Gravitation

Exercise Solutions

Question 1: The spherical balls of mass 10 kg each are paced 10 cm apart. Find the gravitational force of attraction between them.

Solution:

The mass of each ball = m = 10 kg

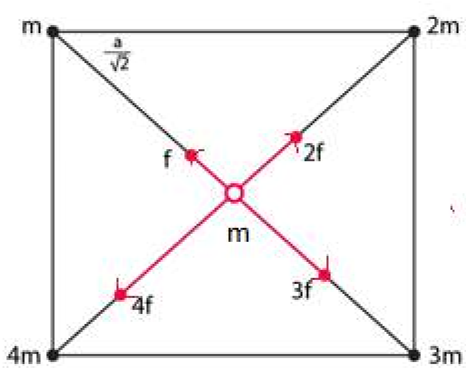
Distance of separation = r = 10 cm or 0.010 cm

Force = $GMm/r^2 = [6.67 \times 10^{-11} \times 10^2]/(0.010)^2$

$= 6.67 \times 10^{-7}$

Question 2: Four particles having masses m, 2m, 3m and 4m are placed at the four corners of a square of edge α. Find the gravitational force acting on a particle of mass m placed at the center.

Solution:



The gravitational force at the center = vector sum of all the forces acting on it.

The distance between the center particle with others, say $r = a/\sqrt{2}$

Force acting between particles of mass m and center particle = $F_m = GMm/r^2 = 2Gm^2/a^2$

Force acting between particles of mass m and center particle = $F_2m = GM(2m)/r^2 = (4Gm^2)/r^2 = 2F_m$

Similarly, we can calculate of mass 3m and 4m along with the center particle:
F_{3m} =3 F_m and F_{4m} = 4 F_m

The net force:

$F_{net} = 2\ F_m \cos\theta = 4\ F_m\ (1/\sqrt{2}) = 2\sqrt{2}\ F_m$

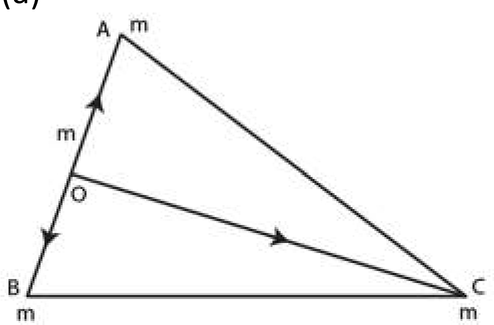
=> $F_{net} = 2\sqrt{2}\ F_m = 2\sqrt{2} \times 2Gm^2/a^2 = [4\sqrt{2}\ Gm^2]/a^2$

Question 3: Three equal masses m are placed at the three corners of an equilateral triangle of side α. Find the force exerted by the system on another particle of mass m placed at

(a) The mid-point of a side,
(b) At the Centre of the triangle.

Solution:

(a)



If "m" is the mid point of a side, then

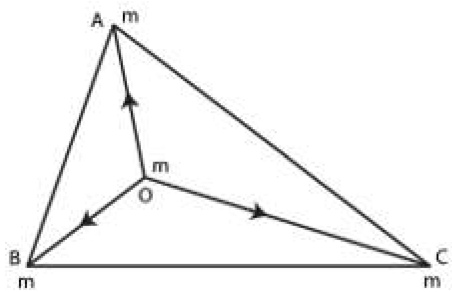
$F_{OA} = 4Gm^2/a^2$ in OA direction
$F_{OB} = 4Gm^2/a^2$ in OB direction

=> $F_{OC} = 4Gm^2/3a^2$ in OC direction

[As, Equal and opposite cancel each other]

So, net gravitational force on m is $4Gm^2/a^2$

(b)



If point "O" is the centroid, then

$F_{OA} = 3Gm^2/a^2$ and $F_{OB} = 3Gm^2/a^2$

So, resultant force is

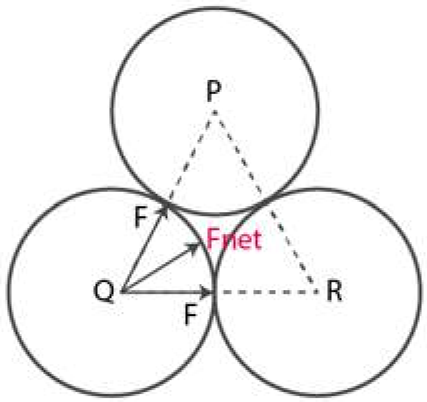
$$= \sqrt{2(\frac{3Gm^2}{a^2})^2 - 2(\frac{3Gm^2}{a^2})^2 \times \frac{1}{2}} = \frac{3Gm^2}{a^2}$$

Since $F_{OC} = 3Gm^2/a^2$
[Equal and opposite to F cancel each other]

=> Net gravitational force become zero.

Question 4: Three uniform spheres each having a mass M and radius α are kept in such a way that each touches the other two. Find the magnitude of the gravitational force on any of the spheres due to the other two.

Solution:
Distance between the centers of two spheres = r= 2a



Force on one sphere due to another = F = $GM^2/4a^2$

Net force = F_{net} = 2F cosθ = 2F cos 30°

= 2 x √3/2 x $GM^2/4a^2$

=> $F_{net} = \sqrt{3}GM^2/4a^2$

Question 5: Four particle of equal masses M move along a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle.

Solution:

Let A, B, C and D are four particles of mass , moving in a circle of radius R.

Force between A and B = $F_{AB} = GM^2/(\sqrt{2}R)^2 = GM^2/2R^2$

Force between A and D = $F_{AD} = GM^2/(\sqrt{2}R)^2 = GM^2/2R^2 = F_{AB}$

Net force in downward direction = $F_D = 2F_{AB} \cos 45^o = \sqrt{2}\, F_{AE}$

Force between A and C = $F_{AC} = GM^2/(2R)^2 = GM^2/4R^2$

Now,

Net force on particle A = $F_{net} = F_D + F_{AC}$

$$F_{net} = \left(\frac{2\sqrt{2}+1}{4}\right)\frac{GM^2}{R^2}$$

For moving along the circle, F_net = mv^2/R

$$\frac{Mv^2}{R} = \left(\frac{2\sqrt{2}+1}{4}\right)\frac{GM^2}{R^2}$$

$$v = \sqrt{\left(\frac{2\sqrt{2}+1}{4}\right)\frac{GM}{R}}$$

Question 6: Find the acceleration due to gravity of the moon at a point 1000 Km above the moon's surface. The mass of the moon is 7.4×10^{22} kg and its radius is 1740 km.

Solution:

Mass of the moon= M = 7.4×10^{22} kg
Radius =R = 1740 km and Distance of the point from surface= R' = 1000 km

Total distance from the center = r = 1740+1000 = 2740 km

Now,
Find Acceleration due to gravity:
$g = GM/r^2$

$$g = \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{(2740 \times 10^3)^2} = 0.65 m/s^2$$

Question 7: Two small bodies of masses 10 kg and 20 kg are kept a distance of 1.0 m apart and released. Assuming that only mutual gravitational forces are acting, find the speed of particles when the separation decreases to 0.5 m.

Solution:
let m_1 and m_2 masses of bodies, where m_1 = 10 kg and m_2 = 20 kg

Initial separation, say r_1 = 10 m and Final separation, say r_2 = 0.5 m
Let v_1 be the initial velocity and v_2 be the final velocity, where $v_1 = v_2 = 0$ m/s

Let us consider v_1', v_2' are the final velocities.

Now,
$m_1 v_1' + m_2 v_2' = 0$

=> $v_1' = -(20/10)v_2' = -2 v_2'$

[From momentum conservation]

Again, from using the conservation of energy:
$PE_{initial} + KE_{initial} = PE_{final} + KE_{final}$

$$=> \frac{Gm_1m_2}{r_1^2} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{Gm_1m_2}{r_2^2} + \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2{v_2'}^2$$

Substituting the values, we get

$$=> v_2'^2 = \frac{2 \times 6.67 \times 10^{-11}}{3} = 44.47 \times 10^{-11} = 2.10 \times 10^{-5} m/s$$

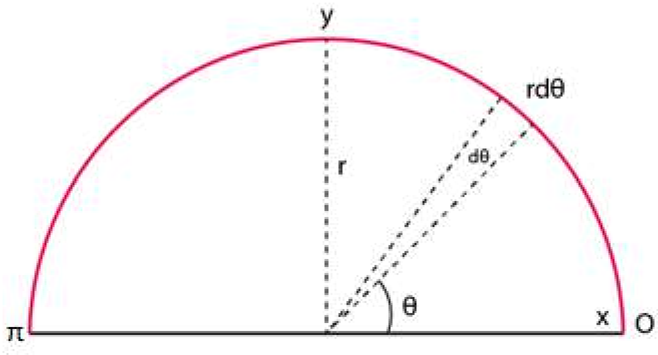
and,

$$v_1' = 2v_2' = 4.20 \times 10^{-5} m/s$$

Question 8: A semicircular wire has a length L and mass M. A particle of mass m is placed at the Centre of the circle. Find the gravitational attraction on the particle due to the wire.

Solution:
Let us take a small element on the wire. The arc length of the element is $r\,d\theta$.
=> Mass of the element = $dM = (M/L)\, r\, d\theta$
Also, $r = L/\pi$

=> $dM = M d\theta/\pi$



The force on particle due to element = dF = GmdM/r^2= (GMmπdθ)/L^2
Therefore,

$$F = \int dF = \int_0^{\pi} \frac{GMm\pi}{L^2} \cos\theta d\theta$$

$$F = \frac{2\pi GM\,m}{L^2}$$

Question 9: Derive an expression for the gravitational field due to a uniform rod of length L and mass M at a point on its perpendicular bisector at a distance d from the center.

Solution:
A small section of rod is at "x" distance mass of the element = dm = (M/L).dx

$dE_1 = [G(dm)]/(d^2+x^2) = dE_2$

So resultant $dE = 2\, dE_1 \sin\theta$

$$2 \times \frac{Gdm}{d^2+x^2} \times \frac{d}{\sqrt{d^2+x^2}} = \frac{2(GM)dx}{L(d^2+x^2)(\sqrt{d^2+x^2})}$$

Now, the total gravitational force:

$$E = \int_0^{\frac{L}{2}} \frac{2Gmddx}{L(d^2 + x^2)^{3/2}}$$

Solving above equation, we get

$E = 2GM/[d \sqrt{(L^2+4d^2)}]$

Question 10: Two concentric spherical shells have masses M_1, M_2 and radii R_1, R_2 ($R_1<R_2$). What is the force exerted by this system on a particle of mass m_1 if it is placed at a distance $(R_1+ R_2)/2$ from the center?

Solution:

The gravitational force on m due to shell of M_2 is zero. Ma is at distance $(R_1 + R_2)/2$

The gravitational force:

$$F = \frac{GM_1m}{r^2} = \frac{GM_1m}{[(R_1 + R_2)/2]^2}$$

$$F = \frac{4GM_1m}{(R_1 + R_2)^2}$$

Question 11: A tunnel is dug along a diameter of the earth. Find the force on a particle of mass m placed in the tunnel at a distance x from the center.

Solution:

Let us assume that tunnel doesn't change the gravitational field distribution of earth. Mass of the sphere:

$$\frac{M'}{\frac{4}{3\pi x^3}} = \frac{M_e}{\frac{4}{3\pi R^3}}$$

or

$$M' = \frac{x^3}{R^3} M_e$$

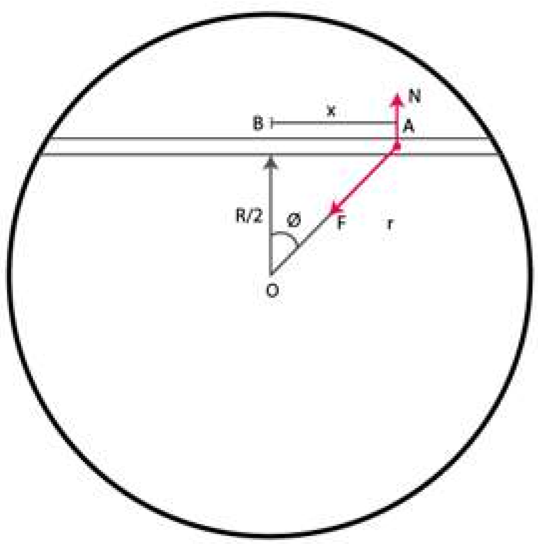
Where M_e is the mass of the earth .

The gravitational force on the particle at distance x,

$F = GMM'/x^2 = GM_e/R^3$

Question 12: A tunnel is dug along a chord of the earth at a perpendicular distance R/2 from the earth's center. The well of the tunnel may be assumed to be frictionless. Find the force exerted by the wall on a particle of mass m when it is at a distance x from the center of the tunnel.

Solution:
Let M_E be the mass of the earth.



From figure,

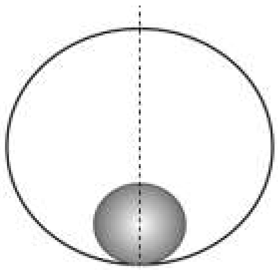
$N = F \cos \phi$

here, $\cos \phi = R/2r$ and $F = GM_E\, mr/R^3$

thus, $N = GM_E\, mr/R^3 \times R/2r$

or $N = GMm/2R^2$

Question 13: A solid sphere of mass m and radius r is placed inside a hollow thin spherical shell of mass M and radius R as shown in figure (below). A particle of mass m' is placed on the line joining the two centers at a distance x from the point of contact of the sphere and the shell. Find the magnitude of the resultant gravitational on this particle due to the sphere and the shell if (a) $r < x < 2r$, (b) $2r < x < 2R$. and (C) $x > 2R$.



Solution:

(a) distance of the particle from the center of solid sphere:

$l = x - r$

Gravitational force on the object:
$F = Gmm'/r^3$

Here, the mass of the sphere "m" and m' is the place at distance x from O.

$\Rightarrow F = Gmm'(x-r)/r^3$

(b) $2r < x < 2R$, then F is due to only sphere

$F = Gmm'/(x-r)^2$

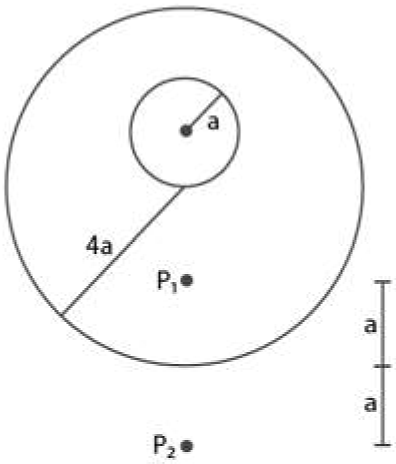
(c) If $x > 2R$, the gravitational force is due to both shell and sphere,

Force due to shell: $F = GMm'/(x-R)^2$

Force due to sphere: $F = GMm'/(x-r)^2$

So, resultant force = $GMm'/(x-R)^2 + GMm'/(x-r)^2$

Question 14: A uniform metal sphere of radius α and mass M is surrounded by a thin uniform spherical shell of equal mass and radius 4α (figure below). The Centre of the shell falls on the surface of the inner sphere. Find the gravitational field at the points P1 and P2 shown in the figure.



Solution:

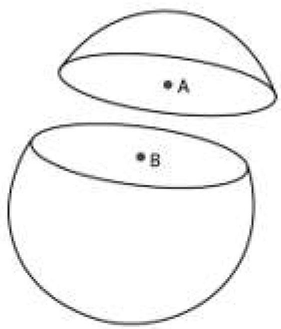
At point P_1
Gravitational force due to sphere = M = $GM/(3a+a)^2 = GM/16a^2$

At point P_2,Gravitational force due to sphere and shell

= $GM/(a+4a+a)^2 + GM/(4a+a)^2$

= $(61/900)\ GM/a^2$

Question 15: A thin spherical shell having uniform density is cut in two parts by a plane and kept separated as shown in figure (below). The point A is the Centre of the plane section of the first part and B is the Centre of the place section of the second part. Show that the gravitational field at A due to the first part is equal in magnitude to the gravitational field at B due to the second part.



Solution:

we know, the field inside the shell is zero. Let the gravitational field at A due to the first part be E and the gravitational field at B due to the second part be E'.

Therefore, $E + E' = 0$

or $E = -E'$

Hence, the fields are equal is magnitude and opposite in direction

Question 16: Two small bodies of masses 2.00 kg and 4.00 Kg are kept at rest at a separation of 2.0 m. Where a particle of mass 0.10 kg should be placed to experience no net gravitational force from these bodies? The particle is placed at this point. What is the gravitational potential energy of the system of three particles with usual reference level?

Solution:
Let the mass of 0.10 kg be at a distance x from 2 kg mass and at the distance of (2-x) from the 4 kg mass.

Force between 0.1 kg mass and 4 kg mass = Force between 0.1 kg mass and 2 kg mass

$(2x0.1)/x^2 = -(4x0.1)/(2-x)^2$

$x = 2/2.414$

or x = 0.83 m from the 2 kg mass.

Now,
The gravitational potential energy is given by

$$V = \sum_{i \neq j} \frac{Gm_i m_j}{r_{ij}} = \frac{G0.1 \times 2}{0.83} + \frac{G0.1 \times 4}{1.17} + \frac{G2 \times 4}{2}$$

$$= 0.24GJ$$

$$= -3.06 \times 10^{-10}$$

Question 17: Three particles of mass m each are placed at the three corners of an equilateral triangle of side α. Find the work which should be done on this system to increase the sides of the triangle to 2α.

Solution:

work done = W = U_f - U_i

Where U_f = Final potential energy and U_i = Initial potential energy

Here, $U_f = -3Gm^2/2a$ and $U_i = -3Gm^2/a$

Now, $W = 3Gm^2/2a$

Question 18: A particle of mass 100 g is kept on the surface of a uniform sphere of mass 10 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle away from the sphere.

Solution:
U_f = Final potential energy = 0
[As the particle is to be taken away, we assume the final point to be approximately at infinite distance]

and U_i = Initial potential energy = $(-GM_s m)/r$

Here, m = Mass of the particle = 100 g or 0.1 kg

M_s = Mass of sphere = 10 kg

And r = radius of sphere = 10 cm or 0.1 m

On putting values, we get

$U_i = -6.67 \times 10^{-10}$

Now, work done = W = $-(U_f - U_i)$

=> W = 6.67×10^{-10} J

Question 19: The gravitational field in a region is given by

$$\vec{E} = (5\ \text{N kg}^{-1})\,\vec{i} + (12\ \text{N kg}^{-1})\,\vec{j}$$

(a)Find the magnitude of the gravitational force acting on a particle of mass 2 kg placed at the origin.
(b)Find the potential at the points (12m, 0) and (0, 5m) if the potential at the origin is taken to be zero.
(c) Find the change in gravitational Potential energy if a particle of mass 2 kg is taken from the origin to the point (12m, 5m).
(d)Find the change in potential energy If the particle is taken from (12m, 0) to (0, 5m).

Solution:

(a) force on the particle

F = mE = 2[5i+12j] = 10i + 24j N

[given mass of the particle = m = 2 kg]

Magnitude of F = 26 N

(b) Potential at (12, 0):

V = -E.r = -12i [5i+12j] = -60 J/kg

Potential at (0, 5):

V = -E.r = -5i [5i+12j] = -60 J/kg

(c) potential energy at (12,5) m:

V = [5i+12j] [2i + 5j] = -120 J/kg

And potential energy at the origin is zero.

Therefore, the change in potential energy is -240 J.

(d) Change in potential energy = 0
[from part (b), potential energy of the particle would be same at both the points.]

Question 20: The gravitational field in a region is given by V= 20N Kg^{-1} (x+y).

(a) Show that the equation is dimensionally correct.
(b) Find the gravitational field at the point (x, y). Leave your answer in terms of the unit vectors i, j, k
(c) Calculate the magnitude of the gravitational force on a particle of mass 500g placed at the origin.

Solution:

(a) V= 20N Kg^{-1} (x+y)

Dimension of V = $[MLT^{-2}]/M \times L = L^2T^{-2}$

Dimension of j/kg = $[ML^2T^{-2}]/M = L^2T^{-2}$

Hence dimensions are correct.

(b)

$$E = -\frac{\partial V}{\partial x} - \frac{\partial V}{\partial y}$$

Or E = -20i – 20j N/kg

E is independent of the coordinate.

(c) Force = mE
= 0.5 x [-20(i+j)]
= -10i – 10j

Magnitude of Force = 10√2 N

Question 21: The gravitational field in a region is given by E = (2i + 3j)Nkg^{-1}. Show that no work is done by the gravitational field whom a particle is moved on the line 3y +2x = 5.

[Hint: if a line y=mx + c makes angle θ with the X – axis, m= tan θ]

Solution:

Electric field = E = 2i + 3j

Angle made by E with the x-axis:

$$\cos\theta = \frac{E.i}{|E||i|} = \frac{(2i+3i).i}{\sqrt{13}} = \frac{2}{\sqrt{13}}$$

$$\sec\theta = \frac{\sqrt{13}}{2}$$

we know,

$$\tan^2\theta = \sec^2\theta - 1$$

$$= \left(\frac{\sqrt{13}}{2}\right)^2 - 1$$

or

$$\tan\theta = \frac{3}{2}$$

Equation of line is $y = -(2/3)x + 5/3$ made angle with x-axis is

$\tan\phi = -2/3$

Now,

$\tan(\phi - \theta) = [\tan\phi - \tan\theta]/[1+\tan\phi \tan\theta]$

= infinity

Now, the angle between the electric filed and the line = $\phi - \theta = 90^o$
Since product of both the slope is -1, the direction of field and the displacement are perpendicular, is done by the particle on the line.

Question 22: Find the height over the earth's surface at which the weight of a body becomes half of its value at the surface.

Solution:
Let h be the height

Therefore, $(1/2)\ Gm/R^2 = GM/(R+h)^2$

or $2R^2 = (R+h)^2$

or $h = (\sqrt{2} - 1)R$

Question 23: What is the acceleration due to gravity on the top of Mount Everest? Mount Everest is the highest mountain peak of the world at the height of 8848 m. The value at sea level is $9.80\ ms^{-2}$.

Solution:
Height of Mount Everest = h = 8848 m or 8.848 km

Acceleration due to gravity at a height h, say g'

$g' = g(1 - 2h/R)$

$= 9.8(1 - 640/(6400x10^3)$

$= 9.799\ m/s^2$

Question 24: Find the acceleration due to gravity in a mine of depth 640 m if the value at the surface is 9.800 ms^{-2}. The radius of the earth is 6400 Km.

Solution:
Let g' be the acceleration due to gravity

$g' = g\ (1 - 2h/R)$

$= 98(1-0.64/6400)$

$= 9.799\ m/s^2$

Question 25: A body is weighed by a spring balance to be 1.000 kg at the North Pole. How much will it weigh at the equator? Accounts for the earth’s rotation only.

Solution:
Let g' be the acceleration due to gravity at equation and that of pole g

Angular velocity of earth = $\omega = 2\pi/T$

$= 2\pi/(24x3600)$ rad/s

Now, acceleration due to gravity at equator:

$g' = g - \omega^2 R$

$= 9.8 - [2\pi/(24x3600)]^2 x 64000$

$= 9.767\ m/s^2$

And the weight at equator = mg' = 1 x 9.767 N = 0.997 kg

Question 26: A body stretches a spring by a particular length at the earth's surface at equator. At what height above the South Pole will it stretch the same spring by the same length? Assume the earth to be spherical.

Solution:

Acceleration due to gravity at equator = $g' = g - \omega^2 R$

Acceleration due to gravity at a height above south pole = $g'' = g(1-2h/R)$

Now $g' = g''$

=> $g - \omega^2 R = g(1-2h/R)$

=> $h = \omega^2 R^2/2g$

Or $h = [4 \pi \times 6400000^2]/[(4\times3600)^2\times2\times9.8]$

= 10 km (approx)

Question 27: At what rate should the earth rotate so that the apparent g at the equator becomes zero? What will be the length of the day in this situation?

Solution:
for apparent g at equator be zero.

$g' = g - \omega^2 R = 0$

or $g = \omega^2 R$

=> $\omega = \sqrt{(g/R)} = \sqrt{(9.8/6400000)}$

= 1.237×10^{-3} rad/s

Now, $T = 2 \pi / \omega$

$= [2x3.14]/[1.237x10^{-3}x3600]$

= 1.4 h (approx.)

Question 28: A pendulum having a bob of mass m is happening in a ship sailing along the equator from east to west. When the ship is stationary with respect to water the tension in the string is T_0.
(a) Find the speed of the ship due to rotation of the earth about its axis.
(b) Find the difference between T_0 and the earth's attraction on the bob.
(c) If the ship sails at speed v. what is the tension in the string? Angular speed of earth's rotation is ω and radius of the earth is R.

Solution:
a) the speed of the ship is equal to earth's rotation when the ship is stationary point.

speed $= \omega R$

(b) tension in the string at the equator

$T_0 = mg' = mg - m\omega^2 R$

$mg – T_0 = m\omega^2 R$
Difference between T_0 and the earth's attraction on the bob.

(c) angular speed of the ship is v/R about its center.

Total angular speed $= \omega' = \omega – v/R$

And $T = mg – m\omega'^2 R$ [tension given]

$=> T = mg – m(\omega – V/R)^2 R$

$=> T = mg – [m\omega^2 + mv^2/R^2 – 2m\omega v/R)R]$

$=> T = mg - m\omega^2 R – mv^2/R + 2m\omega v$

From part (b),

$T_0 = mg' = mg - m\omega^2 R$

$\Rightarrow T = T_0 - mv^2/R + 2m\omega v$

$\Rightarrow T = T_0 + 2m\omega v$
Neglect, mv^2/R, As small quantity.

Question 29: The time taken by the Mars to revolve round the sun is 1.88 years. Find the ratio of average distance between Mars and the sun to that between the earth and the sun.

Solution:
From Kepler's third law, the time period of an orbit is proportional to the cube of the radius of the orbit.

$T^2 \alpha R^3$

$T_m^2/T^2 = R_{MS}^3/R_{SE}^3$

$R_{MS}/R_{SE} = (3.534)^{1/3} = 1.52$

Question 30: The moon takes about 27.3 days to revolve round the earth in a nearly circular orbit of radius 3.84×10^5 Km. Calculate the mass of the earth from these data.

Solution:
For an orbit, the time period:

$T^2 = 4\pi^2 a^3/GM$

[here $a = 3.84 \times 10^5$ km and $T = 27.3 \times 24 \times 3600$ sec]

Or $M = 6.02 \times 10^{24}$ kg

Question 31: A Mars satellite moving in an orbit of radius 9.4×10^3 Km takes 27540 s to complete one revolution. Calculate the mass of Mars.

Solution:

For an orbit, the time period:

$T^2 = 4\pi^2 a^3/GM$

Or $M = 4\pi^2 a^3/G T^2$ ....(1)
Where M = mass of mars.

Here, Radius of mars=a = 9.4×10^3 km or 9.4×10^6 m and Time = T = 27540 s

Now, (1)=> $M = [4\pi^2 (9.4 \times 10^6)^3]/[6.67 \times 10^{-11} \times 27540^2]$

Or $M = 6.5 \times 10^{23}$ kg

Question 32: A satellite of mass 1000 kg is supposed to orbit the earth at a height of 2000 Km above the earth's surface. Find

(a) Its speed in the orbit.
(b) Its kinetic energy.
(c) The potential energy of the earth-satellite system and
(d) Its time period. Mass of the earth = 6×10^{24} Kg.

Solution:

(a) Radius of the orbit = a = 2000 + 6400 = 8400 km or 8.4×10^6 m

Therefore, the speed = $v = \sqrt{(GM)}/a$

$v = \sqrt{[(6.67 \times 10^{-11} \times 6 \times 10^{24})/(8.4 \times 10^6)]}$ = 6.9 km/s (approx.)

(b) $KE = (1/2)mv^2$
Here m = 1000 kg (mass of satellite)

$KE = (1/2) \times 1000 \times 6900^2 = 2.38 \times 10^{10}$

(c) Potential energy at infinity is zero. Hence, the potential energy at a radius, a

PE = -GMm/a

$= [-6.67x10^{-11}x6x10^{24}x1000]/[8.4x10^{6}]$

$= -4.76 \times 10^{10}$

(d) Time period

$T^2 = 4\pi^2a^3/GM$

$= [4\pi^2(8.4x10^6)^3]/[6.67x\ 10^{-11}x6x10^{24}]$

= 2.12 hours

Question 33: (a) Find the radius of the circular orbit of a satellite moving with an angular speed equal to the angular speed of earth's rotation.

(b) If the satellite is directly above the North Pole at some instant, find the time it takes to come over the equatorial plane.
Mass of the earth = 6×10^{24} Kg.

Solution:
(a) Time period of revolution of satellite: T = 24x3600 = 86400 sec

Let "a" be the radius of the orbit.

$T^2 = 4\pi^2a^3/GM$

Or $a^3 = GM\ T^2/4\pi^2$

$= [6.67x\ 10^{-11}x6x10^{24}\ x\ 86400^2]/[\ 4\pi^2]$

$= 7.56 \times 10^{22}$

Or a = 42300 km (approx.)

(b) A complete revolution takes 24 hours, therefore a quarter of revolution is 24/4 = 6 hours

Question 34: What is the true weight of an object in a geostationary satellite that weighed exactly 10.0 N at the North Pole?

Solution:
Weight at north pole, $W_p \alpha 1/R^2$
Let h is distance of the satellite from earth.
Weight of satellite at equator, $W_e \alpha 1/(R+h)^2$

Now, ratio is $W_p/ W_e = (R+h)^2/R^2$

$W_e = [W_pR^2]/[(R+h)^2]$

We are given, $W_p = 10$ N and h = 36000 km [Height of the geostationary satellite,]

Therefore, $W_e = [10x6400^2]/[(6400+36000)^2] = 0.23$ N(approx.)

Question 35: The radius of planet is R_1 and a satellite revolves round it in a circle of radius R_2. The time period of revolution is T. Find the acceleration due to the gravitation of the planet at its surface.

Solution:

The time period of revolution: $T^2 = 4 \pi^2a^3/GM$

Or $GM = 4 \pi^2(R_2)^3/ T^2$

Or $GM/R_1^2 = 4 \pi^2(R_2)^3/ T^2 R_1^2$

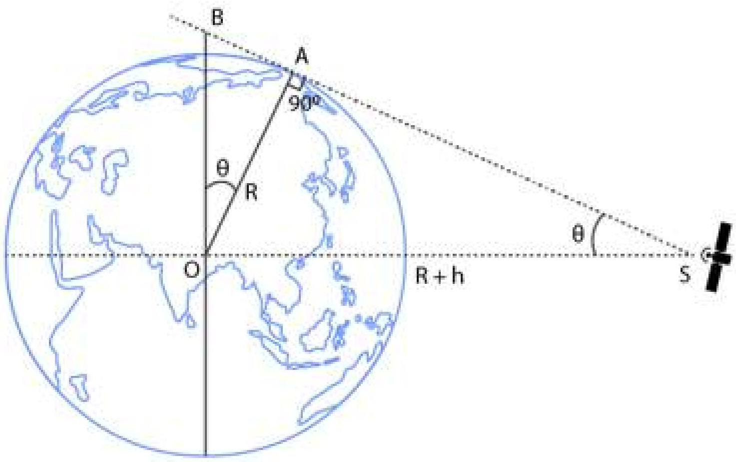
Now, Acceleration due to gravity :

$g = GM/ R_1^2$

or $g = 4 \pi^2(R_2)^3/ T^2 R_1^2$

Question 36: Find the minimum colatitude which can directly receive a signal from a geostationary satellite.

Solution:



form figure, angle BOA = angle OSA

In triangle AOS

sinθ = AO/OS

= R/(R+h)

= 6400/(6400+36000)

= 0.15 (approx)

or θ = $\sin^{-1}(0.15)$

Question 37: A particle is fired vertically upward from the earth's surface and it goes up to a maximum height of 6400 km. Find the initial speed of the particle.

Solution:
Let KE_i be initial KE and PE_i initial potential energy of the system.

$KE_i = (1/2)mv^2$ and $PE_i = -GMm/R$

$KE_f = 0$ [at the maximum height]

And PE at height h is h = 6400 km
And PE_f = -GMm/(R+h)

Now, From conservation of energy, $KE_i + PE_i = KE_f + PE_f$

=> $(1/2)mv^2 – GMm/R = -GMm/(R+h)$

=> $(1/2)mv^2 = GMm[(-R+R+h)/R(R+h)] = GMmh/R(R+h)$

Or $v^2 = 2GMh/R(R+h)$

$= [2x6.67x10^{-11}x6x10^{24}x6400x10^3]/[2x(6400x10^3)^2]$

= 7.9 km/s

Question 38: A particle is fired vertically upward with a speed of 15 km s^{-1}. With what speed will it move in interstellar space. Assume only earth's gravitational field.

Solution:
Let KE_i be initial KE and PE_i initial potential energy of the system.

$KE_i = (1/2)mv_i^2$ and $PE_i = -GMm/R$

Final KE = $KE_f = (1/2)mv_f^2$

Final potential energy = $PE_f = 0$

Using energy conservation, we have
$KE_i + PE_i = KE_f + PE_f$

$$\Rightarrow \frac{1}{2}mv_i^2 - \frac{GMm}{R} = \frac{1}{2}mv_f^2$$

$$\Rightarrow \frac{15^2}{2} - \frac{6.67 \times 10^{-7} \times 6 \times 10^{24}}{6400} = \frac{1}{2}v_f^2$$

$$\Rightarrow v_f = \times\ 10^4 m/s = 10km/_s$$

Question 39: A mass of 6×10^{24} Kg (equal to the mass of the earth) is to be compressed in a sphere in such a way that the escape velocity from its surface is 3×10^8 ms^{-1}. What should be the radius of the sphere?

Solution:

We have, $(1/2)\ mv^2 = GMm/R$

or $R = 2GM/v^2$

$R = [2x6.67x10^{-11}x6x10^{24}]/[(3x10^8)^2]$

R = 9 mm (approx.)