Chapter-05
fourier-Transform

* FT is a mathematical tool for freq. analysis of sig. where as LT is a convenient mathematical tool for ckt analysis.
* FT exists for energy & power signals where as LT also exists for NENP signals. (upto centain extent only)
* In the category of NENP signal unit impulse is the only f^n for which FT also exists.

$$u(t) \xrightarrow{LT} \frac{1}{S} \quad \Big\downarrow S=j\omega\,(FT)$$

$$\frac{1}{j\omega} + \pi\delta(\omega)$$

$$e^{2t}u(t) \xrightarrow{LT} \frac{1}{S-2} \quad \text{(FT does not exists)}$$

* The replacement (S=jω) is used for Laplace to fourier conversion only for absolutely integrable signal.
* Impulse f^n & energy signals are absolutely integrable signals.

<u>fourier Xform →</u>

$$x(t) \rightleftharpoons X(\omega) \rightarrow \text{rad/sec}$$

$$\boxed{X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt}$$

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t} d\omega}$$

<u>Conditions for existance of FT:-</u> (Dirichet's cond^n)

(1.) sig. should have finite no. of maxima & minima over finite interval.

(2) Sig. should have finite no. of discontinuties over finite interval.

(3.) Sig. should be absolutely integrable

i.e. $\boxed{\int_{-\infty}^{\infty} |x(t)|\,dt < \infty}$ → Impulse sig.
→ Energy sig.

* Dirichelt's $cond^n$ are sufficient but not necessary.

Que.→ Cal. FT for sig. $x(t) = e^{-at}u(t), a>0$

Soln→

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty}$$

$$* = \frac{e^{-(a+j\omega)\infty} - e^{0}}{-(a+j\omega)}$$

$$e^{-(a+j\omega)\infty} = e^{-a\infty} \cdot e^{-j\omega\infty}$$

$$= \cancel{e^{-a\infty}}^{\,0} \cdot e^{-j\infty}\ \text{(Undefined)}$$

$$e^{-a\infty} = 0,\ a>0$$

{ $e^{-j\infty} = \cos\infty - j\sin\infty$
These cos & sin are not defined in the given Range.

$$* = \frac{0-1}{-(a+j\omega)}$$

$$\boxed{X(\omega) = \frac{1}{a+j\omega}}$$

* At $t = \pm\infty$, complex exponantials & sinusoidal f^n are undefined.

Properties of FT →

* (1.) Linearity → $a_1 x_1(t) + a_2 x_2(t) \rightleftharpoons a_1 X_1(\omega) + a_2 X_2(\omega)$

* (2.) Time reversal → $x(-t) \rightleftharpoons X(-\omega)$

* (3.) Conjugation → $x^*(t) \rightleftharpoons X^*(-\omega)$

* (4.) Time shifting → $x(t-t_0) \rightleftharpoons X(\omega) e^{-j\omega t_0}$

* (5.) Time scaling → $x(at) \rightleftharpoons \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$, $a \neq 0$

* (6.) Freq. shifting → $e^{-j\omega_0 t} x(t) \rightleftharpoons X(\omega+\omega_0)$

* (7.) Diff. in time → $\frac{d^n x(t)}{dt^n} \rightleftharpoons (j\omega)^n X(\omega)$

* (8.) Integration in time → $\int_{-\infty}^{t} x(t)\,dt \rightleftharpoons \frac{X(\omega)}{j\omega} + \pi X(0) \cdot \delta(\omega)$

  where; $X(0) = X(\omega)\big|_{\omega=0}$

* (9.) Convolution in time → $x(t) * x_2(t) \rightleftharpoons [X_1(\omega) \cdot X_2(\omega)]$

* (10.) multiplication in time → $x_1(t) \cdot x_2(t) \rightleftharpoons \frac{1}{2\pi}[X_1(\omega) * X_2(\omega)]$

  $x_1(t) \cdot x_2(t) \rightleftharpoons X_1(f) * X_2(f)$

* (11.) Diff. in freq. → $t^n x(t) \rightleftharpoons (j)^n \frac{d^n X(\omega)}{d\omega^n}$

* (12.) Parseval's energy theorem → $E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$.

* (13.) Modulation → $x_1(t) \cdot \cos\omega_0 t \rightleftharpoons \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$

  $x(t) \cdot \sin\omega_0 t \rightleftharpoons \frac{j}{2}[X(\omega+\omega_0) - X(\omega-\omega_0)]$

* (14.) Area of time-domain → $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

  $\downarrow \omega = 0$

  $X(0) = \int_{-\infty}^{\infty} x(t) \cdot dt$

Eg:- $x(t) = e^{-at}u(t),\ a>0 \rightleftharpoons X(\omega) = \dfrac{1}{a+j\omega}$

$$\text{area of } x(t) = X(\omega)\Big|_{\omega=0} = \frac{1}{a}$$

* (15) Area under freq. domain →

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, d\omega$$

$$\int_{-\infty}^{\infty} X(\omega)\, d\omega = 2\pi x(0)$$

$$\boxed{\text{Area under } X(\omega) = 2\pi\, x(t)\Big|_{t=0}}$$

Que → $x(t) = e^{at}u(-t) \rightleftharpoons X(\omega) = ?\quad a>0$

Soln →

$$e^{-at}u(-t) \rightleftharpoons X(\omega)$$

$\downarrow (t=-t)$ $\quad\downarrow (\omega=-\omega)$ → time reversal.

$$e^{at}u(-t) \rightleftharpoons \frac{1}{a-j\omega}$$

Que → $y(t) = e^{-a|t|},\ a>0 \rightleftharpoons Y(\omega) = ?$

Soln →

$$y(t) = e^{-a|t|}$$

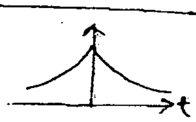
$$= \begin{cases} e^{at}, & t<0 \\ e^{-at}, & t>0 \end{cases}$$

$$= e^{at}u(-t) + e^{-at}u(t)$$

$$Y(\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$Y(\omega) = \frac{2a}{a^2+\omega^2}$$

$$\boxed{e^{-a|t|},\ a>0 \rightleftharpoons \frac{2a}{a^2+\omega^2}}$$



Property of duality →

(1.)

$$x(t) \rightleftharpoons X(\omega)$$

$$(\omega = t) \qquad (t = -\omega)$$

$$X(t) \rightleftharpoons 2\pi\, x(-\omega)$$

$$x(t) \rightleftharpoons X(f)$$

$$(f = t) \qquad (t = -f)$$

$$X(t) \rightleftharpoons x(-f)$$

Q.→ $x(t) = \frac{1}{a+jt} \rightleftharpoons X(\omega) = ?$

Sol^n→

$$x(t) = \frac{1}{a+jt}$$

$$\downarrow (t=\omega)$$

$$e^{-at}u(t), a>0 \rightleftharpoons \frac{1}{a+j\omega}$$

$$(\omega = t) \qquad (t=-\omega)$$

$$\boxed{\frac{1}{a+jt} \rightleftharpoons 2\pi e^{a\omega}u(-\omega),\ a>0}$$

Q.→ $x(t) = \frac{2a}{a^2+t^2} \rightleftharpoons X(\omega) = ?$

Sol^n→

$$x(t) = \frac{2a}{a^2+t^2}$$

$$\downarrow (t=\omega)$$

$$e^{-a|t|}, a>0 \rightleftharpoons \frac{2a}{a^2+\omega^2}$$

$$(\omega = t) \qquad (t=-\omega)$$

$$\frac{2a}{a^2+t^2} \rightleftharpoons 2\pi e^{-a|-\omega|},\ a>0$$

$$\boxed{\frac{2a}{a^2+t^2} \rightleftharpoons 2\pi e^{-a|\omega|},\ a>0}$$

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$$\boxed{A_0\,\delta(t) \rightleftharpoons A_0}$$

Q.→ $x(t) = A_0 \rightleftharpoons X(\omega) = ?$

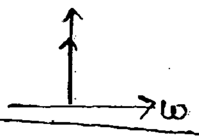
Sol^n→

$$A_0\,\delta(t) \rightleftharpoons A_0$$

$$\omega = t \qquad t = -\omega$$

$$A_0 \rightleftharpoons \cancel{2\pi A_0[\delta(\omega)]}\ 2\pi A_0\,\delta(-\omega)$$

$$\boxed{A_0 = \text{dc signal} \rightleftharpoons 2\pi A_0\,\delta(\omega)}$$



Q → Find $Y(\omega)$ in terms of $X(\omega)$

Soln →

$$x(t) = X(\omega)$$

$$y(t) = Y(\omega)$$

(i) $y(t) = e^{j2t}x(t)$

Soln → $Y(\omega) = X(\omega-2)$ } freq. shifting property

(ii) $y(t) = x(-2t)$

Soln → $Y(\omega) = \frac{1}{2}X\left(\frac{-\omega}{2}\right)$ } time scaling

Q →

(iii) $y(t) = x(2t-3)$

Soln → $y(t) = x(2t-3) = x\left[2\left(t-\frac{3}{2}\right)\right]$

$$x(t) \longrightarrow x(2t) \longrightarrow y(t) = x[2(t-1.5)] = f(t-1.5)$$

$$X(\omega) \quad F(\omega) = \frac{1}{2}X\left(\frac{\omega}{2}\right) \quad Y(\omega) = F(\omega)\,e^{-j1.5\omega}$$

$$= \frac{1}{2}X\left(\frac{\omega}{2}\right)e^{-j1.5\omega}$$

(OR)

$$y(t) = x(2t-3) = x[2(t-1.5)]$$

$$= \frac{1}{2}X\left(\frac{\omega}{2}\right)e^{-j\omega 1.5}$$

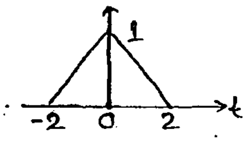
(iv) $y(t) = x(-2t-4)$

Soln → $y(t) = x[-2(t+2)]$

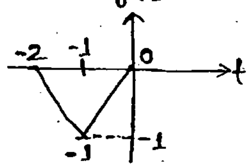
$t_0 = 2$

$$Y(\omega) = \frac{1}{2}X\left(\frac{-\omega}{2}\right)e^{j\omega \cdot 2}$$

(v.) $x(t)$



$y(t)$

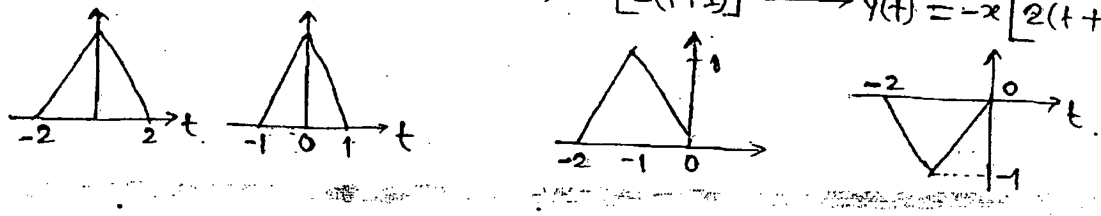


$Y(\omega) = ?$

Sol^n →

$T_{01} = 4 \quad T_{02} = 2$

$x(t) \longrightarrow x(2t) \longrightarrow x[2(t+1)] \longrightarrow y(t) = -x[2(t+1)]$



$$y(t) = -x[2(t+1)]$$

$$Y(\omega) = -\frac{1}{2} X\left(\frac{\omega}{2}\right) e^{j\omega} \qquad (t_0 = 1)$$

Q.→ $y(t) = x(t) * h(t)$ ------ (i)

$g(t) = x(3t) * h(3t)$ ------ (ii)

If $g(t) = Ay(Bt)$ then calculate values of A & B.

Sol^n →

from eq^n (i)

$$Y(\omega) = X(\omega) H(\omega) \text{ ------ (iii)}$$

from eq^n (ii)

$$G(\omega) = \left[\frac{1}{3} X\left(\frac{\omega}{3}\right)\right]\left[\frac{1}{3} H\left(\frac{\omega}{3}\right)\right]$$

$$G(\omega) = \frac{1}{9}\left[X\left(\frac{\omega}{3}\right) H\left(\frac{\omega}{3}\right)\right]$$

$$= \frac{1}{9}\left[Y\left(\frac{\omega}{3}\right)\right] \quad \text{from eq}^n \text{ (iii)}$$

$$= \frac{1}{3}\left[\frac{1}{3} Y\left(\frac{\omega}{3}\right)\right]$$

$$g(t) = \frac{1}{3} y(3t)$$

$$g(t) = Ay(Bt) = \frac{1}{3} y(3t)$$

$$\boxed{A = \frac{1}{3},\ B = 3}$$

2nd method →

$$x(t) * h(t) = y(t)$$

$$x(at) * h(at) = \frac{1}{|a|} y(at)$$

$$\Big\downarrow (a = 3)$$

$$x(3t) * h(3t) = \frac{1}{3} y(3t)$$

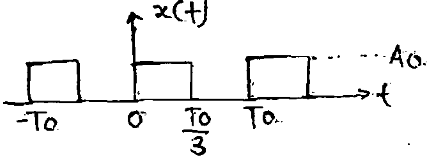
$$g(t) = Ay(Bt)$$

By comparision;

$$\boxed{A = \frac{1}{3} \text{ (or) } B = 3}$$

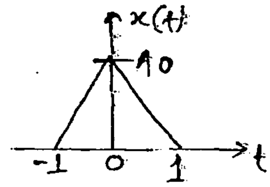
Avg. Value →

(1.)



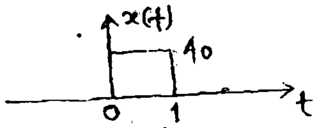
$\boxed{avg. = \frac{Ao}{3}}$

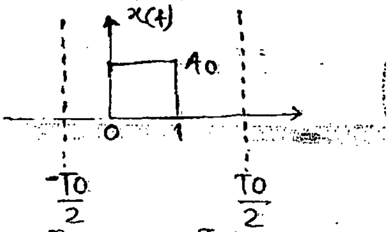
(3.)



$\boxed{avg = Ao}$

(2.)



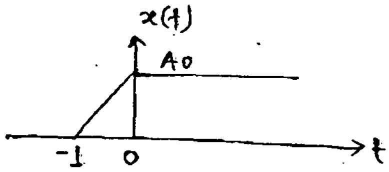


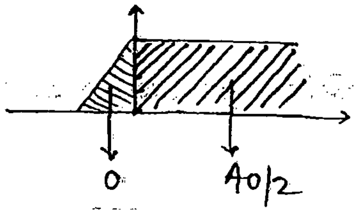
$$Avg. = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} x(t)\,dt$$

$$= \lim_{T_0 \to \infty} \frac{1}{T_0} \int_0^1 A_0\,dt = \frac{A_0}{T_0 \to \infty\, T_0} = 0$$

$\boxed{Avg. = 0}$

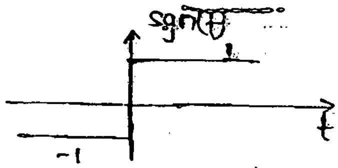
(4.)



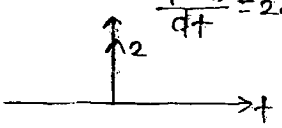


★★★ **For any finite duration pulse avg. value will be = 0**

Q. → $x(t) = sgn(t) \rightleftharpoons X(\omega) = ?$



Sol^n →

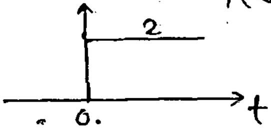


$\boxed{x(t) = sgn(t) \rightleftharpoons X(\omega) = \frac{2}{j\omega}}$

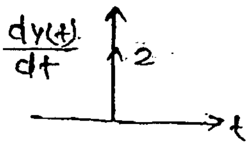
$$\frac{dx(t)}{dt} = 2\delta(t)$$

$\downarrow$ FT

Que.→ $y(t) \rightleftharpoons Y(\omega) = ?$



Soln→



$\rightarrow avg = \frac{2}{2} = 1 = 2\pi\delta(\omega)$

$$\frac{dy(t)}{dt} = 2\delta(t)$$

$$[j\omega]\, Y(\omega) = 2$$

$$\boxed{Y(\omega) = \frac{2}{j\omega}} \times$$

2nd method→

$$y(t) = 1 + x(t)$$

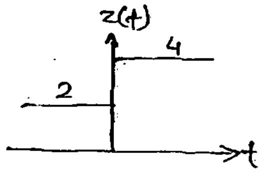
$\downarrow$ (FT)

$$Y(\omega) = 2\pi\delta(\omega) + X(\omega)$$

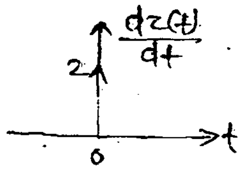
$$= \frac{2}{j\omega} + 2\pi\delta(\omega)$$

$$\boxed{Y(\omega) = \frac{2}{j\omega} + 2\pi\delta(\omega)}$$

Que.→



Soln→



$$\frac{dz(t)}{dt} = 2\delta(t)$$

$$j\omega Z(\omega) = 2$$

$$\boxed{Z(\omega) = \frac{2}{j\omega}} \times$$

$$Avg. = \frac{4+2}{2} = 3$$

$\downarrow$

$3 \times 2\pi\delta(\omega)$

$6\pi\delta(\omega)$

$$\boxed{Z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)}$$

2nd method→

$$z(t) = 3 + x(t)$$

$\downarrow$ FT

$$Z(\omega) = 6\pi\delta(\omega) + \frac{2}{j\omega}X(\omega)$$

$$= X(\omega) + 6\pi\delta(\omega)$$

$$Z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)$$

$$\boxed{Z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)}$$

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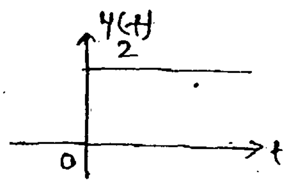
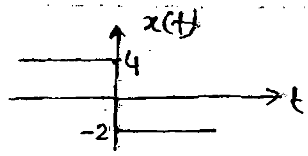


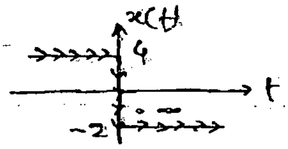
$$2u(t) = \left[\frac{2}{j\omega} + 2\pi\delta(\omega)\right]$$

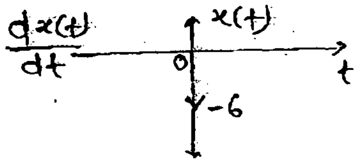
$$\frac{2u(t)}{2} = \frac{1}{2}\left[\frac{2}{j\omega} + 2\pi\delta(\omega)\right]$$

Q.→



Sol^n→





$$\frac{dx(t)}{dt} = -6\delta(t)$$

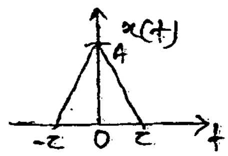
$$j\omega X(\omega) = -6$$

$$X(\omega) = \frac{-6}{j\omega}$$

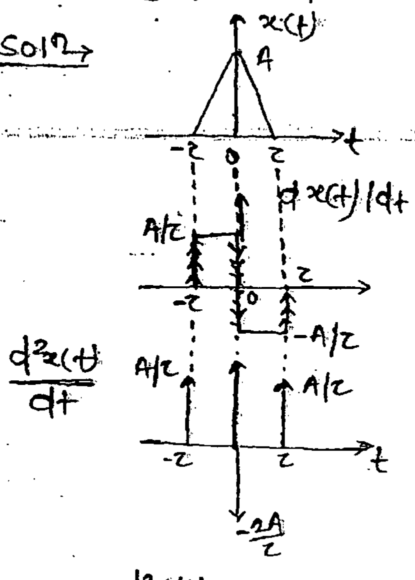
$$Avg. = \frac{4-2}{2} = 1$$

$$\boxed{X(\omega) = \frac{-6}{j\omega} + 2\pi\delta(\omega)}$$

Q.→



Sol^n→

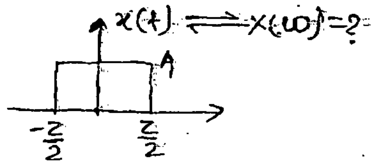


$$\frac{d^2x(t)}{dx} = \frac{A}{\tau}\delta(t+\tau) + \frac{A}{\tau}\delta(t-\tau) - \frac{2A}{\tau}\delta(t)$$

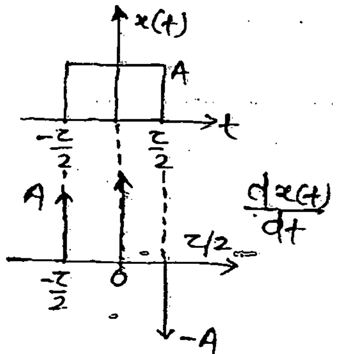
$$\Big\downarrow \text{FT}$$

$$(j\omega)^2 X(\omega) = A\,e^{j\omega\tau},\ A\ e^{-j\omega\tau}\ \ldots$$

Que.→



Sol^n→



$$\frac{dx(t)}{dt} = A\delta\left(t+\frac{\tau}{2}\right) - A\delta\left(t-\frac{\tau}{2}\right)$$

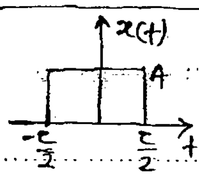
$$j\omega X(\omega) = Ae^{j\omega\tau/2} - Ae^{-j\omega\tau/2}$$

$$X(\omega) = \frac{A}{j\omega}\times\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$$

$$= \frac{A}{j\omega}\times 2j\times\sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{2A}{\cancel{\omega}}\left[\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}\right]\times\left(\frac{\cancel{\omega}\tau}{2}\right)$$

$$= A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$



$$X(\omega) \rightleftharpoons A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$$X(\omega)=\frac{A}{\tau(-\omega^2)}\left[\left(e^{j\omega\tau}+e^{-j\omega\tau}\right)-2\right]$$

$$X(\omega)=\frac{A}{-\tau\omega^2}\left[2\cos\omega\tau-2\right]$$

$$X(\omega)=\frac{2A}{+\tau\omega^2}\left(1-\cos\omega\tau\right)$$

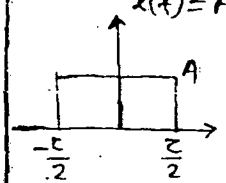
$$=\frac{2A}{\tau\omega^2}2\left[\sin^2\left(\frac{\omega\tau}{2}\right)\right]$$

$$=\frac{2A}{\tau\omega^2}\times 2\,\frac{\sin^2\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)^2}\times\left(\frac{\omega\tau}{2}\right)^2$$

$$\boxed{X(\omega)=A\tau\, sa^2\left(\frac{\omega\tau}{2}\right)}$$

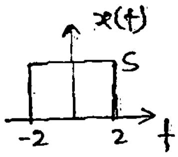
***

$x(t)=A\,\mathrm{rect}\left(\frac{A}{2}\right)$



$$\Longrightarrow X(\omega)=A\tau\, sa\left(\frac{\omega\tau}{2}\right)=\text{Area}\; sa\left(\frac{\omega\,\text{duration}}{2}\right)$$

Q.→

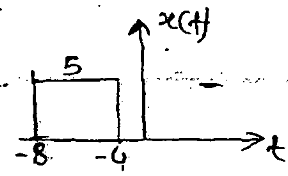


Sol^n→ $X(\omega)=20\,sa\left(\omega\frac{4}{2}\right)$

$$=20\,sa(2\omega)$$

$$\boxed{X(\omega)=20\,sa(2\omega)}$$

Q.→



Sol^n→ $y(t)=x(t+6)$

$$Y(\omega)=X(\omega)e^{j\omega 6}$$

$$=20\,sa(2\omega)\,e^{j\omega 6}$$

$$\boxed{Y(\omega)=20\,sa(2\omega)\,e^{j\omega 6}}$$

$$Y(\omega) = 2\,sa(\omega) = 2\,\mathrm{sinc}\left(\frac{\omega}{\pi}\right)$$

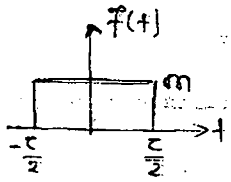
$$= \frac{2\sin\omega}{\omega} = 2\times\cancel{2}\left(\frac{\sin\frac{\omega}{2}}{\omega/2}\right)\times\cos\frac{\omega}{2} = 2\,sa\left(\frac{\omega}{2}\right)\cos\left(\frac{\omega}{2}\right)$$

$$= \cancel{\frac{2\times\sin\frac{\omega}{2}\cos\frac{\omega}{2}}{\omega}} \qquad = 2\,\mathrm{sinc}\left(\frac{\omega}{2\pi}\right)\cdot\cos\left(\frac{\omega}{2}\right)$$

$$\boxed{Y(\omega) = 2\,\mathrm{sinc}\left(\frac{\omega}{2\pi}\right)\cdot\cos\left(\frac{\omega}{2}\right)}$$

Q.→ $x(t) = A_0 sa(t) \rightleftharpoons$ Draw FT $X(\omega)$

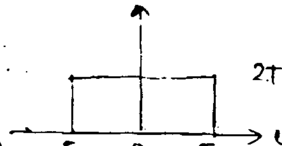
Soln→



$$\rightleftharpoons m\tau\, sa\left(\frac{\omega\tau}{2}\right)$$

$(\omega = t)$ $\quad$ $(t = -\omega)$

$$m\tau\, sa\left(\frac{t\tau}{2}\right) \rightleftharpoons 2\pi f(-\omega)$$



$$2\pi m = \frac{A_0\pi}{k}$$

$$-k = -\frac{\tau}{2} \quad 0 \quad \frac{\tau}{2} = k$$

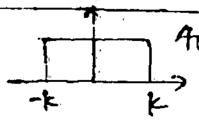
$$m\tau\, sa\left(\frac{t\tau}{2}\right) = A_0\, sa(kt)$$

$$m\tau = A_0\,, \quad k = \frac{\tau}{2}$$

$$2\pi m = 2\pi\cdot\frac{A_0}{\tau} = \frac{2\pi\times A_0}{2k} = \frac{A_0\pi}{k}$$

***

$A_0\, sa(kt) \rightleftharpoons$



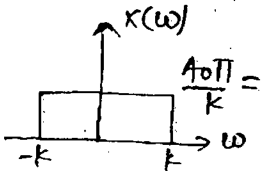
$A_0\pi/k$

$-k \quad k$

Q.→ $x(t) = 3\,sa(4t) \rightleftharpoons X(\omega)$

Soln→ $\quad A_0 = 3 \quad A_0\, sa(kt)$

$k = 4$



$X(\omega)$

$$\frac{A_0\pi}{k} = \frac{3\pi}{4}$$

$-k \quad k$

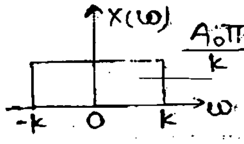
$-4 \quad 4$

1

Q.→ Calculate area & energy of $x(t) = Sa(t)$

Soln →

$x(t) = Sa(t)$



$\frac{A_0\pi}{k} = \pi$

$= \cancel{A_0 Sa(t)} \cdot A_0 Sa(kt)$

$$x(t) = A_0 Sa(kt) = Sa(t)$$

$$A_0 = 1,\ k = 1$$

area under time-domain →

$$\text{area of } x(t) = X(\omega)\big|_{\omega=0}$$

$$\boxed{A = \pi}$$

parseval's energy theorem →

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} \pi^2 d\omega$$

$$\boxed{E = \pi}$$

Q.→ $h(t) \rightleftharpoons H(\omega) = \frac{2\cos\omega \cdot \sin 2\omega}{\omega}$

Find $h(0) = ?$

(a) 1/4 (b) 1/2 (c) 1 (d) 2

Soln →

$$H(\omega) = \frac{2\cos\omega.\sin 2\omega}{\omega}$$

$$= \frac{\sin(3\omega) + \sin\omega}{\omega}$$

$$= \frac{\sin(3\omega)}{\omega} + \frac{\sin\omega}{\omega}$$

$$= \frac{3\sin(3\omega)}{3\omega} + \frac{\sin\omega}{\omega}$$

$$H(\omega) = 3Sa(3\omega) + Sa(\omega)$$

$$= H_1(\omega) + H_2(\omega)$$

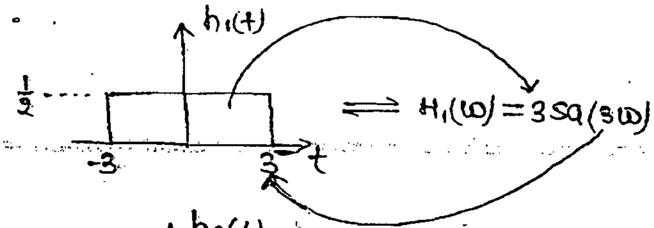
$$h(t) = h_1(t) + h_2(t)$$

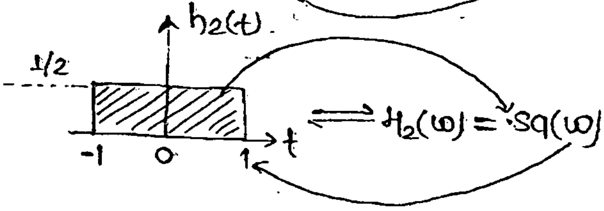
$\big\downarrow t=0$

$$h(0) = h_1(0) + h_2(0)$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$





Area

$x(t) \longrightarrow A$

$x(kt) \longrightarrow \frac{A}{k}$

$k\,x(t) \longrightarrow kA$

$Sa(\omega) \longrightarrow \pi$

$Sa(3\omega) \longrightarrow \pi/3$

$3\,Sa(3\omega) \longrightarrow 3 \times \frac{\pi}{3} = \pi$

2nd method →

Area under Freq. domain

$2\pi h(0) = \text{Area of } H(\omega)$

$h(0) = \dfrac{\text{Area of } H(\omega)}{2\pi}$

$h(0) = \dfrac{2\pi + \pi}{2\pi} = 1$

$\boxed{h(0) = 1}$

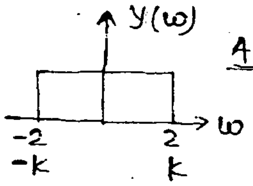
Q → $y(t) = x(t)\cos t \rightleftharpoons Y(\omega) = \begin{cases} 2, & |\omega| \le 2 \\ 0 & \text{otherwise.} \end{cases}$

Find $x(t)$

(a) $\frac{4}{\pi}\frac{\sin t}{t}$ (b) $\frac{2\sin t}{t}$ (c) $4\frac{\sin t}{t}$ (d) $2\pi \cdot \frac{\sin t}{t}$

Solⁿ → **1st method →**

$y(t) = A_0 Sa(kt) \rightleftharpoons$



$\frac{A_0\pi}{k} = 2$

$= \frac{4}{\pi} Sa(2t)$

$k = 2,\ A_0 = 4/\pi$

$= \frac{4}{\pi} \cdot \frac{\sin 2t}{2t} = \left[\frac{4}{\pi} \cdot \frac{2\sin t}{2t}\right] \cdot \cos t$

$= \left[\frac{4}{\pi} \cdot \frac{\sin t}{t}\right] \cdot \cos t$

$= x(t) \cdot \cos t$

2nd method →

area under Freq. domain

$2\pi y(0) = \text{area of } Y(\omega)$

$2\pi y(0) = 8$

$y(0) = \frac{8}{2\pi} = \frac{4}{\pi} = x(0)$

$y(0) = x(0)$ at

Q.→ Find FT of $\cos\omega_0 t$

$$x(t) = \cos\omega_0 t \Longleftrightarrow X(\omega) = ?$$

Sol^n→

$$x(t) = \cos\omega_0 t$$

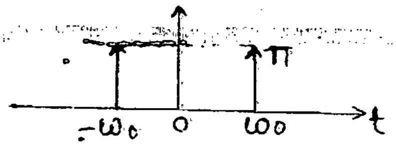
$$x(t) = \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$$

$$X(\omega) = \frac{1}{2}\left[2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right]$$

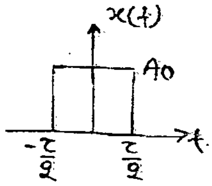
$$X(\omega) = \pi\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\pi$$

$$\left\{\begin{aligned} A_0 &= 2\pi A_0\delta(\omega) \\ &\downarrow A_0 = 1 \\ 1 &= 2\pi\delta(\omega) \\ 1\cdot e^{j\omega_0 t} &= 2\pi\delta(\omega-\omega_0) \\ e^{-j\omega_0 t} &= 2\pi\delta(\omega+\omega_0) \end{aligned}\right.$$

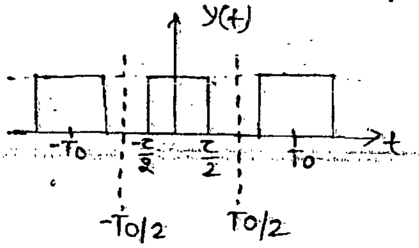
$$\cos\omega_0 t = \pi\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\pi$$



* Calculation of 'c_n' by using FT→



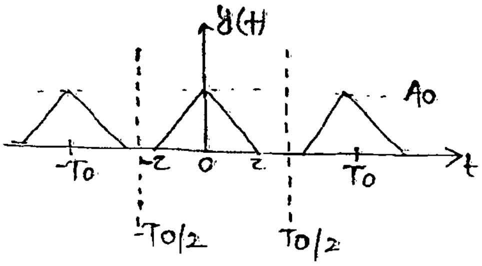
$$X(\omega) = A_0\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$



$$\boxed{c_n = \frac{X(n\omega_0)}{T_0}}$$

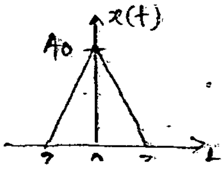
$$X(\omega) = \frac{A_0\tau}{T_0}\, Sa\left(\frac{n\omega_0\tau}{2}\right)$$

Q.→



$c_n = ?$

Sol^n→



$$X(\omega) = A_0\tau\, Sa^2\left(\frac{\omega\tau}{2}\right)$$

$c_n = X(n\omega_0) \ldots A_0\tau \ldots (n\omega_0\tau)$

* FT for periodic signal →

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \qquad X(\omega) = ?$$

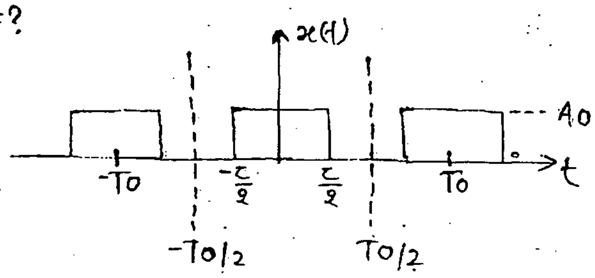
$$1 = 2\pi\,\delta(\omega)$$

$$C_n = 2\pi C_n\,\delta(\omega)$$

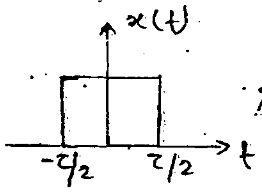
$$C_n e^{jn\omega_0 t} = 2\pi C_n\,\delta(\omega - n\omega_0) \quad \text{(Freq. shifting)}$$

$$\boxed{\sum C_n e^{jn\omega_0 t} = 2\pi \sum_{n=-\infty}^{\infty} C_n\,\delta(\omega - n\omega_0)}$$

Que. → $X(\omega) = ?$



Sol^n →



$$X(\omega) \longrightarrow A_0 \tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

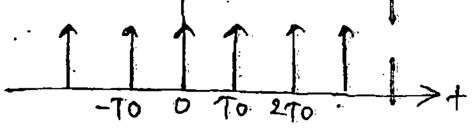
$$C_n = \frac{A_0\tau}{T_0} Sa\left(\frac{n\omega_0\tau}{2}\right)$$

$$X(\omega) = 2\pi \sum C_n\,\delta(\omega - n\omega_0)$$

$$= 2\pi \sum_{n=-\infty}^{\infty} \left[\frac{A_0\tau}{T_0} Sa\left(\frac{n\omega_0\tau}{2}\right)\right] \delta(\omega - n\omega_0)$$

Que. → $X(\omega) = ?$

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$



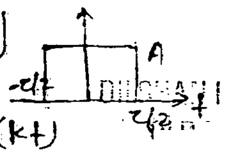
Sol^n →

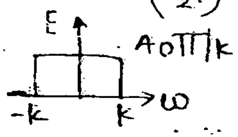
$$X(\omega) = 2\pi \sum C_n\,\delta(\omega - n\omega_0)$$

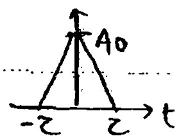
$$= 2\pi \sum \frac{1}{T_0}\,\delta(\omega - n\omega_0)$$

*Important signal →

	$x(t)$	$X(\omega)$
(1.)	$\delta(t)$	1
(2.)	$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
(3.)	$sgn(t)$	$\frac{2}{j\omega}$
(4.)	A_0	$2\pi A_0 \delta(\omega)$
(5.)	$e^{-at}u(t)$, $a>0$	$\frac{1}{a+j\omega}$
(6.)	$e^{-a\lvert t\rvert}u(t)$, $a>0$	$\frac{2a}{a^2+\omega^2}$
(7.)	$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$
(8.)	$\sin\omega_0 t$	$\pi j[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$
(9.)	$A\,rect(\frac{t}{\tau})$	$A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$
(10.)	$A_0\, Sa(kt)$	
(11.)	Periodic sig.	$2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0)$
(12.)	$\sum \delta(t-nT_0)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$
(13.)		$A\tau\, Sa^2\left(\frac{\omega\tau}{2}\right)$
(14.)	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
(15.)	$e^{-j\omega_0 t}$	$2\pi\delta(\omega+\omega_0)$

(9.)


(10.)


(13.)


$\boxed{\text{DATE-21/10/14}}$

*** $x(t)$-$X(\omega)$ pairs →**

$x(t)$	$X(\omega)$
Real	CS
CS	Real
Img.	CAS
CAS	Img
R+E	R+E

R+O	I+O
I+O	R+O

$x(t)$	$X(\omega)$
Continues	Non-periodic
Non-periodic	contineous
Discrete	periodic
periodic	discrete

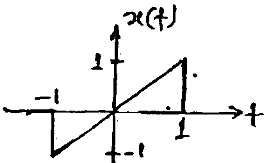
$C + P \longrightarrow D + NP$

$C + NP \longrightarrow C + NP$

$D + P \longrightarrow D + P$

$D + NP \longrightarrow C + P$

Q.1



$X(\omega) = ?$

(a.) $4\pi j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega}\right]$ (c.) $2j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega^2}\right]$

(b.) $4\pi j\left[\frac{\cos\omega}{\omega^2} - \frac{\sin\omega}{\omega}\right]$ (d.) $2j\left[\frac{\cos\omega}{\omega^2} - \frac{\sin\omega}{\omega}\right]$

Sol^n→ For sol^n go through the option. ans.(c)

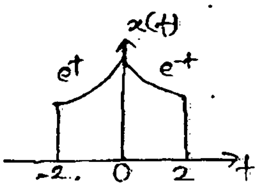
$x(t) \longrightarrow R + O$. $I + O$

$X(\omega) \longrightarrow I + D$

$\frac{\sin\omega}{\omega} = E$

$X(\omega) = 2j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega^2}\right]$

Q.→



$\Longrightarrow X(\omega) = ?$

(a) $2 - 2e^{-2}\sin 2\omega + 2\omega e^{-2}\sin 2\omega$ (b) $2 + 2e^{-2}\cos 2\omega - 2\omega e^{-2}\cos 2\omega$

(c.) $2 + 2e^{-2}\cos 2\omega - 2\omega e^{-2}\sin 2\omega$ (d.) $2 - 2e^{-2}\cos 2\omega + 2\omega e^{-2}\sin 2\omega$

Sol^n → $x(t) = R+E$; $X(\omega) = R+E$

So; option (a) & (b) $\neq R+E$

Area under time domain;

$$X(0) = \int_{-\infty}^{\infty} x(t)dt$$

$$= \int_{-2}^{2} x(t)dt = 2\int_{0}^{2} x(t)dt$$

$$= 2\int_{0}^{2} e^{-t}dt$$

$$= \frac{2[e^{-t}]_0^2}{-1} = 2(1-e^{-2})$$

$$= 2-2e^{-2}$$

Now; put $x(0)$ in the option (C) & (D)

$\boxed{\text{Ans. (d)}}$

Q.→ $f(t) \rightleftharpoons F(\omega)$

$$g(t) = \int_{-\infty}^{\infty} F(u)e^{-jut}du$$

what is the relationship bet^n $f(t)$ & $g(t)$?

(a.) $g(t)$ would always be proportional to $f(t)$.

(b.) $g(t)$ would always be proportional to $f(t)$ is $f(t)$ is an even sig.

(c.) $g(t)$ would proportional to $f(t)$ only if $f(t)$ is sinusoidal f^n

(d.) $g(t)$ would ~~area~~ never to be proportional to $f(t)$.

Sol^n → IFT (inverse FT)

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

$$2\pi f(t) = \int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

$(\omega = u)$

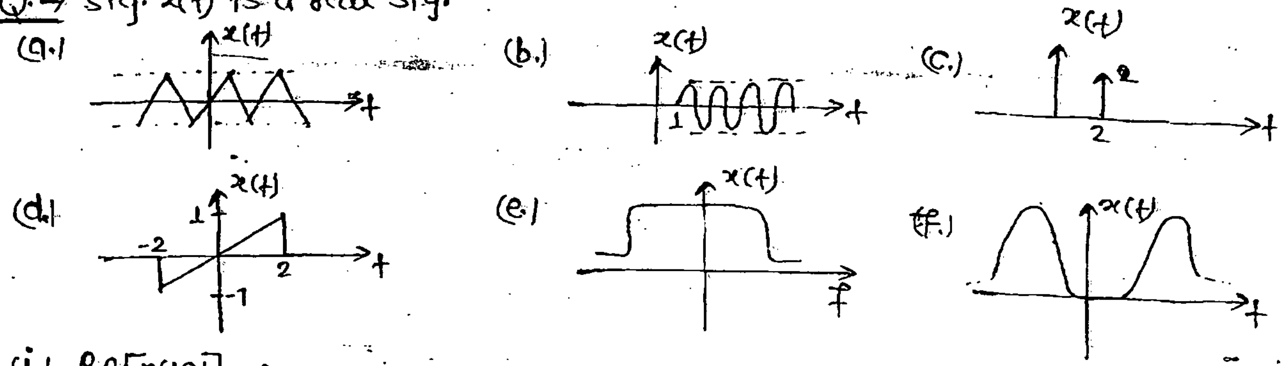
$$2\pi f(t) = \int_{-\infty}^{\infty} F(u)\, e^{jut}du$$

$(t = -t)$

$$2\pi f(-t) = \int_{-\infty}^{\infty} F(u)e^{-jut}du$$

$\boxed{g(t) = 2\pi f(t)}$ If $f(t)$ is even
$f(-t) = f(t)$

Q.→ sig. $x(t)$ is a real sig.



(i.) $Re[X(\omega)] = 0$

(a.) a, d (b) e, f (c) b, c (d) b, d.

Sol^n→

$x(t) \longrightarrow$ Real

$\therefore X(\omega) \longrightarrow CS_o$ (even)

$= \text{Real}[X(\omega)] + j\,\text{Img}[X(\omega)]$

$\downarrow$ even $\qquad \downarrow$ odd

$X(\omega) \rightarrow CS$ & nature is imag. odd.

So $x(t) \rightarrow R + O$.

$\boxed{\text{ans (a.)}}$

(ii) $\int_{-\infty}^{\infty} X(\omega)\,d\omega = 0$

(a) e (b) a, b, c, d, f (c) b, c (d) a, d, e, f.

Sol^n→ Area under freq. domain

$$2\pi x(0) = \int_{-\infty}^{\infty} X(\omega)\,d\omega$$

$$2\pi x(0) = 0$$

$$\boxed{x(0) = 0}$$

$\boxed{\text{ans (b)}}$

(iii) $\int_{-\infty}^{\infty} \omega X(\omega)\, d\omega = 0$

(a.) a, b, c, d, f (b.) b, c, e, f (c.) e (d.) b, c

Sol^n →

$$x(t) \rightleftharpoons X(\omega)$$

$$\frac{dx(t)}{dt} = (j\omega) X(\omega)$$

$$\frac{1}{j}\frac{dx(t)}{dt} = \omega X(\omega)$$

$$y(t) = Y(\omega)$$

area under Freq. domain

$$2\pi y(0) = \int_{-\infty}^{\infty} Y(\omega)\, d\omega$$

$$= \int_{-\infty}^{\infty} \omega X(\omega)\, d\omega$$

$$2\pi y(0) = 0$$

$$y(0) = (0)$$

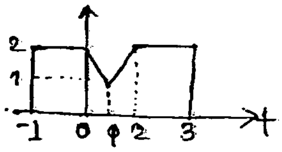
$$y(t)\Big|_{t=0} = 0$$

$$\frac{1}{j}\frac{dx(t)}{dt}\Big|_{t=0} = 0$$

$$\boxed{\frac{dx(t)}{dt} = 0}$$ (slope is zero at the origin)

$\boxed{\text{ans. (b)}}$

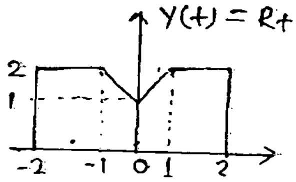
Q → $x(t) \rightleftharpoons X(\omega)$



Find $\angle X(\omega) = ?$

Sol^n →

$Y(t) = R + E \rightleftharpoons Y(\omega) = R + E$



$$x(t) = y(t-1)$$

$$\angle X(\omega) = \angle Y(\omega) + (-\omega)$$

$\because$ $Y(\omega) = R + E$ & $\angle Y(\omega) = 0$