-(a+b+c)+(ab+bc+ca)-abc$

$= 1-6+11-6$

$= 0$

Hence (c) is the correct option.

10. If $x = \frac{a}{b+c}, y = \frac{b}{c+a}, z = \frac{c}{a+b}$

$\frac{1}{x} = \frac{b+c}{a}$

$\frac{1}{y} = \frac{c+a}{b}$

$\frac{1}{z} = \frac{a+b}{c}$

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$

$= \frac{bc(b+c)+ac(a+c)-ab(a+b)}{abc}$

Hence (a) is the correct option.

11. $8x^2 + 25y^3$

$= (2x)^3 + (5y)^3$

$= (2x+5y)+(4x^2+25y^2+10xy)$

Hence (a) is the correct option.

12. If a polynomial $p(x)$ is divided by $x-a$ then $p(a)$ is the remainder

If $p(a) = 0$

$\Rightarrow$ $x-a$ is a factor of $p(x)$ when

$p(x) = x^4 - 3x^2 + 2x + 1$ is divided by $x-1$ then the remainder is

$p(1)$

$p(1)=1^4-3(1)^2+2.2+1$

$1-3+2+1=1$

Hence (b) is the correct option.

13. Let $f(x)=ax^3+4x^2+3x-4$

$g(x)=x^2-4x+a$

When $f(x)$ is divided by $x-3$ the remainder is $f(3)$.

When $g(x)$ is divided by $x-3$ the remainder is $g(3)$

$f(3)=g(3)$

$\Rightarrow$ $a(3)^3+4(3)^2+3(3)-4$

$=3^3-4(3)+a$

$\Rightarrow$ $27a+36+9-4=27-12+a$

$\Rightarrow$ $27a+41=15+a$

$\Rightarrow$ $26a=-26$

$a=-1$

Hence (c) is the correct option.

14. $f(x)=x^4-2x^3+3x^3-ax+b$

If $f(x)$ is divided by $x-1$ then the remainder is $f(1)$ and $f(x)$ is divided by $x+1$ then the remainder is $f(-1)$

$f(1)=1^4-2(1)^3+3(1)^2-a(1)+b=5$

$1-2+3-a+b=5$

$-a+b=5$

$f(-1)=(-1)^4-2(-1)^3$

$+3(-1)^2-a(-1)+b=9$

$1+2+3+a+b=9$

$a+b=3$

$$-a+b=3$$
$$a+b=3$$
$$2b=6$$

$b=3$

$a=0$

then $f(x)=x^4-2x^3+3x^2+3$

when $f(x)$ is divided by $x-2$ then the remainder is $f(2)$

$f(2)=2^4-2.2^3+3(2)^2+3$

$=16-16+12+3$

$=15$

Hence (d) is the correct option.

15. $x^3-3x^2+4x-12$

$x^3-3x^2+4x-12$

x	1	−3	4	−12
	0	3	0	12
	1	0	4	0

$\Rightarrow$ $x^3-3x^2+4x-12=(x-3)(x^2+4)$

Hence, $x-3$ is a factor.

Hence (a) is the correct option.

16. Let $f(x)=3x^2+k$

$x+3$ is a factor of $f(x)$

$\Rightarrow$ $f(-3)=0$

$f(-3)=3(-3)^2+k=0$

$k+27=0$

$k=-27$

Hence (c) is the correct option.

17. x^3+10x^2+ax+6 is exactly divisible by $x-1$ as well as $x-2$

$$x^2-3x+2 \overline{\big)\ \cancel{x^3}+10x^2+ax+6\ \big(}\ x+$$
$$\cancel{x^3}\mp 3x^2 \pm 2x$$
$$\overline{7x^2+(a-2)x+6}$$

If $f(x)$ is divided by $x-a$ then the remainder is $f(a)$

$\Rightarrow$ x^2+10x^2+ax+6 is exactly divisible by

$x-1$ as well as $x-2$ then $a=-37$

Hence (a) is the correct option.

18. If $2x^3+ax^2+11x+a$ is exactly divisible by $2x-1$ then $f\left(\frac{1}{2}\right)=0$

$\Rightarrow$ $f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3$

$+a\left(\frac{1}{2}\right)^2+11\left(\frac{1}{2}\right)+a+3=0$

$\Rightarrow$ $\frac{1}{4}+\frac{a}{4}+\frac{11}{2}+a+3=0$

$\Rightarrow$ $1+a+22+4a+12=0$

$5a+35=0$

$\Rightarrow$ $a=-7$

Hence (b) is the correct option.

19. $x-2$ and $x-\frac{1}{2}$ are factors of px^2+5x+r then

$(x-2)\left(x-\frac{1}{2}\right)=px^2+5x+r$

$x^2-x\left(2+\frac{1}{2}\right)+1=px^2+5x+r$

$\Rightarrow$ $p=1,\ r=1$

$\Rightarrow$ $p=r$

Hence (a) is the correct option.

20. $x^2 - 1$ is a factor of

$ax^4 + bx^3 + cx^2 + dx + a$

$x - 1$ and $x + 1$ are factor of

$ax^4 + bx^3 + cx^2 + dx + c$

$a + b + c + d + e = 0$

and $a + c + e = b + d$

Since If $x + 1$ is a factor of $f(x)$ then the sum of coefficient of all even terms is equal to sum of odd coefficients.

Hence (b) is the correct option.

21. If $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by $3x - 1$ then the remainder is $f\left(\frac{1}{3}\right)$

$$f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 6\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right) - 4$$

$$= \frac{1}{27} - \frac{6}{9} + \frac{2}{3} - 4 = 0$$

$$= \frac{1 - 18 + 18 + 108}{27} = \frac{-107}{27}$$

Hence (c) is the correct option.

22. Let $f(x) = ax^3 + 3x^2 - 13$

$g(x) = 2x^3 - 5x + a$

$f(x)$ is divided by $x + 2$ then the remainder is $f(-2)$ and $g(x)$ divided by $x + 2$ then the remainder is $g(-2)$

$f(-2) = g(-2)$

$a(-2)^3 + 3(-2)^2 - 13$

$= 2(-2)^3 - 5(-2) + a$

$-8a + 12 - 13 = -16 + 10 + a$

$-99 = -5$

$a = \frac{5}{9}$

Hence (b) is the correct option.

23. Let $f(x) = x^2 + 2x^2 - 5ax - 7$

$g(x) = x^3 + ax^2 - 12x + 6$

If $f(x)$ is divided by $x+1$ then the remainder is $f(-1)$

$\Rightarrow \quad f(-1) = R_1$

If $g(x)$ is divided by $x-2$ then the remainder is $g(2)$

$\Rightarrow \quad g(2) = R_2$

$1 + 2(1)^2 - 5a(1) - 7 = R_1$

$\Rightarrow \quad 1 + 2 - 5a - 7 = R_1$

$\Rightarrow \quad -5a - 4 = R_1$

$2^3 + a(2)^2 - 12(2) + 6 - R_2$

$8 + 4a - 24 + 6 = R_2$

$4a - 10 = R_2$

$2R_1 + R_2 = -10a - 8 + 4a - 10 = 6$

$-6a - 18 = 6$

$-6a = 24$

Hence (d) is the correct option.

24. $x^n - y^n$ is always divisible by $x-1$ for all $'n'$ belongs to Natural numbers

Hence (a) is the correct option.

25. $x+3$ is a factor of $3x^2 + kx - 6$

$\Rightarrow \quad 3(-3)^2 + k(-3) - 6 = 0$

$\Rightarrow \quad -27 - 3k - 6 = 0$

$\Rightarrow \quad -3k - 33 = 0$

$k = -11$

Hence (b) is the correct option.

26. Let $f(x)=2x^3+ax^2+11x+a+3$ is exactly divisible by $2x-1$

$\Rightarrow$ $f\left(\frac{1}{2}\right)=0$

$\Rightarrow$ $2\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2+11\left(\frac{1}{2}\right)+a+3=0$

$\frac{1}{4}+\frac{a}{4}+\frac{11}{2}+a+3=0$

$\frac{1+a+22+4a+12}{4}=0$

$5a+35=0$

$\Rightarrow$ $5a=-35$

$a=-7$

Hence (a) is the correct option

27. Let $f(x)=a\left(b^2-c^2\right)$

$+b\left(c^2-a^2\right)+c\left(a^2-b^2\right)$

$\Rightarrow$ $a-b$ is a factor $f(x)$

Hence (a) is the correct option

28. Let $f(x)=x^3+ax^2-2x+a+4$

$x+a$ is a factor of $f(x)$

$\Rightarrow$ $f(-a)=0$

$(-a)^3+a(-a)^2-2(-a)+a+4=0$

$a^3-a^3+2a+a+4=0$

$-2a^3+3a+4=0$

$\Rightarrow$ $a=0$

Hence (b) is the correct option.

29. $x+a$ us a factor of x^n+a^n for any odd positive integer
Hence (b) is the correct option.

30. Let $f(x)=(x-b)^5+(b-a)^5$

$f(a)=(x-b)^5+(b-a)^5$

$\Rightarrow$ $a-b$ is factor of $f(x)$

Hence (a) is the correct option.

SECTION - II
Assertion - Reason Questions

31. $8x^3+125y^3=(2x)^3+(5y)^3$

$=(2x+5y)(4x^2+10xy+25y^2)$

Hence (a) is the correct option.

32. $\left(\frac{x}{y}+\frac{y}{x}\right)^2=\frac{x^2}{y^2}+2.\frac{x}{y}.\frac{y}{x}+\frac{y^2}{x^2}$

$=\frac{x^2}{y^2}+2+\frac{y^2}{x^2}$

Hence (a) is the correct option.

33. $x-3$ is a factor of the polynomial $x^3-3x^2+4x-12$

$$\begin{array}{r|rrrr} 3 & 0 & -3 & 4 & -12 \\ & 0 & 3 & 0 & 12 \\ \hline & 1 & 0 & 4 & \underline{|0} \end{array}$$

Hence (a) is the correct option.

34. $12-9+2+1=$

Sum of even term coefficients = Sum of odd terms

$\Rightarrow$ $x+1$ is a coefficient of the polynomial

$a-b, b-c, c-a$ are factors of

$a\left(b^2-c^2\right)+b\left(c^2-a^2\right)+c\left(a^2-b^2\right)$

Hence (b) is the correct option.

35. $x^4+2x^3-13x^2-14x+24$

Sum of coefficient of all terms

$=1+2-13-14+24=0$

Hence $x-1$ is a factor

1	1	2	-13	-14	24	
	0	1	3	-10	-24	
-2	1	3	-10	-24		0
	0	-2	-2	24		
3	1	+1	-12			0
	0	3	12			
	1	4	0			

Hence

$x^4+2x^3-13x^2-14x+24$

$=(x-1)(x+2)(x-3)(x+4)$

Hence (a) is the correct explination.

36. The highest power of 'n' in an algoric expression is called the degree of the polynomial.

$\Rightarrow$ $3x^2-5x^2+8x+9$ is a polynomial in x of degree 3.

Hence (a) is the correct option.

37. $x^2+6x+5=(x+5)(x+1)$

$\Rightarrow$ $(x+1)$ is a factor of x^2+6x+5

Hence (a) is the correct option.

38. $ax-by+bx-ay$

$=ax+bx-(by+ay)$

$=x(a+b)(x-y)$

$=(a+b)(x-y)$

$\Rightarrow$ $(a+b)$ is factor of

$ax-by+bx-ay$

$a^2-2ab+b^2=(a-b)^2$

Hence (c) is the correct option.

39. $$\frac{\left(a^2-b^2\right)^3+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)}{\left(a-b\right)^3+\left(b-c\right)^3+\left(c-a\right)^3}$$

If $a+b+c=0$ then

$a^3+b^3+c^3=3abc$

$a^2-b^2+b^2-c^2+c^2-a^2=0$

$\Rightarrow$ $\left(a^2-b^2\right)^3+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^2$

$=3\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)$

$a-b+b-c+c-a=0$

$\Rightarrow$ $\left(a-b\right)^3+\left(b-c\right)^3+\left(c-a\right)^3$

$=3(a-b)\ (b-c)(c-a)$

$$\frac{\left(a^2-b^2\right)+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)}{\left(a-b\right)^3+\left(b-c\right)^3+\left(c-a\right)^3}$$

$$=\frac{3\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)}{3\left(a-b\right)\left(b-c\right)\left(c-a\right)}$$

$=(a+b)(b+c)(c+a)$

$x-2y+2y-3z+3z-x=0$

$\Rightarrow$ $\left(x-2y\right)^3+\left(2y-3z\right)^3+\left(3z-x\right)^3$

$=3\left(x-2y\right)\left(2y-3z\right)\left(3z-x\right)$

$\Rightarrow$ $x-2y$ is a factor

Hence (b) is the correct option.

40. If a polynomial $p(x)$ is divided by $ax+b$, the remainder is the value of $p(x)$ at $x=-b/a$ i.e., $p(-b/a)$

$(x-a)\ (x-b)$ is a factor of a polynomial

$p(x)$ if $p(a)=0$ and $p(b)=0$

SECTION - III
Linked Comprehension Type

41. Let $f(x)=ax^2+bx^2+x-6$

$x+2$ is a factor of $f(x)$

$\Rightarrow$ $f(-2)=0$

$f(-2)=a(-2)^3+b(-2)^2+(-2)-6=0$

$-8a+4b-8=0$

$-8a+4b=8$

$-2a+b=2$

When $f(x)$ is divided by $x-2$ then it leaves the remainder 2

i.e., $f(2)=4$

$f(2)=a(2)^3+b(2)^2+2-6=4$

$8a+4b=8$

$2a+b=2$

$-2a+b=2$

$\underline{2a+b=2}$

$2b=4$

$b=2$

$a=0$

Hence (b) is the correct option.

42. $b=2$

Hence (c) is the correct option.

43. If $a=0, b=+2$

then $f(x)=2x^2+x=6$

$2x^2+x=6$

$=2x^2+4x-3x-2$

$=2x(x+2)-3(x+2)$

$=(2x-3)(x+2)$

$x+2$ is a factor

Hence (c) is the correct option.

44. $\frac{x^2}{4y^2}-\frac{2}{3}+\frac{4y^2}{9x^2}$

$=\left(\frac{x}{2y}\right)^2-2.\frac{x}{2y}.\frac{2y}{3x}+\left(\frac{2y}{3x}\right)^2$

$=\left(\frac{x}{2y}-\frac{2y}{3x}\right)^2$

Hence (a) is the correct option.

45. $25(3x-4y)^2-k\left(9x^2-16y^2\right)$ is a

$+16(3x+4y)^2$

perfect square

$\Rightarrow$ $5(3x-4y)^2-2.5.4.(3x-4y)$ is a

$(3x+4y)+4(3x+4y)^2$

perfect square

$\Rightarrow$ $k=2\times5\times4$

$= 40$

Hence (c) is the correct option.

46. If $\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+k$ is a perfect square i.e., $\left(x^2+\frac{1}{x}\right)^2-2-4\left(x+\frac{1}{x}\right)+6$

$\left(x^2+\frac{1}{x}\right)^2-4\left(x+\frac{1}{x}\right)+4=\left(x+\frac{1}{x}-2\right)^2$

$\Rightarrow$ $k=6$

Hence (b) is the correct option.

47. When $f(x)=x^2+4x+5$ is divided by $x-5$ then the remainder is $f(5)$

$f(5)=5^2+4(5)+5$

$=25+20+5$

$=45+5$

$=50$

Hence (a) is the correct option.

48. When $f(x)=x^3+5x-3$ is divided by $2x-1$ then the remainder is $f\left(\frac{1}{2}\right)$

$f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+5\left(\frac{1}{2}\right)-3$

$=\frac{1}{8}+\frac{5}{2}-3$

$=\frac{1+20-2y}{8}=\frac{-3}{8}$

Hence (b) is the correct option.

49. Let

$f(x)=(a-b)x^2+(b-c)x+(c-a)$ is divided by $x-1$ then the remainder is $f(1)$

$f(1)=(a-b)1^2+(b-c)1+(c-a)=0$

Hence (c) is the correct option.

50. $x-y+y-z+z-x=0$

$\Rightarrow$ $(x-y)^3+(y-z)^3+(z-x)^3$

$=3(x-y)(y-z)(z-x)$

$x-y$ is a factor of

$(x-y)^3+(y-z)^3+(z-x)^3$

Hence (a) is the correct option.

51. $p(q-r)+q(r-p)+r(p-q)=0$

$\Rightarrow$ $p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3$

$=3pqr(q-r)(r-p)(p-q)$

Hence $p, p-q$ are the factor of

$p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3$

Hence (a) are correct options.

52. $a^2-b^2+b^2-c^2+c^2-a^2=0$

$\Rightarrow$ $\left(a^2-b^2\right)^3+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^3$

$=3\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)$

$=3(a-b)(a+b)(b-c)$

$(b+c)(c-a)(a+a)$

Hence option (a) are correct options.

53. If $\dfrac{x^2-1}{x}=4$

$\Rightarrow$ $\dfrac{x-1}{x}=4$

$\Rightarrow$ $\dfrac{x^6-1}{x^3}=x^3-\dfrac{1}{x^3}$

$$=\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)$$

$$=4\left(\left(x-\frac{1}{x}\right)^2+2+1\right)$$

$$=4\left(4^2+2+1\right)$$

$$=4(16+3)=76$$

Hence (a) is the correct option.

54. The value of

$$216-144x+108x^2-27x^3$$

at $x=3$ is

$$=216-144\times3+108\left(3^2\right)-27\left(3^3\right)$$

$$=216-372+972-729$$

$$=27$$

Hence (b) is the correct option.

55. $(6a-5b)^3-(3a-4b)^3$

$$-3(3a-b)(6a-5b)(3a-4b)$$

$$=(6a-5b)^3-(3a-4b)^3-3(3a-b)$$

$$(6a-5b)(-3a+4b+6a-5b)$$

$$=\left[(6a-5b)-(3a-4b)\right]^3$$

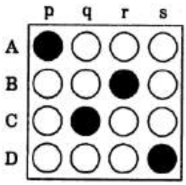
$$=[3a-b]^3$$

If $3a-b=0$

$\Rightarrow$ $(3a-b)^3=0$

Hence (c) is the correct option.

56.



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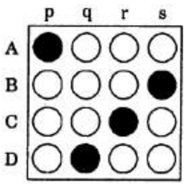

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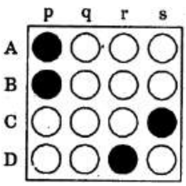
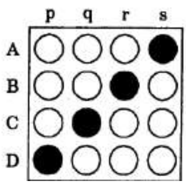


59.



60.