Maharashtra State Board

Class X Mathematics – Algebra – Paper I

Board Paper 2018

Time: 2 hours **Maximum Marks: 40**

Note: - (i) All questions are compulsory.
(ii) Use of calculator is not allowed.

1. Attempt any five of the following sub questions: **5**

(i) Find next two terms of an A.P.
4, 9, 14,.......

(ii) State whether the given equation is quadratic or not. Give reason.
$\frac{5}{4}m^2 - 7 = 0$

(iii) If D_x = 25, D = 5 are the values of the determinants for certain simultaneous equations in x and y, find x.

(iv) If S = {2, 4, 6, 8, 10, 12} and A = {4, 8 , 12}. Find A'.

(v) Write any one solution of equation x + 2y = 7.

(vi) If S_5 = 15 and S_6 = 21 find t_6.

2. Attempt any four of the following sub questions: **8**

(i) Find 'n' if the nth term of the following A.P. is 64:
1, 4, 7 , 10, ...

(ii) If one of the roots of the quadratic equation $x^2 - 10x + k = 0$ is 2, then find the value of k.

(iii) A box contains 20 cards marked with the numbers 1 to 20. One card is drawn at random, where the event A is such that the number on the card is multiple of 3. Write S, n(s), A and n(A).

(iv) Find the value of x – y if 6x + 5y = 17, 5x + 6y = 13.

(v) If the roots of quadratic equation are 5 and −6, form the quadratic equation.

(vi) For a certain frequency distribution mean is 72 and median is 78, find the mode.

3. Attempt any three of the following sub questions: 9

(i) For an A.P., find S_9 if a = 4 and d = 2.

(ii) Solve the following quadratic equation by formula method:
$3x^2 + 2x = 1$

(iii) Solve the following simultaneous equations by using Cramer's rule:
$4x + 3y = 4;$
$3x + 5y = 8$

(iv) A die is thrown, find the probability of the event of getting a square number.

(v) The marks obtained by a student in an examination out of 100 are given below. The total marks obtained in various subjects are as follows:

Subject	**Marks**
Marathi	80
English	80
Science	100
Mathematics	100
Total	360

Represent the above data using pie diagram.

4. Attempt any two of the following sub equations : 8

(i) If $\alpha + \beta = 5$ and $\alpha^3 + \beta^3 = 35$, find the quadratic equation whose roots are α and β.

(ii) Two dice are thrown. Find the probability of getting:
(a) The sum of the numbers on their upper faces is at least 9.
(b) The sum of the numbers on their upper faces is 15.
(c) The number on the upper face of the second die is greater than the number on the upper face of the first die.

(iii) Frequency distribution of daily commission received by 100 salesman is given below :

Daily commission (in Rs.)	**No. of Salesmen**
100 – 120	20
120 – 140	45
140 – 160	22
160 – 180	09
180- 200	04

Find mean daily commission received by salesmen, by assumed mean method.

5. Attempt any two of the following sub questions : 10

(i) A boat takes 10 hours to travel 30 km upstream and 44 km downstream, but it takes 13 hours to travel 40 km upstream and 55 km downstream. Find the speed of the boat in still water and the speed of the stream.

(ii) If the 9^{th} term of an A.P. is zero, then prove that 29^{th} term is double of 19^{th} term.

(iii) Draw histogram and frequency polygon on the same graph paper for the following frequency distribution :

Class	**Frequency**
15 – 20	20
20 – 25	30
25 – 30	50
30 – 35	40
35 – 40	25
40 – 45	25

Maharashtra State Board
Class X Mathematics – Algebra – Paper I
Board Paper – 2018 Solution

1.

i. The given sequence is 4, 9, 14, ...

Here, first term = a = 4

$\Rightarrow t_1 = 4$

$t_2 = 9$

Now, $t_2 - t_1 = 9 - 4 = 5$

$\Rightarrow$ Common difference = d = 5

Hence,

$t_4 = a + (4 - 1)d = 4 + 3 \times 5 = 4 + 15 = 19$

$t_5 = a + (5 - 1)d = 4 + 4 \times 5 = 4 + 20 = 24$

Therefore, the next two terms of the given A.P. are 19 and 24.

ii.

Given equation is $\frac{5}{4}m^2 - 7 = 0$

$\therefore \frac{5m^2 - 28}{4} = 0$

$\therefore 5m^2 - 28 = 0$

Here, the maximum index of variable m is 2.

Comparing with general form of quadratic equation $ax^2 + bx + c$,

we have $a = 5$, $b = 0$ and $c = -28$, which are real numbers and $a \neq 0$.

Hence, it is a quadratic equation in variable m.

iii. $D_x = 25$, $D = 5$

$\therefore x = \frac{D_x}{D} = \frac{25}{5} = 5$

iv. S = {2, 4, 6, 8, 10, 12} and A = {4, 8, 12)

Hence, A' = outcomes in sample space S which are not in A = {2, 6, 10}

v. Given equation is $x + 2y = 7$
Substituting $x = 1$ in the given equation, we get
$1 + 2y = 7$
$\therefore 2y = 7 - 1$
$\therefore 2y = 6$
$\therefore y = 3$
Hence, (1, 3) is a solution of the given equation.

vi. $S_5 = 15$ and $S_6 = 21$
$\therefore t_6 = S_6 - S_5 = 21 - 15 = 6$

2.

i. Given A.P. is 1, 4, 7, 10, ……
$t_n = 64$
Here, $a = 1$
$d = 4 - 1 = 3$
Now, $t_n = a + (n - 1)d$
$\therefore 64 = 1 + (n - 1)3$
$\therefore (n - 1)3 = 63$
$\therefore n - 1 = 21$
$\therefore n = 22$

ii. Given equation is $x^2 - 10x + k = 0$.

$x = 2$ is a root of the given equation.

Thus, it satisfies the given equation.

Hence, substituting $x = 2$ in the given equation, we have

$(2)^2 - 10 \times 2 + k = 0$

$\therefore 4 - 20 + k = 0$

$\therefore -16 + k = 0$

$\therefore k = 16$

iii. Sample space, S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
$\therefore n(S) = 20$
A = Event that the number on the card is a multiple of 3
$\therefore$ A = {3, 6, 9, 12, 15, 18}
Hence, $n(A) = 6$

iv. $6x + 5y = 17$ ....(i)

$5x + 6y = 13$ ....(ii)

Subtracting equation (ii) from equation (i), we have

$x - y = 4$

v.

Let $\alpha = 5$ and $\beta = -6$

$\therefore \alpha + \beta = 5 + (-6) = 5 - 6 = -1$

And, $\alpha\beta = 5 \times (-6) = -30$

Hence, the required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e. $x^2 - (-1)x + (-30) = 0$

i.e. $x^2 + x - 30 = 0$

vi. Let the mode of the data be x.

Then Mean – Mode = 3(Mean – Median)

$\therefore 72 - x = 3(72 - 78)$

$\therefore 72 - x = 3 \times (-6)$

$\therefore 72 - x = -18$

$\therefore x = 72 + 18$

$\therefore x = 90$

Hence, the value of the mode is 90.

3.

i. For an A.P., a = 4 and d = 2

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_9 = \frac{9}{2}[2 \times 4 + 8 \times 2]$

$= \frac{9}{2}[8 + 16]$

$= \frac{9}{2} \times 24$

$= 108$

ii. Given quadratic equation is $3x^2 + 2x = 1$

$\Rightarrow 3x^2 + 2x - 1 = 0$

Comparing with the standard form $ax^2 + bx + c = 0$, we have

$a = 3,\ b = 2,\ c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4 \times 3 \times (-1)}}{2 \times 3}$$

$$= \frac{-2 \pm \sqrt{4 + 12}}{6}$$

$$= \frac{-2 \pm \sqrt{16}}{6}$$

$$= \frac{-2 \pm 4}{6}$$

$\therefore x = \frac{-2+4}{6}$ or $x = \frac{-2-4}{6}$

$\therefore x = \frac{2}{6}$ or $x = -\frac{6}{6}$

$\therefore x = \frac{1}{3}$ or $x = -1$

$\therefore \frac{1}{3}, -1$ are the roots of the given equation.

iii. Given simultaneous equations are

$4x + 3y = 4$

$6x + 5y = 8$

Now,

$$D = \begin{vmatrix} 4 & 3 \\ 6 & 5 \end{vmatrix} = 20 - 18 = 2 \neq 0$$

$$D_x = \begin{vmatrix} 4 & 3 \\ 8 & 5 \end{vmatrix} = 20 - 24 = -4$$

$$D_y = \begin{vmatrix} 4 & 4 \\ 6 & 8 \end{vmatrix} = 32 - 24 = 8$$

$$x = \frac{D_x}{D} = \frac{-4}{2} = -2$$

$$y = \frac{D_y}{D} = \frac{8}{2} = 4$$

So, $x = -2$ and $y = 4$

iv. When a die is thrown, the sample space S is given by

S = {1, 2, 3, 4, 5, 6}

$\Rightarrow$ n(S) = 6

Let A = event of getting a square number

$\Rightarrow$ A = {1, 4}

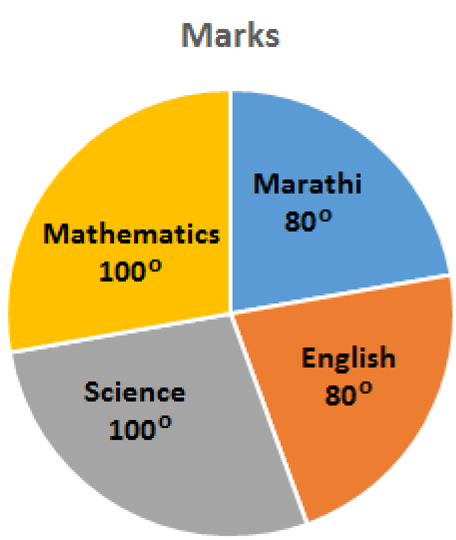
$\Rightarrow$ n(A) = 2

$$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

v.

Subject	Marks	Central Angle
Marathi	80	$\frac{80}{360} \times 360° = 80°$
English	80	$\frac{80}{360} \times 360° = 80°$
Science	100	$\frac{100}{360} \times 360° = 100°$
Mathematics	100	$\frac{100}{360} \times 360° = 100°$
Total	360	360°

The pie diagram is as follows:



4.

i. α and β are the roots of the quadratic equation.

We have $\alpha+\beta=5$ and $\alpha^3+\beta^3=35$

Now,

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$\therefore 35=(5)^3-3\alpha\beta(5)$

$\therefore 35=125-15\alpha\beta$

$\therefore 7=25-3\alpha\beta$ .....(Dividing by 3)

$\therefore 3\alpha\beta=25-7=18$

$\therefore \alpha\beta=6$

Hence, the required quadratic equation is $x^2-(\alpha+\beta)x+\alpha\beta=0$

i.e. $x^2-5x+6=0$

ii. When two dice are thrown, the sample space S is given by

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3),(4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4),(5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

∴ n(S) = 36

Let A be the event that the sum of the numbers on the upper faces is at least 9.

A = {(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)}

∴ n(A) = 10

$\therefore P(A)=\frac{n(A)}{n(S)}=\frac{10}{36}=\frac{5}{18}$

Let B be the event that the sum of the numbers on the upper faces is 15.

This is an impossible event as maximum sum when two dice are rolled is 6 + 6 = 12

∴ P(B) = 0

Let C be the event that the number on the upper face of the 2nd die is greater than the number on the upper face of the first die..

C = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6)}

∴ n(C) = 15

$\therefore P(C)=\frac{n(C)}{n(S)}=\frac{15}{36}=\frac{5}{12}$

iii. We take 150 as the assumed mean

A = 150 and Deviation $d_i = x_i - A = x_i - 150$

Daily Commission (in Rs.)	Class marks x_i	Deviations d_i	No. of Salesmen f_i	$f_i d_i$
100 – 120	110	–40	20	–800
120 – 140	130	–20	45	–900
140 – 160	150	0	22	0
160 – 180 180 –	170	20	09	180
200	190	40	04	160
Total			$\sum f_i = 100$	$\sum f_i d_i = -1360$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{-1360}{100} = -13.6$$

$$\bar{x} = A + \bar{d} = 150 + (-13.6) = 136.4$$

Thus, the mean daily commission received by salesmen Rs. 136.40

5.

i. Let the speed of the boat in still water be x km/h.

Let the speed of the stream be y km/h.

The speed of the boat downstream = speed of the boat + speed of the stream.

∴ The speed of the boat downstream = (x + y) km/h

The speed of boat upstream = speed of boat – speed of stream.

∴ The speed of the boat upstream = (x – y) km/h

From the first condition, time taken by boat:

To travel 30 km upstream = $\frac{30}{x-y}$

To travel 44 km downstream = $\frac{44}{x+y}$

$$\therefore \frac{30}{x-y} + \frac{44}{x+y} = 10 \quad ....(i)$$

From the second condition, time taken by the boat:

To travel 40 km upstream = $\frac{40}{x-y}$

To travel 55 km downstream = $\frac{55}{x+y}$

$$\therefore \frac{40}{x-y} + \frac{55}{x+y} = 13 \quad ....(ii)$$

Substituting, $\frac{1}{x-y}$ for m and $\frac{1}{x+y}$ for n in (i) and (ii)

$30m + 44n = 10 \quad ....(iii)$

$40m + 55n = 13 \quad ....(iv)$

Multiplying (iiii) by 40 and (iv) by 30, we get

$120m + 1760n = 400 \quad ....(v)$

$120m + 1650n = 390 \quad ....(vi)$

Subtracting (vi) from (v), wehave

$110n = 10$

$\Rightarrow n = \frac{10}{110} = \frac{1}{11}$

Substituting n = $\frac{1}{11}$ in (iii),

$30m + 44 \times \frac{1}{11} = 10$

$\Rightarrow 30m + 4 = 10$

$\Rightarrow 30m = 6$

$\Rightarrow m = \frac{6}{30}$

$\Rightarrow m = \frac{1}{5}$

Resubstituting m for $\frac{1}{x-y}$ and n for $\frac{1}{x+y}$

$\frac{1}{x-y} = \frac{1}{5}$

$\therefore x - y = 5 \quad ....(vii)$

$\frac{1}{x+y} = \frac{1}{11}$

$\therefore x + y = 11 \quad ....(viii)$

Adding (vii) and $(viii)$

$2x = 16$

$\Rightarrow x = 8$

$\Rightarrow 8 + y = 11$

$\Rightarrow y = 3$

Thus, the speed of the boat in still water is 8 km/h and the speed of the stream is 3 km/h.

ii. $t_9 = 0$

$\Rightarrow a + 8d = 0$

$\Rightarrow a = -8d \quad ....(i)$

Now, $t_{29} = a + 28d$

$= -8d + 28d \quad ....[From (i)]$

$= 20d$

And, $t_{19} = a + 18d$

$= -8d + 18d \quad ....[From (i)]$

$= 10d$

$\Rightarrow 2 \times t_{19} = 2 \times 10d = 20d$

$\therefore t_{29} = 2 \times t_{19}$

iii. The histogram with frequency polygon is as follows: