# 5. Factorization of Algebraic Expressions

**Exercise 5.1**

**1. Question**

Factorize:

$x^3 + x - 3x^2 - 3$

**Answer**

Given,

$x^3 + x - 3x^2 - 3$

$= x(x^2 + 1) - 3(x^2 + 1)$

$= (x - 3)(x^2 + 1)$

**2. Question**

Factorize:

$a(a+b)3 - 3a^2b(a+b)$

**Answer**

Given,

$a(a + b)^3 - 3a^2b(a + b)$

$= a(a + b)\{(a + b)^2 - 3ab\}$use the identity:$(a + b)^2 = a^2 + b^2 + 2ab$

$= a(a + b)(a^2 + b^2 + 2ab - 3ab)$

$= a(a + b)(a^2 + b^2 - ab)$

**3. Question**

Factorize:

$x(x^3 - y^3) + 3xy - (x-y)$

**Answer**

Given,

$x(x^3 - y^3) + 3xy - (x-y)$

As $(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$

$x(x^3 - y^3) + 3xy - (x-y) = x[(x - y)(x^2 + xy + y^2)] + 3xy - (x-y)$

Take $x(x-y)$ common to get,

$x(x-y)[(x^2 + xy + y^2) + 3y]$

**4. Question**

Factorize:

$a^2x^2 + (ax^2+1)x + a$

**Answer**

Given,

$a^2x^2 + (ax^2 + 1)x + a = a^2x^2 + ax^3 + x + a$

$= ax^2(a+x)+1(x+a)$

$= (ax^2+1)(x+a)$

**5. Question**

Factorize:

$x^2+y-xy-x$

**Answer**

Given,

$x^2 + y - xy - x = x^2 - x + y - xy$

$= x(x-1) + y(1-x)$

$= x(x-1) - y(x-1)$

$= (x-y)(x-1)$

**6. Question**

Factorize:

$x^3-2x^2y+3xy^2-6y^3$

**Answer**

Given,

$x^3 - 2x^2y + 3xy^2 - 6y^3$

$= x^2(x-2y) + 3y^2(x-2y)$

$= (x-2y)(x^2+3y^2)$

**7. Question**

Factorize:

$6ab-b^2+12ac-2bc$

**Answer**

Given,

$6ab-b^2+12ac-2bc = b(6a-b)+2c(6a-b)$

$= (b+2c)(6a-b)$

**8. Question**

Factorize:

$\left(x^2+\dfrac{1}{x^2}\right)-4\left(x+\dfrac{1}{x}\right)+6$

**Answer**

Given,

$\left(x^2+\dfrac{1}{x^2}\right)-4\left(x+\dfrac{1}{x}\right)+6$

$= \left(x^2+\dfrac{1}{x^2}+2\right)-4\left(x+\dfrac{1}{x}\right)+4$

$= \left(x+\dfrac{1}{x}\right)^2-4\left(x+\dfrac{1}{x}\right)+4$

$$= \left\{ \left( x + \frac{1}{x} \right) - 2 \right\}^2 \qquad \text{[ BY applying } (a^2 - 2ab + b^2) = (a-b)^2 \text{ ]}$$

**9. Question**

Factorize:

x(x–2)(x–4)+4x-8

**Answer**

Given,

X(x-2) (x-4)+4x-8 = x(x-2) (x-4)+4(x-2)

= (x-2) (x(x-4)+4)

= (x-2) $(x^2 - 4x + 4)$

= (x-2) $(x-2)^2$

= $(x-2)^3$

**10. Question**

Factorize:

(x+2)$(x^2+25)$-10$x^2$-20x

**Answer**

Given,

(x+2) $(x^2+25)$ - 10$x^2$ - 20x = (x+2) $(x^2+25)$-10x (x+2)

= (x+2) $(x^2+25-10x)$

= (x+2) $(x-5)^2$

**11. Question**

Factorize:

2$a^2$+2$\sqrt{6}$ ab+3$b^2$

**Answer**

Given,

2$a^2$√6ab+3$b^2$ = $(\sqrt{2}a)^2$ +2(√3×√2)ab + $(\sqrt{3}b)^2$

= $(\sqrt{2}a+\sqrt{3}b)^2$

**12. Question**

Factorize:

(a-b+c)$^2$+(b-c+a)$^2$+2(a-b+c)(b-c+a)

**Answer**

Given,

(a-b+c)$^2$+(b-c+a)$^2$+2(a-b+c)(b-c+a)

= $(a-(b-c))^2$ + $(a+(b-c))^2$ + 2(a-(b-c)) (a+(b-c))

[Applying identity: $x^2 + y^2 +2xy = (x + y)^2$ , where x= a-(b-c), y= a+(b-c)]

=$[(a-(b-c)) + (a+(b-c))]^2$=$(2a)^2$=4$a^2$

**13. Question**

Factorize:

$a^2+b^2+2(ab+bc+ca)$

**Answer**

Given,

$a^2+b^2+2(ab+bc+ca)$

$= a^2+b^2+2ab+2bc+2ca$

$= (a+b)^2+2c(a+b)$

$= (a+b)(a+b+2c)$

**14. Question**

Factorize:

$4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$

**Answer**

Given,

$4(x-y)^2 -12(x-y)(x+y)+9(x+y)^2 = 4(x^2-2xy+y^2)-12(x^2 – y^2)+9(x^2+y^2+2xy)$

$= 4x^2-8xy+4y^2-12x^2+12y^2+9x^2+9y+18xy$

$= x^2+25y^2+10xy$

$= (x)^2+(5y)^2+2×x×5y$

$= (x+5y)^2$

**15. Question**

Factorize:

$a^2-b^2+2ab-c^2$

**Answer**

Given,

$a^2-b^2+2ab-c^2 = a^2-(b^2-2bc+c^2)$

$= a – (b-c)^2$

$= (a + (b-c)) (a-(b-c))$

$= (a+b-c) (a-b+c)$

**16. Question**

Factorize:

$a^2+2ab+b^2-c^2$

**Answer**

Given,

$a^2+2ab+b^2-c^2 = (a+b)^2 – c^2$

$= (a+b-c) (a+b+c)$

**17. Question**

Factorize:

$a^2+4b^2-4ab-4c^2$

**Answer**

Given,

$a^2 +4b^2-4ab-4c^2 = (a)^2+ (2b)^2 – 2×a×2b-4c^2$

$= (a-2b)^2 – (2c)^2$

$= (a-2b-2c)(a-2b+2c)$

**18. Question**

Factorize:

$xy^9–yx^9$

**Answer**

Given,

$xy^9 - yx^9 = xy (y^8-x^8)$

$= xy ((y^4)^2 – (x^4)^2)$

$= xy (y^4+x^4) (y^4 - x^4)$

$= xy (y^4+x^4) (y^2-x^2) (y^2+x^2)$

$= xy (y^4+x^4) (y^2+x^2) (y – x) (y+x)$

**19. Question**

Factorize:

$x^4+x^2y^2+y^4$

**Answer**

Given,

$x^4+x^2y^2+y^4 = x^4+2x^2y^2+y^4 – x^2y^2$

$= (x^2y^2)^2 – (xy)^2$

$= (x^2+ y^2 – xy) (x^2+ y^2 + xy)$

**20. Question**

Factorize:

$x^2-y^2-4xz+4z^2$

**Answer**

Given,

$x^2 – y^2 - 4xz + 4z^2 = x^2 – 4xz +4z^2 – y^2$

$= (x)^2 - 2×x×2z+(2z)^2 – y^2$

$= (x-2z)^2 – y^2$

$= (x - 2z - y) (x – 2z + y)$

$= (x – y – 2z) (x + y – 2z)$

**21. Question**

Factorize:

$x^2 + 6\sqrt{2}x + 10$

**Answer**

Given,

$x^2 + 6\sqrt{2}x + 10 = x^2 + \sqrt{2}x + 10$

$= x(x + \sqrt{2}) + 5\sqrt{2}(x + \sqrt{2})$

$= (x + \sqrt{2})(x + 5\sqrt{2})$

**22. Question**

Factorize:

$x^2 - 2\sqrt{2}x - 30$

**Answer**

Given,

$x^2 - 2\sqrt{2}x - 30 = x^2 - 5\sqrt{2}x + 3\sqrt{2}x - 30$

$= x(x - 5\sqrt{2}) + 3\sqrt{2}(x - 5\sqrt{2})$

$= (x - 5\sqrt{2})(x + 3\sqrt{2})$

**23. Question**

Factorize:

$x^2 - \sqrt{3}x - 6$

**Answer**

Given,

$X^2 - \sqrt{3}x - 6 = x^2 - 2\sqrt{3}x + \sqrt{3}x - 6$

$= x(x - 2\sqrt{3}) + \sqrt{3}(x - 2\sqrt{3})$

$= (x + \sqrt{3})(x - 2\sqrt{3})$

**24. Question**

Factorize:

$x^2 + 5\sqrt{5}x + 30$

**Answer**

Given,

$x^2 + 5\sqrt{5}x + 30 = x^2 + 3\sqrt{5}x + 2\sqrt{5}x + 30$

$= x(x + 3\sqrt{5}) + 2\sqrt{5}(x + 3\sqrt{5})$

$= (x + 3\sqrt{5})(x + 2\sqrt{5})$

**25. Question**

Factorize:

$x^2 + 2\sqrt{3}x - 24$

**Answer**

Given,

$x^2 + 2\sqrt{3}x - 24 = x^2 + 4\sqrt{3}x - 2\sqrt{3}x - 24$

$= x(x + 4\sqrt{3}) - 2\sqrt{3}(x + 4\sqrt{3})$

$= (x + 4\sqrt{3})(x - 2\sqrt{3})$

**26. Question**

Factorize:

$2x^2 - \frac{5}{6}x + \frac{1}{12}$

**Answer**

Given,

$2x^2 - \dfrac{5}{6}x + \dfrac{1}{12}$

$= 2x^2 - \dfrac{1}{3}x - \dfrac{1}{2}x + \dfrac{1}{12}$

$= x\left(2x - \dfrac{1}{3}\right) - \dfrac{1}{4}\left(2x - \dfrac{1}{3}\right)$

$= \left(2x - \dfrac{1}{3}\right)\left(x - \dfrac{1}{4}\right)$

**27. Question**

Factorize:

$x^2 + \frac{12}{35}x + \frac{1}{35}$

**Answer**

Given,

$x^2 - \dfrac{12}{36}x + \dfrac{1}{36} = x^2 - \dfrac{1}{7}x - \dfrac{1}{5}x + \dfrac{1}{35}$

$x\left(x + \dfrac{1}{7}\right) + \dfrac{1}{5}\left(x + \dfrac{1}{7}\right)$

$= \left(x - \dfrac{1}{7}\right)\left(x + \dfrac{1}{5}\right)$

**28. Question**

Factorize:

$21x^2 - 2x + \frac{1}{21}$

**Answer**

Given,

$21x^2 - 2x + \dfrac{1}{21} = 21x^2 - x - x + \dfrac{1}{21}$

$= x(21x - 1) - \dfrac{1}{21}(21x - 1)$

$= \left(x - \dfrac{1}{21}\right)(21x - 1)$

**29. Question**

Factorize:

$5\sqrt{5}\,x^2+20x+3\sqrt{5}$

**Answer**

Given,

$5\sqrt{5}x^2+20x+3\sqrt{5} = 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$

$= 5x(\sqrt{5}x+3) + \sqrt{5}\,(\sqrt{5}x+3)$

$= (5x +\sqrt{5})\,(\sqrt{5}x + 3)$

## 30. Question

Factorize:

$2x^2+3\sqrt{5}\,x+531.9(2a-b)^2-4(2a-b)-13$

**Answer**

Given,

$2x^2+3\sqrt{5}x+5 = 2x^2 + 2\sqrt{5}x+ \sqrt{5}x+5$

$= 2x\,(x + \sqrt{5}) +\sqrt{5}\,(x+\sqrt{5})$

$= (2x+\sqrt{5})\,(x+\sqrt{5})$

## 31. Question

Factorize:

$9(2a – b)^2 –4\,(2a – b) – 13$

**Answer**

Given,

$9(2a – b)^2 –4\,(2a – b) – 13$

Let us assume $(2a – b) = x$

$9x^2 – 4x – 13$

$9x^2 – 13x + 9x – 13$

$x(9x – 13)+1\,(9x + 3)$

$(9x – 13)\,(x + 1)$

$[9(2a – b) – 13]\,[2a – b + 1]$

$(18a – 9b – 13)\,(2a – b + 1)$

## 32. Question

Factorize:

$7(x-2y)^2-25(x-2y)+12$

**Answer**

Given,

$7(x – 2y)^2 – 25\,(x – 2y) + 12$

Let $a = (x – 2y)$,

So we have,

$= 7a^2 – 25a + 12$

$= 7a^2 - 21a - 4a + 12$

$= 7a(a - 3) -4 (a - 3)$

$= (7a - 4) (a - 3)$

Put a = (x – 2y)

$= \{7(x - 2y) - 4\} (x - 2y - 3)$

$= (7x - 14y - 4) (x - 2y - 3)$

## 33. Question

Factorize:

$2(x+y)^2-9(x+y)-5$

**Answer**

Given,

$2(x+y)^2 - 9(x+y) - 5,$

$= 2a^2 - 9a - 5$

Let (x+y) = a

$= 2a^2 - 10a + a - 5$

$= 2a (a-5) +1(a-5)$

$= (2a+1) (a-5)$

$= \{2(x+y)+1\}(x+y-5)$

(2x+2y+1) (x+y-5)

## 34. Question

Give possible expressions for the length and breadth of the rectangle having $35y^2+13y-12$ as its area.

**Answer**

We know that,

Area of rectangle = length × breadth

Given,

$35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$= 7y(5y+4) -3 (5y + 4)$

$= (7y - 3) (5y + 4)$

Thus,

Length = (7y – 3), then breadth = (5y + 4)

Length = (5y + 4), then breadth = (7y – 3)

## 35. Question

What are the possible expressions for the dimensions of the cuboid whose volume is $3x^2-12x$.

**Answer**

Given,

We know that,

Volume of cuboids = length × breadth × height

Given,

$3x^2 – 12x = 3x (x-4)$

Thus,

Dimensions of cuboids are –

| Length | breadth | height |
|--------|---------|--------|
| 3 | x | (x-4) |
| X | (x-4) | 3 |
| (x-4) | 3 | x |

## Exercise 5.2

**1. Question**

Factorize each of the following expressions:

$p^3+27$

**Answer**

Given,

$P^3+27$,

$= p^3 + (3)^3$ [∵ $a^3+b^3 = (a+b)(a^2 – 2ab +b^2)$]

$= (p + 3) (p^2 + 9 – 3p)$

**2. Question**

Factorize each of the following expressions:

$y^3+125$

**Answer**

Given,

$y^3+125$,

$= y^3 + (5)^3$ [∵ $a^3+b^3 = (a+b)(a^2 – 2ab +b^2)$]

$= (y + 5) (y^2 – 5y + 25)$

**3. Question**

Factorize each of the following expressions:

$1-27a^3$

**Answer**

Given,

$1 – 27a^3$,

$= 1 – (3a)^3$

$= (1 – 3a) (1+9a^2+3a)$

**4. Question**

Factorize each of the following expressions:

$8x^3y^3+27a^3$

**Answer**

Given,

$8x^3 y^3 + 27a^3$,

$= (2xy)^3 + (3a)^3$

$= (2xy + 3a)(4x^2 y^2 + 9a^2 - 6axy)$

**5. Question**

Factorize each of the following expressions:

$64a^3 - b^3$

**Answer**

Given,

$64a^3 - b^3$,

$= (4a)^3 - (b)^3$

$= (4a - b)(16a^2 + b^2 + 4ab)$

**6. Question**

Factorize each of the following expressions:

$\frac{x^3}{216} - 8y^3$

**Answer**

Given,

$$\frac{x^3}{216} - 8y^3 = \left(\frac{x}{6}\right)^3 - (2y)^3$$

$$= \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + 4y^2 + \frac{xy}{3}\right)$$

**7. Question**

Factorize each of the following expressions:

$10x^4 y - 10xy^4$

**Answer**

Given,

$10x^4 y - 10xy^4$,

$= 10xy(x^3 - y^3)$

$= 10xy(x - y)(x^2 + xy - y^2)$

**8. Question**

Factorize each of the following expressions:

$54x^6 y + 2x^3 y^4$

**Answer**

Given,

$54x^6 y + 2x^3 y^4$,

$= 2x^3 y(27x^3 + y^3)$

$= 2x^3 y \{(3x)^3 + (y)^3\}$

$= 2x^3 y (3x + y) (9x^2 + y^2 – 3xy)$

**9. Question**

Factorize each of the following expressions:

$32a^2+108b^3$

**Answer**

Given,

$32a^3+108b^3$,

$= 4 (8a^3+27b^3)$

$= 4 \{ (2a)^3 + (3b)^3 \}$

$= 4 (2a + 3b) (4a^2 +9b^2 – 6ab)$

**10. Question**

Factorize each of the following expressions:

$(a-2b)^3-512b^3$

**Answer**

Given,

$(a – 2b)^3 – 512b^3$

$= (a – 2b)^3 – (8b)^3$

$= (a – 2b – 8b) \{(a – 2b)^2 +(8b)^2 +(a – 2b) 8b\}$

$= (a – 10b) (a^2 +4b^2 – 4ab +64b^2 +8ab – 16b^2)$

$= (a – 10b) (a^2 +52b^2 +4ab)$

**11. Question**

Factorize each of the following expressions:

$(a+b)^3-8(a-b)^3$

**Answer**

Given,

$(a + b)^3 – \{2(a – b)\}^3$

$= \{(a + b) –2(a - b)\} \{ (a+b)^2 +4(a-b)^2 +2(a+b)(a - b)\}$  [ By using: $x^3 - y^3 = (x - y)(x^2 + y^2 +xy)$

$= (a + b – 2a + 2b)(a^2 + b^2 + 2ab + 4a^2 +4b^2 – 8ab + 2a^2 – 2b^2)$

$= (3b – a) (7a^2 +3b^2 - 6ab)$

**12. Question**

Factorize each of the following expressions:

$(x+2)^3+(x-2)^3$

**Answer**

Given,

$(x+2)^3 + (x - 2)^3 = (x + 2 + x - 2) \{ (x + 2)^2 + (x - 2)^2 – (x + 2)(x - 2) \}$

$= 2x (x^2 + 4 + 4x + x^2 + 4 – 4x – x^2 + 4)$

$= 2x (x^2 + 12)$

**13. Question**

Factorize each of the following expressions:

$8x^2y^3-x^5$

**Answer**

Given,

$8x^2y^3 – x^5$

$= x^2 (8y^3 – x^3)$

$= x^2 \{ (2y)^2 – (x)^3\}$

$= x^2 (2y - x) (4y^2 + x^2 + 2xy)$

**14. Question**

Factorize each of the following expressions:

$1029-3x^3$

**Answer**

Given,

$1029 – 3x^3$

$= 3 (343 – x^3)$

$= 3 \{ (7)^3 – (x)^3\}$

$= 3 (7 - x) (49 +x^2 +7x)$

**15. Question**

Factorize each of the following expressions:

$x^6+y^6$

**Answer**

Given,

$X^6 + y^6 = (x^2)^3 + (y^2)^3$

$= (x^2 + y^2) (x^4 +y^4 – x^2y^2)$

**16. Question**

Factorize each of the following expressions:

$x^3y^3+1$

**Answer**

Given,

$X^3y^3 + 1 = (xy)^3 +(1)^3$

$= (xy + 1) (x^2 y^2 +1 - xy)$

**17. Question**

Factorize each of the following expressions:

$x^4y^4-xy$

**Answer**

Given,

$X^4y^4 - xy = xy(x^3y^3 - 1)$

$= xy\{(xy)^3 - (1)^3\}$

$= xy(xy - 1)(x^2y^2 + 1 + xy)$

**18. Question**

Factorize each of the following expressions:

$a^{12}+b^{12}$

**Answer**

Given,

$a^{12} + b^{12} = (a^4)^3 + (b^4)^3$

$= (a^4 + b^4)(a^3 + b^3 - a^4b^4)$

**19. Question**

Factorize each of the following expressions:

$x^3+6x^2+12x+16$

**Answer**

Given,

$X^3 + 6x^2 12x + 16 = (x^3 + 6x^2 + 12x + 8) + 8$

$= (x+2)^3 + 8 \ [\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)]$

$= (x+2)^3 + (2)^3$

$= (x+2+2)\{(x+2)^2 + 4 - 2(x+2)\}$

$= (x+4)(x^2 + 4 + 4x + 4 - 2x - 4)$

$= (x+4)(x^2 + 2x + 4)$

**20. Question**

Factorize each of the following expressions:

$a^3+b^3+a+b$

**Answer**

Given,

$a^3+b^3+a+b = (a^3 + b^3) + (a+b)$

$= (a+b)(a^2 - ab + b^2) + a + b$

$= (a+b)(a^2 - ab + b^2 + 1)$

**21. Question**

Factorize each of the following expressions:

$a^3 - \frac{1}{a^3} - 2a\frac{2}{a}$

**Answer**

Given,

$$a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$$

$$= a^3 - \frac{1}{a^3} - 2\left(a - \frac{1}{a}\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} + 1\right) - 2\left(a - \frac{1}{a}\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} + 1 - 2\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right)$$

**22. Question**

Factorize each of the following expressions:

$a^3 + 3a^2b^3 + 3ab^2 + b^3 - 8$

**Answer**

Given,

$A^3 + 3a^2 b + 3ab^2 + b^3 - 8 = (a^3 + 3a^2b + 3ab^2 + b^3) - 8$

$= (a+b)^3 - 8$

$= (a+b)^3 - (2)^3$

$= (a+b-2)\{ (a+b)^2 + (2)^2 + 2(a+b) \}$

$= (a+b-2)(a^2 + b^2 + 2ab + 4 + 2a + 2b)$

**23. Question**

Factorize each of the following expressions:

$8a^3 - b^3 - 4ax + 2bx$

**Answer**

Given,

$8a^3 - b^3 - 4ax + 2bx = (2a)^3 - (b)^3 - 2x(2a - b)$

$= (2a - b)(4a^2 + b^2 + 2ab) - 2x(2a - b)$

$= (2a - b)(4a^2 + b^2 + 2ab - 2x)$

**24. Question**

Simplify

(i) $\frac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$

(ii) $\frac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 + 55 \times 55}$

(iii) $\frac{1.2 \times 1.2 \times 1.2 - 0.2 \times 0.2 \times 0.2}{1.2 \times 1.2 + 1.2 \times 0.2 + 0.2 \times 0.2}$

**Answer**

(i) Given,

$$= \frac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$$

$$= \frac{(173)^3 + (127)^3}{(173)^2 - 173 \times 127 + (127)^2}$$

$$= \frac{(173 + 127)\{(173)^2 + (127)^2 - 173 \times 127\}}{(173)^2 - 173 \times 127 + (127)^2} = 300$$

(ii) Given,

$$\frac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 \times 55 \times 55}$$

$$= \frac{(155)^3 - (55)^3}{(155)^2 + 155 \times 55 + (55)^2}$$

$$= \frac{(155 - 55)\{(155)^2 + 155 \times 55 + (55)^2\}}{(155)^2 + 155 \times 55 + (55)^2}$$

$$= (155 - 55) = 100$$

(ii) Given,

$$= \frac{1.2 \times 1.2 \times 1.2 - 0.2 \times 0.2 \times 0.2}{1.2 \times 1.2 + 1.2 \times 0.2 + 0.2 \times 0.2}$$

$$= \frac{(1.2)^3 - (0.2)^3}{(1.2)^2 + 1.2 \times 0.2 + (0.2)^2}$$

$$= \frac{(1.2 - 0.2)\{(1.2)^2 + (0.2)^2 + 1.2 \times 0.2\}}{(1.2)^2 + 1.2 \times 0.2 \times (0.2)^2}$$

$$= (1.2 - 0.2) = 1$$

## Exercise 5.3

**1. Question**

Factorize:

$64a^3 + 125b^3 + 240a^2b + 300ab^2$

**Answer**

Given,

$64a^3 + 125b^3 + 240a^2b + 300ab^2$,

$= (4a)^3 + (5b)^3 + 60ab (4a + 5b)$

$= (4a)^3 + (5b)^3 + 3 \times 4a \times 5b (4a + 5b)$

$= (4a + 5b)^3$

$= (4a + 5b)(4a + 5b)(4a + 5b)$

**2. Question**

Factorize:

$125x^3 - 27y^3 - 225x^2y + 125xy^2$

**Answer**

Given,

$125x^3 - 27y^3 - 225x^2y + 125xy^2$,

$= (5x)^3 - (3y)^3 - 45xy (5x - 3y)$

$= (5x)^3 - (3y)^3 - 3×5x×3y (5x - 3y)$

$= (5x - 3y)^3$

$= (5x - 3y) (5x - 3y) (5x - 3y)$

**3. Question**

Factorize:

$\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$

**Answer**

Given,

$$\frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x,$$

$$= \left(\frac{2}{3}x\right)^3 + (1)^3 + 2x\left(\frac{2}{3}x + 1\right)$$

$$= \left(\frac{2}{3}x\right)^3 + (1)^3 + 3×\frac{2}{3}x×1\left(\frac{2}{3}x + 1\right)$$

$$= \left(\frac{2}{3}x + 1\right)^3$$

$$= \left(\frac{2}{3}x + 1\right)\left(\frac{2}{3}x + 1\right)\left(\frac{2}{3}x + 1\right)$$

**4. Question**

Factorize:

$8x^3+27y^3+36x^2y+54xy^2$

**Answer**

Given,

$8x^3+27y^3+36x^2y+54xy^2$,

$= (2x)^3+(3y)^3 +18xy (2x +3y)$

$= (2x)^3+(3y)^3 +3×2x×3y (2x +3y)$

$= (2x+3y)^3$

$= (2x +3y) (2x +3y) (2x +3y)$

**5. Question**

Factorize:

$a^3-3a^2b+3ab^2-b^3+8$

**Answer**

Given,

$a^3 - 3a^2b + 3ab^2 - b^3 + 8$,

$= \{(a)^3 - (b)^3 -3ab (a-b)\} +8$

$= (a-b)^3 + (2)^3$

= (a – b +2) {(a - b)$^2$ +(2)$^2$ +2(a - b)}

= (a - b +2)(a$^2$ -2ab +b$^2$ +4+2a – 2b)

**6. Question**

Factorize:

$x^3+8y^3+6x^2y+12xy^2$

**Answer**

Given,

$x^3+8y^3+6x^2y+12xy^2$,

= (x)$^3$+(2y)$^3$ +6xy (x +2y)

= (x)$^3$+(2y)$^3$ +3×x×2y (x +2y)

= (x+2y)$^3$

= (x +2y) (x +2y) (x +2y)

**7. Question**

Factorize:

$8x^3+y^3+12x^2y+6xy^2$

**Answer**

Given,

$8x^3+y^3+12x^2y+6xy^2$,

= (2x)$^3$+(y)$^3$ +6xy (2x +y)

= (2x)$^3$+(y)$^3$ +3×2x×y (2x +y)

= (2x+y)$^3$

= (2x +y) (2x +y) (2x +y)

**8. Question**

Factorize:

$8a^3+27b^3+36a^2b+54ab^2$

**Answer**

Given,

$8a^3+27b^3+36a^2b+54ab^2$,

= (2a)$^3$+(3b)$^3$ +18ab (2a + 3b)

= (2a)$^3$ + (3b)$^3$ +3×2a×3b (2a +3b)

= (2a +3b)$^3$

= (2a +3b) (2a + 3b) (2a +3b)

**9. Question**

Factorize:

$8a^3-27b^3-36a^2b+54ab^2$

**Answer**

Given,

$8a^3 - 27b^3 - 36a^2b + 54ab^2$,

$= (2a)^3 - (3b)^3 - 18ab\ (2a - 3b)$

$= (2a)^3 - (3b)^3 - 3 \times 2a \times 3b\ (2a - 3b)$

$= (2a - 3b)^3$

$= (2a - 3b)\ (2a - 3b)\ (2a - 3b)$

**10. Question**

Factorize:

$x^3 - 12x(x-4) - 64$

**Answer**

Given,

$x^3 - 12x(x-4) - 64$,

$= (x)^3 - 12x\ (x - 4) - (4)^3$

$= (x)^3 - (4)^3 - 3 \times x \times 4\ (x - 4)$

$= (x - 4)^3$

$= (x - 4)\ (x - 4)\ (x - 4)$

**11. Question**

Factorize:

$a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$

**Answer**

Given,

$a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$,

$= (ax)^3 - 3abx(ax-b) - (b)^3$

$= (ax)^3 - (b)^3 - 3abx\ (ax-b)$

$= (ax-b)^3$

$= (ax-b)\ (ax-b)\ (ax-b)$

## Exercise 5.4

**1. Question**

Factorize each of the following expressions:

$a^3 + 8b^3 + 64c^3 - 24abc$

**Answer**

Given,

$= a^3 + 8b^3 + 64c^3 - 24abc$

This can be written in form

$= a^3 + 8b^3 + 64c^3 - 24abc = (a)^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \text{ By formula}$

Hence,

$= a^3 + 8b^3 + 64c^3 - 24abc = (a + 2b + 4c)\{(a)^2 + (2b)^2 + (4c)^2 - a \times 2b - 2b \times 4c - 4c \times a)$

$= (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca)$

Thus the required factors of $a^3 + 8b^3 + 64c^3 - 24abc$ is $(a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca)$

## 2. Question

Factorize each of the following expressions:

$x^3 - 8y^3 + 27z^3 + 18xyz$

**Answer**

Given,

$= x^3 - 8y^3 + 27z^3 + 18xyz$

This can be written in form,

$= x^3 - 8y^3 + 27z^3 + 18xyz = (x)^3 + (-2y)^3 + (3z)^3 - 3 \times x \times (-2y) \times 3z$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

$= x^3 - 8y^3 + 27z^3 + 18xyz = \{x + (-2y) + 3z\}\{(x)^2 + (-2y)^2 + (3z)^2 - (x)(-2y) - (-2y)(3z) - (3z)(x)\}$

$= (x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$

Thus the required factors of $x^3 - 8y^3 + 27z^3 + 18xyz$ is $(x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$

## 3. Question

Factorize each of the following expressions:

$27x^3 - y^3 - z^3 - 9xyz$

**Answer**

Given,

$= 27x^3 - y^3 - z^3 - 9xyz$

This can be written in form ,

$= 27x^3 - y^3 - z^3 - 9xyz = (3x)^3 + (-y)^3 + (-z)^3 - 3(3x)(-y)(-z)$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

So ,

$= 27x^3 - y^3 - z^3 - 9xyz = \{3x + (-y) + (-z)\}\{(3x)^2 + (-y)^2 + (-z)^2 - (3x)(-y) - y(-z) - z(3x)\}$

$= (3x - y - z)(9x^2 + y^2 + z^2 + 3xy - yz + 3zx)$

Thus the factors of $27x^3 - y^3 - z^3 - 9xyz$ is $(3x - y - z)(9x^2 + y^2 + z^2 + 3xy - yz + 3zx)$

## 4. Question

Factorize:

$\frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$

**Answer**

Given,

$$= \frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz$$

This can be written in form ,

$$= \frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz = \left(\frac{1}{3}x\right)^3 + (-y)^3 + (5z)^3 - 3\left(\frac{1}{3}x\right)(-y)(5z)$$

$$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca) \text{ By formula}$$

$$= \frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz = \left\{\left(\frac{1}{3}x\right) + (-y) + (5z)\right\}\left\{\left(\frac{1}{3}x\right)^2 + (-y)^2 + (5z)^2 - \left(\frac{1}{3}x\right)(-y) - y\right)(5z) - (5z)\left(\frac{1}{3}x\right)$$

$$= \left(\frac{1}{3}x - y + 5z\right)\left(\frac{1}{9}x^2 + y^2 + 25z^2 + \frac{1}{3}xy + 5yz - \frac{5}{3}zx\right)$$

Thus the factors of $\frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz$ is $\left(\frac{1}{3}x - y + 5z\right)\left(\frac{1}{9}x^2 + y^2 + 25z^2 + \frac{1}{3}xy + 5yz - \frac{5}{3}zx\right)$

## 5. Question

Factorize each of the following expressions:

$8x^3+27y^3-216z^3+108xyz$

### Answer

Given,

$$= 8x^3 + 27y^3 - 216z^3 + 108xyz$$

This can be written in form ,

$$= 8x^3 + 27y^3 - 216z^3 + 108xyz = (2x)^3 + (3y)^3 + (-6z)^3 - 3(2x)(3y)(-6z)$$

$$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca) \text{ By formula}$$

$$= 8x^3 + 27y^3 - 216z^3 + 108xyz = \{(2x) + (3y) + (-6z)\}\{(2x)^2 + (3y)^2 + (-6z)^2 - (2x)(3y) - (3y)((-6z) - (-6z)(2x)\}$$

$$= (2x + 3y - 6z)(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$$

Thus the factors of

$8x^3 + 27y^3 - 216z^3 + 108xyz$ is $(2x + 3y - 6z)(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$

## 6. Question

Factorize each of the following expressions:

$125+8x^3-27y^3+90xy$

### Answer

Given,

$$= 125 + 8x^3 - 27y^3 + 90xy$$

This can be written in form ,

$$= 125 + 8x^3 - 27y^3 + 90xy = (5)^3 + (2x)^3 + (-3y)^3 - 3(5)(2x)(-3y)$$

$$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca) \text{ By formula}$$

$$= 125 + 8x^3 - 27y^3 + 90xy = \{5 + 2x - 3y\}\{(5)^2 + (2x)^2 + (-3y)^2 - (5)(2x) - (2x)(-3y) - (-3y)(5)\}$$

$$= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$$

Thus the factors of $125 + 8x^3 - 27y^3 + 90xy$ is $(5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$

**7. Question**

Factorize:

$(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$

**Answer**

Given,

$= (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3$

Let $a = (3x - 2y), b = (2y - 4z), c = (4z - 3x)$

$= (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3 = a^3 + b^3 + c^3$

Here,

$= a + b + c = (3x - 2y + 2y - 4z + 4z - 3x) = 0$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

Hence,

$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 = 3abc$

$= (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3 = 3(3x - 2y)(2y - 4z)(4z - 3x)$

**8. Question**

Factorize each of the following expressions:

$(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

**Answer**

Given,

$= (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3$

Let $a = (2x - 3y), b = (4z - 2x), c = (3y - 4z)$

Then,

$= (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3 = a^3 + b^3 + c^3$

Here,

$= a + b + c = 2x - 3y + 4z - 2x + 3y - 4z = 0$

Hence,

$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 = 3abc$

$= (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3 = 3(2x - 3y)(4z - 2x)(3y - 4z) \cdot$

**9. Question**

Factorize each of the following expressions:

$\left(\dfrac{x}{2} + y + \dfrac{z}{3}\right)^3 + \left(\dfrac{x}{3} - \dfrac{2y}{3} + y\right)^3 + \left(-\dfrac{5x}{6} - \dfrac{y}{3} - \dfrac{4z}{3}\right)^3$

**Answer**

Given,

$$= \left(\tfrac{x}{2}+y+\tfrac{z}{3}\right)^3 + \left(\tfrac{x}{3}-\tfrac{2y}{3}+z\right)^3 + \left(-\tfrac{5x}{6}-\tfrac{y}{3}-\tfrac{4z}{3}\right)^3$$

Let $a = \left(\tfrac{x}{2}+y+\tfrac{z}{3}\right), b = \left(\tfrac{x}{3}-\tfrac{2y}{3}+z\right), c = \left(-\tfrac{5x}{6}-\tfrac{y}{3}-\tfrac{4z}{3}\right)$

Then ,

$$= \left(\tfrac{x}{2}+y+\tfrac{z}{3}\right)^3 + \left(\tfrac{x}{3}-\tfrac{2y}{3}+z\right)^3 + \left(-\tfrac{5x}{6}-\tfrac{y}{3}-\tfrac{4z}{3}\right)^3 = a^3 + b^3 + c^3$$

Here ,

$$= a+b+c = \left(\tfrac{x}{2}+y+\tfrac{z}{3}\right) + \left(\tfrac{x}{3}-\tfrac{2y}{3}+z\right) + \left(-\tfrac{5x}{6}-\tfrac{y}{3}-\tfrac{4z}{3}\right) = 0$$

$$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca) \text{ By formula}$$

Hence ,

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2+b^2+c^2-ab-bc-ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= \left(\tfrac{x}{2}+y+\tfrac{z}{3}\right)^3 + \left(\tfrac{x}{3}-\tfrac{2y}{3}+z\right)^3 + \left(-\tfrac{5x}{6}-\tfrac{y}{3}-\tfrac{4z}{3}\right)^3 = 3\left(\tfrac{x}{2}+y+\tfrac{z}{3}\right)\left(\tfrac{x}{3}-\tfrac{2y}{3}+z\right)\left(-\tfrac{5x}{6}-\tfrac{y}{3}-\tfrac{4z}{3}\right).$$

## 10. Question

Factorize each of the following expressions:

$(a-3b)^3+(3b-c)^3+(c-a)^3$

**Answer**

Given,

$$=(a-3b)^3 + (3b-c)^3 + (c-a)^3$$

Let $x = (a-3b), y = (3b-c), z = (c-a)$

Then,

$$= (a-3b)^3 + (3b-c)^3 + (c-a)^3 = x^3 + y^3 + z^3$$

Here ,

$$= x+y+z = a-3b+3b-c+c-a = 0$$

$$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca) \text{ By formula}$$

Hence,

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2+b^2+c^2-ab-bc-ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= (a-3b)^3 + (3b-c)^3 + (c-a)^3 = 3(a-3b)(3b-c)(c-a)$$

## 11. Question

Factorize each of the following expressions:

$2\sqrt{2}\,a^3+3\sqrt{3}\,b^3+c^3-3\sqrt{6}\,abc$

**Answer**

Given,

$$= 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$$

This can be written in form ,

$$= 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc = (\sqrt{2}a)^3 + (\sqrt{3}b)^3 + (c)^3 - 3(\sqrt{2}a)(\sqrt{3}b)(c)$$

And , $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

Hence,

$$= 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc = \{(\sqrt{2}a) + (\sqrt{3}b) + c)\}\{(\sqrt{2}a)^2 + (\sqrt{3}b)^2 + (c)^2 - (\sqrt{2}a)(\sqrt{3}b) - (\sqrt{3}b)(c) - (c)(\sqrt{2}a)\}$$

$$= \sqrt{2}a + \sqrt{3}b + c)(2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}\,bc - \sqrt{2}ca$$

## 12. Question

Factorize each of the following expressions:

$3\sqrt{3}\,a^3 - b^3 - 5\sqrt{5}\,c^3 - 3\sqrt{15}\,abc$

### Answer

Given,

$$= 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc$$

This can be written in form .

$$= 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc = (\sqrt{3}a)^3 + (-b)^3 + (-\sqrt{5}c)^3 - 3(\sqrt{3}a)(-b)(-\sqrt{5}c)$$

And , $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$$= 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc = \{(\sqrt{3}a) + (-b) + (-\sqrt{5}c)\}\{(\sqrt{3}a)^2 + (-b)^2 + (-\sqrt{5}c)^2 - (\sqrt{3}a)(-b) - b)(-\sqrt{5}c) - (-\sqrt{5}c)(\sqrt{3}a)$$

$$= (\sqrt{3}a - b - \sqrt{5}c)\,(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ca)$$

## 13. Question

Factorize each of the following expressions:

$8x^3 - 125y^3 + 180xy + 216$

### Answer

Given,

$$= 8x^3 - 125y^3 + 180xy + 216$$

This can be written in form ,

$$= 8x^3 - 125y^3 + 180xy + 216 = (2x)^3 + (-5y)^3 + 6^3 - 3(2x)(-5y)(6)$$

And , $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

Hence ,

$$= 8x^3 - 125y^3 + 180xy + 216 = \{2x + (-5y) + 6\}\{(2x)^2 + (-5y)^2 + 6^2 - (2x)(-5y) - (-5y)(6) - (6)(2x)\}$$

$$= (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$$

Thus the factors of $8x^3 - 125y^3 + 180xy + 216$ is

$(2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$

## 14. Question

Factorize each of the following expressions:

$2\sqrt{2}\,a^3 + 16\sqrt{2}\,b^3 + c^3 - 12abc$

**Answer**

Given,

$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$

This can be written in form ,

$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = \left(\sqrt{2}a\right)^3 + \left(2\sqrt{2}b\right)^3 + c^3 - 3\left(\sqrt{2}a\right)\left(2\sqrt{2}b\right)(c)$

And , $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

Hence ,

$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = \left\{\sqrt{2}a + 2\sqrt{2}b + c\right\}\left\{\left(\sqrt{2}a\right)^2 + \left(2\sqrt{2}b\right)^2 + c^2 - \left(\sqrt{2}a\right)\left(2\sqrt{2}b - \left(2\sqrt{2}b\right)(c) - (c)\left(\sqrt{2}a\right)\right)\right\}$

$= \left(\sqrt{2}a + 2\sqrt{2}b + c\right)\left(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ca\right)$

Thus the factors of $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$ is $\left(\sqrt{2}a + 2\sqrt{2}b + c\right)\left(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ca\right)$

## 15. Question

Find the value of $x^3 + y^3 - 12xy + 64$, when $x + y = -4$.

**Answer**

Given,

$= x^3 + y^3 - 12xy + 64$

$= x + y = -4$ Given

$= x + y + 4 = 0$

This can be written in form ,

$= x^3 + y^3 - 12xy + 64 = x^3 + y^3 + 4^3 - 3(x)(y)(4)$

And , $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

$= x^3 + y^3 - 12xy + 64 = (x + y + 4)\{(x)^2 + (y)^2 + 4^2 - xy - 4y - 4x\}$

$= 0 \times (x^2 + y^2 + 16 - xy - 4y - 4x)$

$= 0$

## 16. Question

Multiply:

(i) $x^2 + y^2 + z^2 - xy + xz + yz$ by $x + y - z$

(ii) $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $x - 2y - z$

(iii) $x^2 + 4y^2 + 2xy + -3x + 6y + 9$ by $x - 2y + 3$

(iv) $9x^2 + 25y^2 + 15xy + 12x - 20y + 16$ by $3x - 5y + 4$

**Answer**

(i) Given,

$= x^2 + y^2 + z^2 - xy + xz + yz$

Multiply the above expression by ( x+ y – z)

$= (x + y - z)(x^2 + y^2 + z^2 - xy + xz + yz)$

$= \{x + y + (-z)\}\{(x)^2 + (y)^2 + (-z)^2 - xy + yz + zx\}$

$= x^3 + y^3 + (-z)^3 - 3.x.y(-z)$

$= x^3 + y^3 - z^3 + 3xyz$

(ii) Given,

$= x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$

Multiply above expression by (x-2y-z)

Then ,

$= (x - 2y - z)(x^2 + 4y^2 + z^2 + 2xy + xz - 2yz)$

$= \{x + (-2y) + (-z)\}\{x^2 + 4y^2 + z^2 + 2xy - 2yz + zx\}$

By formula...

$= x^3 - 8y^3 - z^3 - 6xyz$

iii) we have

$= x^2 + 4y^2 + 2xy - 3x + 6y + 9$

(iii) Given,

$= x^2 + 4y^2 + 2xy - 3x + 6y + 9$

Multiply above equation by ( x- 2y +3 )

$= (x - 2y + 3)(x^2 + 4y^2 + 2xy - 3x + 6y + 9)$

$= \{x - 2y + 3\}\{x^2 + 4y^2 + 9 + 2xy + 6y - 3x\}$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

$= x^3 - 8y^3 + 27 + 18xy$

(iv) Given,

$= 9x^2 + 25y^2 + 15xy + 12x - 20y + 16$

Multiply above equation by (3x – 5y +4)

We got,

$= (3x - 5y + 4)(9x^2 + 25y^2 + 15xy + 12x - 20y + 16) = \{3x + (-5y) + 4\}\{(3x)^2 + (-5y)^2 + 4^2 - 3x(-5y) - (-5y)(4) - (4)(3x)\}$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

$= (3x)^3 + (-5y)^3 + (4)^3 - 3.3x(-5y)(4)$

$= 27x^3 - 125y^3 + 64 + 180xy$

## CCE - Formative Assessment

**1. Question**

Factorize: $x^4 + x^2 + 25$.

**Answer**

We have,

First we rewrite the question,

$x^4 + x^2 + 25 = (x^2)^2 + 2.x^2.5 + 5^2 - 9x^2$

$\qquad = \{(x^2)^2 + 2.x^2.5 + 5^2\} - (3x)^2$ [By using $a^2 + 2ab + b^2 = (a + b)^2$]

$\qquad = \{x^2 + 5\}^2 - (3x)^2$ \qquad [ By using $a^2 - b^2 = (a + b)(a - b)$

$\qquad = (x^2 + 5 + 3x)(x^2 + 5 - 3x)$

Thus , the factors of $x^4 + x^2 + 25$ are $(x^2 + 5 + 3x)(x^2 + 5 - 3x)$.

## 2. Question

Factorize: $x^2 - 1 - 2a - a^2$.

**Answer**

We have ,

$= x^2 - 1 - 2a - a^2$

Taking -1 as common from last three terms

$= x^2 - (1 + 2a + a^2)$

$= x^2 - (1^2 + 2 \times 1 \times a + a^2)$

$= x^2 - (1 + a)^2 \ [a^2 - b^2 = (a + b)(a - b)]$

$= (x + 1 + a)(x - 1 - a)$

Thus the factors of $x^2 - 1 - 2a - a^2$ are $(x + 1 + a)(x - 1 - a)$.

## 3. Question

If a + b + c = 0, then write the value of $a^3 + b^3 + c^3$.

**Answer**

We have,

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

When ( a + b + c) = 0 Given

$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3abc = 0$

$= a^3 + b^3 + c^3 = 3abc.$

## 4. Question

If $a^2 + b^2 + c^2 = 20$, and $a + b + c = 0$, find $ab + bc + ca$.

**Answer**

We have ,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ By formula

$= a^2 + b^2 + c^2 = 20$ Given

$= (a + b + c) = 0$ Given

Then,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= (0)^2 = 20 + 2(ab + bc + ca)$

$= 2(ab + bc + ca) = -20$

$= (ab + bc + ca) = -10$

## 5. Question

If a + b + c = 9 and ab + bc + ca = 40, find $a^2 + b^2 + c^2$.

**Answer**

We have ,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ By formula

$= a + b + c = 9$ Given

$= ab + bc + ca = 40$ Given

Then,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= (9)^2 = a^2 + b^2 + c^2 + 2 \times 40$

$= a^2 + b^2 + c^2 = 81 - 80$

$= a^2 + b^2 + c^2 = 1$

## 6. Question

If $a^2 + b^2 + c^2 = 250$ and ab + bc + ca = 3, find a + b + c.

**Answer**

We have,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ By formula

$= a^2 + b^2 + c^2 = 250$ Given

$= ab + bc + ca = 3$ Given

Then,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= (a + b + c)^2 = 250 + 2 \times 3$

$= (a + b + c)^2 = 256$

$= (a + b + c) = \sqrt{256} = \pm 16$.

## 7. Question

Write the value of: $25^3 - 75^3 + 50^3$.

**Answer**

We have,

$= 25^3 - 75^3 + 50^3$

Let a = 25 , b = -75 , c = 50 ,

Then the expression becomes as ,

$= 25^3 - 75^3 + 50^3 = a^3 + b^3 + c^3$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

Here , $(a + b + c) = 25 + (-75) + 50 = 0$

Hence,

$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 = 3abc$

$= 25^3 - 75^3 + 50^3 = 3 \times 25 \times -75 \times 50$

$= 25^3 - 75^3 + 50^3 = -281250.$

## 8. Question

Write the value of: $48^3 - 30^3 - 18^3$.

### Answer

We have,

$= 48^3 - 30^3 - 18^3$ Given

Let a = 48 , b = -30 , c = -18

Then the expression becomes ,

$= 48^3 - 30^3 - 18^3 = a^3 + b^3 + c^3$

$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

Here,

$= (a + b + c) = 48 + (-30) + (-18) = 0$

Hence,

$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 = 3abc$

$= 48^3 - 30^3 - 18^3 = 3 \times 48 \times -30 \times -18$

$= 48^3 - 30^3 - 18^3 = 77760.$

## 9. Question

Write the value of: $\left(\dfrac{1}{2}\right)^3 + \left(\dfrac{1}{3}\right)^3 - \left(\dfrac{5}{6}\right)^3$

### Answer

We have ,

$= \left(\dfrac{1}{2}\right)^3 + \left(\dfrac{1}{3}\right)^3 - \left(\dfrac{5}{6}\right)^3$ Given

Let $a = \dfrac{1}{2}$ , $b = \dfrac{1}{3}$ , $c = -\dfrac{5}{6}$ ,then the expression becomes

$= \left(\dfrac{1}{2}\right)^3 + \left(\dfrac{1}{3}\right)^3 - \left(\dfrac{5}{6}\right)^3 = a^3 + b^3 + c^3$

Here,

$= a + b + c = \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{5}{6} = 0$

Hence ,

$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2+b^2+c^2-ab-bc-ca)$

$= a^3 + b^3 + c^3 = 3abc$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = 3 \times \frac{1}{2} \times \frac{1}{3} \times -\frac{5}{6}$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = -\frac{5}{12}$

## 10. Question

Write the value of: $30^3 + 20^3 - 50^3$.

**Answer**

We have ,

$= 30^3 + 20^3 - 50^3$ given

Let $a = 30, b = 20, c = -50$ , then the expression becomes,

$= 30^3 + 20^3 - 50^3 = a^3 + b^3 + c^3$

Here ,

$= a + b + c = 30 + 20 - 50 = 0$

Hence,

$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2+b^2+c^2-ab-bc-ca)$

$= a^3 + b^3 + c^3 = 3abc$

$= 30^3 + 20^3 - 50^3 = 3 \times 30 \times 20 \times -50$

$= 30^3 + 20^3 - 50^3 = -90000$

## 1. Question

The factors of $a^2-1-2x-x^2$ are

A. (a-x+1)(a-x-1)

B. (a+x-1)(a-x+1)

C. (a+x+1)(a-x-1)

D. none of these

**Answer**

We have,

$= a^2 - 1 - 2x - x^2$

$= a^2 - (1 + 2x + x^2) = a^2 - (1+x)^2$

$= (a - 1 - x)(a + 1 + x) \; [a^2 - b^2 = (a+b)(a-b)]$

Thus, the factors of $a^2 - 1 - 2x - x^2$ are $(a - 1 - x)(a + 1 + x)$.

## 2. Question

The factors of $x^4 + x^2 = 25$ are

A. $(x^2+3x+5)(x^2-3x+5)$

B. $(x^2+3x+5)(x^2+3x-5)$

C. $(x^2+x+5)(x^2-x+5)$

D. none of these

**Answer**

We have,

$= x^4 + x^2 + 25$

Adding and subtracting $9x^2$ in the equation

$= x^4 + 25 + x^2 + 9x^2 - 9x^2$

$= x^4 + 25 + 10x^2 - 9x^2 = (x^2 + 5)^2 - (3x)^2$

$= (x^2 + 5 - 3x)(x^2 + 5 + 3x)$

Thus the factors of $(x^4+x^2+25)$ are $(x^2 + 5 - 3x)(x^2 + 5 - 3x)$.

### 3. Question

The factors of $x^2+4y^2+4y-4xy-2x-8$ are

A. $(x-2y-4)(x-2y+2)$

B. $(x-y+2)(x-4y-4)$

C. $(x+2y-4)(x+2y+2)$

D. none of these

**Answer**

We have ,

$= x^2 + 4y^2 + 4y - 4xy - 2x - 8$

$= x^2 + 4y^2 - 4xy + 4y - 2x - 8 = (x - 2y)^2 - 2(x - 2y) - 8$

Let a = ( x-2y) , then the expression becomes ,

$= x^2 + 4y^2 + 4y - 4xy - 2x - 8 = a^2 - 2a - 8$

$= a^2 - 4a + 2a - 8$

$= a(a-4) + 2( a -4)$

$= (a - 4)(a + 2)$

Put a = ( x -2y)

$= x^2 + 4y^2 + 4y - 4xy - 2x - 8 = (x - 2y - 4)(x - 2y + 2)$

Thus the factors of $x^2 + 4y^2 + 4y - 4xy - 2x - 8$ are $(x - 2y - 4)(x - 2y + 2)$

### 4. Question

The factors of $x^3-x^2y-xy^2+y^3$ are

A. $(x+y)(x^2-xy+y^2)$

B. $(x+y)(x^2+xy+y^2)$

C. $(x+y)^2(x-y)$

D. $(x-y)^2(x+y)$

**Answer**

We have,

$= x^3 - x^2y - xy^2 + y^3$

$= x^3 + y^3 - x^2y - xy^2 = (x+y)^3 - 3x^2y - 3xy^2 - x^2y - xy^2$

As $x^3 + y^3 = (x+y)^3 - 3x^2y - 3xy^2$

$= (x+y)^3 - 4xy(x+y)$

$= (x+y)\{(x+y)^2 - 4xy)$

$= (x+y)(x^2 + y^2 + 2xy - 2xy)$

$= (x+y)(x^2 + y^2 - 2xy) = (x+y)(x-y)^2$

**5. Question**

The factors of $x^3-1+y^3+3xy$ are

A. $(x-1+y)(x^2+1+y^2+x+y-xy)$

B. $(x+y+1)(x^2+y^2+1-xy-x-y)$

C. $(x-1+y)(x^2-1-y^2+x+y+xy)$

D. $3(x+y-1)(x^2+y^2-1)$

**Answer**

We have ,

$= x^3 - 1 + y^3 + 3xy$

$= (x)^3 + (-1)^3 + (y)^3 - 3(x)(-1)(y)$

$= (x-1+y)(x^2+1+y^2+x+y-xy)\{(a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2 - ab-bc-ca)\}$

Thus the factors of $x^3 - 1 + y^3 + 3xy$ are $(x-1+y)(x^2+1+y^2+x+y-xy)$

**6. Question**

The factors of $8a^2+b^3-6ab+1$ are

A. $(2a+b-1)(4a^2+b^2+1-3ab-2a)$

B. $(2a-b+1)(4a^2+b^2-4ab+1-2a+b)$

C. $(2a+b+1)(4a^2+b^2+1-2ab-b-2a)$

D. $(2a-1+b)(4a^2+1-4a-b-2ab)$

**Answer**

We have ,

$= 8a^3 + b^3 - 6ab + 1$

$= (2a)^3 + (b)^3 - 3(2a)(b)(1) + (1)^3$ try to make $(a^3+b^3+c^3-3abc)$

$= (2a+b+1)(4a^2+b^2+1-2ab-b-2a)$

Thus the factors of $8a^3 + b^3 - 6ab + 1$ are $(2a+b+1)(4a^2+b^2+1-2ab-b-2a)$ .

**7. Question**

$(x+y)^3-(x-y)^3$ can be factorized as:

A. $2y(3x^2+y^2)$

B. $2x(3x^2+y^2)$

C. $2y(3y^2+x^2)$

D. $2x(x^2+3y^2)$

**Answer**

We have ,

$=(x+y)^3-(x-y)^3$

Applying formulas,

$= x^3+y^3+3x^2y+3xy^2-(x^3-y^3-3x^2y+3xy^2)$

$= x^3+y^3+3x^2y+3xy^2-x^3+y^3+3x^2y-3xy^2$

$= 2y^3+6x^2y$

$= 2y(y^2+3x^2)$

Thus the factors of $(x+y)^3-(x-y)^3$ are $2y(y^2+3x^2)$

## 8. Question

The factors of $x^2$-7x+6are

A. x(x-6)(x-1)

B. $(x^2$-6)(x-1)

C. (x+1)(x+2)(x-3)

D. (x-1)(x+3)(x-2)

**Answer**

We have,

$= x^3-7x-6$

Adding and subtracting 1 in the equation

$= x^3-7x-6-1+1 = x^3+1^3-7x-7$

$= (x+1)(x^2+1+x)-7(x+1)$

$=(x+1)(x^2-6+x)$

$=(x+1)(x^3-3x+2x+x) = (x+1)(x+2)(x-3)$

Thus the factors of $x^3-7x-6$ are $(x+1)(x+2)(x-3)$.

## 9. Question

The expression (a-b)$^3$+(b-c)$^3$+(c-a)$^3$ can be factorized as:

A. (a-b)(b-c)(c-a)

B. 3(a-b)(b-c)(c-a)

C. -3(a-b)(b-c)(c-a)

D. (a+b+c)($a^2$+$b^2$+$c^2$-ab-bc-ca)

**Answer**

We have,

Let $x = (a-b), y = (b-c), z(c-a)$

So, $x+y+z = a-b+b-c+c-a$

If a+b+c = 0 , then,

$= a^3 + b^3 + c^3 = 3abc$

$= (a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a).$

## 10. Question

The expression $x^4+4$ can be factorized as

A. $(x^2+2x+2)(x^2-2x+2)$

B. $(x^2+2x+2)(x^2+2x-2)$

C. $(x^2-2x-2)(x^2-2x+2)$

D. $(x^2+2)(x^2-2)$

**Answer**

We have ,

$= x^4 + 4$

$= x^4 + 4x^2 + 4 - 4x^2$

$=(x^2 + 2)^2 - 4x^2$

$= (x^2 + 2)^2 - (2x)^2 \, [x^2 - y^2 = (x-y)(x+y)]$

$= (x^2 + 2 + 2x)(x^2 + 2 - 2x)$

## 11. Question

If 3x=a+b+c, then the value of $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$ is

A. a+b+c

B. (a-b)(b-c)(c-a)

D. 0

D. none of these

**Answer**

We have,

$= 3x = a+b+c$

Let $a_1 = x - a, b_1 = x - b, c_1 = x - c$

So, $a_1 + b_1 + c_1 = x - a + x - b + x - c = 3x - (a+b+c)$

$= 3x - 3x = 0 \, [\, a+b+c = 3x \,]$ given

$= (x-a)^3 + (x-b)^3 + (x-c)^3 = 3(x-a)(x-b)(x-c)$

Now, $(x-a)^3 + (x-b)^3 + (x-c)^3 - 3(x-a)(x-b)(x-c) = 0.$

## 12. Question

If $(x+y)^3-(x-y)^3-6y(x^2-y^2)=ky^2$ then k=

A. 1

B. 2

C. 4

D. 8

**Answer**

We have,

$$= (x+y)^3 - (x-y)^3 - 6y(x^2 - y^2) = ky^3$$

$$= (x+y-x+y)^3 + 3(x+y)(x-y)(x+y-x+y) - 6y(x^2 - y^2) = ky^3$$

$$= 2y^3 + 6y(x^2 - y^2) - 6y(x^2 - y^2) = ky^3$$

$$= 8y^3 = ky^3$$

$$= k = 8.$$

**13. Question**

If $x^3 - 3x^2 + 3x - 7 = (x+1)(ax^2 + bx + c)$, then a+b+c=

A. 4

B. 12

C. -10

D. 3

**Answer**

We have,

$$= x^3 - 3x^2 + 3x - 7 = (x+1)(ax^2 + bx + c)$$

$$= x^3 - 3x^2 + 3x - 7 = ax^3 + bx^2 + cx + ax^2 + bx + c$$

$$= x^3 - 3x^2 + 3x - 7 = ax^3 + (a+b)x^2 + (b+c)x + c$$

By compairing both sides ,

= a = 1

= a + b = -3

= b + c = 3

= c = -7

Thus , a + (b + c) = 1+3 = 4.

**14. Question**

The value of $\dfrac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$ is

A. 2

B. 3

C. 2.327

D. 2.273

**Answer**

We have,

$$= \dfrac{2.3^3 - 0.027}{2.3^2 + 0.69 + 0.09}$$

$$= \frac{2.3^3 - 0.027}{2.3^2 + 0.69 + 0.09} = \frac{2.3^3 - 0.3^3}{2.3^2 + 0.3^2 + 2.3 \times 0.3} \quad [a^3 - b^3 = (a-b)(a^2 + b^2 + ab)]$$

Hence,

$$= \frac{\{(a-b)(a^2 + b^2 + ab)\}}{a^2 + b^2 + ab} = a - b$$

= ( 2.3 – 0.3 ) = 2 .

### 15. Question

The value of $\dfrac{(0.013)^3 + (0.007)^3}{(0.013)^2 - 0.013 \times 0.013 \times 0.007 + (0.007)^2}$ is

A. 0.006

B. 0.02

C. 0.0091

D. 0.00185

### Answer

We have,

$$= \frac{0.013^3 + 0.007^3}{0.013^2 - 0.013 \times 0.007 + 0.007^2} \quad [ a^3 + b^3 = (a+b)(a^2 + b^2 - ab)]$$

Hence,

$$= \frac{\{(a+b)(a^2 + b^2 - ab)\}}{a^2 + b^2 - ab} = a + b$$

= 0.013 + 0.007 = 0.020