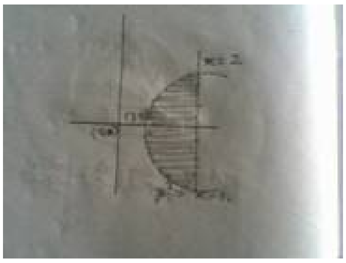
# Area and Volume of Curve

---

**Q.1.** Draw a rough sketch of the curve $y^2 + 1 = x$, $x \le 2$. Find the area enclosed by the curve and the line $x = 2$.

**Solution : 1**



Fig

We have , $y^{2} + 1 = x$ , $x \le 2$ ,

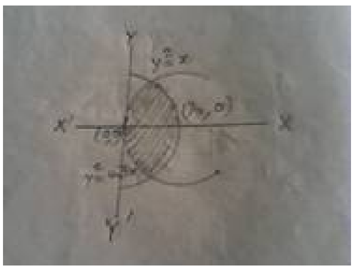
Or, $y^{2} = x - 1$ .

Required area = 2 (1, 2) ∫ √(x − 1) dx

= {2[(x − 1)](1, 2)}/(3/2) = 2 × (2/3) [(1)3/2] = 4/3 sq. units.

**Q.2.** Find the area enclosed by the curve $y^2 = x$ and $y^2 = 4 - 3x$.

**Solution : 2**



Fig

We have , $y^{2} = x$ and $y^{2} = 4 - 3x$ ,

Solving these two equations simultaneously ,

x = 4 − 3x => 4x = 4 => x = 1 and y = ±1.

The points are (0, 0), (4/3, 0), (1, 1) and (1 , − 1) .

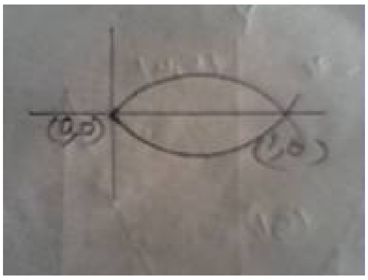
The required area = 2[01∫√x dx + $1^{4/3}$∫√(4 − 3x) dx ]

$= 2[\{(2x^{3/2})/3\}01 + \{2(4 - 3x)^{3/2}/(-3 \times 3)\}1^{4/3}]$

$= 2[2/3 + 2/9]\ 2[(6 + 2)/9] = 16/9$ sq. units.

**Q.3.** The region bounded by the curve $y^2 = x(x - 1)^2$, the x –axis and the lines $x = 1$, $x = 0$ , is rotated through four right angles about the x-axis. Calculate the volume of the solid of revolution so formed.

**Solution : 3**




Fig

Curve $y^2 = x(x - 1)^2$

Volume $= \pi 0\int 1\ y^2\ dx = \pi 0\int 1\ x(x - 1)\ dx$

$= \pi 0\int 1\ (x^3 - 2x^2 + x)\ dx$

$= \pi[x4/4 - 2x3/3 + x^2/2]0^1$

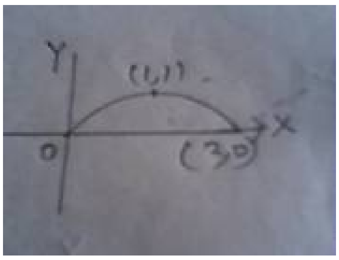
$= \pi[1/4 - 2/3 + 1/2]$

$= \pi[(3 - 8 + 6)/12]$

$= \pi/12$ cubic unit.

**Q.4.** Calculate the area bounded by the curve $y = x(2 - x)$ and the line $x = 0$ , $y = 0$ , $x = 2$. This area is rotated through four right angles about the x-axis. Calculate the volume of the solid so formed.

**Solution : 4**




Fig

Area = $\int_0^2 x(2 - x)dx = \int_0^2 (2x - x^2)dx$

= $[2x^2/2 - x^3/3]_0^2$

= 4 – 8/3 = 4/3 sq. units.

Volume = $\pi \int_0^2 y^2 dx$

= $\pi \int_0^2 x^2(2 - x)^2\, dx$

= $\pi \int_0^2 (4x^2 - 4x^3 + x^4)\, dx$

= $\pi[4x^3/3 - 4x^4/4 + x^5/5]_0^2$

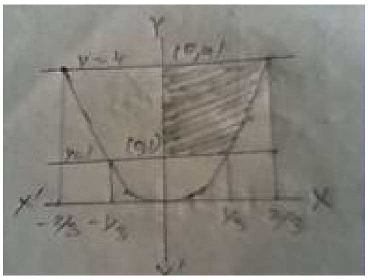
= $\pi[32/3 - 16/1 + 32/5]$

= $\pi[(160 - 240 + 96)/15]$

= $16/15\pi$ cubic units.

**Q.5.** Sketch and shade the area of the region lying in the first quadrant and bound by y = 9x2, x = 0, y = 1 and y = 4.Find the area of the shaded region.

**Solution : 5**



Fig

We have $y = 9x^2$,

at x = 0 , $y = 9(0)^2 = 0$ and at y = 4, x = √(4/9) = ±2/3, at y = 1, x = √(1/9) = ±1/3.

Atrea of shaded region = $\int_1^4 \{(y^{1/2})/3\}.dy$ [As, ∫x.dy = ∫√(y/9).dy = ∫(√y)/3 .dy ]

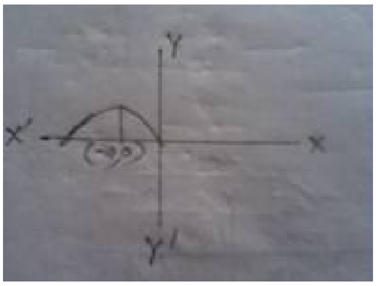
= $1/3[(y^{3/2}(3/2)]$

= $2/9[4^{3/2} - 1^{3/2}]$

= 2/9[8 – 1] = 14/9 = 1.55 sq.unit.

**Q.6.** Show that the area included between the x-axis and the curve $a^2y = x^22(x + a)$ is $a^2/12$.

**Solution : 6**




Fig

We have $a^2y = x^2(x + a)$

When $y = 0$ , $x = -a$ .

Therefore, Area = $-a\int^{0} ydx = -a\int 0[(x^3 + ax^2)/a^2]dx$

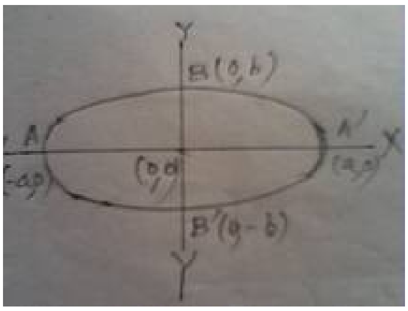
$= 1/a^2[x^4/4 + ax^3/3]-a^0$

$= 1/a^2[(0 + 0) - (a^4/4 - a^4/3)$

$= a^2/12$ units .

**Q.7.** Find the volume of the solid obtained by revolving the ellipse $x^2/a^2 + y^2/b^2 = 1$ about the axis of x.

**Solution : 7**




Fig

If the curve $y = f(x)$ revolves round x-axis, then the volume generated = $\int \pi y^2.dx$

Here we have $x^2/a^2 + y^2/b^2 = 1$

Or, $y^2/b^2 = 1 - x^2/a^2$

Or, $y^2 = b^2/a^2.(a^2 - x^2)$

Therefore, Volume of the solid = $\int \pi.b^2/a^2.(a^2 - x^2).dx$

$= (\pi\times b^2)/a^2 \; -a\int a(a^2 - x^2).dx$

$= (\pi b^2)/a^2\ [a^2x - x^3/3]$ a–a

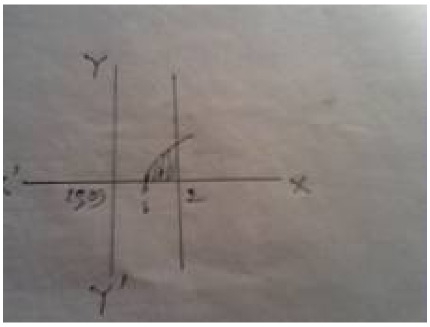
$= (\pi b^2)/a^2[a^3 - a^3/3 + a^3 - a^3/3]$

$= \{(\pi b2)/a^2\}\times(4/3)a^3$

$= (4/3)\pi b^2a$ cu. unit.

**Q.8.** Calculate the area of the figure bounded by the curve y = log x, the straight line x = 2 and the x-axis.

**Solution : 8**




Fig

We have, y = log x, when x = 1, y = log 1 = 0 , when x = 2, y = log 2 = 0.6931.

Area of shaded region = $_1\int^2$ ydx = $_1\int^2$log x dx

= $_1\int^2$1. log x dx = x log x − $_1\int^2$ (1) dx

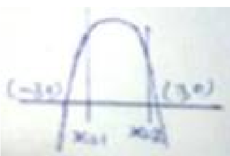
= [x log x − x ]$_1^2$

= [2log $^2$ − 2] − [log 1 − 1 ]

= 2 log $^2$ − 1.

**Q.9.** Draw a rough sketch of the curve $x^2 + y = 9$ and find the area enclosed by the curve, the x-axis and the line x + 1 = 0 and x − 2 = 0.

**Solution : 9**



Fig

Required area = $-1\int^{2}(9 - x^{2})dx = [9x - x^{3}/3]-12$

= [(18 − 8/3) − (−9 + 1/3)]

= [27 − 3] = 24 sq. units.

**Q.10.** Find the area of the figure bounded by the graphs of the functions $y = x^2$ and $y = 2x - x^2$.

**Solution : 10**

We have, $y = x^{2}$ and $y = 2x - x^{2}$ ;

Therefore, $x^{2} = 2x - x^{2}$

Or, $2x^{2} - 2x = 0$

Or, 2x(x − 1) = 0

Therefore , x = 0 ; 1.

When x = 0 , y = 0 and when x = 1 , $y = (1)^{2} = 1$.

Now , y = x2 represents a parabola and its vertex is (0, 0).

And $y = 2x - x^{2}$ => $y - 1 = 2x - x^{2} - 1 = 0 - (x^{2} - 2x + 1) = -(x - 1)^{2}$.

This represents a parabola with vertex (1, 1).

Area of the common region = $0\int^{1}[(2x - x^{2}) - x^{2}]\, dx = 0\int^{1}(2x - x^{2})dx - 0\int 1x^{2}dx$

= $[x^{2} - x^{3}/3]01 - [x^{3}/3]01$

= (1 − 1/3) − (1/3)

= 2/3 − 1/3 = 1/3 sq. units.