July Test (Mathematics)

(2020-21)

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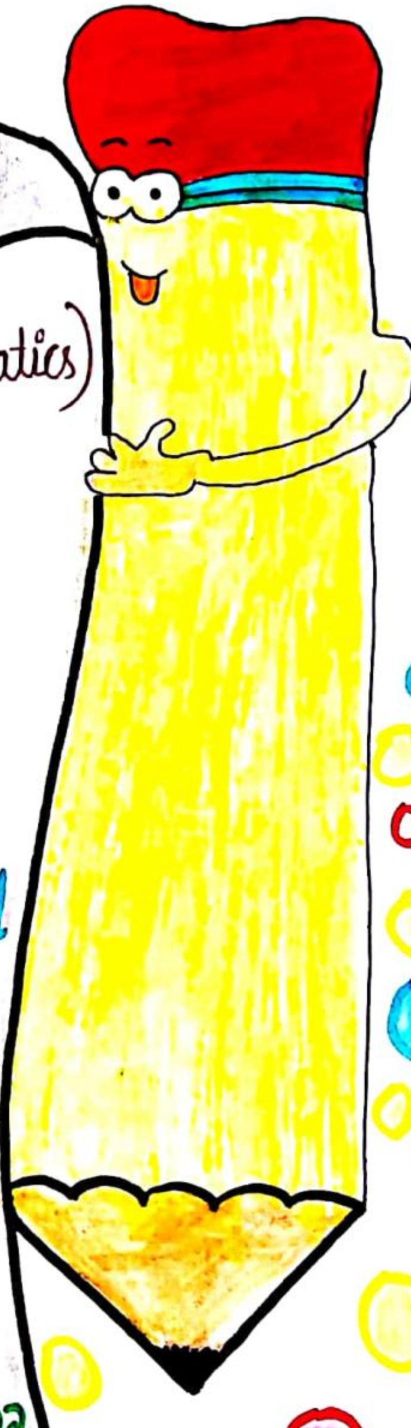

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Ques) 1. If matrix A is of order 3x4, then each row of matrix contains elements.

(A) 4 (B) 12 (C) 3 (D) 7

Sol. ⇒ 4.

2.) If $A = \begin{bmatrix} k & 10 \\ 3 & 5 \end{bmatrix}$ is a singular matrix, then the value of k is equal to.

(A) 0 (B) 6 (C) ÷6 (D) 1/6

Sol. ⇒ we are given

$$A = \begin{bmatrix} k & 10 \\ 3 & 5 \end{bmatrix}$$

Since, A is singular matrix

$$\therefore |A| = 0$$

$$\therefore \begin{vmatrix} k & 10 \\ 3 & 5 \end{vmatrix} = 0$$

$$\therefore (5k - 30) = 0$$

$$5k = 30$$

$$\boxed{k = 6}$$

(C) $\boxed{6}$

3.) If $\begin{vmatrix} 2x & 3 \\ -5 & x \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ -5 & 8 \end{vmatrix}$, the negative value of x.

(A) ±4 (B) −4 (C) 4 (D) 0

Sol. ⇒ we are given

$$\begin{vmatrix} 2x & 3 \\ -5 & x \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ -5 & 8 \end{vmatrix}$$

$$\therefore (2x^2 + 15) = (32 + 15)$$

$$\therefore 2x^2 = 32 + \cancel{15} - \cancel{15}$$

$\therefore 2x^2 = 32$

$\therefore x^2 = 16$

$\therefore x = \pm 4$

for negative value of x

$\therefore \boxed{x = -4}$

(B) -4

4.) $\frac{d}{dx}(\sin^{-1}x + \cos^{-1}x)$ is equal to:

(A) $\frac{1}{\sqrt{1-x^2}}$ (B) $\frac{2}{\sqrt{1-x^2}}$ (C) 0 (D) $\sqrt{1-x^2}$

Sol. $\Rightarrow$ Let $y = \sin^{-1}x + \cos^{-1}x$

$\therefore$ Diff. w.r.to x

$\therefore \frac{dy}{dx} = \cancel{\frac{1}{\sqrt{1-x^2}}} + \left(\cancel{\frac{-1}{\sqrt{1-x^2}}}\right)$

$\therefore \frac{dy}{dx} = 0$

(C) $\boxed{0}$

5) If $y = \sin^{-1}x$, then $\frac{dy}{dx}$ is:

(A) $\frac{1}{\sqrt{1-x^2}}$ (B) $\frac{-1}{\sqrt{1-x^2}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-1}{1+x^2}$

Sol. $\Rightarrow$ $\frac{1}{\sqrt{1-x^2}}$

6) If $y = 3x^2 + x - 100$, then $\frac{dy}{dx}$ is:

(A) $6x$ (B) $6x+1$ (C) $6x+1-100$ (D) x^3+x^2-100

Sol. $\Rightarrow$ we are given

$y = 3x^2 + x - 100$

$\therefore$ Diff. w.r.to x

$\frac{dy}{dx} = 6x + 1 - 0$

$\therefore \frac{dy}{dx} = 6x + 1$

(B) $\boxed{6x+1}$

7) The Principle Value of $\tan^{-1}(-1)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{5\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $-\frac{\pi}{4}$

Sol. $\Rightarrow$ $\boxed{-\frac{\pi}{4}}$

(8) The Principle Value of $\cos^{-1}(\cos\frac{7\pi}{6})$ is:

(A) $\frac{7\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{5\pi}{6}$ (D) $-\frac{\pi}{6}$

Sol. $\Rightarrow$ we are given

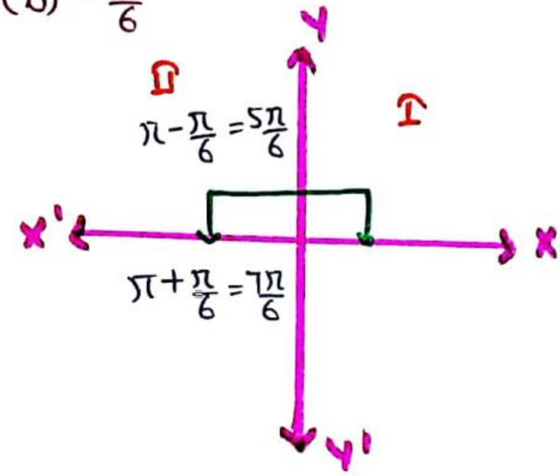
$= \cos^{-1}(\cos\frac{7\pi}{6})$

$= \cos^{-1}(\cos(\pi - \frac{\pi}{6}))$

$= \cos^{-1}(\cos(\frac{5\pi}{6}))$

$= \frac{5\pi}{6}$

(C) $\boxed{\frac{5\pi}{6}}$



(9). Construct a 2×3 matrix whose elements are given by $a_{ij} = i+2j$.

Sol. $\Rightarrow$ Let $A = [a_{ij}]$ be a matrix of order 2×3

where $[a_{ij}] = i+2j$

$$\therefore A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times3}$$

$\therefore a_{11} = 1+2 = 3$; $a_{21} = 2+2 = 4$

$a_{12} = 1+4 = 5$; $a_{22} = 2+4 = 6$

$a_{13} = 1+6 = 7$; $a_{23} = 2+6 = 8$

$$\therefore A = \begin{bmatrix} 3 & 5 & 7 \\ 4 & 6 & 8 \end{bmatrix}_{2\times3}$$

(10) Find $\frac{dy}{dx}$ when $x = at^2$, $y = 2at$.

Sol. $\Rightarrow$ we are given

$x = at^2$, $y = 2at$

Now, $x = at^2$

$\therefore$ diff. w.r.to t

$$\therefore \frac{dx}{dt} = 2at$$

$$\therefore y = 2at$$

$\therefore$ Diff. w.r. to, t

$$\frac{dy}{dt} = 2a$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{\cancel{2a}}{\cancel{2a}t}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{t}}$$

(11) Prove that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$

Sol. $\Rightarrow$ we have to prove

$$\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$$

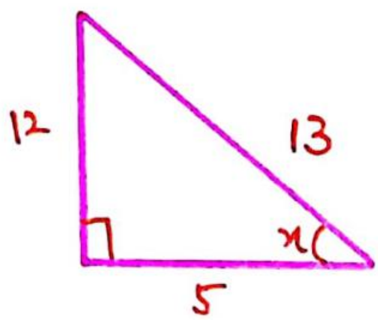
Now,

(i) $\sin^{-1}\left(\frac{12}{13}\right)$

Let $x = \sin^{-1}\left(\frac{12}{13}\right)$

$\therefore \sin x = \frac{12}{13}$

$\therefore \tan x = \frac{12}{5}$

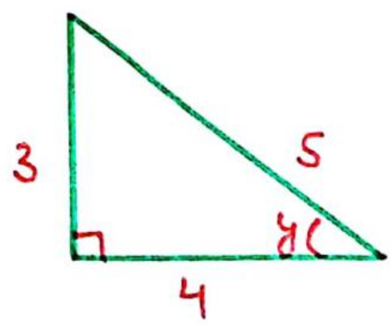


(ii) $\cos^{-1}\left(\frac{4}{5}\right)$

Let $y = \cos^{-1}\left(\frac{4}{5}\right)$

$\therefore \cos y = \frac{4}{5}$

$\therefore \tan y = \frac{3}{4}$



(iii) $\tan^{-1}\left(\frac{63}{16}\right)$

Let $\tan^{-1}\left(\frac{63}{16}\right) = z$

$\therefore \tan z = \frac{63}{16}$

Now,

$\therefore \tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan x \tan y}$

$\therefore \tan(x+y) = \frac{\frac{12}{5} + \frac{3}{4}}{1 - \left(\frac{12}{5}\right)\left(\frac{3}{4}\right)}$

$\therefore \tan(x+y) = \frac{48+15}{20-36}$

$\therefore \tan(x+y) = -\frac{63}{16}$

$\therefore \tan(x+y) = -\tan z$

$\therefore \tan(x+y) = \tan(-z)$ or $\tan(\pi - z)$

$\therefore x+y = -z$ or $x+y = \pi - z$

But, $x+y \neq -z$ $\{\because x, y, z$ are +ve$\}$

$\therefore x+y+z = \pi$

$\therefore \sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$

Hence Result is proved.

OR

Ques) Solve $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

Sol. ⇒ we are given

$$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$$

$$\therefore \tan^{-1}\left(\frac{2x+3x}{1-(2x)(3x)}\right) = \frac{\pi}{4}, \quad \text{Provided } (2x)(3x)<1$$

$$\therefore \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}, \quad \text{Provided } 6x^2<1$$

$$\therefore \frac{5x}{1-6x^2} = \tan\left(\frac{\pi}{4}\right)$$

$$\therefore \frac{5x}{1-6x^2} = 1$$

$$\therefore 5x = 1-6x^2$$

$$\therefore 6x^2+5x-1=0$$

$$\therefore 6x^2+6x-x-1=0$$

$$\therefore 6x(x+1)-(x+1)=0$$

$$\therefore (6x-1)(x+1)=0$$

Either, $(6x-1)=0$ or $(x+1)=0$

$\therefore \boxed{x=\frac{1}{6}}$ $\qquad$ $\boxed{x=-1}$

for, $x=\frac{1}{6}$	for $x=-1$
$6x^2<1$	$6x^2<1$
$6\left(\frac{1}{6}\right)^2<1$	$6(-1)^2<1$
$\cancel{6}\left(\frac{1}{\cancel{36}_{6}}\right)<1$	$6(1)<1$
$\frac{1}{6}<1$	$6<1$
(True)	(False)

$\therefore$ Rejecting $x=-1$

$$\therefore \boxed{x=\frac{1}{6}}$$

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(12.) If $y = x^{\tan x} + (\tan x)^x$, then find $\frac{dy}{dx}$.

Sol. ⇒ we are given

$$y = x^{\tan x} + (\tan x)^x$$

$$\therefore y = u + v \quad \text{--- (1)}$$

where $u = x^{\tan x}$, $v = (\tan x)^x$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{--- (2)}$$

$$u = x^{\tan x}$$

Taking log both side

$$\log u = \log(x^{\tan x})$$

$$\log u = \tan x \log x$$

Diff. w.r.to x

$$\frac{1}{u}\frac{du}{dx} = \tan x \times \frac{1}{x} + \log x \times \sec^2 x$$

$$\therefore \frac{1}{u}\frac{du}{dx} = \frac{\tan x}{x} + \log x \sec^2 x$$

$$\frac{du}{dx} = u\left\{\frac{\tan x}{x} + \log x \cdot \sec^2 x\right\}$$

$$\frac{du}{dx} = x^{\tan x}\left\{\frac{\tan x}{x} + \log x \cdot \sec^2 x\right\}$$

$$v = (\tan x)^x$$

Taking log both side

$$\log v = \log(\tan x)^x$$

$$\log v = x \cdot \log(\tan x)$$

$\therefore$ Diff. w.r.to x

$$\frac{1}{v}\frac{dv}{dx} = x \times \frac{1}{\tan x} \cdot \sec^2 x + \log(\tan x) \cdot 1$$

$$\frac{dv}{dx} = v\left\{\frac{x}{\tan x} \cdot \sec^2 x + \log(\tan x)\right\}$$

$$\frac{dv}{dx} = (\tan x)^x\left\{\frac{x}{\tan x} \cdot \sec^2 x + \log(\tan x)\right\}$$

from (2) we get,

$$\therefore \frac{dy}{dx} = x^{\tan x}\left\{\frac{\tan x}{x} + \log \cdot \sec^2 x\right\} + (\tan x)^x\left\{\frac{x}{\tan x} \cdot \sec^2 x + \log(\tan x)\right\}$$

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