Lines and Angles

Complementary and Supplementary Angles

In some cases, pairs of angles show some special properties. Complementary and supplementary angles are examples of such pairs of angles, whose sum of measures exhibit a relationship.

So, the two important definitions of complementary and supplementary angles are as follows.

“If the sum of the measures of two angles is 90°, then the two angles are said to be complement to each other or complementary angles”.
“If the sum of the measures of two angles is 180°, then the two angles are said to be supplement to each other or supplementary angles.”

Let us solve some examples related to complementary and supplementary angles to understand the concept better.

Example 1:

Find the complement of the following angles.

52° and 75°

Solution:

Complement of 52° = 90° – 52°

= 38°

Complement of 75° = 90° – 75°

= 15°

Example 2:

Find the supplement of the following angles.

100° and 36°

Solution:

Supplement of 100° = 180° − 100°

= 80°

Supplement of 36° = 180° − 36°

= 144°

Example 3:

Can two acute angles be supplementary angles?

Solution:

No, two acute angles cannot be supplementary angles. The measure of an acute angle is less than 90°. Therefore, the sum of the measures of two acute angles is always less than 180°.

Example 4:

Write True or False.

(i) The opposite angles of a square are complementary angles.

(ii) Two obtuse angles can be supplementary angles.

Solution:

(i) False, as each angle of a square is a right angle.

Sum of two opposite angles = 90° + 90°

= 180°

(ii) False, because the measure of an obtuse angle is greater than 90°. Therefore, the sum of the measures of two obtuse angles cannot be 180°.

Example 5:

An angle measures four times its supplementary angle. Find the measures of both the angles.

Solution:

Let the measure of the supplementary angle of the given angle be $x°$ then the measure of the given angle will be $4x°$.
According to the definition of supplementary angles, we obtain

$4x° + x° = 180°$

$\Rightarrow 5x° = 180°$

$\Rightarrow x° = 36°$

$\therefore 4x° = 4 \times 36°$

$\Rightarrow 4x° = 144°$

Thus, the measures of the given angles are 144° and 36°.

Example 6:

The measure of an angle is 6° more than the twice of its complementary angle. Find the measures of both the angles.

Solution:

Let the measure of the complementary angle of the given angle be $x°$ then the measure of the given angle will be $2x° + 6°$.
According to the definition of complementary angles, we obtain

$2x° + 6° + x° = 90°$

$\Rightarrow 3x° + 6° = 90°$

$\Rightarrow 3x° = 84°$

$\Rightarrow x° = 28°$

$\therefore 2x° + 6° = 2 \times 28° + 6°$

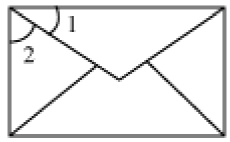
$\Rightarrow 2x° + 6° = 56° + 6°$

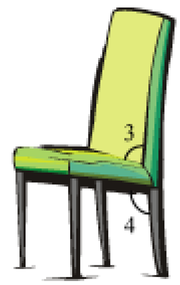
$\Rightarrow 2x° + 6° = 62°$

Thus, the measures of the given angles are 62° and 28°.

Adjacent Angles

Look at the following figures. You will see that some angles are marked in the figures as 1, 2, 3, and 4.



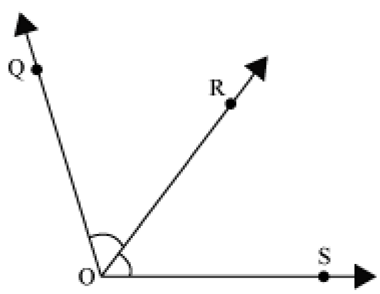


What did you notice?

In the figure of the envelope, ∠1 and ∠2 are adjacent angles. Similarly, in the figure of the chair, ∠3 and ∠4 are adjacent angles.

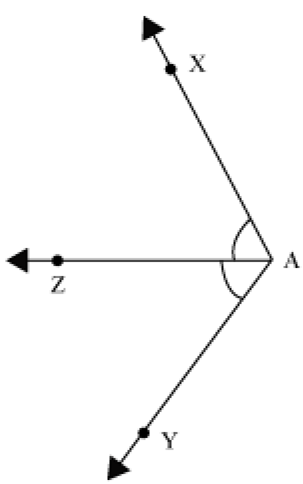
Some more adjacent angles are shown below.

(i)



∠QOR and ∠ROS

(ii)



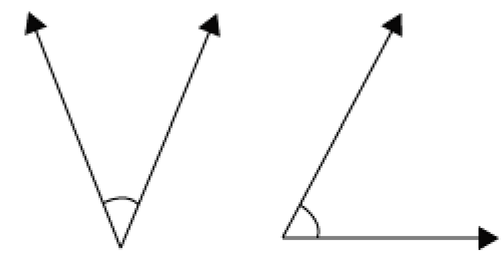
∠XAZ and ∠ZAY

Now, let us discuss some examples based on the above concept.

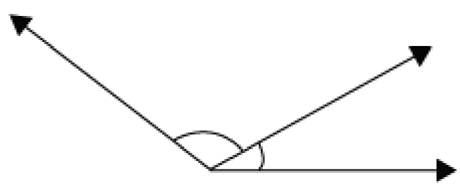
Example 1:

Find out the pairs of adjacent angles from the following pairs of angles.

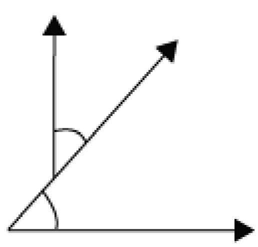
(i)



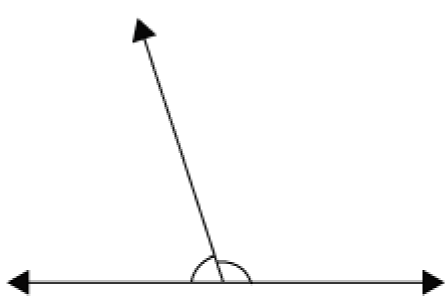
(ii)



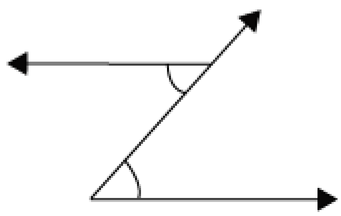
(iii)



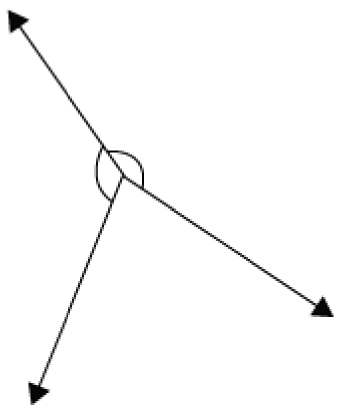
(iv)



(v)



(vi)



Solution:

The pairs of angles in figures **(ii), (iv),** and **(vi)** are adjacent angles. These angles share a common vertex and a common side but no common internal points.

In figure **(i),** the two angles are totally different.

In figures **(iii**) and **(v),** the given angles do not share a common vertex.

Hence, the pairs of angles in figures **(i), (iii),** and **(v)** are not adjacent angles.

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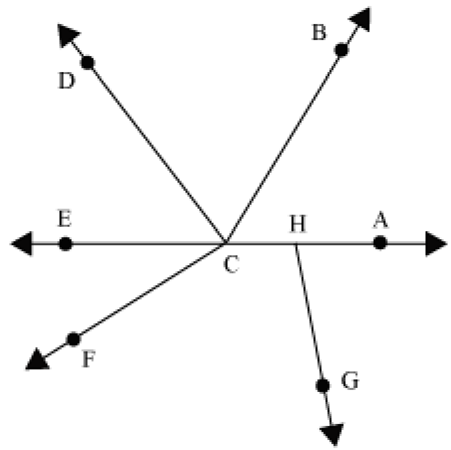
Example 2:

Find out whether the following statements are correct or incorrect.



1. **∠ACB is adjacent to ∠DCE.**

2. **∠ECF is adjacent to ∠BCE.**

3. **∠BCF is not adjacent to ∠BCA.**

4. **∠GHA is adjacent to ∠DCF.**

Solution:

1. False. The angles do not share a common side.

2. True. The angles share a common vertex and a common side but no common internal points.

3. False. ∠BCF is adjacent to ∠BCA.

4. False. The angles share neither a common side nor a common vertex.

Example 3:

The sum and the difference of two adjacent angles is 99° and 5° respectively. Find the measures of the two angles.

Solution:

Let the measure of one angle be x.

Since the sum of the measures of the angles is 99°, the measure of the other angle will be $99° - x$.

Now, we know that the difference between the angles is 5°.

$\therefore x - (99° - x) = 5°$

$\Rightarrow x - 99° + x = 5°$

$\Rightarrow 2x - 99° = 5°$

By adding 99° to both sides, we get

$2x - 99° + 99° = 5° + 99°$

$\Rightarrow 2x = 104°$

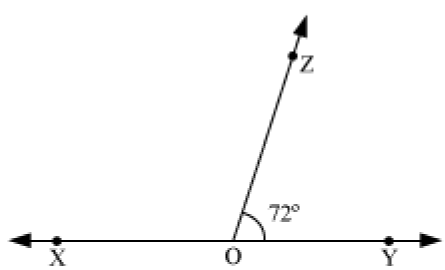
i.e., $x = \frac{104}{2}$

Hence, $x = 52°$

Hence, the measure of one angle is 52° and the measure of other angle is 99° − 52° = 47°.

Linear Pair of Angles

A ray OZ stands on a line XY such that ∠ZOY = 72°, as shown in the following figure.

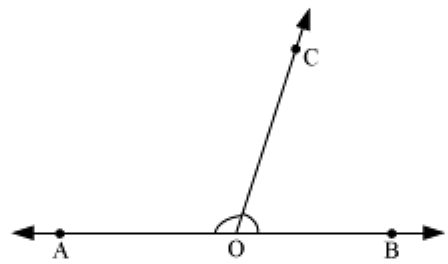


Can we find ∠XOZ?

We can find ∠XOZ using the concept of linear pair.

Therefore, first of all let us know about the linear pair of angles.

"**A linear pair is a pair of adjacent angles and whose non-common sides are opposite rays.**"

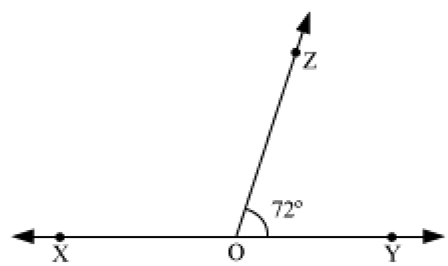


In the above figure, ∠AOC and ∠BOC form a linear pair. Their non-common arms form a straight line.

In other words, we can say, **"When a ray stands on a line, the two angles thus obtained form a linear pair".**

One very important property of a linear pair of angles is that **the sum of measures of linear pair of angles is equal to 180°.**

Now, let us solve the above given example using this property.



Here, we have to find ∠XOZ.

Now, ∠XOZ and ∠ZOY form a linear pair. Therefore,

$m\angle XOZ + m\angle ZOY = 180°$

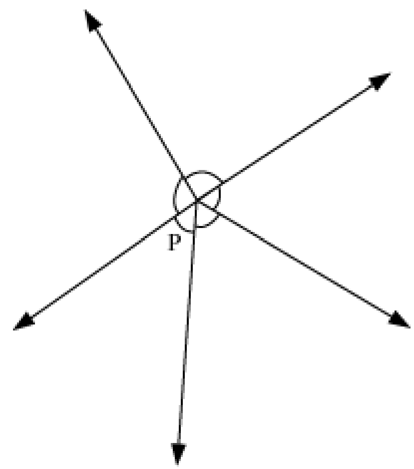
$m\angle XOZ + 72° = 180°$

$m\angle XOZ = 180° - 72°$

$m\angle XOZ = 108°$

We can also state this property as "**The sum of angles lying on a straight line is equal to 180°".**

Now consider the following figure:



In this figure, five angles have a common vertex, which is point P. In other words, the five angles make a complete turn and therefore the sum of these five angles will be equal to 360°. This is true no matter how many angles make a complete turn.

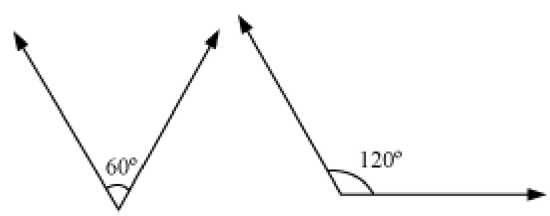
"The sum of angles around a point is equal to 360°".

Let us solve some examples to understand the above discussed concept better.

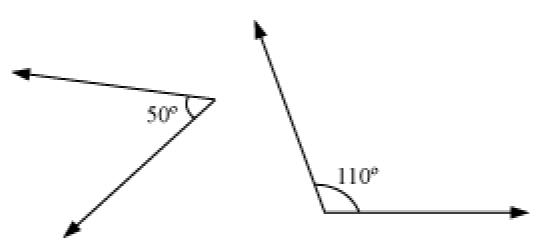
Example 1:

Which of the following pairs of angles forms a linear pair?

(i)



(ii)



Solution:

(i) Sum of measures of angles = 60° + 120°

= 180°

Thus, the given pair forms a linear pair of angles.

(ii) Sum of the measures of angles = 50° + 110°

= 160°

Thus, the given pair does not form a linear pair of angles.

Example 2:

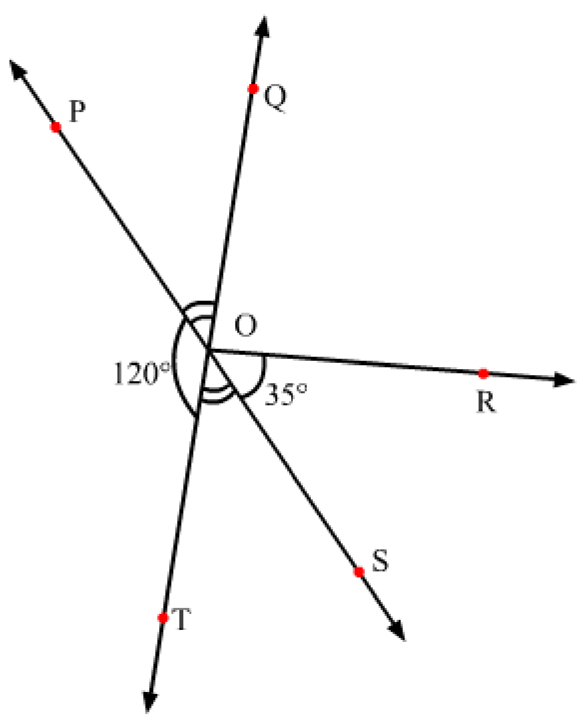
Can two acute angles form a linear pair of angles?

Solution:

Two acute angles cannot be supplementary angles as the sum of the measures of two acute angles is less than 180°. Therefore, two acute angles cannot form linear pair of angles.

Example 3:

In the given figure, ∠POQ = ∠SOT. Find the measure of ∠QOR.



Solution:

In the given figure, ∠POT and ∠SOT form a linear pair.

Therefore,

∠POT + ∠SOT = 180°

⇒ ∠SOT = 180° − 120° = 60°

Now, it is given that ∠SOT = ∠POQ

∴∠POQ = 60°

Now, we know that the sum of angles around a point is equal to 360°. Therefore,

∠POT + ∠SOT + ∠SOR + ∠QOR + ∠POQ = 360°

⇒ 120° + 60° + 35° + ∠QOR + 60° = 360°

⇒ ∠QOR = 360° − 275°

⇒ ∠QOR = 85°

Example 4:

Let $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ be three rays such that $\overrightarrow{OC}$ is between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. If ∠BOC + ∠COA = 180°, then prove that A, O, B are collinear, that is, they lie on the same straight line.

Solution:

Given: $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are three rays. ∠BOC and ∠COA are adjacent angles formed by the rays such that ∠BOC + ∠COA = 180°.

To Prove: A, O, B all lie on the same line.
Construction:
Extend $\overline{AO}$ to D such that A, O, D all lie on the same line AD.
Proof:

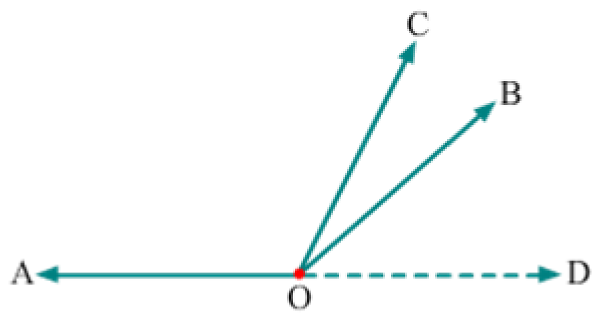


Fig (1)

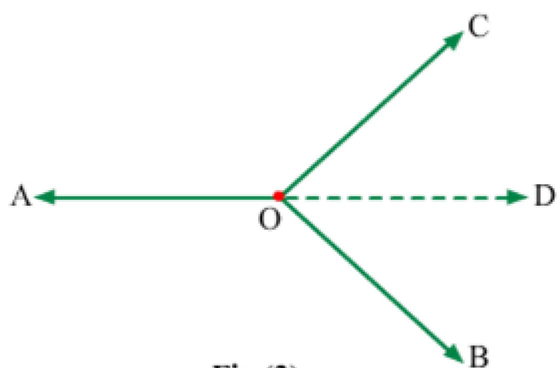


Fig (2)

Since AD is a line,

∴ ∠DOC + ∠COA = 180° ...(1)

Given, ∠BOC + ∠COA = 180° ...(2)

From (1) and (2), we have

∠DOC + ∠COA = ∠BOC + ∠COA

⇒ ∠DOC = ∠BOC

Now, there are two possibilities:

1. $\overrightarrow{OB}$ lies between $\overrightarrow{OD}$ and $\overrightarrow{OC}$ (fig 1)
2. $\overrightarrow{OD}$ lies between $\overrightarrow{OB}$ and $\overrightarrow{OC}$ (fig 2)

From the first possibility, we have

∠BOC = ∠DOC = ∠DOB + ∠DOC⇒∠DOB = 0

From the second possibility, we have

∠DOC = ∠BOC = ∠BOD + ∠DOC ⇒∠BOD = 0

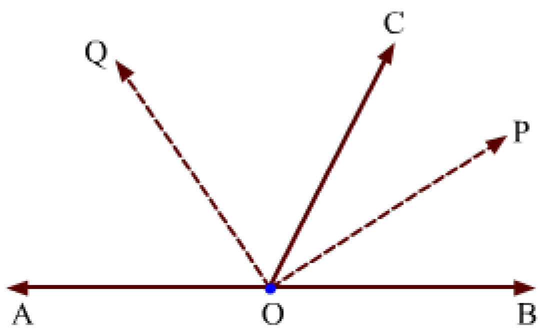
Thus, the angle between the rays $\overrightarrow{OB}$ and $\overrightarrow{OD}$ is zero. Hence, the rays $\overrightarrow{OB}$ and $\overrightarrow{OC}$ coincides. This means that the points B and O are on the same line AD. Thus, A, O, B are collinear.

Example 5:

Let AB be a straight line and $\overrightarrow{OC}$ be any ray standing on it. If $\overrightarrow{OP}$ is the bisector of ∠BOC and $\overrightarrow{OQ}$ is the bisector of ∠COA, then prove that ∠POQ = 90°.

Solution:

Given: $\overrightarrow{OP}$ is the bisector of ∠BOC and $\overrightarrow{OQ}$ is the bisector of ∠COA.
To prove: ∠POQ = 90°
Proof:



Since $\overrightarrow{OP}$ is the bisector of ∠BOC, we have

$$\angle POC = \frac{1}{2}\angle BOC \quad ...(1)$$

Since $\overrightarrow{OQ}$ is the bisector of ∠COA, we have

$$\angle COQ = \frac{1}{2}\angle COA \quad ...(2)$$

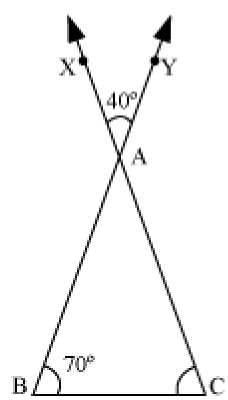
Adding (1) and (2), we have

$$\angle POQ = \frac{1}{2}(\angle BOC + \angle COA)$$

$$\Rightarrow \angle POQ = \frac{1}{2}\times 180° = 90°$$

Vertically Opposite Angles

Let us consider the following figure.



Can you find the measure of ∠ACB?

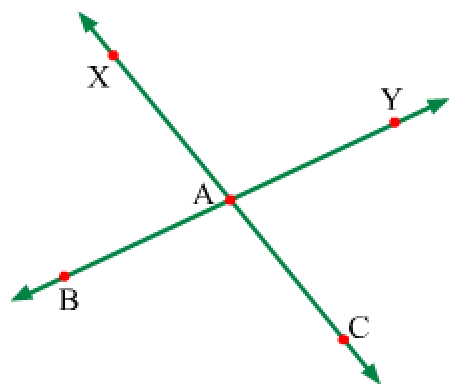
In the given figure, one angle of ΔABC is given as 70° and the other two angles are unknown. We know that the sum of all the three angles of a triangle is 180°. We can find ∠ACB, if we can find ∠BAC.

Therefore, first of all, we have to find ∠BAC. We can easily do so by using the concept of vertically opposite angles.

Therefore, let us know about vertically opposite angles and its property.

Let us prove the property discussed in the video.

Given: Two lines XC and YB intersecting each other at a point A.



To prove: ∠XAY = ∠BAC and ∠XAB = ∠CAY

Proof: Since lines XC and YB intersect each other, we have

∠XAB + ∠CAB = 180° ...(1) (Linear pair)

Also, ∠CAB + ∠CAY = 180° ...(2) (Linear pair)

From (1) and (2), we have

⇒ ∠XAB + ∠CAB = ∠CAB + ∠CAY

⇒ ∠XAB = ∠CAY

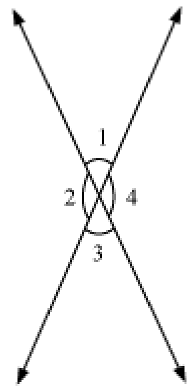
Similarly, we can easily prove that ∠XAY = ∠BAC.

Therefore, if two lines intersect each other then vertically opposite angle are equal.

Let us solve some more examples to understand the concept better.

Example 1:

In the given figure, if ∠1 = 75°, then find ∠2, ∠3, and ∠4.



Solution:

∠1 and ∠4 form a linear pair of angles.

∴ ∠1 + ∠4 = 180°

⇒ 75° + ∠4 = 180°

⇒ ∠4 = 180° − 75°

⇒ ∠4 = 105°

Now, ∠1 and ∠3 are vertically opposite angles,

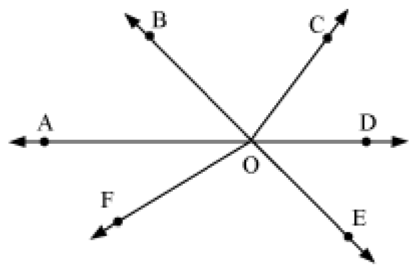
∴ ∠1 = ∠3 = 75°

Again, ∠2 and ∠4 are vertically opposite angles,

∴ ∠2 = ∠4 = 105°

Example 2:

In the given figure, name the pairs of vertically opposite angles.



Solution:

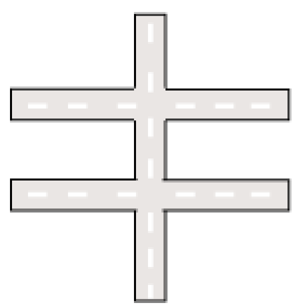
There are two pairs of vertically opposite angles in the given figure.

(1) ∠AOB and ∠DOE

(2) ∠BOD and ∠AOE

Transversal on Two Lines

Suppose there are two parallel roads in a city and a third road is cutting the two roads as shown in the figure below:

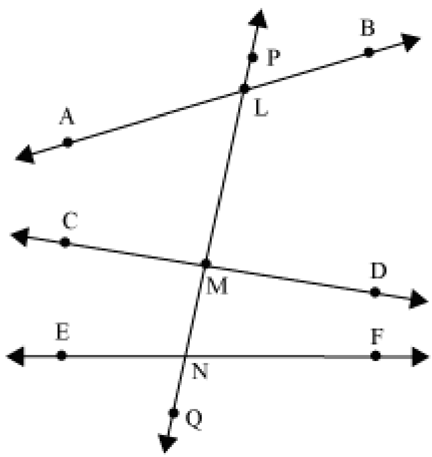


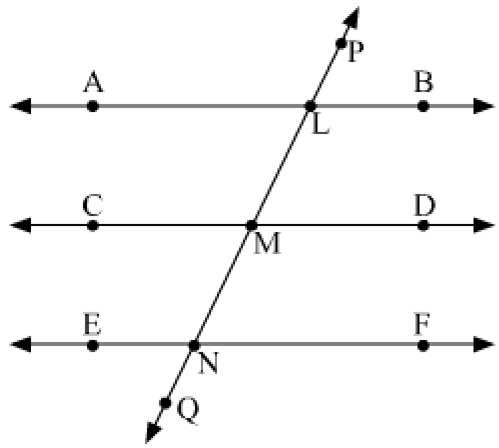
We can see that the third road is intersecting the two roads at two distinct points. We call this as a **transversal** to the other two roads.

The line which intersects two or more lines at distinct points is called transversal to the lines.

Also, the part of the transversal which lie between the the lines is known as intercept.

For example, consider the following figures.

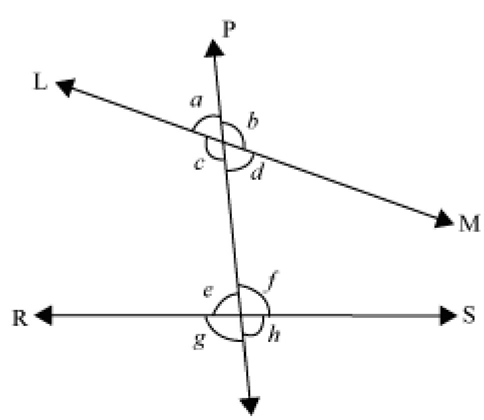


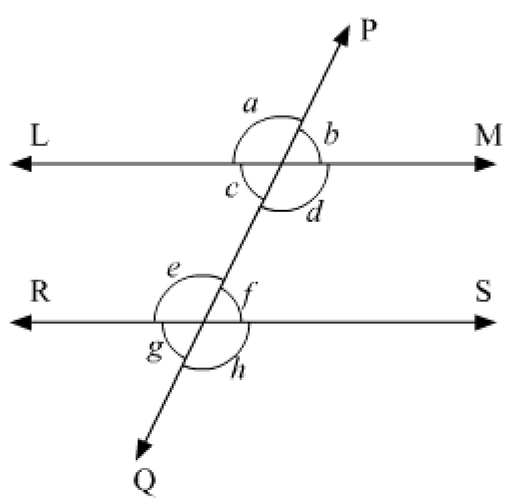


In the above figures, we can see that a line $\overleftrightarrow{PQ}$ intersects the lines $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ (in second figure, $\overleftrightarrow{AB} \| \overleftrightarrow{CD} \| \overleftrightarrow{EF}$) at three distinct points L, M, and N respectively. Therefore, $\overleftrightarrow{PQ}$ **is a transversal to the lines $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$.**

Also, LM is the intercept of the transversal $\overleftrightarrow{PQ}$ between $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Similarly, MN is the intercept of the transversal $\overleftrightarrow{PQ}$ between $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$.

Now, let us discuss about the **angles made by a transversal** with the lines, with which it intersects.





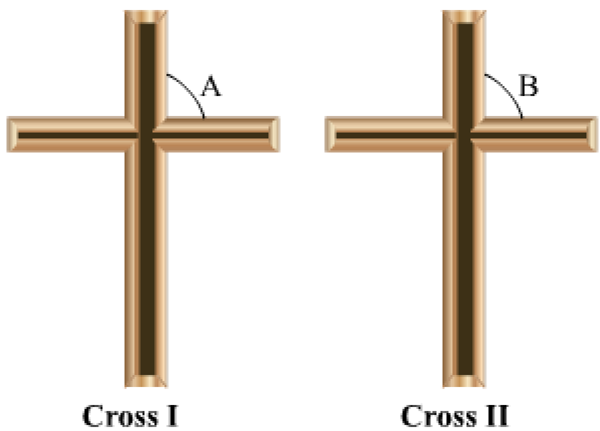
In the above figures, we can see that the line $\overleftrightarrow{PQ}$ intersects two lines $\overleftrightarrow{LM}$ and $\overleftrightarrow{RS}$ (in second figure, $\overleftrightarrow{LM} \parallel \overleftrightarrow{RS}$) at two distinct points. Therefore, $\overleftrightarrow{PQ}$ is a transversal to the lines $\overleftrightarrow{LM}$ and $\overleftrightarrow{RS}$.

The transversal $\overleftrightarrow{PQ}$ forms eight angles named as ∠*a*, ∠*b*, ∠*c*, ∠*d*, ∠*e*, ∠*f*, ∠*g*, and ∠*h* with the lines $\overleftrightarrow{LM}$ and $\overleftrightarrow{RS}$. These angles have special names.

Corresponding Angles Axiom

Corresponding Angles in Real Life

Consider the two crosses shown in the given figure.



In cross I, the right arm makes ∠A with the head of the cross; in cross II, the right arm makes ∠B with the head of the cross. Thus, both ∠A and ∠B are at the same position in the two crosses. Such angles are called **corresponding angles**.

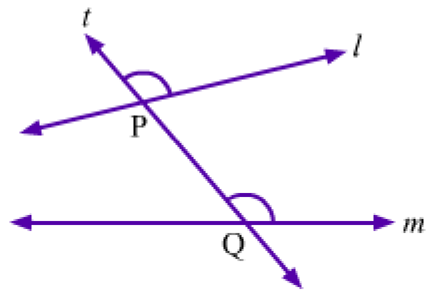
Now, suppose the right arm of cross I is joined to the left arm of cross II. What we obtain is a transversal cutting across a pair of lines to form different pairs of corresponding angles (like ∠A and ∠B). So, we can say that the relation between the angles in the so formed corresponding pairs of angles is decided by the relation between the lines cut by the transversal. This is defined by the **corresponding angles axiom**.

In this lesson, we will discuss the above axiom and its converse. We will also crack some problems based on the same.

Concept Builder

A transversal is a line that intersects two or more lines in the same plane at different points.

Look, for example, at the following figure.



In the figure, line *t* intersects lines *l* and *m* at two different points P and Q respectively; so, *t* is a transversal. The marked angles are corresponding angles.

Corresponding Angle Axiom and Its Converse

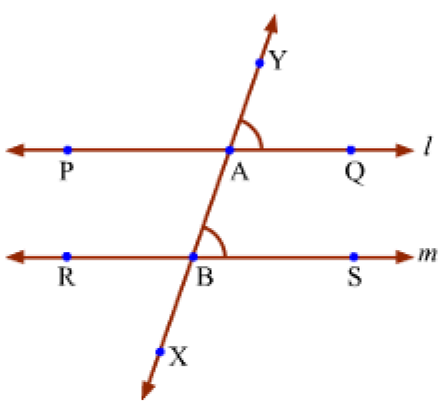
Proof of Corresponding Angle Axiom

When a transversal crosses two parallel lines, each pair of corresponding angles is equal (or congruent).

Given: Transversal XY of coplanar lines *l* and *m* such that *l* || *m*

To prove: ∠YAQ = ∠ABS

Proof:



l || *m* (Given)

∴ ∠PAB = ∠ABS ...(1) (Alternate angles)

And, ∠PAB = ∠YAQ ...(2) (Vertically opposite angles)

From (1) and (2), we obtain

∠YAQ = ∠ABS

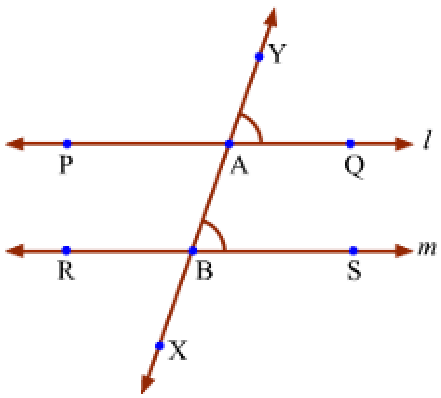
Proof of Converse of Corresponding Angle Axiom

If a pair of corresponding angles is equal (or congruent) then the lines are parallel to each other.

Given: Transversal XY of coplanar lines *l* and *m* such that ∠YAQ = ∠ABS

To prove: *l* || *m*

Proof:



∠YAQ = ∠ABS ...(1) (Given)

And, ∠YAQ = ∠PAB ...(2) (Vertically opposite angles)

From (1) and (2), we obtain

∴ ∠PAB = ∠ABS

∴ *l* || *m* (Using alternate angle axiom)

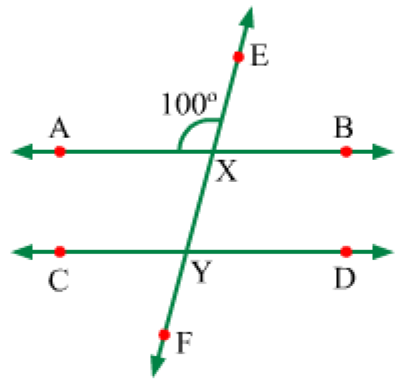
Note: If one pair of corresponding angles is equal (or congruent) then all the pairs of corresponding angles are equal.

Solved Examples

Easy

Example 1:

In the given figure, if AB and CD are parallel lines, then find the measure of ∠CYX.



Solution:

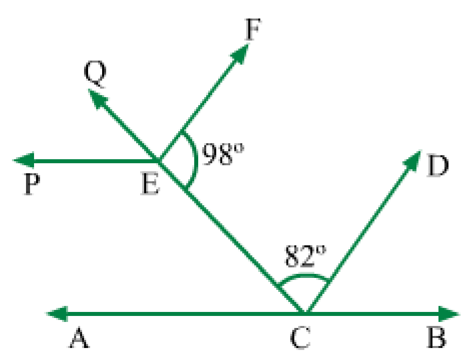
In the figure, ∠AXE and ∠CYX are corresponding angles made by the transversal EF on lines AB and CD respectively.

Using the corresponding angles axiom, we obtain:

∠CYX = ∠AXE = 100°

Example 2:

In the given figure, prove that EF||CD.



Solution:

In the give figure, ∠QEF and ∠FEC form a linear pair.

⇒ ∠QEF + ∠FEC = 180°

⇒ ∠QEF + 98° = 180°

⇒ ∴ ∠QEF = 82°

It can be seen that ∠QEF and ∠ECD are corresponding angles made by the transversal QC on lines EF and CD respectively.

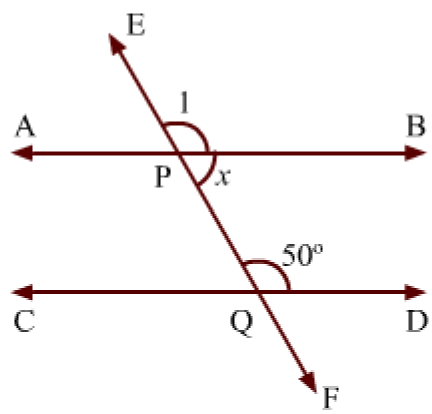
Now, ∠QEF = ∠ECD = 82°

∴ EF||CD (By the converse of the corresponding angles axiom)

Medium

Example 1:

In the given figure, line segments AB and CD are parallel. Find the value of *x*.



Solution:

It is given that AB is parallel to CD and EF is the transversal.

∴ ∠1 = ∠DQP = 50° (Corresponding angles)

Now, ∠1 + x = 180° (Angles forming a linear pair)

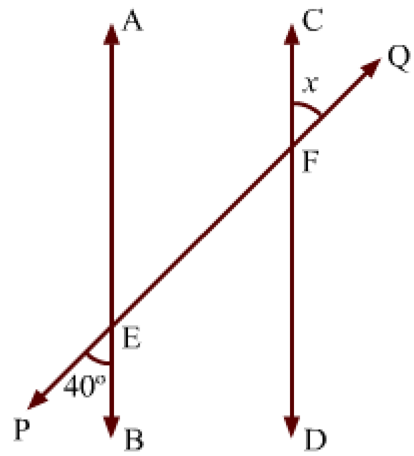
⇒ 50° + x = 180°

⇒ x = 180° − 50°

⇒ ∴ x = 130°

Example 2:

In the given figure, line segments AB and CD are parallel. What is the value of *x*?



Solution:

It is given that AB is parallel to CD and PQ is the transversal.

∠PEB = 40° (Given)

∠AEF = ∠PEB = 40° (Vertically opposite angles)

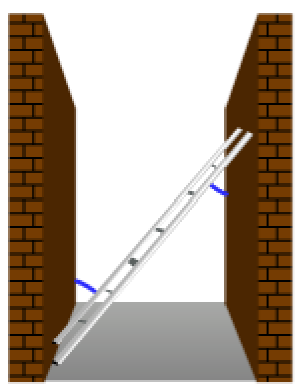
x = ∠AEF (By the corresponding angles axiom)

⇒ ∴ x = 40°

Alternate Angle Axiom

Introduction to Alternate Angles

We can find examples of alternate angles in daily life. These angles play an important role in various situations. Look, for example, at the given figure.

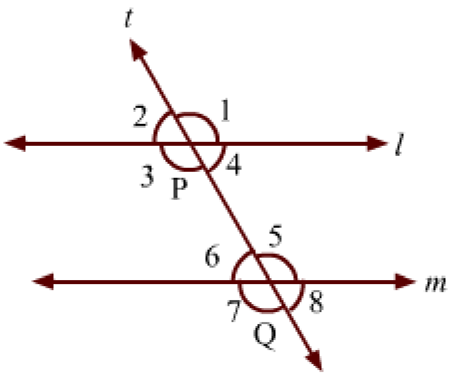


In the figure, the ladder resting between the two walls resembles a transversal joining two lines. The angles marked in the figure are **alternate angles**. These angles exhibit a special property that is defined by the **alternate angles axiom**.

In this lesson, we will study about the above axiom and its converse. We will also solve some examples based on the same.

Alternate Angles Axiom

Consider the given figure in which line *t* is a transversal intersecting two parallel lines *l* and *m* at points P and Q respectively.



In the figure, there are two pairs of alternate angles lying outside the parallel lines, i.e., ∠1 and ∠7, and ∠2 and ∠8. These angles are called **alternate exterior angles**. Also, there are two pairs of alternate angles lying between the parallel lines, i.e., ∠3 and ∠5, and ∠4 and ∠6. These angles are called **alternate interior angles**. The alternate angles made by a transversal on parallel lines have a special property which is stated as follows:

If a transversal intersects two parallel lines, then the angles in each pair of alternate angles are equal.

This property is known as the alternate angles axiom.

So, by using the alternate angles axiom, we can say the following for the given figure.

$\angle 1 = \angle 7$, $\angle 2 = \angle 8$, $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$

The converse of this axiom is also true. It states that:

If a transversal intersects two lines such that the angles in a pair of alternate angles are equal, then the two lines are parallel.

The alternate angles axiom and its converse are helpful in solving many problems in geometry as well as in real life.

Did You Know?

Playfair's axiom

If *m* is a line and P is any point which does not lie on this line, then there is only one line parallel to line *m* through point P.

For example:

In the given figure, line *l* is the only line parallel to line *m* through point P.

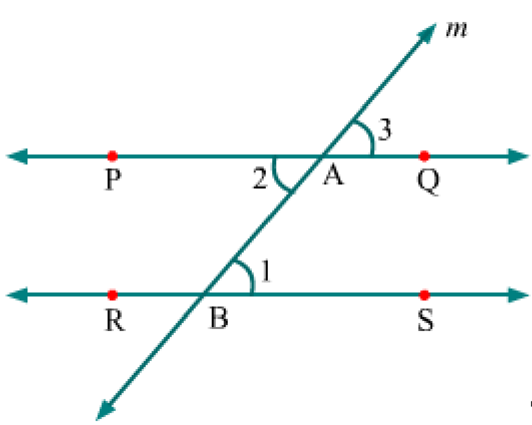
Proof of the Converse of the Alternate Angles Axiom

Statement of the converse of the alternate angles axiom

If a transversal intersects two lines such that the angles in a pair of alternate angles are equal, then the two lines are parallel.

Let us prove this statement.

Given: The transversal *m* intersects lines PQ and RS at points A and B respectively, such that the alternate angles 1 and 2 are equal.



To prove: PQ||RS

Proof: From the figure, we have

$\angle 2 = \angle 3$ (Vertically opposite angles)

$\angle 1 = \angle 2$ (Given)

$\Rightarrow \angle 1 = \angle 3$

$\angle 1$ and $\angle 3$ are corresponding angles; so, by using the converse of the corresponding angles axiom, it can be said that PQ and RS are parallel or PQ||RS.

Note: If one pair of alternate angles is equal (or congruent) then the other pair of alternate angles is also equal (or congruent).

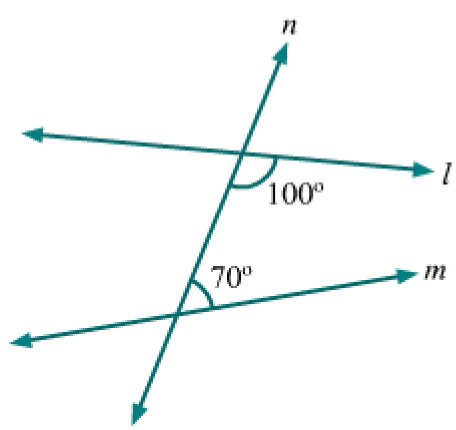
An Example on the Alternate Angles Axiom

Watch this video to understand the alternate angles axiom with the help of an example.

Whiz Kid

The great mathematician Euclid gave a few important results about geometry which are known as Euclid's postulates. The fifth postulate (or the parallel postulate) is about transversals. This postulate states that *if the sum of the interior angles on the same side of a transversal is less than two right angles, then the lines cut by the transversal must intersect when extended indefinitely*.

For example:



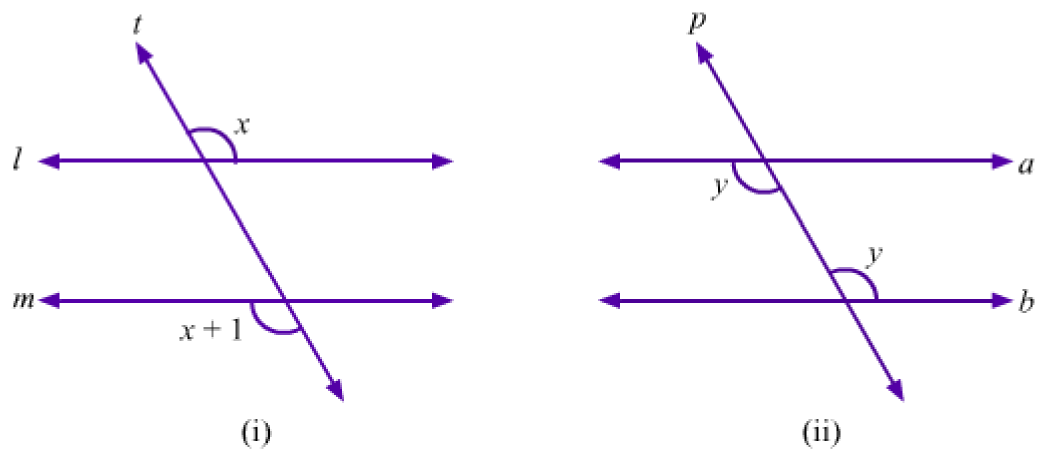
In the given figure, the sum of the interior angles on the right side of the transversal *n* is 170° which is less than two right angles or 180°. So, lines *l* and *m* will intersect when extended indefinitely on the same side as the given interior angles.

Solved Examples

Easy

Example 1:

In which case are the lines cut by the transversal parallel to each other?



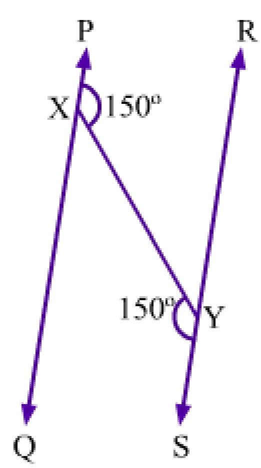
Solution:

In figure (i), the alternate exterior angles formed by the transversal *t* with lines *l* and *m* are not equal as $x \neq x + 1$. Therefore, lines *l* and *m* are not parallel to each other.

In figure (ii), the alternate interior angles formed by the transversal *p* with lines *a* and *b* are equal as $y = y$. Therefore, line *a* is parallel to line *b*.

Example 2:

Is PQ parallel to RS in the given figure?



Solution:

In the figure, we are given ∠PXY and ∠SYX.

These are the alternate interior angles made by PQ and RS with the transversal XY.

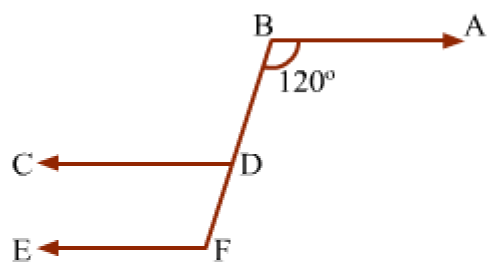
Now, ∠PXY = ∠SYX = 150°

Therefore, by the converse of the alternate angles axiom, we have PQ||RS.

Medium

Example 1:

In the given figure, AB is parallel to CD and CD is parallel to EF. It is given that ∠ABD = 120°. Show that AB is parallel to EF.



Solution:

It is given that ∠ABD = 120°.

Also, AB is parallel to CD.

∴ ∠BDC = ∠ABD = 120° (Pair of alternate interior angles between parallel lines)

We know that CD is parallel to EF.

∴ ∠BFE = ∠BDC = 120° (By the corresponding angles axiom)

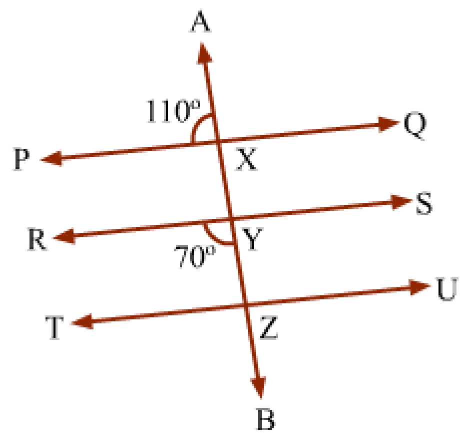
⇒ ∠BFE = ∠ABD

Now, ∠ABD and ∠BFE form a pair of alternate interior angles with respect to lines AB and EF.

Thus, by using the converse of the alternate angles axiom, we obtain AB||EF.

Example 2:

If RS||TU, then prove that PQ||TU.



Solution:

We are given that RS||TU and AB is the transversal.

Now, the interior angles on the same side of the transversal are supplementary.

So, ∠RYB + ∠TZA = 180°

⇒ ∠TZA = 180° − 70°

⇒ ∴ ∠TZA = 110°

It is given that ∠PXA = 110°.

∴ ∠QXB = 110° (because ∠PXA and ∠QXB are vertically opposite angles)

⇒ ∠QXB = ∠TZA

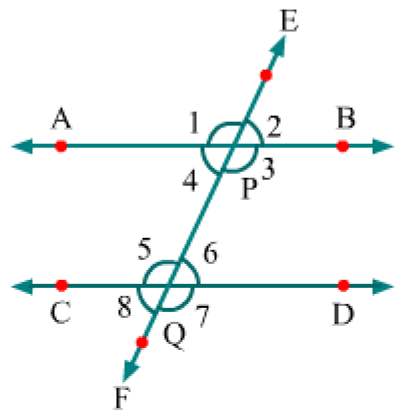
Now, ∠QXB and ∠TZA form a pair of alternate interior angles with respect to lines PQ and TU.

Thus, by using the converse of the alternate angles axiom, we obtain PQ||TU.

Hard

Example 1:

In the following figure, AB and CD are parallel to each other and EF is the transversal intersecting AB and CD at points P and Q respectively. If ∠APE = 110°, then find all the angles formed at points P and Q.



Solution:

In the given figure, we have the angles at points P and Q numbered from 1 to 8. Also, we have ∠APE = ∠1 = 110°.

Now,

∠3 = ∠1 = 110° (Vertically opposite angles)

∠5 = ∠3 = 110° (Alternate interior angles between parallel lines)

∠7 = ∠5 = 110° (Vertically opposite angles)

∠1 and ∠2 form a linear pair.

So, ∠1 + ∠2 = 180°

⇒ 110° + ∠2 = 180°

⇒ ∴ ∠2 = 70°

Now,

∠4 = ∠2 = 70° (Vertically opposite angles)

∠6 = ∠4 = 70° (Alternate interior angles between parallel lines)

∠8 = ∠6 = 70° (Vertically opposite angles)

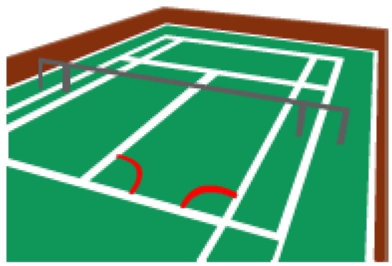
Thus, we have the angles around points P and Q as follows:

∠1 = ∠3 = ∠5 = ∠7 = 110° and ∠2 = ∠4 = ∠6 = ∠8 = 70°

Interior Angles on The Same Side of The Transversal

Experiencing Interior Angles on the Same Side of a Transversal

Consider the following figures of a tennis court and a house.

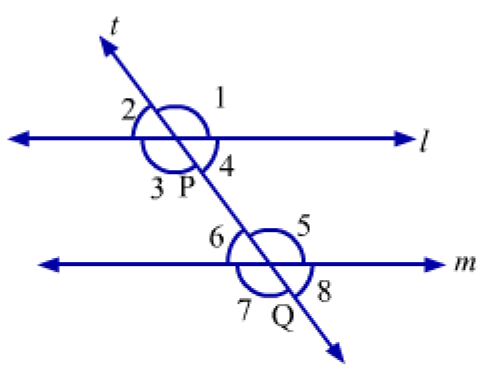


Transversals on parallel lines can be seen easily in the two figures. In each figure, the marked angles are interior angles lying on the same side of a transversal. Examples of such angles can be seen in many of the things that surround us.

In this lesson, we will discuss the property of interior angles on the same side of a transversal and solve some problems based on the same.

Property of Interior Angles on the Same Side of a Transversal

Consider the given figure.



In the figure, the transversal *t* intersects two parallel lines *l* and *m* at points P and Q respectively. Four angles are formed around each point. These angles have been numbered from 1 to 8.

Now, ∠3 and ∠6 form a pair of interior angles lying on the same side of the transversal *t*. ∠4 and ∠5 is another such pair of interior angles. The property exhibited by these types of angles is stated as follows:

If a transversal intersects two parallel lines, then the angles in a pair of interior angles on the same side of the transversal are supplementary.

$\therefore \angle 3 + \angle 6 = 180°$ and $\angle 4 + \angle 5 = 180°$

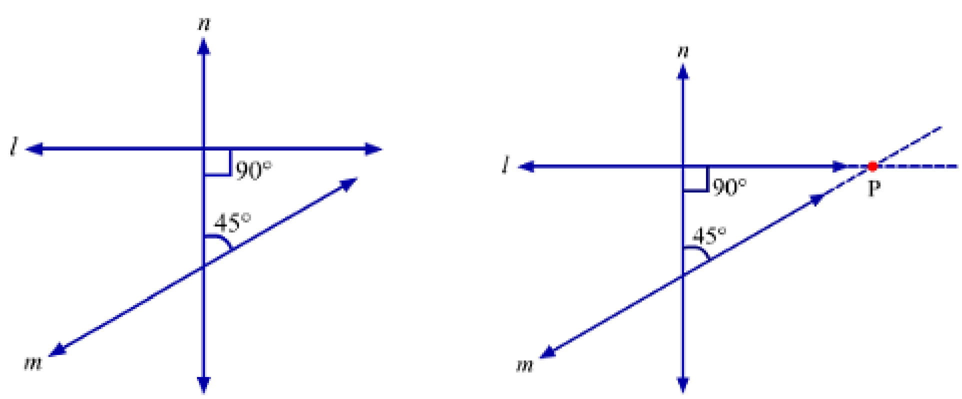
The converse of this property is also true. It states that:

If a transversal intersects two lines such that the interior angles on the same side of the transversal are supplementary, then the lines intersected by the transversal are parallel.

Whiz Kid

If the sum of the interior angles on the same side of a transversal is less than two right angles or 180°, then the lines cut by the transversal must intersect when extended along that side of the transversal.

For example:



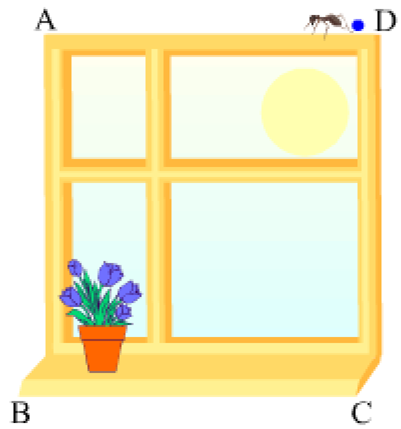
In first figure, the sum of the interior angles on the right side of transversal *n* is 90° + 45° = 135° < 180°. So, lines *l* and *m* will intersect when extended along the right side of the transversal *n*, as is shown in second figure.

Solved Examples

Easy

Example 1:

An ant is moving on a window frame by following the path DABC. If edge AD is parallel to edge BC, then what angle will the ant have to move along in order to reach point C from point D?



Solution:

In the window frame, edges AD and BC are parallel to each other and edge AB is the transversal. So,∠A and ∠B are interior angles lying on the same side of the transversal AB.

From point D to point C, the ant will move along a total angle that is the sum of ∠A and ∠B.

We know that if a transversal intersects two parallel lines, then the angles in a pair of interior angles on the same side of the transversal are supplementary.

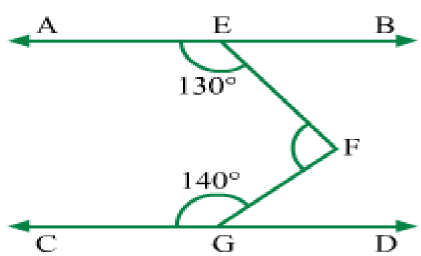
∴ ∠A + ∠B = 180°

Hence, the ant will have to move along an angle of 180° to reach point C from point D.

Medium

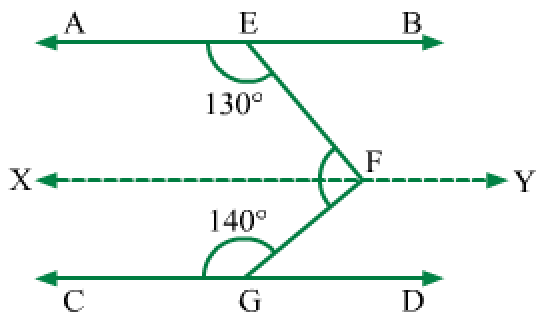
Example 1:

In the given figure, AB and CD are parallel lines. Find the measure of ∠EFG.



Solution:

Construction: Draw a line XY passing through point F and parallel to lines AB and CD.



Now, we have EF as the transversal on the parallel lines AB and XY. Similarly, we have GF as the transversal on the parallel lines CD and XY.

∠AEF and ∠EFX are interior angles on the same side of the transversal EF.

$\therefore \angle AEF + \angle EFX = 180°$

$\Rightarrow 130° + \angle EFX = 180°$

$\Rightarrow \angle EFX = 180° - 130°$

$\Rightarrow \therefore \angle EFX = 50°$

Similarly,

$\angle CGF + \angle GFX = 180°$

$\Rightarrow 140° + \angle GFX = 180°$

$\Rightarrow \angle GFX = 180° - 140°$

$\Rightarrow \therefore \angle GFX = 40°$

From the figure, we have:

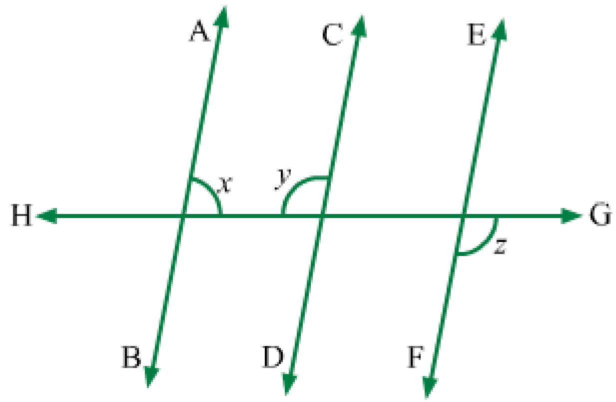
$\angle EFG = \angle EFX + \angle GFX$

$\Rightarrow \angle EFG = 40° + 50°$

$\Rightarrow \therefore \angle EFG = 90°$

Example 2:

In the given figure, AB||CD, CD||EF and *x* : *z* = 2 : 3. What are the measures of *x*, *y* and *z*?



Solution:

It is given that AB||CD, CD||EF and GH is the transversal on these pairs of parallel lines.

Also, $x : z = 2 : 3$

$$\Rightarrow \frac{x}{z} = \frac{2}{3}$$

Let $x = 2a$ and $z = 3a$

Now,

$y = z$ (Exterior alternate angles formed by GH on CD and EF)

$x + y = 180°$ (Interior angles on the same side of GH)

$\Rightarrow x + z = 180°$

$\Rightarrow 2a + 3a = 180°$

$\Rightarrow 5a = 180°$

$\Rightarrow \therefore a = 36°$

So, we have $x = 2 \times 36° = 72°$ and $z = 3 \times 36° = 108°$

Also, $y = z = 108°$

Activity Time

Follow these steps to verify the property of interior angles lying on the same side of a transversal.

- Take a chart paper.
- Draw two parallel lines.
- Draw four or five transversals, each intersecting the parallel lines at two different points.
- Make a list of the pairs of interior angles formed on the same side of each transversal.
- Measure each angle in the list using a protractor.
- Find the sum of the angles in each listed pair of angles.
- Check whether each pair of angles consists of supplementary angles or not.
- Write the result common to all the transversals.

This activity proves the property that interior angles on the same side of a transversal lying on two parallel lines are supplementary.