Differential Equations
Short Answer Type Questions

1. Find the differential equation of the family of curves $y = Ae^{2x} + B.e^{-2x}$.

Sol. $y = Ae^{2x} + B.e^{-2x}$.

$\frac{dy}{dx} = 2Ae^{2x} - 2B.e^{-2x} and \frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x}$

Thus, $\frac{d^2y}{dx^2} = 4y \, i.e., \frac{d^2y}{dx^2} - 4y = 0.$

2. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

Sol. $\frac{dy}{dx} = \frac{y}{x} \quad \Rightarrow \frac{dy}{y} = \frac{dx}{x} \quad \Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$

$\Rightarrow log\ y = log\ x + log\ c \ \Rightarrow \ y = cx$

3. Given that $\frac{dy}{dx} = ye^x$ and $x = 0,\ y = e$. Find the value of y when $x = 1$.

Sol. $\frac{dy}{dx} = ye^x \Rightarrow \int \frac{dy}{y} \ = \ \int e^x dx \quad \Rightarrow \quad \log y = e^x + c$

Substituting $x = 0$ *and* $y = e$ we get $log\ e = e^0 + c,$ i.e., $c = 0$ $(\because log\ e = 1)$

Therefore, $log\ y = e^x$.

Now, substituting x = 1 in the above, we get $log\ y = e \Rightarrow y = e^e$.

4. Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$

Sol. The equation is of the type $\frac{dy}{dx} + Py = Q$, which is a linear differential equation.

Now I.F. $\int \frac{1}{x} dx = e^{\log x} = x.$

Therefore, solution of the given differential equation is

$y.x = \int x\, x^2 dx$, $i.e.\ yx = \frac{x^4}{4} + c$

Hence, $y = \frac{x^3}{4} + \frac{c}{x}$.

5. Find the differential equation of the family of lines through the origin.

Sol. Let $y = mx$ be the family of lines through origin. Therefore, $\frac{dy}{dx} = m$.

Eliminating m, we get $y = \frac{dy}{dx}.x \ or\ x\frac{dy}{dx} - y = 0.$

6. Find the differential equation of all non-horizontal lines in a plane.

Sol. The general equation of all non-horizontal lines in a plane is $ax + by = c$, where $a \neq 0$.

Therefore, $a\frac{dx}{dy} + b = 0.$

Again, differentiating both sides w.r.t. y, we get

$$a\frac{d^2x}{dy^2}=0 \Rightarrow \frac{d^2x}{dy^2}=0.$$

7. Find the equation of a curve whose tangent at any point on it, different from origin, has slope $y+\frac{y}{x}$.

Sol. Given $\frac{dy}{dx}=y+\frac{y}{x}=y\left(1+\frac{1}{x}\right)$

$$\Rightarrow \frac{dy}{y}=\left(1+\frac{1}{x}\right)dx$$

Integrating both sides, we get

$$log\ y=x+log\ x+c \ \Rightarrow \log\left(\frac{y}{x}\right)=x+c$$

$$\Rightarrow \frac{y}{x}=e^{x+c}=e^x.e^c \ \Rightarrow \frac{y}{x}=k.e^x$$

$$\Rightarrow y = kx\ .\ e^x.$$

Long Answer Type Questions

8. Find the equation of a curve passing through the point (1, 1) if the perpendicular distance of the origin from the normal at any point $P(x,\ y)$ of the curve is equal to the distance of P form the $x-axis$.

Sol. Let the equation of normal at $P(x, y)\ be\, Y-y=\frac{-dx}{dy}(X-x), ie.,$

$$Y+X\frac{dx}{dy}-\left(y+x\frac{dx}{dy}\right)=0 \ ...(1)$$

Therefore, the length of perpendicular from origin to (1) is $\dfrac{y+x\frac{dx}{dy}}{\sqrt{1+\left(\frac{dx}{dy}\right)^2}}$ ...(2)

Also, distance between $P\ and\ x-axis\ is\ |y|$. Thus, we get

$$\frac{y+x\frac{dx}{dy}}{\sqrt{1+\left(\frac{dx}{dy}\right)^2}}=|y|$$

$$\Rightarrow \left(y+x\frac{dx}{dy}\right)^2=y^2\left[1+\left(\frac{dx}{dy}\right)^2\right] \Rightarrow \frac{dx}{dy}\left[\frac{dx}{dy}\left(x^2-y^2\right)+2xy\right]=0 \Rightarrow \frac{dx}{dy}=0$$

$$or\ \frac{dx}{dy}=\frac{2xy}{y^2-x^2}$$

Case I: $\frac{dx}{dy} = 0 \Rightarrow dx = 0$

Integrating both sides, we get $x = k$, Substituting $x = 1$, we get $k = 1$.

Therefore, $x = 1$ is the equation of curve (not possible, so rejected).

Case II: $\frac{dx}{dy} = \frac{2x\,y}{y^2 - x^2} \Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$. Substituting $y = vx$, we get

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2vx^2} \Rightarrow x.\frac{dv}{dx} - \frac{v^2 - 1}{2v} - v$$

$$= \frac{-(1+v^2)}{2v} \Rightarrow \frac{2v}{1+v^2}dv = \frac{-dx}{x}$$

Integrating both sides, we get

$$log\left(1+v^2\right) = -log\,x + log\,c \Rightarrow log\left(1+v^2\right)\left(x\right) = log\,c\left(1+v^2\right)x = c$$

$\Rightarrow x^2 + y^2 = cx.$ Substituting $x = 1,\ y = 1,$ we get $c = 2$.

Therefore, $x^2 + y^2 - 2x = 0$ is the required equation.

9. Find the equation of a curve passing through $\left(1, \frac{\pi}{4}\right)$ if the slope of the tangent to the curve at any point $P\left(x,\ y\right)$ is $\frac{y}{x} - \cos^2\frac{y}{x}$.

Sol. According to the given condition

$$\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x} \quad ...(i)$$

This is a homogeneous differential equation. Substituting $y = vx,$ we get

$$v + x\frac{dv}{dx} = v - \cos^2 v \Rightarrow x\frac{dv}{dx} = -\cos^2 v$$

$$\Rightarrow \sec^2 v\,dv = -\frac{dx}{x} \Rightarrow \tan v = -\log x + c$$

$$\Rightarrow \tan\frac{y}{x} + \log x = c \quad ...(ii)$$

Substituting $x = 1, y = \frac{\pi}{4}$, we get $c = 1$. Thus, we get

$tan\left(\frac{y}{x}\right) + \log x = 1,$ which is the required equation.

10. Solve $x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$ and $x = 1, y = \frac{\pi}{2}$

Sol. Given equation can be written as

$$x^2\frac{dy}{dx} - xy = 2\cos^2\left(\frac{y}{2x}\right), x \neq 0.$$

$$\Rightarrow \frac{x^2\frac{dy}{dx} - xy}{2\cos^2\left(\frac{y}{2x}\right)} = 1 \Rightarrow \frac{\sec^2\left(\frac{y}{2x}\right)}{2}\left[x^2\frac{dy}{dx} - xy\right] = 1$$

Dividing both sides by x^3, we get

$$\frac{\sec^2\left(\frac{y}{2x}\right)}{2}\left[\frac{x\frac{dy}{dx}-y}{x^2}\right]=\frac{1}{x^3} \Rightarrow \frac{d}{dx}\left[\tan\left(\frac{y}{2x}\right)\right]=\frac{1}{x^3}$$

Integrating both sides, we get

$$\tan\left(\frac{y}{2x}\right)=\frac{-1}{2x^2}+k.$$

Substituting $x=1, y=\frac{\pi}{2}$, we get

$k=\frac{3}{2}$, therefore, $\tan\left(\frac{y}{2x}\right)=-\frac{1}{2x^2}+\frac{3}{2}$ is the required solution.

11. State the type of the differential equation for the equation.
$xdy-ydx=\sqrt{x^2+y^2}dx$ **and solve it.**

Sol. Given equation can be written as $xdy=\left(\sqrt{x^2+y^2}+y\right)dx$, *i.e.*,

$$\frac{dy}{dx}+\frac{\sqrt{x^2+y^2}+y}{x} \quad ...(1)$$

Clearly RHS of (1) is a homogeneous function of degree zero. Therefore, the given equation is a homogeneous differential equation. Substituting $y=vx$, we get form (1)

$$v+x\frac{dv}{dx}=\frac{\sqrt{x^2+v^2x^2}+vx}{x} \quad i.e.\ v+x\frac{dv}{dx}=\sqrt{1+v^2}+v$$

$$x\frac{dv}{dx}=\sqrt{1+v^2} \Rightarrow \frac{dv}{\sqrt{1+v^2}}=\frac{dx}{x} \quad ...(2)$$

Integrating both sides of (2), we get

$$\log(v+\sqrt{1+v^2})=\log x+\log c \Rightarrow v+\sqrt{1+v^2}=cx$$

$$\Rightarrow \frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}=cx \Rightarrow y+\sqrt{x^2+y^2}=cx^2$$

Objective Type Questions

Choose the correct answer form the given four options in each of the Examples 12 to 21.

12. The degree of the differential equation $\left(1+\frac{dy}{dx}\right)^3=\left(\frac{d^2y}{dx^2}\right)^2$ **is**

(A) 1
(B) 2
(C) 3
(D) 4

Sol. The correct answer is (B).

13. **The degree of the differential equation** $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x^2\log\left(\frac{d^2y}{dx^2}\right)$ **is**

(A) 1
(B) 2
(C) 3
(D) Not defined

Sol. Correct answer is (D). The given differential equation is not a polynomial equation in terms of its derivatives, so its degree is not defined.

14. **The order and degree of the differential equation** $\left[1+\left(\frac{dy}{dx}\right)^2\right]^2=\frac{d^2y}{dx^2}$ **respectively, are**

(A) 1, 2
(B) 2, 2
(C) 2, 1
(D) 4, 2

Sol. Correct answer is (C).

15. **The order of the differential equation of all circles of given radius** a **is:**

(A) 1
(B) 2
(C) 3
(D) 4

Sol. Correct answer is (B). Let the equation of given family be $(x-h)^2+(y-k)^2=a^2$.

It has two arbitrary constants h and k. Therefore, the order of the given differential equation will be 2.

16. **The solution of the differential equation** $2x.\frac{dy}{dx}-y=3$ **represents a family of**

(A) straight lines
(B) circles
(C) parabolas
(D) ellipses

Sol. Correct answer is (C). Given equation can be written as

$\frac{2dy}{y+3}=\frac{dx}{x} \Rightarrow 2\log(y+3)=\log x+\log c$

$\Rightarrow (y+3)^2=cx$ which represents the family of parabolas.

17. **The integrating factor of the differential equation** $\frac{dy}{dx}(x\log x)+y=2\log x$ **is**

(A) e^x
(B) $log\ x$
(C) $log(log\ x)$
(D) x

Sol. Correct answer is (B). Given equation can be written as $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}$.

Therefore, I.F. $= e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x.$

18. **A solution of the differential equation** $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ **is**

(A) $y = 2$
(B) $y = 2x$
(C) $y = 2x - 4$
(D) $y = 2x^2 - 4$

Sol. Correct answer is (C).

19. **Which of the following is not a homogeneous function of** x *and* y **.**

(A) $x^2 + 2xy$
(B) $2x - y$
(C) $\cos^2\left(\frac{y}{x}\right) + \frac{y}{x}$
(D) $sin\, x - cos\, y$

Sol. Correct answer is (D).

20. **Solution of the differential equation** $\frac{dx}{x} + \frac{dy}{y} = 0$ **is**

(A) $\frac{1}{x} + \frac{1}{y} = c$
(B) $log\, x . log\, y = c$
(C) $xy = c$
(D) $x + y = c$

Sol. Correct answer is (C). From the given equation, we get $log\, x + log\, y = log\, c$ giving $xy = c$.

21. **The solution of the differential equation** $x\frac{dy}{dx} + 2y = x^2$ **is**

(A) $y = \frac{x^2 + c}{4x^2}$
(B) $y = \frac{x^2}{4} + c$
(C) $y = \frac{x^4 + c}{x^2}$
(D) $y = \frac{x^4 + c}{4x^2}$

Sol. Correct answer (D). $I.F. = e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$. Therefore, the solution is $y . x^2 = \int x^2 .x dx = \frac{x^4}{4} + k,\ i.e.,\ y = \frac{x^4 + c}{4x^2}.$

22. Fill in the blanks of the following:

(i) **Order of the differential equation representing the family of parabolas** $y^2 = 4ax$ **is** __________.

Sol. One; a is the only arbitrary constant.

(ii) The degree of the differential equation $\left(\frac{dy}{dx}\right)^3+\left(\frac{d^2y}{dx^2}\right)^2=0$ **is _____________.**

Sol. Two; since the degree of the highest order derivative is two.

(iii) The number of arbitrary constants in a particular solution of the differential equation $tan\ x\ dx+tan\ y\ dy=0$ **is _____________.**

Sol. Zero; any particular solution of a differential equation has no arbitrary constant.

(iv) $F(x,y)=\frac{\sqrt{x^2+y^2}+y}{x}$ **is a homogeneous function of degree _______________.**

Sol. Zero.

(v) An appropriate substitution to solve the differential equation

$\frac{dx}{dy}=\frac{x^2\log\left(\frac{x}{y}\right)=x^2}{xy\log\left(\frac{x}{y}\right)}$ **is _______.**

Sol. $x=vy$.

(vi) Integrating factor of the differential equation $x\frac{dy}{dx}-y=\sin x$ **is ____________.**

Sol. $\frac{1}{x}$; given differential equation can be written as $\frac{dy}{dx}-\frac{y}{x}=\frac{\sin x}{x}$ and therefore

$I.F.=e^{\int-\frac{1}{x}dx}=e^{-\log x}=\frac{1}{x}.$

(vii) The general solution of the differential equation $\frac{dy}{dx}=e^{x-y}$ **is _____________.**

Sol. $e^y=e^x+c$ from given equation, we have $e^y dy=e^x dx$.

(viii) The general solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=1$ **is ____________.**

Sol. $xy=\frac{x^2}{2}+c; I.F.=e^{\int\frac{1}{x}dx}=e^{\log x}=x$ and the solution is $y.x=\int x.1dx=\frac{x^2}{2}+C.$

(ix) The differential equation representing the family of curves y = *A*sin*x* + *B*cos*x* is___.

Sol. $\frac{d^2y}{dx^2}+y=0;$ Differentiating the given function w.r.t. x successively, we get

$\frac{dy}{dx}=A\cos x-B\sin x$

$and\ \frac{d^2y}{dx^2}=-A\sin x-B\cos x$

$\Rightarrow\frac{d^2y}{dx^2}+y=0$ is the differential equation.

(x) $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1(x\neq0)$ **when written in the form** $\frac{dy}{dx}+Py=Q,$ **then P =**

________.

Sol. $\frac{1}{\sqrt{x}}$; the given equation can be written as

$$\frac{dy}{dx}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}} \text{ i.e. } \frac{dy}{dx}+\frac{y}{\sqrt{x}}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

This is a differential equation of the type $\frac{dy}{dx}+Py=Q.$

23. State whether the following statements are True or False.

(i) Order of the differential equation representing the family of ellipses having centre at origin and foci on x-axis is two.

Sol. True, since the equation representing the given family is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$ which has two arbitrary constants.

(ii) Degree of the differential equation $\sqrt{1+\frac{d^2y}{dx^2}}=x+\frac{dy}{dx}$ is not defined.

Sol. True, because it is not a polynomial equation in its derivatives.

(iii) $\frac{dy}{dx}+y=5$ is a differential equation of the type $\frac{dy}{dx}+Py=Q$ but it can be solved using variable separable method also.

Sol. True

(iv) $F(x,y)=\dfrac{y\cos\left(\frac{y}{x}\right)+x}{x\cos\left(\frac{y}{x}\right)}$ is not a homogeneous function.

Sol. True, because $f(\lambda x,\lambda y)=\lambda^{o}f(x,y).$

(v) $F(x,y)=\dfrac{x^2+y^2}{x-y}$ is a homogeneous function of degree 1.

Sol. Ture, because $f(\lambda x,\lambda y)=\lambda^{1}f(x,y).$

(vi) Integrating factor of the differential equation $\frac{dy}{dx}-y=\cos x$ is e^x.

Sol. False, because I.F $=e^{\int -1dx}=e^{-x}.$

(vii) The general solution of the differential equation $x\left(1+y^2\right)dx+y\left(1+x^2\right)dy=0$ is $\left(1+x^2\right)\left(1+y^2\right)=k$.

Sol. True, because given equation can be written as

$$\frac{2x}{1+x^2}dx=\frac{-2y}{1+y^2}dy$$

$$\Rightarrow \quad \log(1+x^2)=-\log(1+y^2)+\log k$$

$$\Rightarrow \quad (1+x^2)(1+y^2)=k$$

(viii) The general solution of the differential equation $\frac{dy}{dx}+y\sec x=\tan x$ is $y\left(sec\,x-tan\,x\right)=sec\,x-tan\,x+x+k$.

Sol. False, since I.F. $= e^{\int \sec x dx} = e^{\log(\sec x + \tan x)} = \sec x + \tan x$, the solution is,

$y(\sec x + \tan x) = \int (\sec x + \tan x) \tan x dx$

$= \int \left(\sec x \tan x + \sec^2 x - 1\right) dx = \sec x + \tan x - x + k$

(ix) $x + y = tan^{-1} y$ **is a solution of the differential equation** $y^2 \dfrac{dy}{dx} + y^2 + 1 = 0$

Sol. True, $x + y = tan^{-1} y \Rightarrow 1 + \dfrac{dy}{dx} = \dfrac{1}{1+y^2} \dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx}\left(\dfrac{1}{1+y^2} - 1\right) = 1, i.e., \dfrac{dy}{dx} = \dfrac{-(1+y^2)}{y^2}$ which satisfies the given equation.

(x) $y = x$ **is a particular solution of the differential equation** $\dfrac{d^2 y}{dx^2} - x^2 \dfrac{dy}{dx} + xy = x.$

Sol. False, $y = x$ because does not satisfy the given differential equation.

Differential Equations

Objective Type Quesitons

Choose the correct answer from the given four options in each of the Exercises from 34 to 75 (M.C.Q)

34. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ is:

(A) 1
(B) 2
(C) 3
(D) Not defined

Sol. (D) The degree of the differential equation is not defined because when we expand $\sin\left(\frac{dy}{dx}\right)$ we get an infinite series in the increasing power of $\frac{dy}{dx}$.
Therefore its degree is not defined.

35. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

(A) 4
(B) $\frac{3}{2}$
(C) Not defined
(D) 2

Sol. (D) Given that $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$

On squaring both sides, we get

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

So, the degree of differential equation is 2.

36. The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{\frac{1}{4}}+x^{\frac{1}{5}}=0,$ respectively, are

(A) 2 and not defined
(B) 2 and 2
(C) 2 and 3
(D) 3 and 3

Sol. (A) Given that, $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/4} = -x^{\frac{1}{5}}$

$$\Rightarrow \left(\frac{dy}{dx}\right)^{1/4} = -\left(x^{1/5}+\frac{d^2y}{dx^2}\right)$$

On squaring both sides, we get

$$\Rightarrow \left(\frac{dy}{dx}\right)^{\frac{1}{2}} = \left(x^{\frac{1}{5}} + \frac{d^2y}{dx^2}\right)^2$$

Again, on squaring both sides, we have

$$\frac{dy}{dx} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^4$$

order=2, degree=4

37. **If** $y = e^{-x}\left(A\cos x + B\sin x\right)$, **then** y **is a solution of**

(A) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

(B) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

(C) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$

(D) $\frac{d^2y}{dx^2} + 2y = 0$

Sol. (C) Given that, $y = e^{-x}\left(A\cos x + B\sin x\right)$,

On differentiating both sides w.r.t., x we get

$$\frac{dy}{dx} = -e^{-x}\left(A\cos x + B\sin x\right) + e^{-x}\left(-A\sin x + B\cos x\right)$$

$$\frac{dy}{dx} = -y + e^{-x}\left(-A\sin x + B\cos x\right)$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{-dy}{dx} + e^{-x}\left(-\cos x - B\sin x\right) - e^{-x}\left(-A\sin x + B\cos x\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-dy}{dx} - y - \left[\frac{dy}{dx} + y\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{dy}{dx} - y - \frac{dy}{dx} - y$$

$$\Rightarrow \frac{d^2y}{dx^2} = -2\frac{dy}{dx} - 2y$$

$$\Rightarrow \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

38. **The differential equation for** $y = A\cos\alpha x + B\sin\alpha x$, **where A and B are arbitrary constants is**

(A) $\frac{d^2y}{dx^2} - \alpha^2 y = 0$

(B) $\frac{d^2y}{dx^2} + \alpha^2 y = 0$

(C) $\frac{d^2y}{dx^2} + \alpha y = 0$

(D) $\frac{d^2y}{dx^2} - \alpha y = 0$

Sol. (B) Given, $y = A cos\alpha + B\sin\alpha$

$\Rightarrow \frac{dy}{dx} = -\alpha A\sin\alpha x + \alpha B cos\alpha x$

Again, differentiating both sides w.r.t. x, we get

$\frac{d^2y}{dx^2} = -A\alpha^2\cos\alpha x - \alpha^2 B\sin\alpha x$

$\Rightarrow \frac{d^2y}{dx^2} = -\alpha^2\left(A\cos\alpha x - B\sin\alpha x\right)$

$\Rightarrow \frac{d^2y}{dx^2} = -\alpha^2 y$

$\Rightarrow \frac{d^2y}{dx^2} + \alpha^2 y = 0$

39. Solution of differential equation $xdy - ydx = 0$ **represents:**

(A) a rectangular hyperbola
(B) parabola whose vertex is at origin
(C) straight line passing through origin
(D) a circle whose centre is at origin

Sol. (C) Given that, $xdy - ydx = 0$

$\Rightarrow xdy = ydx$

$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$

On integrating both sides, we get

$\log y = \log x + \log C$

$\Rightarrow \log y = \log Cx$

$\Rightarrow y = Cx$

Which is a straight line passing through origin.

40. Integrating factor of the differential equation $\cos x\frac{dy}{dx} + y\sin x = 1$ **is:**

(A) $cos\, x$
(B) $tan\, x$
(C) $sec\, x$
(D) $sin\, x$

Sol. (C) Given that, $\cos x\frac{dy}{dx} + y\sin x = 1$

$\Rightarrow \frac{dy}{dx} + y\tan x = \sec x$

Here, $P = tan\, x\ and\ Q = sec\, x$

$IF = e^{\int pdx} = e^{\int \tan xdx} = e^{\log\sec x}$

$\therefore = \sec x$

41. Solution of the differential equation $\tan y\sec^2 xdx + \tan x\sec^2 ydy = 0$ **is:**

(A) $tan\, x + tan\, y = k$
(B) $tan\, x - tan\, y = k$
(C) $\frac{\tan x}{\tan y} = k$
(D) $tan\, x.tan\, y = k$

Sol. (D) Given that, $\tan y \sec^2 x dx + \tan x \sec^2 y dy = 0$

$\Rightarrow \tan y \sec^2 x dx = -\tan x \sec^2 y dy$

$\Rightarrow \frac{\sec^2 x}{\tan x} dx = \frac{-\sec^2 y}{\tan y} dy \quad ...(i)$

On integrating both sides, we have

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$

Put $tan\, x = t$ in LHS integral, we get

$\sec^2 x dx = dt \Rightarrow \sec^2 x dx = dt$

and $\tan y = u$ *in* RHS integral, *we get*

$\sec^2 y dy = du$

On substituting these values in Eq. (i), we get

$\int \frac{dt}{t} = -\int \frac{du}{u}$

$\log t = -\log u + \log k$

$\Rightarrow \log(t.u) = l\log k$

$\Rightarrow \log(\tan x \tan y) = \log k$

$\Rightarrow \tan x \tan y = k$

42. Family $y = Ax + A^3$ of curves is represented by the differential equation of degree:

(A) 1
(B) 2
(C) 3
(D) 4

Sol. (A) Given that,

$y = Ax + A^3$

$\Rightarrow \frac{dy}{dx} = A$

[we can differential above equation only once because it has only one arbitrary constant]

$\therefore \; Degree = 1$

43. Integrating factor of $\frac{xdy}{dx} - y = x^4 - 3x$ is:

(A) x
(B) $log\, x$
(C) $\frac{1}{x}$

(D) $-x$

Sol. (C) Given that $\frac{xdy}{dx} - y = x^4 - 3x$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$

Here, $P = -\frac{1}{x}, Q = x^3 - 3$

$\therefore IF = e^{\int Pdx} = e^{-\int \frac{1}{x}dx} = e^{-\log x}$

$= \frac{1}{x}$

44. Solution of $\frac{dy}{dx} - y = 1, y(0) = 1$ **is given by**

(A) $xy = -e^x$

(B) $xy = -e^{-x}$

(C) $xy = -1$

(D) $y = 2e^x - 1$

Sol. (B) Given that

$\frac{dy}{dx} - y = 1,$

$\Rightarrow \frac{dy}{dx} = 1 + y$

$\Rightarrow \frac{dy}{1+y} = dx$

On integrating both sides, we get

$\log(1+y) = x + C \quad ...(i)$

When $x = 0$ *and* $y = -1,$ then

$\log 2 = 0 + c$

$\Rightarrow C = \log 2$

The required solution is

$\log(1+y) = x + \log 2$

$\Rightarrow \log\left(\frac{1+y}{2}\right) = x$

$\Rightarrow \frac{1+y}{2} = e^x$

$\Rightarrow 1 + y = 2e^x$

$\Rightarrow y = 2e^x - 1$

45. The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$ **when** $y(1) = 2$ **is:**

(A) none

(B) one

(C) two

(D) infinite

Sol. (B) Given that, $\frac{dy}{dx}=\frac{y+1}{x-1}$

$\Rightarrow \frac{dy}{y+1}=\frac{dx}{x-1}$

On integrating both sides, we get

$\log(y+1)=\log(x-1)-\log C$

$C(y+1)=(x-1)$

$\Rightarrow C=\frac{x-1}{y+1}$

When $x=1$ and $y=2,$ $then$ $C=0$

So, the required solution is $x-1=0$

Hence, only one solution exists.

46. Which of the following is a second order differential equation?

(A) $(y')^2+x=y^2$

(B) $y'y''+y=sin\,x$

(C) $y'''+(y'')^2+y=0$

(D) $y'=y^2$

Sol. (B) The second order differential equation is $y'y''+y=sin\,x$.

47. Integrating factor of the differential equation $(1-x^2)\frac{dy}{dx}-xy=1$ **is**

(A) $-x$

(B) $\frac{x}{1+x^2}$

(C) $\sqrt{1-x^2}$

(D) $\frac{1}{2}\log(1-x^2)$

Sol. (C) Given that, $(1-x^2)\frac{dy}{dx}-xy=1$

$\Rightarrow \frac{dy}{dx}-\frac{x}{1-x^2}y=\frac{1}{1-x^2}$

Which is a linear differential equation.

$\therefore\ IF=e^{-\int\frac{x}{1-x^2}dx}$

Put $1-x^2=t\Rightarrow -2xdx=dt\Rightarrow xdx=-\frac{dt}{2}$

Now, $IF=e^{\frac{1}{2}\int\frac{dt}{t}}=e^{\frac{1}{2}\log t}=e^{\frac{1}{2}\log(1-x^2)}=\sqrt{1-x^2}$

48. $tan^{-1}x+tan^{-1}y=c$ **is the general solution of the differential equation:**

(A) $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

(B) $\frac{dy}{dx}=\frac{1+x^2}{1+y^2}$

(C) $\left(1+x^2\right)dy+\left(1+y^2\right)dx=0$

(D) $\left(1+x^2\right)dx+\left(1+y^2\right)dy=0$

Sol. (C) Given that, $\tan^{-1}x+\tan^{-1}y=C$

On differentiating w.r.t. x, we get

$\frac{1}{1+x^2}+\frac{1}{1+x^2}.\frac{dy}{dx}=0$

$\Rightarrow\frac{1}{1+y^2}.\frac{dy}{dx}=-\frac{1}{1+x^2}$

$\Rightarrow\left(1+x^2\right)dy+\left(1+y^2\right)dx=0$

49. The differential equation $y\frac{dy}{dx}+x=c$ represents:

(A) Family of hyperbolas
(B) Family of parabolas
(C) Family of ellipses
(D) Family of circles

Sol. (D) Given that, $y\frac{dy}{dx}+x=c$

$\Rightarrow y\frac{dy}{dx}=C-x$

$\Rightarrow yd\ y=\left(C-x\right)dx$

On integrating both sides, we get

$\frac{y^2}{2}=Cx-\frac{x^2}{2}+k$

$\Rightarrow\frac{x^2}{2}+\frac{y^2}{2}=Cx+K$

$\Rightarrow\frac{x^2}{2}+\frac{y^2}{2}-Cx=k$

which represent family of circles.

50. The general solution of $e^x cosy\ dx-e^x siny\ dy=0$ is:

(A) $e^x cos\ y=k$

(B) $e^x siny\ =\ k$

(C) $e^x\ =\ k\ cosy$

(D) $e^x\ =\ k\ siny$

Sol. (A) Given that, $e^x cosy\ dx-e^x siny\ dy=0$

$\Rightarrow e^x cosy\ dx=e^x sinydy\ =\ 0$

$\Rightarrow\frac{dx}{dy}=\tan y$

$\Rightarrow dx=\tan ydy$

On integrating both sides, we get

$x = \log \sec y + C$

$\Rightarrow x - C = \log \sec y$

$\Rightarrow sec\ y = e^{x-C}$

$\Rightarrow sec\ y = e^{x} e^{-C}$

$\Rightarrow \frac{1}{\cos y} = \frac{e^{x}}{e^{C}}$

$\Rightarrow e^{x} \cos y = e^{C}$

$\Rightarrow e^{x} \cos y = K \qquad \left[where, K = e^{C} \right]$

51. **The degree of the differential equation** $\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^3 + 6y^5 = 0$ **is:**

(A) 1
(B) 2
(C) 3
(D) 5

Sol. (A) $\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^3 + 6y^5 = 0$

We know that, the degree of a differential equation is exponent highest of order derivative.

$\therefore\ Degree = 1$

52. **The solution of** $\frac{dy}{dx} + y = e^{-x},\ y(0) = 0$ **is:**

(A) $y = e^{x}(x-1)$

(B) $y = xe^{-x}$

(C) $y = xe^{-x} + 1$

(D) $y = (x+1)e^{-x}$

Sol. (B) Given that, $\frac{dy}{dx} + y = e^{-x}$

Here, $P = 1,\ Q = e^{-x}$

$IF = e^{\int P dx} = e^{\int dx} = e^{x}$

The general solution is

$y.e^{x} = \int e^{-x} e^{x} dx + C$

$\Rightarrow y.e^{x} = \int dx + C$

$\Rightarrow y.e^{x} = x + C \ \ ...(i)$

When $x = 0\ and\ y = 0,$ then

$0 = 0 + C \Rightarrow 0$

Eq. (i) becomes $y.e^{x} = x$

$\Rightarrow\ y = xe^{-x}$

53. **Integrating factor of the differential equation $\frac{dy}{dx}+y\tan x-\sec x=0$ is:**

(A) $cos\, x$

(B) $sec\, x$

(C) $e^{cos\, x}$

(D) $e^{sec\, x}$

Sol. (B) Given that, $\frac{dy}{dx}+y\tan x-\sec x=0$

Here, $P=\tan x, Q=\sec x$

$IF=e^{\int Pdx}=e^{\int \tan x dx}$

$=e^{(\log \sec x)}$

$=\sec x$

54. **The solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is:**

(A) $y=tan^{-1}x$

(B) $y-x=k(1+xy)$

(C) $x=tan^{-1}y$

(D) $tan(xy)=k$

Sol. (B) Given that, $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

$\Rightarrow \frac{dy}{1+y^2}=\frac{dx}{1+x^2}$

On integrating both sides, we get

$\tan^{-1}y=\tan^{-1}x+C$

$\Rightarrow \tan^{-1}y-\tan^{-1}x=C$

$\Rightarrow \tan^{-1}\left(\frac{y-x}{1+xy}\right)=C$

$\Rightarrow \frac{y-x}{1+xy}=\tan C$

$\Rightarrow y-x=\tan C(1+xy)$

$\Rightarrow y-x=K(1+xy)$

Where, $k=tan\, C$

55. **The integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{x}$ is:**

(A) $\frac{x}{e^x}$

(B) $\frac{e^x}{x}$

(C) xe^x

(D) e^x

Sol. (B) Given that,

$\frac{dy}{dx}+y=\frac{1+y}{x}$

$\Rightarrow \frac{dy}{dx}=\frac{1+y}{x}-y$

$\Rightarrow \frac{dy}{dx}=\frac{1+y-xy}{x}$

$\Rightarrow \frac{dy}{dx}=\frac{1}{x}+\frac{y(1-x)}{x}$

$\Rightarrow \frac{dy}{dx}-\left(\frac{1-x}{x}\right)y=\frac{1}{x}$

Here, $P=\frac{-(1-x)}{x}, Q=\frac{1}{x}$

$IF=e^{\int Pdx}=e^{-\int \frac{1-x}{x}dx}=e^{\int \frac{x-1}{x}dx}$

$=e^{\int\left(1-\frac{1}{x}\right)dx}$

$=e^{\int x-\log x}$

$=e^x.e^{\log\left(\frac{1}{x}\right)}$

$=e^x.\frac{1}{x}$

56. $y=ae^{mx}+be^{-mx}$ **satisfies which of the following differential equation?**

(A) $\frac{dy}{dx}+my=0$

(B) $\frac{dy}{dx}-my=0$

(C) $\frac{d^2y}{dx^2}-m^2y=0$

(D) $\frac{d^2y}{dx^2}+m^2y=0$

Sol. (C) Given that, $y=ae^{mx}+be^{-mx}$

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}$ = $\mathrm{m}ae^{mx}$ - bme^{-mx}

Again, differentiating both sides w.r.t. x, we get

$\frac{d^2y}{dx^2}=\mathrm{m}^2ae^{mx}+bm^2e^{-mx}$

$\Rightarrow \frac{d^2y}{dx^2}=\mathrm{m}^2\left(ae^{mx}+be^{-mx}\right)$

$\Rightarrow \frac{d^2y}{dx^2}=m^2y$

$\Rightarrow \frac{d^2 y}{dx^2} - m^2 y = 0$

57. **The solution of the differential equation** $cos\, x\ sin\, y\ dx + sin\, x\ cos\, y\ dy = 0$ **is:**

(A) $\frac{\sin x}{\sin y} = C$

(B) $sin\, x\ sin\, y = \mathrm{C}$

(C) $sin\, x + sin\, y = \mathrm{C}$

(D) $cos\, x\ cos\, y = \mathrm{C}$

Sol. (B) Given differential equation is

$cosx\ siny\ dx\ +\ sinx\ cosy\ dy\ = 0$

$\Rightarrow cosx\ siny\ dx = -\ sinx\ cosy\ dy$

$\Rightarrow \frac{\cos x}{\sin x} dx = -\frac{\cos y}{\sin y} dy$

$\Rightarrow \cot x dx = -\cot y dy$

On integrating both sides, we get

$\log \sin x = -\log \sin y + \log C$

$\Rightarrow \log \sin x \sin y = \log C$

$\Rightarrow \sin x . \sin y = C$

58. **The solution of** $x\frac{dy}{dx} + y = e^x$ **is:**

(A) $y = \frac{e^x}{x} + \frac{k}{x}$

(B) $y = xe^x + cx$

(C) $y = xe^x + k$

(D) $x = \frac{e^y}{y} + \frac{k}{y}$

Sol. (A) Given that, $x\frac{dy}{dx} + y = e^x$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$

Which is a linear differential equation.

$\therefore IF = e^{\int \frac{1}{x} dx} = e^{(\log x)} = x$

The general solution is $y.x = \int \left(\frac{d^x}{x} . x \right) dx$

$\Rightarrow y.x = \int e^x dx$

$\Rightarrow y.x = e^x + k$

$\Rightarrow y = \frac{e^x}{x} + \frac{k}{x}$

59. **The differential equation of the family of curves** $x^2 + y^2 - 2ay = 0$**, where** a **is arbitrary constant, is:**

(A) $(x^2 - y^2)\frac{dy}{dx} = 2xy$

(B) $2(x^2 + y^2)\ \frac{dy}{dx} = xy$

(C) $2(x^2 - y^2)\ \frac{dy}{dx} = xy$

(D) $(x^2 + y^2)\ \frac{dy}{dx} = 2xy$

Sol. (A) Given equation of curve is

$x^2 + y^2 - 2ay = 0$

$\Rightarrow \frac{x^2 + y^2}{y} = 2a$

On differentiating both sides w.r.t. x, we get

$$\frac{y\left(2x + 2y\frac{dy}{dx}\right) - \left(x^2 + y^2\right)\frac{dy}{dx}}{y^2} = 0$$

$\Rightarrow 2xy + 2y^2\frac{dy}{dx} - \left(x^2 + y^2\right)\frac{dy}{dx} = 0$

$\Rightarrow \left(2y^2 - x^2 - y^2\right)\frac{dy}{dx} = -2xy$

$\Rightarrow\ \left(y^2 - x^2\right)\frac{dy}{dx} = -2xy$

$\Rightarrow\ \left(x^2 - y^2\right)\frac{dy}{dx} = 2xy$

60. Family $y = Ax + A^3$ of curves will correspond to a differential equation of order

(A) 3

(B) 2

(C) 1

(D) Not defined

Sol. (C) Given family of curves is $y = Ax + A^3$

$\Rightarrow \frac{dy}{dx} = A$

Replacing A by $\frac{dy}{dx}$ in Eq. (i), we get

$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$

$\therefore\ Order = 1$

61. The general solution of $\frac{dy}{dx} = 2x\ e^{x^2 - y}$ is:

(A) $e^{x^2 - y} = c$

(B) $e^{-y} + e^{x^2} = c$

(C) $e^y = e^{x^2} + c$

(D) $e^{x^2+y} = c$

Sol. (C) Given that, $\frac{dy}{dx} = 2x\ e^{x^2-y} = 2xe^{x^2}.e^{-y}$

$\Rightarrow e^y \frac{dy}{dx} = 2xe^{x^2}$

$\Rightarrow e^y dy = 2xe^{x^2} dx$

On integrating both sides, we get

$\int e^y dy = 2\int xe^{x^2} dx$

Put $x^2 = t$ *in* RHS integral, *we get*

$2xdx = dt$

$\int e^y dy = \int e^t dt$

$\Rightarrow e^y = e^t + C$

$\Rightarrow e^y = e^{x^2} + C$

62. The curve for which the slope of the tangent at any point is equal to the ratio of the abscissa to the ordinate of the point is:

(A) an ellipse

(B) parabola

(C) circle

(D) rectangular hyperbola

Sol. (D) Slope of tangent to the curve $= \frac{dy}{dx}$

and ratio of abscissa to the ordinate $= \frac{x}{y}$

According to the question, $= \frac{dy}{dx} = \frac{x}{y}$

$yd\ y = xd\ x$

On integrating both sides, we get

$\frac{y^2}{2} = \frac{x^2}{2} + C$

$\Rightarrow \frac{y^2}{2} - \frac{x^2}{2} = C \Rightarrow y^2 - x^2 = 2C$

Which is an equation of rectangular hyperbola.

63. The general solution of the differential equation $\frac{dy}{dx}\ e^{\frac{x^2}{2}} + xy$ **is:**

(A) $y = ce^{\frac{-x^2}{2}}$

(B) $y = ce^{\frac{x^2}{2}}$

(C) $y = (x+c)e^{\frac{x^2}{2}}$

(D) $y=(c-x)e^{\frac{x^2}{2}}$

Sol. (C) Given that, $\frac{dy}{dx}\ \ e^{x^2/2}+xy$

$\Rightarrow \frac{dy}{dx}-xy=e^{x^2/2}$

Here, $P=-x, Q=e^{x^2/2}$

$\therefore\ IF=e^{\int -xdx}=e^{-x^2/2}$

The general solution is

$ye^{-x^2/2}=\int e^{-x^2/2}-e^{x^2/2}dx+C$

$\Rightarrow ye^{-x^2/2}=\int 1dx+C$

$\Rightarrow y.e^{-x^2/2}=x+C$

$\Rightarrow$ y=x$e^{x^2/2}+Ce^{+x^2/2}$

$\Rightarrow$ y=$(x+C)$e$^{x^2/2}$

64. The solution of the equation $(2y-1)dx-(2x+3)dy=0$ **is:**

(A) $\frac{2x-1}{2y+3}=k$

(B) $\frac{2y+1}{2x-3}=k$

(C) $\frac{2x+3}{2y-1}=k$

(D) $\frac{2x-1}{2y-1}=k$

Sol. (C) Given that, $(2y-1)dx-(2x+3)dy=0$

$\Rightarrow (2y-1)dx=(2x+3)dy$

$\Rightarrow \frac{dx}{2x+3}=\frac{dy}{2y-1}$

On integrating both sides, we get

$\frac{1}{2}\log.(2x+3)=\frac{1}{2}\log(2y-1)+\log C$

$\Rightarrow \frac{1}{2}\left[\log.(2x+3)-\log(2y-1)\right]=\log C$

$\Rightarrow \frac{1}{2}\log\left(\frac{2x+3}{2y-1}\right)=\log C$

$\Rightarrow \left(\frac{2x+3}{2y-1}\right)^{1/2}=C$

$\Rightarrow \frac{2x+3}{2y-1}=C^2$

$\Rightarrow \frac{2x+3}{2y-1} = k,\ Where\ K = C^2$

65. The differential equation for which $y = acos\,x + bsin\,x$ is a solution, is:

(A) $\frac{d^2y}{dx^2} + y = 0$

(B) $\frac{d^2y}{dx^2} - y = 0$

(C) $\frac{d^2y}{dx^2} + (a+b)y = 0$

(D) $\frac{d^2y}{dx} + (a-b)y = 0$

Sol. (A) Given that, $= a\cos x + b\sin x$

On differentiating both side w.r.t. x, we get

$\frac{dy}{dx} = -a\sin x + b\cos x$

Again, differentiating both sides w.r.t. x, we get

$\frac{d^2y}{dx^2} = -a\sin x + b\cos x$

$\Rightarrow \frac{d^2y}{dx^2} = -y$

$\Rightarrow \frac{d^2y}{dx^2} + y = 0$

66. The solution of $\frac{dy}{dx} + y = e^{-x},\ y(0) = 0$ is:

(A) $y = e^{-x}(x-1)$

(B) $y = xe^{x}$

(C) $y = xe^{-x} + 1$

(D) $y = xe^{-x}$

Sol. (40) Given that, $\frac{dy}{dx} + y = e^{-x}$

Which is a linear differential equation.

Here, $P = 1\ and\ Q = e^{-x}$

$IF\ = e^{fdx} = e^{x}$

The general solution is

$y.e^{x} = \int e^{-x}.e^{x}dx + c$

$\Rightarrow ye^{x} = \int dx + C$

$\Rightarrow ye^{x} = x + C\ \ ...(i)$

When $x = 0$ *and* $y = 0$ then, $0 = 0 + C\ \Rightarrow C = 0$

Eq. (i) becomes $y.e^{x} = x \Rightarrow y = xe^{-x}$

67. The order and degree of the differential equation

$\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$ **are:**

(A) 1, 4

(B) 3, 4

(C) 2, 4

(D) 3, 2

Sol. (D) Given that $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$

$\therefore\ Order = 3$

and degree=2

68. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right] = \frac{d^2y}{dx^2}$ **are:**

(A) $2, \frac{3}{2}$

(B) 2, 3

(C) 2, 1

(D) 3, 4

Sol. (C) Given that, $\left[1 + \left(\frac{dy}{dx}\right)^2\right] = \frac{d^2y}{dx^2}$

$\therefore Order = 2\ and\ degree = 1$

69. The differential equation of the family of curves $y^2 = 4a(x+a)$ **is:**

(A) $y^2 = 4\frac{dy}{dx}\left(x + \frac{dy}{dx}\right)$

(B) $2y\frac{dy}{dx} = 4a$

(C) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

(D) $2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2 - y$

Sol. (D) Given that, $y^2 = 4a(x+a)$ ...(i)

On differentiating both sides w.r.t. x, we get

$2y\frac{dy}{dx} = 4a \Rightarrow 2y\frac{dy}{dx} = 4a$

$y\frac{dy}{dx} = 2a \Rightarrow a = \frac{1}{2}y\frac{dy}{dx}$ ...(ii)

On putting the value of a from Eq. (ii) in Eq. (i), We get

$y^2 = 2y\frac{dy}{dx} = \left(x + \frac{1}{2}y\frac{dy}{dx}\right)$

$$\Rightarrow y^2 = 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow 2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2 - y = 0$$

70. Which of the following is the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$?

(A) $y = (Ax+B)e^x$

(B) $y = (Ax+B)e^{-x}$

(C) $y = Ae^x + Be^{-x}$

(D) $y = A\cos x + B\sin x$

Sol. (A) Given that, $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$D^2y - 2Dy + y = 0,$

Where, $D = \frac{d}{dx}$

$(D^2 - 2D + 1)y = 0$

The auxiliary equation is $m^2 - 2m + 1 = 0$

$(m-1)^2 = 0 \Rightarrow m = 1, 1$

Since, the roots are real and equal.

$\therefore\ CF = (Ax+B)e^x \ \Rightarrow\ y = (Ax+B)e^x$

[since, if roots of Auxiliary equation are real and equal say (m), then $CF = (C_1x + C_2)e^{mx}$]

71. General solution of $\frac{dy}{dx} + y\tan x = \sec x$ is:

(A) $y \sec x = \tan x + c$

(B) $y \tan x = \sec x + c$

(C) $\tan x = y \tan x + c$

(D) $x \sec x = \tan y + c$

Sol. (A) Given differential equation is

$\frac{dy}{dx} + y\tan x = \sec x$

which is a linear differential equation

Here, $P = \tan x, Q = \sec x,$

$\therefore$ IF$= e^{\int \tan x dx} = e^{\log|\sec x|} = \sec x$

The general solution is

$y.\sec x = \int \sec x.\sec x + C$

$\Rightarrow y.\sec x = \int \sec^2 x\, dx + C$

$\Rightarrow y.\sec x = \tan x + C$

72. **Solution of the differential equation** $\frac{dy}{dx}+\frac{y}{x}=\sin x$ **is:**

(A) $x(y+cos\,x)=sin\,x+c$

(B) $x(y-cos\,x)=sin\,x+c$

(C) $xy\;cos\,x=sin\,x+c$

(D) $x(y+cos\,x)=cos\,x+c$

Sol. (A) Given differential equation is

$\frac{dy}{dx}+y\frac{1}{x}=\sin x$

Which is liner differential equations.

Here, $P=\frac{1}{x}\;and\;Q=\sin x$

$\therefore\;IF=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

The general solution is

$y.x=\int x.\sin x\,dx+C\;\;...(i)$

Take $I=\int x\sin xdx$

$-x\cos x-\int -\cos x\,dx$

$=-x\cos x+\sin x$

Put the value of l in Eq. (i), we get

$xy=-x\cos x+\sin x+C$

$\Rightarrow x(y+\cos x)=\sin x+C$

73. **The general solution of the differential equation** $(e^x+1)\,ydy=(y+1)\,e^x dx$ **is:**

(A) $(y+1)=k(e^x+1)$

(B) $y+1=e^x+1+k$

(C) $y=log\{k(y+1)(e^x+1)\}$

(D) $y=\log\left\{\frac{e^x+1}{y+1}\right\}+k$

Sol. (C) Given differential equation

$(e^x+1)\,ydy=(y+1)\,e^x dx$

$\Rightarrow\frac{dy}{dx}=\frac{e^x(1+y)}{(e^x+1)\,y}\Rightarrow\frac{dx}{dy}=\frac{(e^x+1)\,y}{e^x(1+y)}$

$\Rightarrow\frac{dx}{dy}=\frac{e^x y}{e^x(1+y)}+\frac{y}{e^x(1+y)}$

$\Rightarrow\frac{dx}{dy}=\frac{y}{1+y}+\frac{y}{(1+y)\,e^x}$

$\Rightarrow\frac{dx}{dy}=\frac{y}{1+y}\left(1+\frac{1}{e^x}\right)$

$$\Rightarrow \frac{dx}{dy} = \frac{y}{1+y}\left(\frac{e^x+1}{e^x}\right)$$

$$\Rightarrow \left(\frac{y}{1+y}\right)dy = \left(\frac{e^x}{e^x+1}\right)dx$$

On integrating both sides, we get

$$\int \frac{y}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\int \frac{1+y-1}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\Rightarrow \int 1dy - \int \frac{1}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\Rightarrow y - \log(1+y) + \log(1+e^x) + \log k$$

$$\Rightarrow y = \log(1+y) + \log(1+e^x) + \log(k)$$

$$\Rightarrow y = \log\{k(1+y)(1+e^x)\}$$

74. **The solution of the differential equation** $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ **is:**

(A) $y = e^{x-y} - x^2 e^{-y} + c$

(B) $e^y - e^x = \frac{x^3}{3} + c$

(C) $e^x + e^y = \frac{x^3}{3} + c$

(D) $e^x - e^y = \frac{x^3}{3} + c$

Sol. (B) Given that, $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$$\Rightarrow \frac{dy}{dx} = e^x e^{-y} + x^2 e^{-y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^x + x^2}{e^y}$$

$$\Rightarrow e^y dy = (e^x + x^2)dx$$

On integrating both sides, we get

$$\int e^y dy = \int (e^x + x^2)dx$$

$$\Rightarrow e^y = e^x + \frac{x^3}{3} + C$$

$$\Rightarrow e^y - e^x = \frac{x^3}{3} + C$$

75. **The solution of the differential equation** $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ **is:**

(A) $y\left(1+x^2\right)=c+tan^{-1}x$

(B) $\frac{y}{1+x^2}=c+\tan^{-1}x$

(C) $y\ log\left(1+x^2\right)=c+tan^{-1}x$

(D) $y\left(1+x^2\right)=c+sin^{-1}x$

Sol. (A) Given that, $\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$

Here, $P=\frac{2x}{1+x^2}\ and\ Q=\frac{1}{\left(1+x^2\right)^2}$

Which is a linear differential equation.

$\therefore\ IF=e^{\int\frac{2x}{1+x^2}dx}$

Put $1+x^2=t\Rightarrow 2x\,dx=dt$

$\therefore\ IF=e^{\int\frac{dt}{t}}=e^{\log t}=e^{\log\left(1+x^2\right)}=1+x^2$

The general solution is

$y.\left(1+x^2\right)=\int\left(1+x^2\right)\frac{1}{\left(1+x^2\right)^2}+C$

$\Rightarrow y\left(1+x^2\right)=\int\frac{1}{1+x^2}dx+C$

$\Rightarrow y\left(1+x^2\right)=\tan^{-1}x+C$

76. Fill in the blanks of the following (i to xi)

(i) The degree of the differential equation $\frac{d^2y}{dx^2}+e^{\frac{dy}{dx}}=0$ **is ________.**

Sol. Given differential equation is

$\frac{d^2y}{dx^2}+e^{\frac{dy}{dx}}=0$

Degree of this equation is not defined.

(ii) The degree of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=x$ **is ________.**

Sol. Given differential equation is $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=x$

So, degree of this equation is two.

(iii) The number of arbitrary constants in the general solution of a differential equation of order three is ________.

Sol. There are three arbitrary constants.

(iv) $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{1}{x}$ **is an equation of the type ________.**

Sol. Given differential equation is $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{1}{x}$

The equation of the type $\frac{dy}{dx}+Py=Q$

(v) General solution of the differential equation of the type $\frac{dy}{dx}+P_1x=Q_1$ **is given by _________.**

Sol. Given differential equation is $\frac{dy}{dx}+P_1x=Q_1$

The general solution is

$$x.IF=\int Q(IF)\,dy+C \;\; i.e.,\; xe^{\int Pdy}=\int Q\left\{e^{\int pdy}\right\}dy+C$$

(vi) The solution of differential solution $\frac{xdy}{dx}+2y=x^2$ **is _________.**

Sol. Given differential equation is $x\frac{dy}{dx}+2y=x^2 \Rightarrow \frac{dy}{dx}+\frac{2y}{x}=x$

This equation of the form $\frac{dy}{dx}+Py=Q.$

$$\therefore\; IF=e^{\int\frac{2}{x}dx}=e^{2\log x}=x^2$$

The general solution is

$$yx^2=\int x.x^2dx+C$$

$$\Rightarrow yx^2=\frac{x^4}{4}+C$$

$$\Rightarrow y=\frac{x^2}{4}+Cx^{-2}$$

(vii) The solution of $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ **is _________.**

Sol. Given differential equation is $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$

$$\Rightarrow \frac{dy}{dx}+\frac{2xy}{1+x^2}-\frac{4x^2}{1+x^2}=0$$

$$\Rightarrow \frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{4x^2}{1+x^2}$$

$$\therefore\; IF=e^{\int\frac{2x}{1+x^2}dx}$$

Put $1+x^2=t\Rightarrow 2xdx=dt$

$$\therefore\; IF=e^{\int\frac{dt}{t}}=e^{\log t}=e^{\log(1+x^2)}=1+x^2$$

The general solution is

$$y.(1+x^2)=\int(1+x^2)\frac{.4x^2}{(1+x^2)}dx+C$$

$$\Rightarrow (1+x^2)\,y=\int 4x^2dx+C$$

$$\Rightarrow (1+x^2)\,y=4\frac{x^3}{3}+C$$

$$\Rightarrow y=\frac{4x^3}{3(1+x^2)}+C(1+x^2)^{-1}$$

(viii) The solution of the differential equation $ydx+(x+xy)dy=0$ **is ________.**

Sol. Given differential equation is

$$\Rightarrow ydx+(x+xy)dy=0$$

$$\Rightarrow ydx+x(1+y)dy=0$$

$$\Rightarrow \frac{dx}{-x}=\left(\frac{1+y}{y}\right)dy$$

$$\Rightarrow \int\frac{1}{x}dx=-\int\left(\frac{1}{y}+1\right)dy \text{ [on integrating]}$$

$$\Rightarrow \log(x)=-\log(y)-y+\log A$$

$$\log(x)+\log(y)+y=\log A$$

$$\log(xy)+y=\log A$$

$$\Rightarrow \log xy+\log e^y=\log A$$

$$\Rightarrow xye^y=A$$

$$\Rightarrow xy=Ae^{-y}$$

(ix) General solution of $\frac{dy}{dx}+y=\sin x$ **is ________.**

Sol. Given differential equation is $\frac{dy}{dx}+y=\sin x$

$$IF=\int e^{1dx}=e^x$$

The general solutions is

$$y.e^x=\int e^x\sin x\,dx+C \quad ...(i)$$

Let $\int e^x \sin xdx$

$$I=\sin xe^x-\int\cos xe^x dx$$

$$=\sin xe^x-\cos xe^x+\int(-\sin x)e^x dx$$

$$2I=e^x(\sin x-\cos x)$$

$$I=\frac{1}{2}e^x(\sin x-\cos x)$$

From Eq. (i).

$$y.e^x=\frac{x}{2}(\sin x-\cos x)+C$$

$$\Rightarrow y=\frac{1}{2}(\sin x-\cos x)+C.e^{-x}$$

(x) The solution of differential equation $cot\ y\ dx=xdy$ **is ________.**

Sol. Given differential equation is

$$cot\ y\ dx=xdy$$

$\Rightarrow \frac{1}{x}dx = \tan y dy$

On integrating both sides, we get

$\Rightarrow \int \frac{1}{x}dx = \int \tan y dy$

$\Rightarrow \log(x) = \log(\sec y) + \log C$

$\Rightarrow \log\left(\frac{x}{\sec y}\right) = \log C$

$\Rightarrow \frac{x}{\sec y} = C$

$\Rightarrow x = C \sec y$

(xi) The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is ________.

Sol. Given differential equation is

$\frac{dy}{dx} + y = \frac{1+y}{x}$

$\frac{dy}{dx} + y = \frac{1}{x} + \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} + y\left(1 - \frac{1}{x}\right) = \frac{1}{x}$

$\therefore IF = e^{\int\left(1-\frac{1}{x}\right)dx}$

$= e^{x-\log x}$

$= e^x.e^{-\log x} = \frac{e^x}{x}$

77. State True of False for the following:

(i) Integrating factor of the differential of the form $\frac{dy}{dx} + p_1 x = Q_1$ is given by $e^{\int p_1 dy}$.

Sol. True

Given differential equation,

$\frac{dx}{dy} + P_1 x = Q_1$

$\therefore IF = e^{\int p_1 dy}$.

(ii) Solution of the differential equation of the type $\frac{dy}{dx} + p_1 x = Q_1$ is given by $x.I.F. = \int (I.F) \times Q_1 dy$.

Sol. True

(iii) Correct substitution for the solution of the differential equation of the type $\frac{dy}{dx} = f(x, y)$ where $f(x, y)$ is a homogeneous function of zero degree is y=vx.

Sol. True

(iv) Correct substitution for the solution of the differential equation of the type $\frac{dy}{dx} = g(x, y)$ where $g(x, y)$ is a homogeneous function of the degree zero is x=vy..

Sol. True

(v) Number of arbitrary constants in the particular solution of a differential equation of order two is two.

Sol. False

There is no arbitrary constant in the particular solution of a differential equation.

(vi) The differential equation representing the family of circles $x^2+(y-a)^2=a^2$ will be of order two.

Sol. False

We know that, order of differential equation= number of arbitrary constant.

Here, number of arbitrary constant=1.

So, Order is one.

(vii) The solution of $\frac{dy}{dx}=\left(\frac{y}{x}\right)^{\frac{1}{3}}$ is $y^{\frac{2}{3}}-x^{\frac{2}{3}}=c.$

Sol. True

Given differential equation $\frac{dy}{dx}=\left(\frac{y}{x}\right)^{1/3}$

$\Rightarrow \frac{dy}{dx}=\frac{y^{1/3}}{x^{1/3}}$

$\Rightarrow y^{-1/3}dy=x^{-1/3}dx$

On integrating both sides, we get

$\int y^{-1/3}\ dy=\int x^{-1/3}dx$

$\Rightarrow \frac{y^{-1/3+1}}{\frac{-1}{3}+1}=\frac{x^{-1/3+1}}{\frac{-1}{3}+1}+C'$

$\Rightarrow \frac{3}{2}y^{2/3}=\frac{3}{2}x^{2/3}+C'$

$\Rightarrow y^{2/3}-x^{2/3}=C' \quad \left[where, \frac{2}{3}C'=C\right]$

(viii) Differential equation representing the family of curves $y=e^x(A\cos x+B\sin x)$ is $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

Sol. True

Given that, $y=e^x(A\cos x+B\sin x)$

On differentiating w.r.t. x, we get

$\frac{dy}{dx}=e^x(-A\sin x+B\cos x)+e^x(A\cos x+B\sin x)$

$$\Rightarrow \frac{dy}{dx} - y = e^x\left(-A\sin x + B\cos x\right)$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x\left(-A\cos x - B\sin x\right) + e^x\left(-A\sin x + B\cos x\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = \frac{dy}{dx} - y$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

(ix) The solution of the differential equation $\frac{dy}{dx} = \frac{x+2y}{x}$ is $x + y = kx^2$.

Sol. True

Given that, $\frac{dy}{dx} = \frac{x+2y}{x}$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{2}{x}.y$$

$$\Rightarrow \frac{dy}{dx} - \frac{2}{x}y = 1$$

$$IF = e^{\int \frac{-2}{x}dx} = e^{-2\log x} = x^{-2}$$

The differential solution.

$$y.x^{-2} = \int x^{-2}.1dx + k$$

$$\Rightarrow \frac{y}{x^2} = \frac{x^{-2+1}}{-2+1} + k$$

$$\Rightarrow \frac{y}{x^2} = \frac{-1}{x} + k$$

$$\Rightarrow y = -x + kx^2$$

$$\Rightarrow x + y = kx^2$$

(x) Solution of $\frac{xdy}{dx} = y + x\tan\frac{y}{x}$ is $\sin\left(\frac{y}{x}\right) = cx$

Sol. True

Given differential equation,

$$\frac{xdy}{dx} = y + x\tan\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right) \quad ...(i)$$

$$Put\ \frac{y}{x} = v\ i.e.\ \ y = vx$$

$$\Rightarrow \frac{dx}{x} = v + \frac{xdv}{dx}$$

On substituting these values in Eq. (i), we get

$$\frac{xdv}{dx} + v = v + \tan v$$

$\Rightarrow \frac{dx}{x} = \frac{dv}{\tan v}$

On integrating both sides, we get

$\int \frac{1}{x} dx = \int \frac{1}{\tan v} dx$

$\Rightarrow \log(x) = \log(\sin v) + \log C'$

$\Rightarrow \log\left(\frac{x}{\sin v}\right) = \log C'$

$\Rightarrow \frac{x}{\sin v} = C'$

$\Rightarrow \sin v = Cx \quad \left[where, C = \frac{1}{C'}\right]$

$\Rightarrow \sin \frac{y}{x} = Cx$

(xi) The differential equation of all non-horizontal lines in a plane is $\frac{d^2 x}{dy^2} = 0$.

Sol. True.

Let any non-horizontal line in a plane is given by

$y = mx + c$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = m$

Again, differentiating w.r.t. x, we get

$\frac{d^2 y}{dx^2} = 0$

Differential Equations
Short Answer Type Questions

1. Find the solution of $\frac{dy}{dx} = 2^{y-x}$.

Sol. Given that, $\frac{dy}{dx} = 2^{y-x}$

$\Rightarrow \frac{dy}{dx} = \frac{2^y}{2^x} \quad \left[\because a^{m-n} = \frac{a^m}{a^n}\right]$

$\Rightarrow \frac{dy}{2^y} = \frac{dx}{2^x}$

On integrating both sides, we get

$\int 2^{-y} dy = \int 2^{-x} dx$

$\Rightarrow \frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + c$

$\Rightarrow -2^{-y} + 2^{-x} = +C \log 2$

$\Rightarrow 2^{-x} - 2^{-y} = -C \log 2$

$\Rightarrow 2^{-x} - 2^{-y} = k \ [where,\ k = +C\, log 2]$

2. Find the differential equation of all non-vertical lines in a plane.

Sol. Since, the family of all non-vertical line is y=mx+c, where $m \neq \tan \frac{\pi}{2}$.

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = m$

again, differentiating w.r.t x, we get

$\frac{d^2 y}{dx^2} = 0$

3. Given that $\frac{dy}{dx} = e^{-2y}$ and y = 0 when x = 5.

Find the value of $x \ when \ y = 3$.

Sol. Given that, $\frac{dy}{dx} = e^{-2y} \Rightarrow \frac{dy}{e^{-2y}} = dx$

$\Rightarrow \int e^{2y} dy = \int dx \Rightarrow \frac{e^{2y}}{2} = x + C \ \ ...(i)$

When $x = 5 \ and \ y = 0,$ then substituting these values in Eq. (i), we get

$\frac{e^0}{2} = 5 + C$

$\Rightarrow \frac{1}{2} = 5 + C \Rightarrow C = \frac{1}{2} - 5 = -\frac{9}{2}$

Eq. (i) becomes $e^{2y} = 2x - 9$

when y=3, then $e^6 = 2x - 9 \Rightarrow 2x = e^6 + 9$

$\therefore \quad x = \frac{(e^6 + 9)}{2}$

4. Solve the differential equation $(x^2-1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}$.

Sol. Given differential equation is

$$(x^2-1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{x^2-1}\right)y = \frac{1}{(x^2-1)^2}$$

Which is a linear differential equation.

On Comparing it with $\frac{dy}{dx} + Py = Q,$ we get

$$P = \frac{2x}{x^2-1}, Q = \frac{1}{(x^2-1)^2}$$

$$IF = e^{\int pdx} = e^{\int\left(\frac{2x}{x^2-1}\right)^{dx}}$$

Put $x^2 - 1 = t \Rightarrow 2xdx = dt$

$$\therefore \ IF = e^{\int \frac{dt}{t}} = e^{\log t} = t = (x^2-1)$$

The complete solution is

$$y.IF = \int Q.IF + k$$

$$\Rightarrow y.(x^2-1) = \int \frac{1}{(x^2-1)^2}.(x^2-1)dx + k$$

$$\Rightarrow y.(x^2-1) = \int \frac{dx}{(x^2-1)}$$

$$\Rightarrow y.(x^2-1) = \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + k$$

5. Solve the differential equation $\frac{dy}{dx} + 2xy = y$**.**

Sol. Given that, $\frac{dy}{dx} + 2xy = y$

$$\Rightarrow \frac{dy}{dx} + 2xy - y = 0$$

$$\Rightarrow \frac{dy}{dx} + (2x-1)y = 0$$

Which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q, we\ get$

$$P = (2x-1), Q = 0$$

$$IF = e^{\int pdx} = e^{\int (2x-1)dx}$$

$= e^{\left(\frac{2x^2}{2} - x\right)} = e^{x^2 - x}$

The complete solution is

$y.e^{x^2-x} = \int Q.e^{x^2-x} = 0 + C$

$\Rightarrow y.e^{x^2-x} = 0 + C$

$\Rightarrow y = Ce^{x-x^2}$

6. Find the general solution of $\frac{dy}{dx} + ay = e^{mx}$**.**

Sol. Given differential equation is $\frac{dy}{dx} + ay = e^{mx}$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q, we\ get$

$P = a,\ Q = e^{mx}$

$IF = e^{\int Pdx} = e^{\int adx} = e^{ex}$

The general solution is

$y.e^{ax} = \int e^{mx}.e^{ax}dx + C$

$\Rightarrow y.e^{ax} = \int e^{(m+a)x}dx + C$

$\Rightarrow y.e^{ax} = \frac{e^{(m+a)x}}{(m+a)} + C$

$\Rightarrow (m+a)y = \frac{e^{(m+a)x}}{e^{ax}} + \frac{(m+a)C}{e^{ax}}$

$\Rightarrow (m+a)y = \frac{e^{(m+a)x}}{e^{ax}} + \frac{(m+a)C}{e^{ax}}$

$\Rightarrow (m+a)y = e^{ax} + Ke^{-ax}$ $\quad [\because\ k = (m+a)C]$

7. Solve the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$**.**

Sol. Given differential equation $\frac{dy}{dx} + 1 = e^{x+y}$ ...(i)

On substituting $x + y = t,$ we get

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

Eq. (i) becomes $\frac{dt}{dx} = e^t$

$\Rightarrow\ \mathrm{e}^{-1}dt = dx$

$\Rightarrow -e^{-t} = x + C$

$\Rightarrow \frac{-1}{e^{x+y}} = x + C$

$\Rightarrow -1 = (x + C)e^{x+y}$

$\Rightarrow (x+C)e^{x+y}+1=0$

8. **Solve:** $ydx - xdy = x^2 ydx.$

Sol. Given that, $ydx - xdy = x^2 ydx$

$\Rightarrow \frac{1}{x^2} - \frac{1}{xy}.\frac{dy}{dx} = 1 \quad \left[dividing\ throughout\ by\ X^2 ydx\right]$

$\Rightarrow -\frac{1}{xy}.\frac{dy}{dx} + \frac{1}{x^2} - 1 = 0$

$\Rightarrow \frac{dy}{dx} - \frac{xy}{x^2} + xy = 0$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} + xy = 0$

$\Rightarrow \frac{dy}{dx} + \left(x - \frac{1}{x}\right)y = 0$

Which is linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q, we\ get$

$P = \left(x - \frac{1}{x}\right), Q = 0$

$If = e^{\int Pdx}$

$= e^{\int\left(x-\frac{1}{x}\right)}dx$

$= e^{\frac{x^2}{2} - \log x}$

$= e^{\frac{x^2}{x}}, e^{-\log x}$

$= \frac{1}{x}e^{\frac{x^2}{2}}$

The general solution is

$y.\frac{1}{x}e^{x^2/2} = \int 0.\frac{1}{x}e^{x^2/2}dx + C$

$\Rightarrow y.\frac{1}{x}e^{x^2/2} = C \quad \Rightarrow y = Cxe^{-x^2/2}$

9. **Solve the differential equation** $\frac{dy}{dx} = 1 + x + y^2 + xy^2$**, when y = 0, x = 0.**

Sol. Given that, $\frac{dy}{dx} = 1 + x + y^2 + xy^2$

$\Rightarrow \frac{dy}{dx} = (1+x) + y^2(1+x)$

$\Rightarrow \frac{dy}{dx} = (1+y^2)(1+x)$

$\Rightarrow \frac{dy}{1+y^2} = (1+x)dx$

On integrating both sides, we get

$$\tan^{-1} y = x + \frac{x^2}{2} + K \quad .....(i)$$

When y=0 and x=0. then substituting these values in Eq. (i), we get

$$\tan^{-1}(0) = 0 + 0 + K$$

$\Rightarrow$ K=0

$$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2}$$

$$\Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$$

10. Find the general solution of $(x+2y^3)\frac{dy}{dx} = y.$

Sol. Given that, $(x+2y^3)\frac{dy}{dx} = y$

$$\Rightarrow y.\frac{dx}{dy} = x + 2y^3$$

$$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 2y^2 \qquad \text{[Dividing throughout by y]}$$

$$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y^2$$

Which is a linear differential equation.

On comparing it with $\frac{dx}{dy} + Px = Q,$ we get

$$P = -\frac{1}{y}, Q = 2y^2$$

$$IF = e^{\int -\frac{1}{y}dy} = e^{-\int \frac{1}{y}dy}$$

$$\therefore \qquad = e^{-\log y} = \frac{1}{y}$$

The general solution is $x.\frac{1}{y} = \int 2y^2.\frac{1}{y}dy + C$

$$\Rightarrow \frac{x}{y} = \frac{2y^2}{2} + C$$

$$\Rightarrow \frac{x}{y} = y^2 + C$$

$$\Rightarrow x = y^3 + Cy$$

11. If $y(x)$ **is a solution of** $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$ **and** $y(0)=1,$ **then find the value of** $y\left(\frac{\pi}{2}\right).$

Sol. Given that, $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx}=-\cos x$

$\Rightarrow \frac{dy}{1+y}=-\frac{\cos x}{2+\sin}dx$

On integrating both sides, we get

$\int\frac{1}{1+y}dy=-\int\frac{\cos x}{2+\sin}dx$

$\Rightarrow \log(1+y)=-\log(2+\sin x)+\log C$

$\Rightarrow \log(1+y)+\log(2+\sin x)=\log C$

$\Rightarrow \log(1+y)(2+\sin x)=\log C$

$\Rightarrow (1+y)(2+\sin x)=C$

$\Rightarrow 1+y=\frac{C}{2+\sin x}$

$\Rightarrow y=\frac{C}{2+\sin x}-1 \quad ...(i)$

When $x=0$ *and* $y=1$ then

$1=\frac{C}{2}-1$

$\Rightarrow C=4$

On putting $C=4$ in Eq. (i), we get

$$Y=\frac{4}{2+\sin x}-1$$

$\therefore\ Y\left(\frac{\pi}{2}\right)=\frac{4}{2+\sin\frac{\pi}{2}}-1=\frac{4}{2+1}-1$

$=\frac{4}{3}-1=\frac{1}{3}$

12. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt}-ty=1$ and $y(0)=-1$, then show tha $y(1)=-\frac{1}{2}$.

Sol. Given that,

$(1+t)\frac{dy}{dt}-ty=1$

$\Rightarrow \frac{dy}{dt}-\left(\frac{t}{1+t}\right)y=\frac{1}{1+t}$

Which is a linear differential equation.

On Comparing it with $\frac{dy}{dt}+Py=Q$, we get

$P=-\left(\frac{t}{1+t}\right), Q=\frac{1}{1+t}$

$IF=e^{-\int\frac{t}{1+t}dt}=e^{-\int\left(1-\frac{1}{1+t}\right)dt=e^{-[t-\log(1+t)]}}$

$=e^{-t}.e^{\log(1+t)}$

$= e^{-t}(1+t)$

The general solution is

$$y(t).\frac{(1+t)}{e^t} = \int \frac{(1+t).e^{-1}}{(1+t)} dt + C$$

$$\Rightarrow y(t).\frac{e^{-t}}{(-1)}.\frac{e^t}{1+t} + C', where\ C' = \frac{Ce^t}{1+t}$$

$$\Rightarrow y(t) = -\frac{1}{1+t} + C'$$

When $t = 0\ and\ y = -1,$ then

$-1 = -1 + C' \Rightarrow C' = 0$

$$y(t) = -\frac{1}{1+t} \Rightarrow y(1) = -\frac{1}{2}$$

13. Form the differential equation having $y = (sin^{-1}x)^2 + Acos^{-1}x + B,$ where $A\ and\ B$ are arbitrary constants, as its general solution.

Sol. Given that, $y = (sin^{-1}x)^2 + Acos^{-1}x + B,$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}} + \frac{(-A)}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1}x - A$$

Again, differentiating w.r.t. x, we get

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{-2x}{2\sqrt{1+x^2}} = \frac{2}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}.\sqrt{1-x^2}\frac{dy}{dx} = 2$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

Which is the required differential equation.

14. From the differential equation of all circles which pass through origin and whose centres lie on y-axis.

Sol. It is given that, circles pass through origin and their centreslie on Y-axis. Let (O, k) be the centre of the circle and radius is k.

So, the equation of circle is

$$(x-0)^2 + (y-k)^2 = k^2$$

$$\Rightarrow x^2 + (y-k)^2 = k^2$$

$$\Rightarrow x^2 + y^2 - 2ky = 0$$

$$\Rightarrow \frac{x^2 + y^2}{2y} = k \ \ ...(i)$$

On differentiating Eq. (i) w.r.t. x, we get

$$\frac{2y\left(2x+2y\frac{dy}{dx}\right)-\left(x^2+y^2\right)\frac{2dy}{dx}}{4y^2}=0$$

$$\Rightarrow 4y\left(x+y\frac{dy}{dx}\right)-2\left(x^2+y^2\right)\frac{dy}{dx}=0$$

$$\Rightarrow 4xy+4y^2\frac{dy}{dx}-2\left(x^2+y^2\right)\frac{dy}{dx}=0$$

$$\Rightarrow \left[4y^2-2\left(x^2+y^2\right)\right]\frac{dy}{dx}+4xy=0$$

$$\Rightarrow \left(4y^2-2x^2-2y^2\right)\frac{dy}{dx}+4xy=0$$

$$\Rightarrow \left(2y^2-2x^2\right)\frac{dy}{dx}+4xy=0$$

$$\Rightarrow \left(y^2-x^2\right)\frac{dy}{dx}+2xy=0$$

$$\Rightarrow \left(x^2-y^2\right)\frac{dy}{dx}-2xy=0$$

15. Find the equation of a curve passing through origin and satisfying the differential equation $(1+x^2)\frac{dy}{dx}+2xy=4x^2$.

Sol. Given that, $(1+x^2)\frac{dy}{dx}+2xy=4x^2$

$$\Rightarrow \frac{dy}{dx}+\frac{2x}{1+x^2}.y=\frac{4x^2}{1+x^2}$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx}+Py=Q, we\ \ get$

$$P=\frac{2x}{1+x^2}, Q=\frac{4x^2}{1+x^2}$$

$$\therefore\ IF=e^{\int Pdx}=e^{\int\frac{2x}{1+x^2}}dx$$

Put $1+x^2=t \Rightarrow\ 2xdx=dt$

$$IF=1+x^2=e^{\int\frac{dt}{t}}=e^{\log t}=e^{\log\left(1+x^2\right)}$$

The general solution is

$$y.\left(1+x^2\right)=\int\frac{4x^2}{1+x^2}\left(1+x^2\right)dx+C$$

$$\Rightarrow y.\left(1+x^2\right)=\int 4x^2dx+C$$

$$\Rightarrow y.\left(1+x^2\right)=4\frac{x^3}{3}+C\ \ ...(i)$$

Since, the curve passes through origin, then substituting

$x = 0 \text{ and } y = 0 \text{ in Eq.}(i), \text{we get}$

$C = 0$

The required equation of curve is

$$y\left(1+x^2\right) = \frac{4x^3}{3}$$

$$\Rightarrow y = \frac{4x^3}{3\left(1+x^2\right)}$$

16. Solve: $x^2 \dfrac{dy}{dx} = x^2 + xy + y^2$**.**

Sol. Given that, $x^2 \dfrac{dy}{dx} = x^2 + xy + y^2$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$$

Let $f(x, y) = 1 + \dfrac{y}{x} + \dfrac{y^2}{x^2}$

$$f(\lambda x, \lambda y) = 1 + \frac{\lambda y}{\lambda x} + \frac{\lambda^2 y^2}{\lambda^2 x^2}$$

$$f(\lambda x, \lambda y) = \lambda^0 \left(1 + \frac{y}{x} + \frac{y^2}{x^2}\right)$$

$$= \lambda^0 f(x, y)$$

Which is homogeneous expression of degree 0.

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

On substituting these values in Eq. (i), we get

$$\left(v + x\frac{dv}{dx}\right) = 1 + v + v^2$$

$$\Rightarrow x\frac{dv}{dx} = 1 + v + v^2 - v$$

$$\Rightarrow x\frac{dv}{dx} = 1 + v^2$$

$$\Rightarrow \frac{dv}{1+v^2} = \frac{dx}{x}$$

On integrating both sides, we get

$$\tan^{-1} v = \log|x| + C$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \log|x| + C$$

17. Find the general solution of the differential equation

$$(1+y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0.$$

Sol. Given, differential equation is

$$\left(1+y^2\right) + \left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$$

$$\Rightarrow \left(1+y^2\right) = -\left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx}$$

$$\left(1+y^2\right)\frac{dx}{dy} = -x + e^{\tan^{-1} y}$$

$$\Rightarrow \left(1+y^2\right)\frac{dx}{dy} + x = e^{\tan^{-1} y}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1} y}}{1+y^2} \quad \left[\text{dividing throughout by}\left(1+y^2\right)\right]$$

Which is a linear differential equation.

On comparing it with $\frac{dx}{dy} + Px = Q,$ we get

$$P = \frac{1}{1+y^2}, Q = \frac{e^{\tan^{-1} y}}{1+y^2}$$

$$IF = e^{\int Pdy} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1} y}$$

The general solution is $x.e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2}.e^{\tan^{-1} y}dy + C$

$$\Rightarrow x.e^{\tan^{-1} y} = \int \frac{\left(e^{\tan^{-1} y}\right)^2}{1+y^2}.dy + C$$

Put $\tan^{-1} y = t \Rightarrow \frac{1}{1+y^2}dy = dt$

$$\therefore \; x.\mathrm{e}^{\tan^{-1} y} = \int e^{2t}dt + C$$

$$\Rightarrow \; \mathrm{x.e}^{\tan^{-1} y} = \frac{1}{2}e^{2\tan^{-1} y} + C$$

$$\Rightarrow \; 2\mathrm{xe}^{\tan^{-1} y} = e^{2\tan^{-1} y} + 2C$$

$$\Rightarrow \; 2\mathrm{xe}^{\tan^{-1} y} = e^{2\tan^{-1} y} + K \; [\because K = 2C]$$

18. Find the general solution of $y^2dx + \left(x^2 - xy + y^2\right)dy = 0$ **.**

Sol. Given, differential equation is

$$y^2dx + \left(x^2 - xy + y^2\right)dy = 0$$

$$\Rightarrow y^2dx = -\left(x^2 - xy + y^2\right)dy$$

$$\Rightarrow y^2\frac{dx}{dy} = -\left(x^2 - xy + y^2\right)$$

$$\Rightarrow \frac{dx}{dy} = -\left(\frac{x^2}{y^2} - \frac{x}{y} + 1\right) \; ...(i)$$

Which is a homogeneous differential equation.

Put $\frac{x}{y} = v \; or \; x = vy$

$\Rightarrow \dfrac{dx}{dy}=v+y\dfrac{dv}{dy}$

On substituting these values in Eq. (i), we get

$v+y\dfrac{dv}{dy}=-\left[v^2-v+1\right]$

$\Rightarrow y\dfrac{dv}{dy}=-v^2+v-1-v$

$\Rightarrow y\dfrac{dv}{dy}=-v^2-1 \Rightarrow \dfrac{dv}{v^2+1}=-\dfrac{dy}{y}$

On integrating both sides, we get

$\tan^{-1}(v)=-\log y+C$

$\Rightarrow \tan^{-1}\left(\dfrac{x}{y}\right)+\log y=C \quad \left[\because\ v=\dfrac{x}{y}\right]$

19. **Solve:** $(x+y)(dx-dy)=dx+dy$. **[Hint: Substitute** $x+y=z$ **after separating** dx ***and*** dy**]**

Sol. Given differential equation is

$(x+y)(dx-dy)=dx+dy$

$\Rightarrow (x+y)\left(1-\dfrac{dy}{dx}\right)=1+\dfrac{dy}{dx} \quad ...(i)$

Put $x+y=z$

$\Rightarrow 1+\dfrac{dy}{dx}=\dfrac{dz}{dx}$

On substituting these values in Eq. (i), we get

$z\left(1-\dfrac{dz}{dx}+1\right)=\dfrac{dz}{dx}$

$\Rightarrow z\left(2-\dfrac{dz}{dx}\right)=\dfrac{dz}{dx}$

$\Rightarrow 2z-z\dfrac{dz}{dx}-\dfrac{dz}{dx}=0$

$\Rightarrow 2z-(z+1)\dfrac{dz}{dx}=0$

$\Rightarrow \dfrac{dz}{dx}=\dfrac{2z}{z+1}$

$\Rightarrow \left(\dfrac{z+1}{z}\right)dz=2dx$

On integrating both sides, we get

$\int\left(1+\dfrac{1}{z}\right)dz=2\int dx$

$\Rightarrow z+\log z=2x-\log C$

$\Rightarrow (x+y)+\log(x+y)=2x-\log C \quad [\because z=x+y]$

$\Rightarrow 2x-x-y=\log C+\log(x+y)$

$\Rightarrow x-y=\log\left|C(x+y)\right|$

$\Rightarrow e^{x-y}=C(x+y)$

$\Rightarrow (x+y)=\frac{1}{C}e^{x-y}$

$\Rightarrow x+y=Ke^{x-y} \qquad \left[\because\ K=\frac{1}{C}\right]$

20. Solve: $2(y+3)-xy\frac{dy}{dx}=0,$ **given that** $y(1)=-2$**.**

Sol. Given that, $2(y+3)-xy\frac{dy}{dx}=0,$

$\Rightarrow 2(y+3)=xy\frac{dy}{dx}$

$\Rightarrow 2\frac{dx}{x}=\left(\frac{y}{y+3}\right)dy$

$\Rightarrow 2.\frac{dx}{x}=\left(\frac{y+3-3}{y+3}\right)dy$

$\Rightarrow 2.\frac{dx}{x}=\left(1-\frac{3}{y+3}\right)dy$

On integrating both sides, we get

$2\log x=y-3\log(y+3)+C........(i)$

When $x=1\ and\ \ y=-2, then$

$2\log 1=-2-3\log(-2+3)+C$

$\Rightarrow 2.0=-2-3.0+C$

$\Rightarrow C=2$

On substituting the value of C in Eq. (i), we get

$2\log x=y-3\log(y+3)+2$

$\Rightarrow\ 2\log x+3\log(y+3)=y+2$

$\Rightarrow \log x^2+\log(y+3)^3=(y+2)$

$\Rightarrow \log x^2(y+3)^3=y+2$

$\Rightarrow x^2(y+3)^3=e^{y+2}$

21. Solve the differential equation $dy=cos\,x(2-y\,cosec\,x)\,dx$ **given that y = 2 when** $x=\frac{\pi}{2}$.

Sol. Given differential equation,

$dy=\cos x(2-y\cos ec\,x)dx$

$\Rightarrow \frac{dy}{dx}=\cos x(2-y\cos ec\ x)$

$\Rightarrow \frac{dy}{dx}=2\cos x-y\cos ec\ x.\cos x$

$$\Rightarrow \frac{dy}{dx} = 2\cos x - y\cot x$$

$$\Rightarrow \frac{dy}{dx} + y\cot x = 2\cos x$$

Which is a liner differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q,$ we get

$P = \cot x, Q = 2\cos x$

$IF \ = e^{\int Pdx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

The general solution is

$y.\sin x = \int 2\cos x.\sin x dx + C$

$\Rightarrow y.\sin x = \int \sin 2x dx + C \ [\because \sin 2x = 2\sin x \cos x]$

$\Rightarrow y.\sin x = -\frac{\cos 2x}{2} + C \ ......(i)$

When $x = \frac{\pi}{2}$ $and \ \ y = 2, then$

$$2.\sin\frac{\pi}{2} = -\frac{\cos\left(2\times\frac{\pi}{2}\right)}{2} + C$$

$$\Rightarrow 2.1 = +\frac{1}{2} + C$$

$$\Rightarrow 2 - \frac{1}{2} = C \Rightarrow \frac{4-1}{2} = C \ \Rightarrow C = \frac{3}{2}$$

On substituting the value of C in Eq. (i), we get

$$y\sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$$

22. Form the differential equation by eliminating $A \ and \ B$ **in** $Ax^2 + By^2 = 1$**.**

Sol. Given differential equation is $Ax^2 + By^2 = 1$

On differentiating both sides w.r.t. x, we get

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$\Rightarrow 2By\frac{dy}{dx} = -2Ax$$

$$\Rightarrow By\frac{dy}{dx} = -Ax \Rightarrow \frac{y}{x}.\frac{dy}{dx} = -\frac{A}{B}$$

Again, differentiating w.r.t. x, we get

$$\frac{y}{x}.\frac{d^2y}{dx^2} + \frac{dy}{dx}.\left(\frac{x\frac{dy}{dx} - y}{x^2}\right) = 0$$

$$\Rightarrow \frac{y}{x}\cdot\frac{d^2y}{dx^2}+\frac{x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right)}{x^2}=0$$

$$\Rightarrow xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right)=0$$

$$\Rightarrow xyy''+x(y')^2-yy'=0$$

23. Solve the differential equation $(1+y^2)\,tan^{-1}x\;dx+2y(1+x^2)\,dy=0$ **.**

Sol. Given differential equation is

$$(1+y^2)\tan^{-1}x dx+2y(1+x^2)dy=0$$

$$\Rightarrow (1+y^2)\tan^{-1}x dx=-2y(1+x^2)dy$$

$$\Rightarrow \frac{\tan^{-1}x dx}{1+x^2}=-\frac{2y}{1+y^2}dy$$

On integrating both sides, we get

$$\int\frac{\tan^{-1}x}{1+x^2}dx=-\int\frac{2y}{1+y^2}dy$$

Put $\tan^{-1}x=t\ in\ LHS, we\ get$

$$\frac{1}{1+x^2}dx=dt$$

and put $1+y^2=u\ in\ RHS, we\ get$

$$2ydy=du$$

$$\Rightarrow \int t\,dt=-\int\frac{1}{u}du\Rightarrow\frac{t^2}{2}=-\log u+C$$

$$\Rightarrow \frac{1}{2}\left(\tan^{-1}x\right)^2=-\log\left(1+y^2\right)+C$$

$$\Rightarrow \frac{1}{2}\left(\tan^{-1}x\right)^2+\log\left(1+y^2\right)=C$$

24. Find the differential equation of system of concentric circles with centre $(1,\ 2)$ **.**

Sol. The family of concentric circles with centre $(1,\ 2)$ and radius a is given by

$$(x-1)^2+(y-2)^2=a^2$$

$$\Rightarrow x^2+1-2x+y^2+4-4y=a^2$$

$$\Rightarrow x^2+y^2-2x-4y+5=a^2\quad ...(i)$$

On differentiating Eq. (i) w.r.t. x, we get

$$2x+2y\frac{dy}{dx}-2-4\frac{dy}{dx}=0$$

$$\Rightarrow (2x-4)\frac{dy}{dx}+2x-2=0$$

$$\Rightarrow (y-2)\frac{dy}{dx}+(x-1)=0$$

<u>Long Answer Type Questions</u>

25. **Solve:** $y+\frac{d}{dx}(xy)=x(\sin x+\log x)$

Sol. Given differential equation is

$$y+\frac{d}{dx}(xy)=x(\sin x+\log x)$$

$$\Rightarrow y+x\frac{d}{dx}+y=x(\sin x+\log x)$$

$$\Rightarrow x\frac{dy}{dx}+2y=x(\sin x+\log x)$$

$$\Rightarrow \frac{dy}{dx}+\frac{2}{x}y=\sin x+\log x$$

Which is a linear differential equation.

On comparing it with $\frac{dy}{dx}+Py=Q,$ we get

$$P=\frac{2}{x}, Q=\sin x+\log x$$

$$IF=e^{\int\frac{2}{x}dx}=e^{2\log x}=x^2$$

The general solution is

$$y.x^2=\int(\sin x+\log x)x^2dx+C$$

$$\Rightarrow y.x^2=\int(x^2\sin x+x^2\log x)dx+C$$

$$\Rightarrow y.x^2=\int x^2\sin xdx+\int x^2\log xdx+C$$

$$\Rightarrow y.x^2=I_1+I_2+C \quad ...(i)$$

Now, $I_1=\int x^2\sin xdx$

$$=x^2(-\cos x)+\int 2x\cos xdx$$

$$=-x^2\cos x+\left[2x(\sin x)-\int 2\sin xdx\right]$$

$$I_1=-x^2\cos x+2x\sin x+2\cos x \quad ...(ii)$$

And $I_2=\int x^2\log xdx$

$$=\log x.\frac{x^3}{3}-\int\frac{1}{x}.\frac{x^3}{3}dx$$

$$=\log x.\frac{x^3}{3}-\frac{1}{3}\int x^2dx$$

$$=\log x.\frac{x^3}{3}-\frac{1}{3}.\frac{x^3}{3} \quad ...(iii)$$

On substituting of value of I_1 *and* I_2 in Eq. (i), we get

$$y.x^2=-x^2\cos x+2x\sin x+2\cos x+\frac{x^3}{3}\log x-\frac{1}{9}x^3+C$$

$$\therefore y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x}{3}\log x - \frac{x}{9} + Cx^{-2}$$

26. **Find the general solution of** $(1+tan\ y)(dx-dy)+2xdy=0.$

Sol. Given differential equation is $(1+tan\ y)(dx-dy)+2xdy=0.$

On dividing throughout by dy, we get

$$(1+\tan y)\left(\frac{dx}{dy}-1\right)+2x=0$$

$$\Rightarrow (1+\tan y)\frac{dx}{dy}-(1+\tan y)+2x=0$$

$$\Rightarrow (1+\tan y)\frac{dx}{dy}+2x=(1+\tan y)$$

$$\Rightarrow \frac{dx}{dy}+\frac{2x}{1+\tan y}=1$$

Which is a linear differential equation.

On comparing it with $\frac{dx}{dy}+Px=Q$ we get

$$P=\frac{2}{1+\tan y}, Q=1$$

$$IF = e^{\int \frac{2}{1+\tan y}dy} = e^{\int \frac{2\cos y}{\cos y+\sin y}dy}$$

$$= e^{\int \frac{\cos y+\sin y+\cos y-\sin y}{\cos y+\sin y}dy}$$

$$= e^{\int \left(1+\frac{\cos y-\sin y}{\cos y+\sin y}\right)dy} = e^{y+\log(\cos y+\sin y)}$$

$$= e^y.(\cos y+\sin y) \quad \left[\because\ e^{\log x}=x\right]$$

The general solution is

$$x.e^y(\cos y+\sin y)=\int 1.e^y(\cos y+\sin y)dy+C$$

$$\Rightarrow x.e^y(\cos y+\sin y)=\int e^y(\sin y+\cos y)dy+C$$

$$\Rightarrow x.e^y(\cos y+\sin y)=e^y\sin y+C \quad \left[\because \int e^x\{f(x)+f'(x)\}dx=e^x f(x)\right]$$

$$\Rightarrow x(\sin y+\cos y)=\sin y+Ce^{-y}$$

27. **Solve:** $\frac{dy}{dx}=\cos(x+y)+\sin(x+y)$**. [Hint: Substitute** $x+y=z$**]**

Sol. Given, $\frac{dy}{dx}=\cos(x+y)+\sin(x+y)$

Put $x+y=z$

$$\Rightarrow 1+\frac{dy}{dx}=\frac{dz}{dx}$$

On substituting these values in Eq. (i), we get

$$\left(\frac{dz}{dx}-1\right)=\cos z+\sin z$$

$$\Rightarrow \frac{dz}{dx}=(\cos z+\sin z+1)$$

$$\Rightarrow \frac{dz}{\cos z+\sin z+1}=dx$$

on integrating both sides, we get

$$\Rightarrow \int \frac{dx}{\cos z+\sin z+1}=\int 1dx$$

$$\Rightarrow \int \frac{dz}{\frac{1-\tan^2 z/2}{1+\tan^2 z/2}+\frac{2\tan z/2}{1+\tan^2 z/2}+1}=\int dx$$

$$\Rightarrow \int \frac{dz}{\frac{1-\tan^2 z/2+2\tan z/2+1+\tan^2 z/2}{(1+\tan^2 z/2)}}=\int dx$$

$$\Rightarrow \int \frac{(1+\tan^2 z/2)dz}{2+2\tan^2 z/2}=\int dx$$

$$\Rightarrow \int \frac{\sec^2 z/2dz}{2(1+\tan z/2)}=\int dx$$

Put $1+\tan z/2=t \Rightarrow \left(\frac{1}{2}\sec^2 z/2\right)dz=dt$

$$\Rightarrow \int \frac{dt}{t}=\int dx$$

$$\Rightarrow \log|t|=x+C$$

$$\Rightarrow \log|1+\tan z/2|=x+C$$

$$\Rightarrow \log\left|1+\tan\frac{(x+y)}{2}\right|=x+C$$

28. Find the general solution of $\frac{dy}{dx}-3y=\sin 2x$.

Sol. Given, $\frac{dy}{dx}-3y=\sin 2x$

which is a linear differential equation.

On Comparing it with $\frac{dy}{dx}+Py=Q$ we get

$P=-3, Q=\sin 2x$

$IF=e^{-3\int dx}=e^{-3x}$

The general solution is

$y.e^{-3x}=\int \underset{I}{\sin 2x}\,\underset{II}{e^{-3}}\,dx$

Let $y.e^{-3x}=I$

$\therefore \quad I=\int \underset{II}{e^{-3x}}\,\underset{I}{\sin 2x}$

$$\Rightarrow I = \sin 2x\left(.\frac{e^{-3x}}{-3}\right) - \int 2\cos 2x\left(\frac{e^{-3x}}{-3}\right)dx + C_1$$

$$\Rightarrow I = -\frac{1}{3}e^{-3x}\sin 2x + \frac{2}{3}\int \underset{II}{e^{-3x}}\underset{I}{\cos} 2x\,dx + C_1$$

$$\Rightarrow I = -\frac{1}{3}e^{-3x}\sin 2x + \frac{2}{3}\left(\cos 2x\frac{e^{-3x}}{-3} - \int(-2\sin 2x)\frac{e^{-3x}}{-3}dx\right) + C_1 + C_2$$

$$\Rightarrow I = -\frac{1}{3}e^{-3x}\sin 2x - \frac{2}{9}\cos 2xe^{-3x} - \frac{4}{9}I + C' \quad [where, C' = C_1 + C_2]$$

$$\Rightarrow I + \frac{4l}{9}2 = +e^{-3x}\left(-\frac{1}{3}\sin 2x - \frac{2}{9}\cos 2x\right) + C'$$

$$\Rightarrow \frac{13}{9}I = e^{-3x}\left(-\frac{1}{3}\sin 2x - \frac{2}{9}\cos 2x\right) + C'$$

$$\Rightarrow I = \frac{9}{13}e^{-3x}\left(-\frac{1}{3}\sin 2x - \frac{2}{9}\cos 2x\right) + C \quad \left[where, C = \frac{9C}{13}\right]$$

$$\Rightarrow I = \frac{3}{13}e^{-3x}\left(-\sin 2x - \frac{2}{3}\cos 2x\right) + C$$

$$\Rightarrow \frac{3}{13}e^{-3x}\frac{(-3\sin 2x - 2\cos 2x)}{3} + C$$

$$\Rightarrow = \frac{e^{-3x}}{13}(-3\sin 2x - 2\cos 2x) + C$$

$$\Rightarrow I = -\frac{e^{-3x}}{13}(2\cos 2x + 3\sin 2x) + C$$

On substituting the value of I in Eq. (i), we get

$$y.e^{-3x} = -\frac{e^{-3x}}{13}(2\cos 2x + 3\sin 2x) + C$$

$$\Rightarrow y = -\frac{1}{13}(2\cos 2x + 3\sin 2x) + Ce^{3x}$$

29. Find the equation of a curve passing through (2, 1) if the slope of the tangent to the curve at any point (x, y) is $\frac{x^2 + y^2}{2xy}$.

Sol. It is given that, the slope of tangent to the curve at point $(x, y)\ is\ \frac{x^2 + y^2}{2xy}$

$$\therefore \left(\frac{dy}{dx}\right)_{(x,y)} = \frac{x^2 + y^2}{2xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y} + \frac{y}{x}\right) \quad ...(i)$$

Which is homogeneous differential equation.

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

On substituting these values in Eq. (i), we get

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1}{v} + v\right)$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1+v^2}{v}\right)$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\Rightarrow \frac{2v}{1-v^2}dv = \frac{dx}{x}$$

On integrating both sides, we get

$$\int \frac{2v}{1-v^2}dv = \int \frac{dx}{x}$$

Put $1 - v^2 = t$ in LHS, we get

$$-2v dv = dt$$

$$\Rightarrow -\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow -\log t = \log x + \log C$$

$$\Rightarrow -\log\left(1 - v^2\right) = \log x + \log C$$

$$\Rightarrow -\log\left(1 - \frac{y^2}{x^2}\right) = \log x + \log C$$

$$\Rightarrow -\log\left(\frac{x^2 - y^2}{x^2}\right) = \log x + \log C$$

$$\Rightarrow \log\left(\frac{x^2}{x^2 - y^2}\right) = \log x + \log C$$

$$\Rightarrow \frac{x^2}{x^2 - y^2} = Cx$$

Since, the curve passes through the point (2,1)

$$\therefore \frac{(2)^2}{(2)^2 - (1)^2} = C(2) \Rightarrow C = \frac{2}{3}$$

So, the required solution is $2\left(x^2 - y^2\right) = 3x.$

30. Find the equation of the curve through the point $(1,\ 0)$ if the slope of the tangent to the curve at any point $(x,\ y)$ *is* $\frac{y-1}{x^2+x}$.

Sol. It is given that, slope of tangent to the curve at any point $(x,\ y)$ is $\frac{y-1}{x^2+x}$.

$\therefore \left(\frac{dy}{dx}\right)_{(x,y)} = \frac{y-1}{x^2+x}$

$\Rightarrow \frac{dy}{dx} = \frac{y-1}{x^2+x}$

$\Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x}$

On integrating both sides, we get

$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x}$

$\Rightarrow \int \frac{dy}{y-1} = \int \frac{dx}{x(x+1)}$

$\Rightarrow \int \frac{dy}{y-1} = \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$

$\Rightarrow \log(y-1) = \log x - \log(x+1) + \log C$

$\Rightarrow \log(y-1) = \log\left(\frac{xC}{x+1}\right)$

Since, the given curve passes through point (1,0)

$\therefore 0-1 = \frac{1.C}{1+1} \Rightarrow C = -2$

The particular solution is $y-1 = \frac{-2x}{x+1}$

$\Rightarrow (y-1)(x+1) = -2x$

$\Rightarrow (y-1)(x+1) + 2x = 0$

31. Find the equation of a curve passing through origin if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abscissa and ordinate of the point.

Sol. Slope of tangent to the curve= $\frac{dy}{dx}$

and difference of abscissa and ordinate $= x - y$

According to the question, $\frac{dy}{dx} = (x-y)^2$ ...(i)

Put $x - y = z$

$\Rightarrow 1 - \frac{dy}{dx} = \frac{dz}{dx}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dz}{dx}$

On substituting these values in Eq. (i), we get

$1 - \frac{dz}{dx} = z^2 \Rightarrow 1 - z^2 = \frac{dz}{dx}$

$\Rightarrow dx = \frac{dz}{1-z^2}$

On integrating both sides, we get

$\int dx = \int \frac{dz}{1-z^2}$

$\Rightarrow$ x$= \frac{1}{2}\log\left|\frac{1+z}{1-z}\right| + C$

$\Rightarrow$ tx$= \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| + C$ ...(iii)

Since, the curve passes through the origin.

$\therefore$ 0$= \frac{1}{2}\log\left|\frac{1+0-0}{1-0+0}\right| + C$

$\Rightarrow$ C=0

On substituting the value of C in Eq. (ii), we get

x$= \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right|$

$\Rightarrow$ 2x$= \log\left|\frac{1+x-y}{1-x+y}\right|$

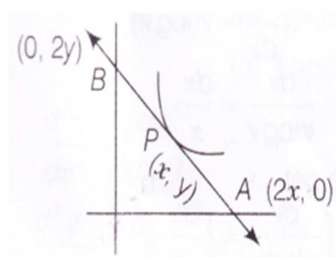
$\Rightarrow$ $e^{2x} = \left|\frac{1+x-y}{1-x+y}\right|$

$\Rightarrow (1-x+y)e^{2x} = 1+x-y$

32. Find the equation of a curve passing through the point $(1, 1)$. If the tangent drawn at any point $P(x, y)$ on the curve meets the co-ordinate axes at A *and* B such that P is the mid-point of AB.

Sol. The below figure obtained by the given information

Let the coordinate of the point P *is* (x, y). It is given that, P is mid-point of AB.



So, the coordinates of points A and B are $(2x, 0)$ *and* $(0, 2y)$, respectively.

$\therefore$ *Slope of* $AB = \frac{0-2y}{2x-0} = -\frac{y}{x}$

Since, the segment AB is a tangent to the curve at P.

$\therefore \frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow \frac{dy}{y} = -\frac{dx}{x}$

On integrating both sides, we get

$\log y = -\log x + \log C$

$\log y = \log\frac{C}{x}$ ...(i)

Since, the given curve passes through $(1, 1)$.

$\therefore$ log1=log$\frac{C}{1}$

$\Rightarrow$ 0=logC

$\Rightarrow$ c=1

$\therefore$ $\log y = \log \frac{1}{x}$

$\Rightarrow$ $y = \frac{1}{x}$

$\Rightarrow$ xy=1

33. **Solve:** $x\frac{dy}{dx} = y\ (\log y - \log x + 1)$

Sol. Given, $x\frac{dy}{dx} = y\ (\log y - \log x + 1)$

$\Rightarrow x\frac{dy}{dx} = y\ \log\left(\frac{y}{x} + 1\right)$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right) \quad ...(i)$

Which is a homogeneous equation.

Put $\frac{y}{x} = v$ *or* $y = vx$

$\therefore \ \frac{dy}{dx} = v + x\frac{dv}{dx}$

On substituting these values in Eq. (i), we get

$v + x\frac{dv}{dx} = v(\log v + 1)$

$\Rightarrow x\frac{dv}{dx} = v(\log v + 1 - 1)$

$\Rightarrow x\frac{dv}{dx} = v(\log v)$

$\Rightarrow \frac{dv}{v\log v} = \frac{dx}{x}$

On integrating both sides, we get

$\int \frac{dv}{v\log v} = \int \frac{dx}{x}$

On putting $\log v = u$ in LHS integral, we get

$\frac{1}{v}.dv = du$

$\int \frac{du}{u} = \int \frac{dx}{x}$

$\Rightarrow \ \log u = \log x + \log C$

$\Rightarrow \log u = \log C\ x$

$\Rightarrow u = Cx$

$\Rightarrow \log v = Cx$

$\Rightarrow \log\left(\frac{y}{x}\right) = Cx$