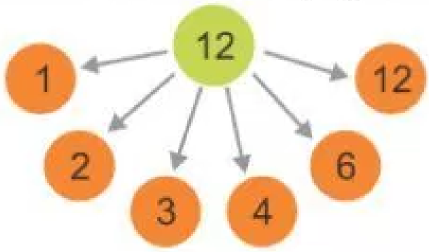
7. Factors and Multiples

Finding Factors And Multiples

FACTORS AND MULTIPLES

Factors: The numbers which completely divide the given number such that the remainder is zero. A number can have finite number of factors.
Example: Factors of the number 12 are: 1, 2, 3, 4, 6 and 12



Multiples: The numbers obtained by multiplying the given number by a natural number. A number can have infinite number of multiples.
Example:

Multiples of 6 are :	
6 × 2	12
6 × 3	18
6 × 4	24
....	....

Mr Sharma wanted to withdraw Rs. 1000 from his bank account to purchase books for his children. The cashier gave him 10 hundred-rupee notes, i.e.,
Rs. 10 × 100 = Rs. 1000
Mr Sharma got the required amount. But the cashier could also give the same amount in the following ways:

- 200 five-rupee notes

  $= \mathbf{200} \times ₹\ 5$

  $= ₹\ 1000$

- 100 ten-rupee notes

  $= \mathbf{100} \times ₹\ 10$

  $= ₹\ 1000$

- 50 twenty-rupee notes

  $= \mathbf{50} \times ₹\ 20$

  $= ₹\ 1000$

- 20 fifty-rupee notes

  $= \mathbf{20} \times ₹\ 50$

  $= ₹\ 1000$

- 2 five hundred-rupee notes

  $= \mathbf{2} \times ₹\ 500$

  $= ₹\ 1000$

- 1 thousand-rupee note

  $= \mathbf{1} \times ₹\ 1000$

  $= ₹\ 1000$

Here, we observe that in each case Mr Sharma got the same amount of Rs. 1000. These numbers 1, 2, 5, 10, 20, 50, 100, 200, 500, and 1000 are factors of 1000. Hence, 1000 is a multiple of these numbers.
Here we will discuss only the natural numbers, that is positive integers.
If $a = b \times c$, we say b and c are factors of a and a is a multiple of c and b.

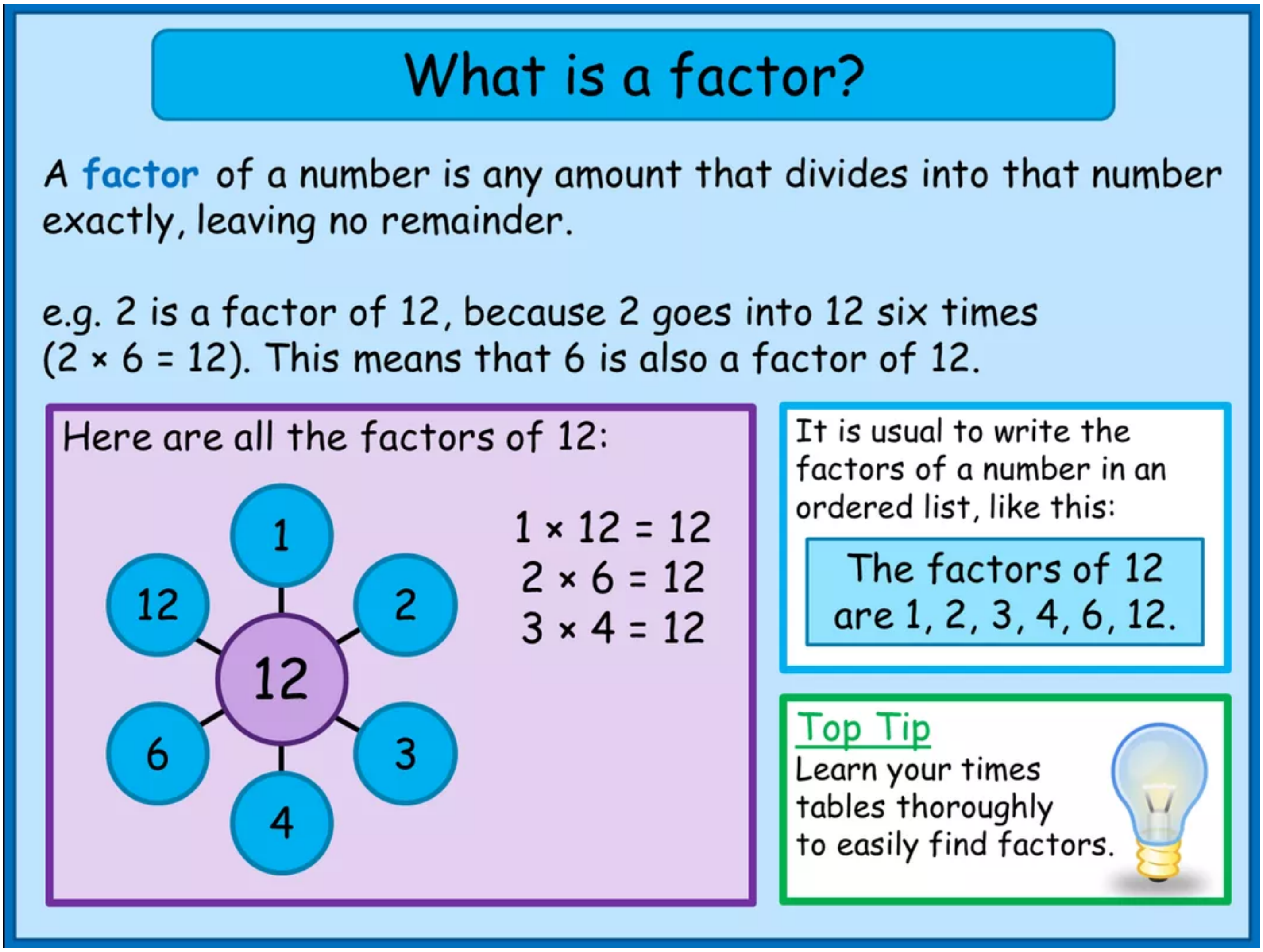
Factors

Factor: A number which divides a given number exactly (without leaving any remainder) is called a factor of the given number.

Example: Factors of 12

$12 = 1 \times 12$
$12 = 2 \times 6$
$12 = 3 \times 4$

Here, 1, 2, 3, 4, 6, and 12 are factors of 12.



Properties of Factors

1. Every non-zero number is a factor of itself.
   Examples: 5 is a factor of 5. ($5 \div 5 = 1$)
   12 is a factor of 12. ($12 \div 12 = 1$)
2. 1 is a factor of every number.
   Examples: 1 is a factor of 5. ($5 \div 1 = 5$)
   1 is a factor of 12. ($12 \div 1 = 12$)
3. Every non-zero number is a factor of 0.
   Example: 5 and 12 are factors of 0 because
   $0 \div 5 = 0$ and $0 \div 12 = 0$
4. The factors of a number are finite.

Multiples

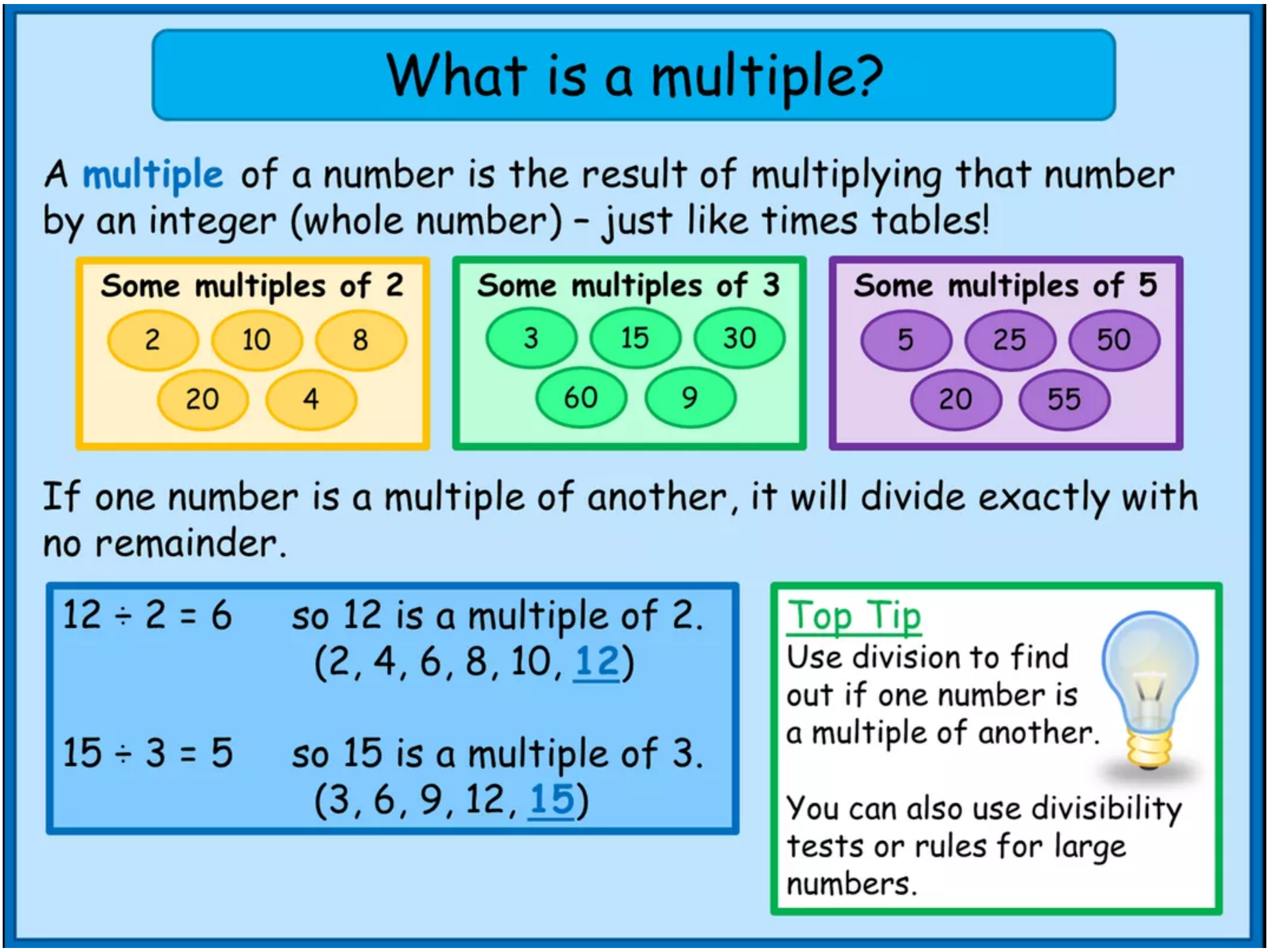
Multiple: A multiple of any natural number is a number formed by multiplying it by another natural number.

Example: Multiples of 6 are $6 \times 1 = 6$; $6 \times 2 = 12$; $6 \times 3 = 18$; $6 \times 4 = 24$
Here, 6,12,18,24 are multiples of 6.

Example: Let us find the LCM and HCF of 24 and 36.

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24
Factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18, 36
Here, the highest common factor is 12.
∴ HCF = 12
Multiples of 24 = 24, 48, 72, 96,…
Multiples of 36 = 36, 72, 108,…

Here, the lowest common multiple is 72.
∴ LCM = 72



Properties of Multiples

1. Every number is a multiple of itself.
   Examples
   (a) 3 × 1 = 3; 3 is the multiple of 3
   (b) 7 × 1 = 7; 7 is the multiple of 7
2. Every number is the multiple of 1.
   Examples
   (a) 1 × 3 = 3; 3 is the multiple of 1
   (b) 1 × 7 = 7; 7 is the multiple of 1
3. The multiples of a number are infinite (unlimited).

Even numbers: A number which is a multiple of 2 is called an even number.

Example: 2, 4, 6, 8, 10,...

Odd numbers: A number which is not a multiple of 2 is called an odd number.

Example: 1, 3, 5, 7, 9, 11,...

Prime numbers: A number which is greater than 1, and has exactly two factors (1 and the number itself) is called a prime number.

Example: Factors of 2 = 1, 2
Factors of 3 = 1, 3
Factors of 5 = 1, 5
Factors of 7 = 1, 7

Factors of 11 = 1, 11
Here, 2, 3, 5, 7, 11 etc. are all prime numbers.

Composite numbers: A number, which is greater than 1 and has more than two factors is called a composite number.

Examples: Here,
Factors of 4 = 1, 2, 4
Factors of 6 = 1, 2, 3, 6
Factors of 8 = 1, 2, 4, 8
Factors of 9 = 1, 3, 9
Factors of 10 = 1, 2, 5

FINDING PRIME NUMBERS FROM 1 TO 100

We can find the prime numbers from 1 to 100 by following these steps (given by the Greek mathematician Eratosthenes).

Step 1: Prepare a list of numbers from 1 to 100. Step 2: As 1 is neither prime nor composite number, cross it out.
Step 3: Encircle '2' as a prime number and cross out all its other multiples.
Step 4: Encircle '3' as a prime number and cross out all its other multiples.
Step 5: Encircle '5' as a prime number and cross out all its other multiples.
Step 6: Continue this process till all the numbers are either encircled or crossed out.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

All the encircled numbers are **prime numbers** and the crossed out numbers (except 1) are **composite numbers**.
Numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97 are the prime numbers between 1 and 100.
This is called the **'Sieve of Eratosthenes'**.

Twin primes: Two prime numbers having a difference of 2 are known as twin primes.

Example: (3, 5), (5,7), (11,13), (17,19), etc are twin primes.

Co-primes: Two numbers are said to be co-primes if they have no common factor other than 1. In other words, two natural numbers are co-primes if their HCF is 1.

Example: (2, 3), (3, 4), (5, 6), (7, 8), and so on.

Example 1: Is 16380 a multiple of 28?

Solution: To check whether 16380 is a multiple of 28 or not, we have to divide 16380 by 28. If the remainder becomes zero, then it is a multiple of the number.

28)16380 (585
140
―――
238
224
―――
140
140
―――
0

So, 16380 = 28 × 585, hence 16380 is a multiple of 28.

Example 2: Express 29 as the sum of three odd prime numbers.

Solution: 29 = 19 + 7 + 3
All 19, 7, and 3 are odd prime numbers.

DIVISIBILITY TESTS FOR 2, 3, 4, 5, 6, 7, 8, 9, 10, AND 11

If we want to know that a number is divisible by another number, we generally perform the actual division and see whether the remainder is zero or not. This process is time-consuming for division of large numbers. Therefore, to cut short our efforts, some divisibility tests of different numbers are given below.

Test of Divisibility by	Condition	Example
2	A number is divisible by 2, if its ones digit is 0, 2, 4, 6 or 8.	1372, 468, 500, 966 are divisible by 2, since their ones digit is 2, 8, 0 and 6 respectively.
3	A number is divisible by 3, if the sum of its digits is divisible by 3.	In 1881, the sum of digits is 1 + 8 + 8 + 1 = 18 which is divisible by 3. So 1881 is divisible by 3.
4	A number is divisible by 4, if the number formed by the last two digits is divisible by 4.	30776, 63784, 864 are all divisible by 4. Since last two digits of the numbers, i.e., 76, 84, and 64 are divisible by 4.
5	A number is divisible by 5, if its ones digit is either 5 or 0.	675, 4320, 145 all are divisible by 5 because their ones digit is 5 or 0.
6	A number is divisible by 6, if the number is divisible by 2 and 3.	In 5922, ones digit is 2, so it is divisible by 2. The sum of digits in 5922 is 5 + 9 + 2 + 2 = 18, which is divisible by 3. So, 5922 is divisible by 6.
7	A number is divisible by 7, if the difference between twice the last digit and the number formed by other digits is either 0 or a multiple of 7.	In number 2975, it is observed that the last digit in 2975 is 5. So, 297 – (2 x 5) = 287, which is a multiple of 7. Hence, 2975 is divisible by 7.
8	A number is divisible by 8, if the number formed by its last three digits is divisible by 8.	In 213456, the last three digits are 456 which is divisible by 8. So, the number 213456 is divisible by 8.
9	A number is divisible by 9, if the sum of its digits is divisible by 9.	In 538425, the sum of the digits are (5 + 3 + 8 + 4 + 2 + 5) = 27 which is divisible by 9. So, 538425 is divisible by 9.
10	A number is divisible by 10, if the digit at ones place of the number is 0.	The numbers 980, 63990 are all divisible by 10 because their ones digit is 0.

11	A number is divisible by 11, if the difference between the sum of digits at odd places and the sum of digits at even places is either 0 or a multiple of 11.	In number 27896, the sum of the digits at odd places are (2 + 8 + 6) = 16. The sum of the digits at even places are (7 + 9) = 16. Their difference is 16 – 16 = 0. So, the number 27896 is divisible by 11.

Example 3: Test whether 72148 is divisible by 8 or not?

Solution: Here, the number formed by the last three digits is 148, which is not divisible by 8.
So, 72148 is not divisible by 8.

Example 4: Test whether 8050314052 is divisible by 11 or not?

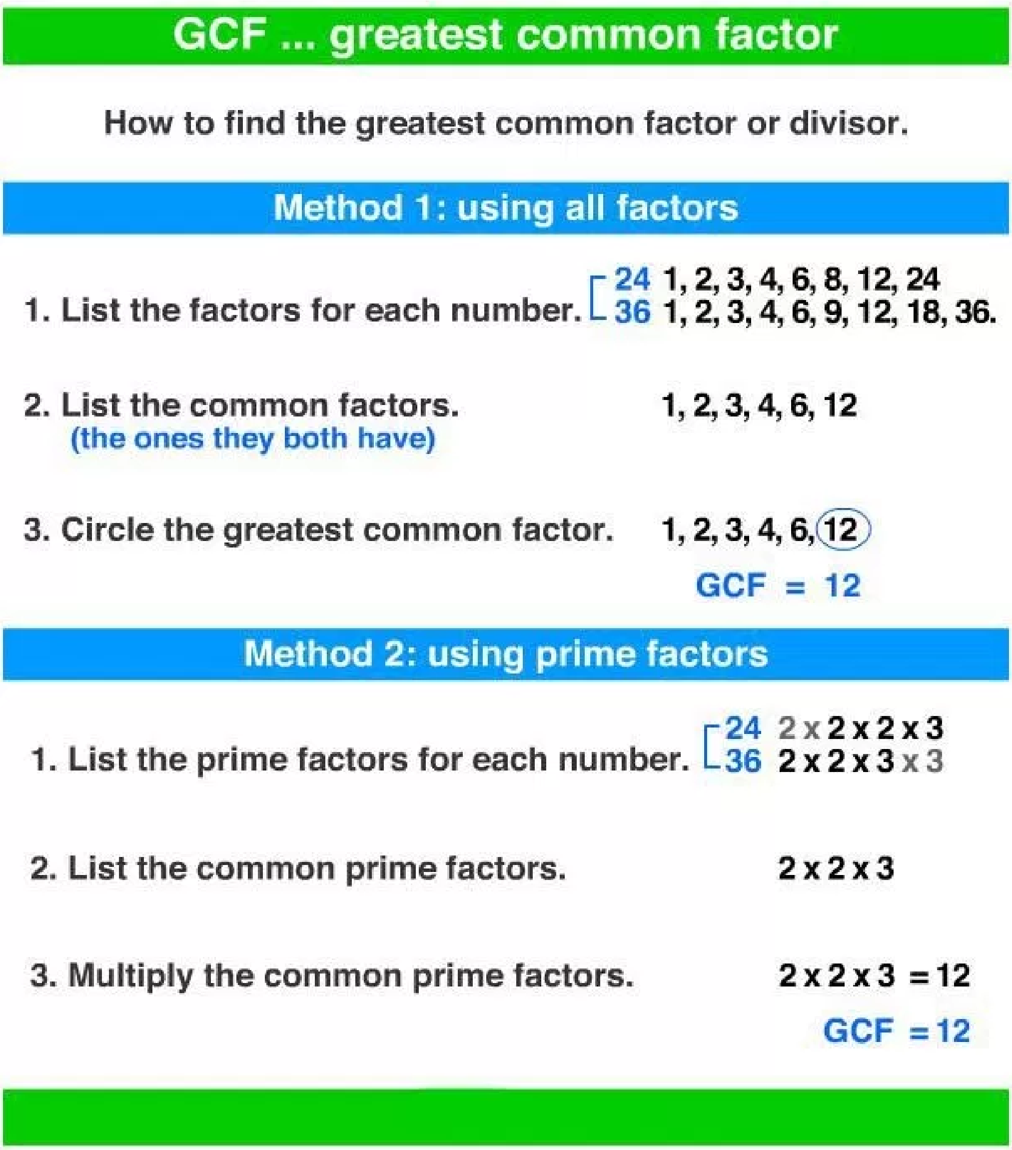
Solution: The sum of the digits at even places = 8 + 5 + 3 + 4 + 5 = 25
The sum of digits at the odd places = 0 + 0 + 1 + 0 + 2 = 3
Difference = 25 – 3 = 22 22 is divisible by 11.
So, the number 8050314052 is divisible by 11.

Common Factors

Common Factors

When two integers are multiplied together, the answer is called a **product.**
The integers that were multiplied together are called the **factors** of the product.
3 • 6 = 18
(3 and **6** are factors of **18)**

The **greatest common factor** of two (or more) integers is the largest integer that is a factor of both (or all) numbers.



Consider the numbers 18, 24, and 36.

The greatest common factor is 6.

(6 is the largest integer that will divide evenly into all three numbers)

The greatest common factor, (GCF), of two (or more) monomials is the product of the greatest common factor of the numerical coefficients (the numbers out in front) and the highest power of every variable that is a factor of each monomial.

Example: Consider $10x^2y^3$ and $15xy^2$

The greatest common factor is $5xy^2$.

The largest factor of 10 and 15 is 5.

The highest power of x that is contained in both terms is x.
The highest power of y that is contained in both terms is y^2 .

When factoring polynomials, first look for the largest monomial which is a factor of each term of the polynomial. Factor out (divide each term by) this largest monomial.

Example 1: Factor: $4x + 8y$

The largest integer that will divide evenly into 4 and 8 is 4. Since the terms do not contain a variable (x or y) in common, we cannot factor any variables.
The greatest common factor is 4. Divide each term by 4.

Answer: $4(x + 2y)$

Example 2: Factor: $15x^2y^3 + 10xy^2$

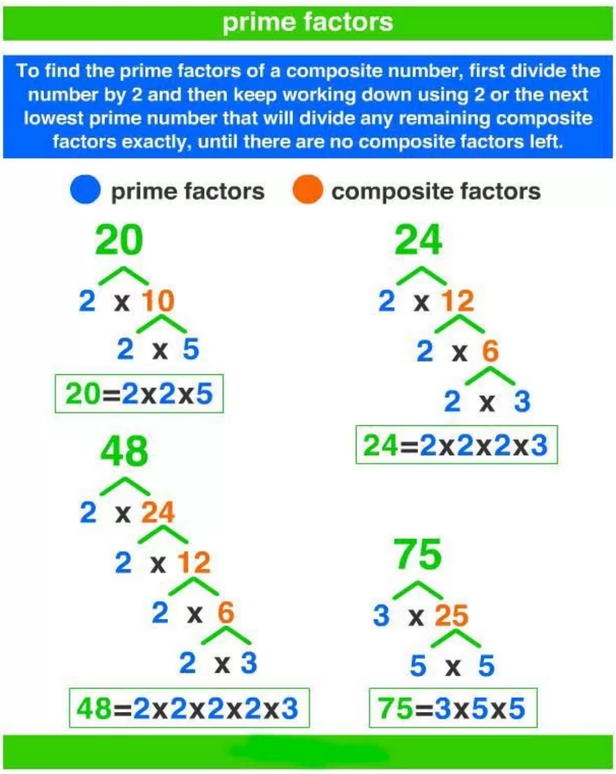
The largest integer that will divide evenly into 15 and 10 is 5. The largest power of x present in both terms is x.
The largest power of y present in both term is y^2.
The GCF is $5xy^2$. Divide each term by the GCF.

Answer: $5xy^2(3xy + 2)$

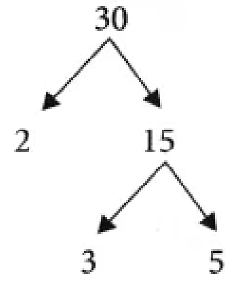
How to Find The Prime Factors Using Factor Tree

PRIME FACTORISATION

Prime factorisation is the process by which a composite number is rewritten as the product of prime factors.



Example 1: Find out the prime factorisation of 30.
First we will see whether the given number is divisible by a least prime number.
Yes, it is, because the digit at its ones place is 0.
30 = 2 × 15
We have, 15 = 3 × 5



So, the factors of 30 are
∴ 30 = 2 × 3 × 5
2, 3, and 5 are prime factors of 30.

Example 2: Let us consider another number 56.
56 = 2 × 28 = 2 × 2 × 14 = 2 × 2 × 2 × 7
2 and 7 are prime factors of 56.

Prime factorisation of a bigger number using short division method

Let us Explain it by taking an example.
Example 1: Express 256 in prime factorisation.
Divide 256 starting from the smallest prime number which can divide it. Repeat the process till the quotient is no more divisible by the prime number.

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

256 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

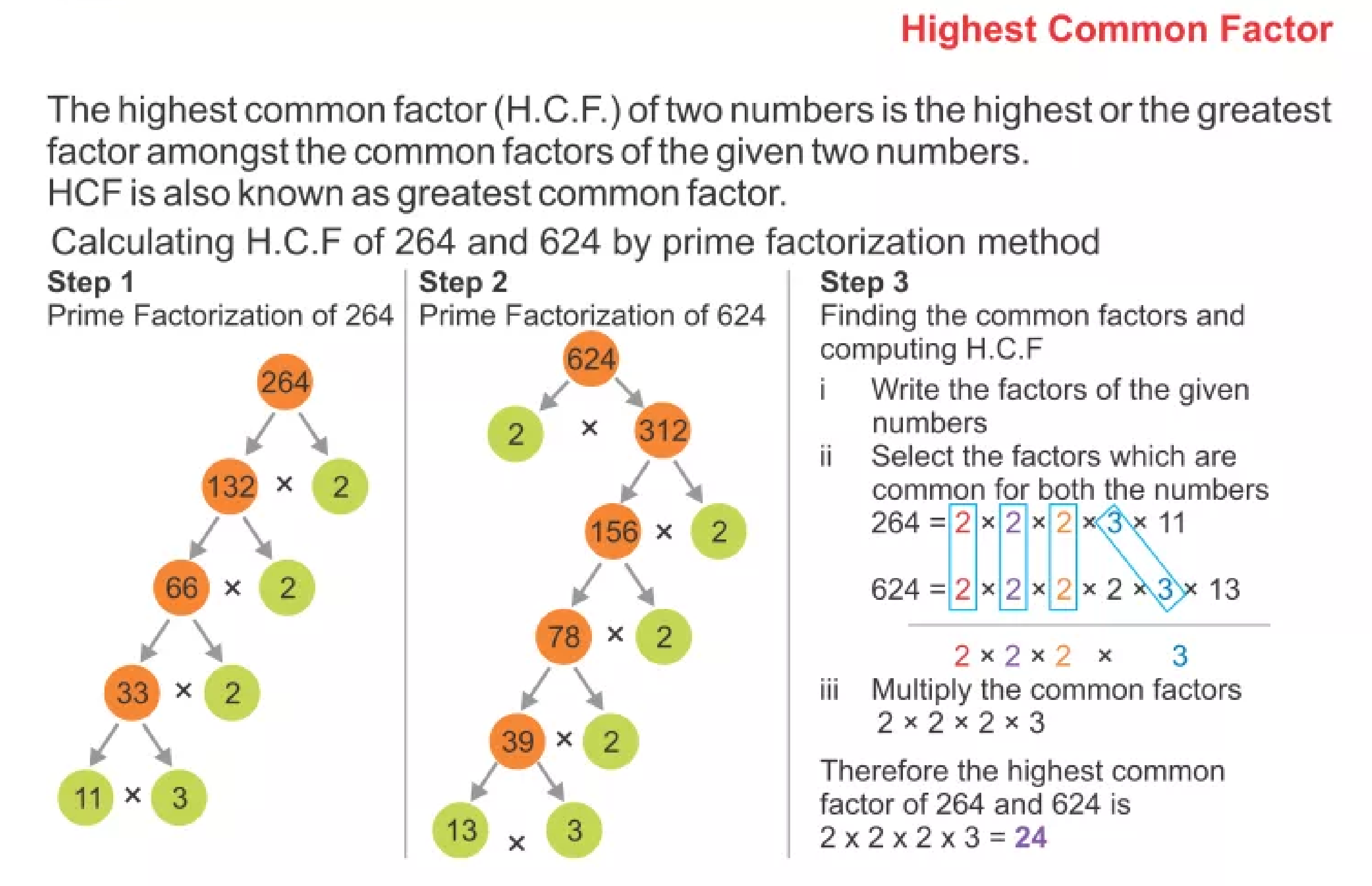
Example 2: Express 540 in prime factorisation.

2	540
2	270
3	135
3	45
3	15
5	5
	1

540 = 2 × 2 × 3 × 3 × 3 × 5

HIGHEST COMMON FACTOR (HCF)

Rita and Rina went to a stationery shop. Rita purchased 2 pencils, 2 pens, and 1 eraser. Rina purchased 2 pencils, 1 scale, and 1 pen. The common stationery bought by both are pencils and pens. Out of these, common stationery with maximum number is pencil (2). Thus, HCF is 2 pencils.
Highest common factor of two natural numbers is the largest common factor, or divisor of the given natural numbers. In other words, HCF is the greatest element of the set of common factors of the given numbers.



Example: Let us consider two numbers 45 and 63.

Factors of 45 = 1, 3, 5, 9, 15, 45

Factors of 63 = 1, 3, 7, 9, 21, 63

Common factors of 45 and 63 = 1, 3, 9
Highest common factor = 9.
So, the HCF of 45 and 63 is 9.

HCF by prime factorisation method

Let us consider two numbers 72 and 48.
To find prime factorisation, we have to follow these steps.
Step 1: Find the prime factorisation of both the numbers.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Prime factors of

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Step 2: Find the common prime factors of the given numbers.
Common factors = 2, 2, 2, 3
Step 3: Multiply all the common factors to find out the HCF.
∴ HCF = 2 × 2 × 2 × 3
= 24
The HCF of two or more numbers is the greatest common factor of all the given numbers.

HCF by long division method

To find HCF using long division method of two
numbers, follow the steps given below.

Step 1: Divide the greater number by smaller number.

Step 2: Take remainder as divisor and the divisor as dividend.

Step 3: Continue the process till you get 0 as the remainder.

Step 4: The last divisor will be the required HCF of the given numbers.

Example 1: Find the HCF of 198 and 360 using the long division method.

Solution:

$$\begin{array}{l} 198)\,360\,(1 \\ \quad\;\; \underline{198} \\ \quad\;\; 162)\,198\,(1 \\ \qquad\qquad \underline{162} \\ \qquad\qquad 36)\,162\,(4 \\ \qquad\qquad\qquad \underline{144} \\ \qquad\qquad\qquad 18)\,36\,(2 \\ \qquad\qquad\qquad\qquad \underline{36} \\ \qquad\qquad\qquad\qquad \times \end{array}$$

Here, the last divisor is 18.
So, HCF of 198 and 360 = 18.

Example 2: Find the greatest number which Exactly divides the numbers 280 and 1245, leaving remainders 4 and 3 respectively.

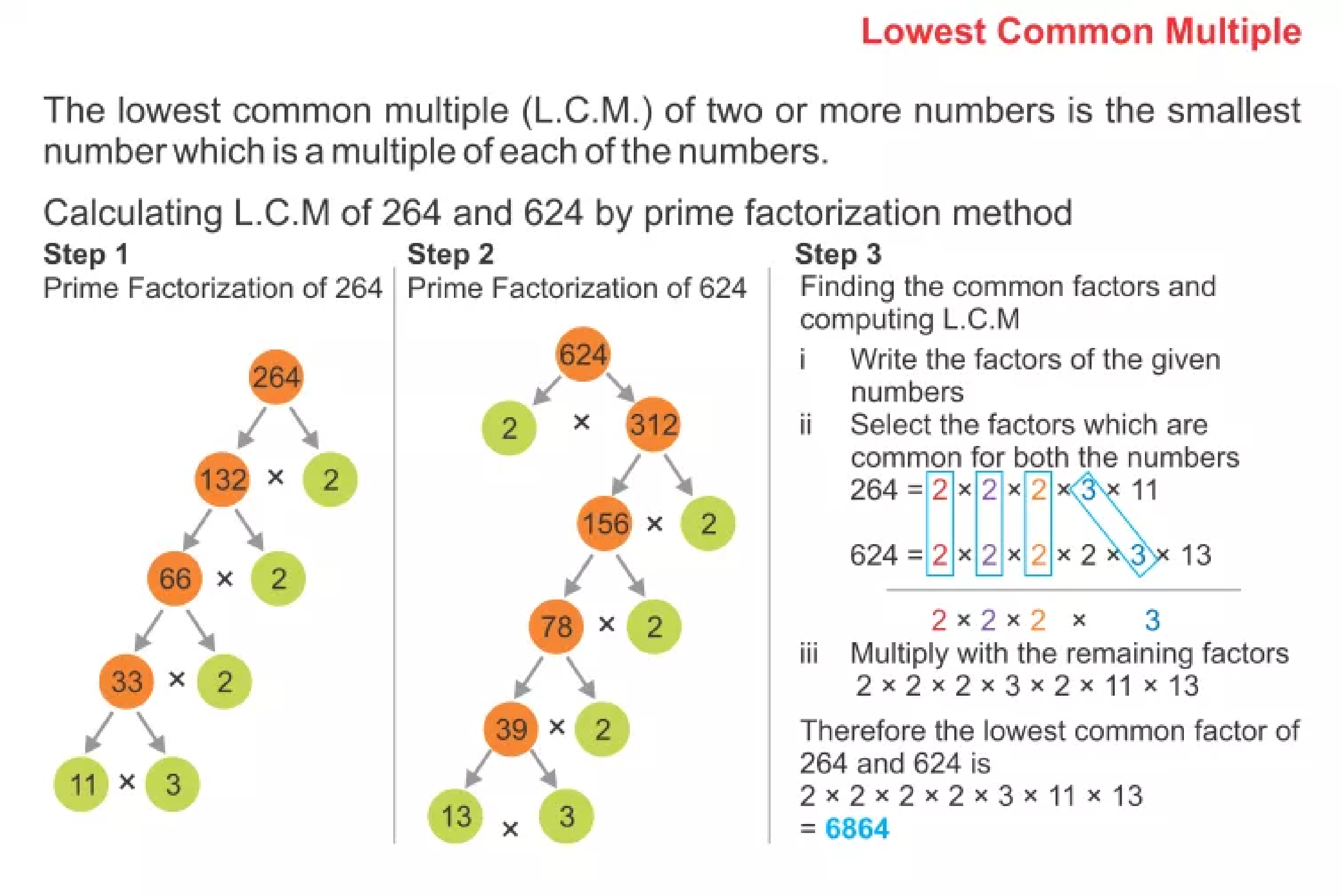
Solution: Since 4 and 3 are the remainders when 280 and 1245 are divided by the required number.
∴ 280 – 4 = 276 and 1245 – 3 = 1242 will be Exactly divisible by the required number.
We find the HCF of 276 and 1242.

276)1242 (4
1104
138) 276 (2
276
×

The HCF of276 and 1242 = 138
So, the required number is 138.

LOWEST COMMON MULTIPLE OR LEAST COMMON MULTIPLE (LCM)

Teena jogs every third day and Meena jogs every fifth day. They are both jogging today. After how many days will they jog together again?
Teena will jog on 3rd day, 6th day, 9th day,…
Meena will jog on 5th day, 10th day, 15th day,…
For Teena, multiples of 3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33,…
For Meena, multiples of 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45,…
This means, they will jog together after 15 days,
30 days, 45 days, etc. Therefore, 15, 30, 45,… are common multiples of 3 and 5 but the least (lowest) common multiple of 3 and 5 is 15. Hence, after 15 days, they will jog together again.
Least common multiple (LCM) of two natural numbers a and b is the smallest natural number which is a multiple of both a and b.
Since it is a multiple, it can be divided by a and b without leaving a remainder.



Example 1: Find the LCM of 4, 8, and 12.

Solution: Multiples of 4 = 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48,…

Multiples of 8 = 8, 16, 24, 32, 40, 48, 56, 64, 72,…
Multiples of 12 = 12, 24, 36, 48, 60, 72, 84,…
Common multiples = 24,48, 72

Lowest common multiple = 24
So, the LCM of 4, 8,12 is 24.

Example 2: Find the LCM of 25 and 30.

Solution: Multiple of 25 = 25, 50, 75, 100, 125, 150, 175, 200

Multiples of 30 = 30, 60, 90, 120, 150, 180, 210, 240
Common multiples of 25 and 30 = 150, 300,…
Least common multiple =150
So, the LCM of 25 and 30 is 150.

Finding LCM by prime factorisation method

To find the LCM by prime factorisation method, we follow the following steps:

Step 1: Express the given numbers as the product of prime numbers.

Step 2: Count the maximum number of times each factor appears then multiply them.

Step 3: The product of those factors is the least common multiple (LCM).

Example 1: Find the LCM of 28, 44, and 132 by the prime factorisation method.

Solution:

2	28
2	14
7	7
	1

2	44
2	22
11	11
	1

2	132
2	66
3	33
11	11
	1

Prime factorisation of 28 = 2 × 2 × 7
Prime factorisation of 44 = 2 × 2 × 11
Prime factorisation of 32 = 2 × 2 × 3 × 11
Here 2 appears twice.
3, 7, and 11 appear once.
∴ LCM = 2 × 2 × 3 × 7 × 11 = 924

Example 2: Find the LCM of 72, 90, and 108 by factorisation method.

Solution:

2	72
2	36
2	18
3	9
3	3
	1

2	90
3	45
3	15
5	5
	1

2	108
2	54
3	27
3	9
3	3
	1

Prime factorisation of 72 =2 × 2 × 2 × 3 × 3
Prime factorisation of 108 = 2 × 2 × 3 × 3 × 3
Here, 2 appears three times, 3 appears three times, and 5 appears once.
∴ LCM = 2 × 2 × 2 × 3 × 3 × 3 × 5
= 1080

LCM by common division method

To find the LCM by common division method, we follow these steps.

Step 1: Arrange the numbers in a row separated by commas.

Step 2: Choose a least prime number that divides at least one of the given numbers.

Step 3: Divide the numbers by the number chosen in step 2 and carry forward the undivided numbers.

Step 4: Repeat the process till the number left in the last row is 1.

Step 5: Multiply all the prime divisors to get the LCM.

Example 1: Find the LCM of 102,170, and 136 by common division method.

Solution:

2	102, 170, 136
2	51, 85, 68
2	51, 85, 34
3	51, 85, 17
5	17, 85, 17
17	17, 17, 17
	1, 1, 1

LCM = 2 × 2 × 2 × 3 × 5 × 17 = 2040

Example 2: Find the LCM of 11, 22, 24, and 36.

Solution:

2	11, 22, 24, 36
2	11, 11, 12, 18
3	11, 11, 6, 9
11	11, 11, 2, 3
2	1, 1, 2, 3
3	1, 1, 1, 3
	1, 1, 1, 1

Prime factorisation of 90 = 2 × 3 × 3 × 5
LCM = 2 × 2 × 3 × 11 × 2 × 3 = 792

Example 3: Find the least number which when divided by 20,24, and 36, leaves a remainder of 18 in each case.

Solution: The least number which is exactly divisible by 20, 24, and 36 is the LCM of these numbers. We first find the LCM of 20, 24, and 36.

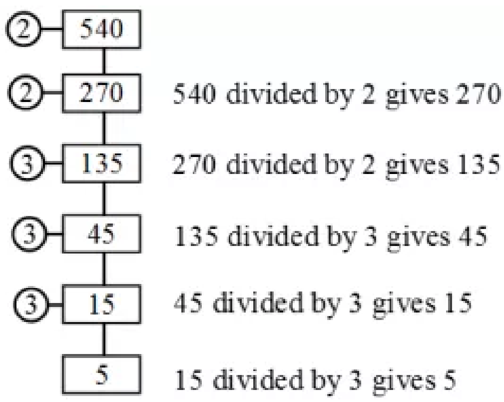
2	20, 24, 36
2	10, 12, 18
3	5, 6, 9
2	5, 2, 3
3	5, 1, 3
5	5, 1, 1
	1, 1, 1

∴ LCM = 2 × 2 × 3 × 2 × 3 × 5 = 360
But, the required number is a number that leaves a remainder of 18 in each case.
That means the required number is 18 more than the LCM.
∴ Required number = 360 + 18 = 378

Prime Factors Using Factor Tree Example Problems With Solutions

Example 1: Find the prime factors of 540

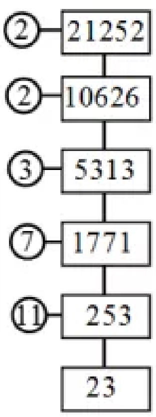
Sol.



∴ 5 is a prime number and so cannot be further divided by any prime number
540 = 2 × 2 × 3 × 3 × 3 × 5 = $2^2 \times 3^3 \times 5$

Example 2: Find the prime factors of 21252

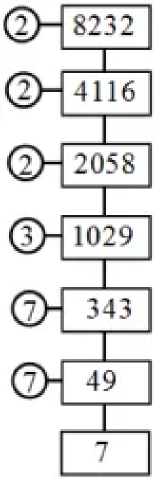
Sol.



∴ 21252 = 2 × 2 × 3 × 7 × 11 × 23
= **2^2 × 3 × 11 × 7 × 23.**

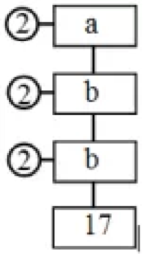
Example 3: Find the prime factors of 8232

Sol.



∴ 8232 = 2 × 2 × 2 × 3 × 7 × 7 × 7
= $\mathbf{2^3 \times 3 \times 7^3}$.

Example 4: Find the missing numbers a, b and c in the following factorisation:



Can you find the number on top without finding the other ?

Sol. c = 17 × 2 = 34
b = c × 2 = 34 × 2 = 68 and
a = b × 2 = 68 × 2 = 136
i.e., a = 136, b = 68 and c = 34.
Yes, we can find the number on top without finding the others.
Reason: The given numbers 2, 2, 2 and 17 are the only prime factors of the number on top and so the number on top = 2 × 2 × 2 × 17 = 136

How To Find The HCF And LCM Using Prime Factorisation Method

Find The HCF And LCM using Prime Factorisation Method

Relation between two numbers and their HCF and LCM

Consider two numbers 18 and 24.
Prime factorisation of 18 = 2 × 3 × 3
Prime factorisation of 24 = 2 × 2 × 2 × 3
So, HCF = 2 × 3 = 6
LCM = 2 × 2 × 2 × 3 × 3 = 72
Product of HCF and LCM = 6 × 72 = 432
Product of given numbers = 18 × 24 = 432
The product of LCM and HCF of two natural numbers is equal to the product of the given natural numbers.
∴ Product of given numbers = HCF × LCM of given numbers
For any two positive integers:
Their LCM. × their HCF. = Product of the number

(i) LCM = $\frac{\text{Product of the numbers}}{\text{HCF}}$

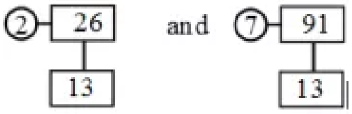
(ii) HCF = $\frac{\text{Product of the numbers}}{\text{LCM}}$

(iii) One number = $\frac{\text{H.C.F.} \times \text{L.C.M.}}{\text{Other number}}$

Finding HCF And LCM using Prime Factorisation Method Example Problems With Solutions

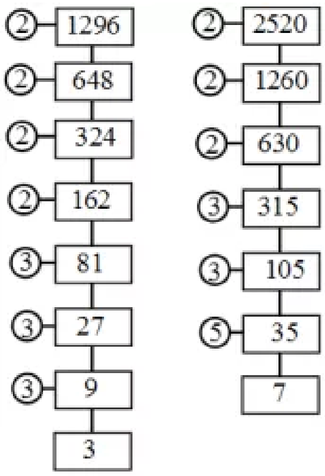
Find the L.C.M. and H.C.F. of the following pairs of integers by applying the Fundamental theorem of Arithmetic method i.e., using the prime factorisation method.

Example 1: 26 and 91
Sol. Since, 26 = 2 × 13 and, 91 = 7 × 13



L.C.M. = Product of each prime factor with highest powers. = 2 × 13 × 7 = 182.
i.e., **L.C.M.** (26, 91) = 182.
H.C.F. = Product of common prime factors with lowest powers. = 13.
i.e., **H.C.F** (26, 91) = 13.
Product of given two numbers = 26 × 91 = 2366
and, product of their **L.C.M.** and **H.C.F.** = 182 × 13 = 2366
Product of L.C.M and H.C.F of two given numbers = Product of the given numbers

Example 2: 1296 and 2520
Sol. Since, 1296 = 2 × 2 × 2 × 2 × 3 × 3 × 3 × 3 = $2^4 \times 3^4$
and, 2520 = 2 × 2 × 2 × 3 × 3 × 5 × 7 = $2^3 \times 3^2 \times 5 \times 7$



L.C.M. = Product of each prime factor with highest powers
= $2^4 \times 3^4 \times 5 \times 7$ = **45,360**
i.e., **L.C.M.** (1296, 2520) = 45,360
H.C.F. = Product of common prime factors with lowest powers = $2^3 \times 3^2$ = 8 × 9 = 72
i.e., H.C.F. (1296, 2520) = **72.**
Product of given two numbers = 1296 × 2520 = 3265920
and, product of their **L.C.M.** and **H.C.F.** = 45360 × 72 = 3265920
L.C.M. (1296, 2520) × H.C.F. (1296, 2520)
= 1296 × 2520 = 3265920

Example 3: 17 and 25
Sol. Since, 17 = 17
and, 25 = 5 × 5 = 5^2
L.C.M. = 17 × 5^2 = 17 × 25 = 425
and, **H.C.F.** = Product of common prime factors with lowest powers = 1, as given numbers do not have

any common prime factor.
The given numbers 17 and 25 do not have any common prime factor. Such numbers are called co-prime numbers and their H.C.F. is always equal to 1 (one), whereas their L.C.M. is equal to the product of the numbers.
But in case of two co-prime numbers also, the product of the numbers is always equal to the product of their L.C.M. and their H.C.F.
As, in case of co-prime numbers 17 and 25;
H.C.F. = 1; L.C.M. = 17 × 25 = 425;
product of numbers = 17 × 25 = 425
and product of their H.C.F. and L.C.M. = 1 × 425 = 425.

Example 4: Given that H.C.F. (306, 657) = 9, find L.C.M. (306, 657)
Sol. H.C.F. (306, 657) = 9 means H.C.F. of
306 and 657 = 9
Required L.C.M. (306, 657) means required L.C.M. of 306 and 657.
For any two positive integers;
their L.C.M. = $\frac{\text{Product of the numbers}}{\text{H.C.F.}}$
i.e., L.C.M. (306, 657) = $\frac{306 \times 657}{9}$ = 22,338.

Example 5: Given that L.C.M. (150, 100) = 300, find H.C.F. (150, 100)
Sol. L.C.M. (150, 100) = 300
⇒ L.C.M. of 150 and 100 = 300
Since, the product of number 150 and 100
= 150 × 100
And, we know :
H.C.F. (150, 100) = $\frac{\text{Product of 150 and 100}}{L.C.M.(150,100)}$
= $\frac{150 \times 100}{300}$ = 50.

Example 6: The H.C.F. and L.C.M. of two numbers are 12 and 240 respectively. If one of these numbers is 48; find the other numbers.
Sol. Since, the product of two numbers
= Their H.C.F. × Their L.C.M.
⇒ One no. × other no. = H.C.F. × L.C.M.
⇒ Other no. = $\frac{12 \times 240}{48}$ = 60.

Example 7: Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 + 5 are composite numbers.
Sol. Since,
7 × 11 × 13 + 13 = 13 × (7 × 11 + 1)
= 13 × 78 = 13 × 13 × 3 × 2;
that is, the given number has more than two factors and it is a composite number.
Similarly, 7 × 6 × 5 × 4 × 3 + 5
= 5 × (7 × 6 × 4 × 3 + 1)
= 5 × 505 = 5 × 5 × 101
∴ The given no. is a composite number.