CBSE Class 12 - Mathematics
Sample Paper 09 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. Show that the function $f : R \to R$ given by $f(x) = x^3$ is injective.

OR

Show that the function $f : R \rightarrow R : f(x) = x^5$ is one-one and onto.

2. State the reason for the relation R on the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.

OR

Determine whether the relation is reflexive, symmetric and transitive:

Relation R in the set A of human beings in a town at a particular time given by

R = {(x, y) : x and y live in the same locality}

3. Let the relation R be defined on N by aRb if 2a + 3b = 30. Then write R as a set of ordered pairs.

4. Let A = $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

Show that $(A^T)^T = A$

5. Construct a 2×3 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by:

$a_{ij} = \frac{(i+j)^2}{2}$

OR

Write the order of each of the following matrices:

F = [6]

6. Let A be a 3×3 square matrix such that A (adj A) = 2 I, where I is the identity matrix.

Write the value of |adj A|

7. **Find the value of** $\int \sin^3 x \cos x dx$

OR

Evaluate $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx.$

8. Let f(x) be a continuous function such that the area bounded by the curve y=f(x), x-axis and the lines x=0 and x=a is $\frac{a^2}{2} + \frac{a}{2} sin\ a + \frac{\pi}{2}\ cos\ a$, then find $f(\frac{\pi}{2})$

9. Write order and degree (if defined) of the differential equation: $(3x + 5y)\,dy - 4x^2\,dx = 0$.

OR

Find the order and degree (if any) of the differential equation given below: $(y'')^2 + \cos y' =$

0.

10. At what point of the curve y - x does the tangent make an angle of 45° with the x-axis?

11. Show that the function $f(x) = x^2$ is

  a. strictly increasing on $[0, \infty]$

  b. strictly decreasing on $[0, \infty]$

  c. neither strictly increasing nor strictly decreasing on R.

12. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

13. Find the Cartesian equation of the plane $\overrightarrow{r}.[(5-2t)\hat{i} + (3-t)\hat{j} + (25+t)\hat{k}] = 15$

14. Find the vector equation for the line passing through the points (-1, 0, 2) and (3, 4, 6).

15. If $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, find x, y and z.

16. 10 coins are tossed simultaneously. Find the probability of getting at least 4 heads.

Section - II

17. **Matrices/Determinant:** In linear algebra, the **determinant** is a scalar value that can be computed from the elements of a square matrix and encodes certain properties of the linear transformation described by the matrix. The **determinant** of a matrix A is denoted det(A) or |A|. Using determinants/ Matrices calculate the following:

Ram purchases 3 pens, 2 bags, and 1 instrument box and pays ₹ 41. From the same shop, Dheeraj purchases 2 pens, 1 bag, and 2 instrument boxes and pays ₹29, while Ankur purchases 2 pens, 2 bags, and 2 instrument boxes and pays ₹44.

Read the above information and answer the following questions:

  i. Find the cost of one pen.

    a. ₹ 2

    b. ₹ 5

    c. ₹ 10

    d. ₹15

ii. What are the cost of one pen and one bag?

a. ₹ 12

b. ₹ 15

c. ₹ 17

d. ₹25

iii. What is the cost of one pen & one instrument box?

a. ₹ 7

b. ₹ 12

c. ₹ 17

d. ₹25

iv. What is the cost of one bag & one instrument box?

a. ₹ 20

b. ₹ 25

c. ₹ 10

d. ₹15

v. Find the cost of one pen, one bag, and one instrument box.

a. ₹ 22

b. ₹ 25

c. ₹ 20

d. ₹24

18.

Deepal left from his village on weekend. First, he travelled d_1 displacement up to a temple. After this, he left for the zoo and travelled d_2 displacement. After this he left for shopping in a mall - Total driving time of Deepal from village to Mall was 1.5 hr.

If d_1 = (6, 8) d_2 = (3, 4) and d_3 = (7, 12) km

i. What is the total displacement from village to Mall?

a. 30 km
b. 20 km
c. 25 km
d. 40 km

ii. What is the speed of Deepal from Village to Mall?

a. 30 km/hr
b. 20 km/hr
c. 25 km/hr
d. 35 km/hr

iii. What is the Displacement from Village to Zoo?

a. 20 km
b. 10 km
c. 15 km
d. 25 km

iv. What is the angle in degree in Final displacement?

a. $\tan^{-1}(\frac{3}{4})$
b. $\tan^{-1}(\frac{4}{3})$
c. 45^{o}
d. 60^{o}

v. What is the displacement from temple to Mall?

a. 40 km
b. 30 km
c. 1 km
d. 20 km

Part - B Section - III

19. Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$

20. Find the value of k in order that the points (5, 5), (k, 1) and (10, 7) are collinear.

OR

If A is a non-singular matrix, prove that $(\text{adj } A)^{-1} = \frac{1}{|A|}A$.

21. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ by expanding the product to obtain a single

polynomial.

22. Find the points of local maxima or local minima, if any, using the first derivative test. Also find the local maximum or local minimum values, as the case may be: f(x) = $x\sqrt{1-x}, x > 0.$

23. If $\int_0^a \sqrt{x}\,dx = 2a\int_0^{\pi/2} \sin^3 x\,dx$, find the value of integral $\int_a^{a+1} x dx$

OR

Evaluate: $\int x^2 \cos x\, dx$

24. Using integration, find the area of the region given below: $\{(x,y) : 0 \leqslant y \leqslant x^2+1, 0 \leqslant y \leqslant x+1, 0 \leqslant x \leqslant 2\}$

25. Solve the differential equation: $x \cos^2 y\, dx = y \cos^2 x\, dy$

26. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.

27. Show that the points whose position vectors are $-2\hat{i}+3\hat{j},\ \hat{i}+2\hat{j}+3\hat{k}$ and $7\hat{i}+9\hat{k}$ are collinear.

28. A and B are two independent events. The probability that A and B occur is 1/6 and the probability that neither of them occurs is 1/3. Find the probability of occurrence of two events.

OR

A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of 6 successes?

Section - IV

29. Let A ={x ∈ R : 1 ≤ x ≤1} = B. Show that f : A → B given by f(x) = $x|x|$ is a bijection.

30. If $f(x) = \frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}$ for $x \neq \frac{\pi}{4}$, find the value which can be assigned to f(x) at x = $\pi/4$ so that the function f(x) becomes continuous everywhere in $[0, \pi/2]$

31. Verify Rolle's theorem for the function on the indicated interval:f(x) = $\log(x^2 + 2) - \log 3$ on [–1, 1]

OR

If $x^x + y^x = 1$, prove that $\frac{dy}{dx} = -\left\{\frac{x^x(1+\log x)+y^x\cdot\log y}{x\cdot y^{(x-1)}}\right\}$

32. A rectangular sheet of fix perimeter with sides having their lengths in the ratio 8:15 is

converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed square is 100 square units, the resulting box has maximum volume. Find the length of the sides of the rectangular sheet.

33. Evaluate the definite integral: $\int_1^2 e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right)dx$

34. If A_n is the area bounded by the curve y = $(tan\ x)^n$ and the lines x = 0, y = 0 and $x = \frac{\pi}{4}$. Then, prove that for n > 2, $A_n + A_{n+2} = \frac{1}{n+1}$

OR

The slope of tangent to a curve y=f(x) at (x, f(x)) is 2x+1. If the curve passes through the point (1,2), then find the area bounded by the curve, the X-axis and the line x=1.

35. In the differential equation show that it is homogeneous and solve it: $x^2dy + y(x + y)\,dx = 0$.

Section - V

36. Three bags contain a number of red and white balls as follows:

Bag 1 : 3 red balls, Bag 2 : 2 red balls and 1 white ball and Bag 3 : 3 white balls.

The probability that bag a will be chosen and a ball is selected from it is $\frac{1}{6}$. What is the probability that

i. a red ball will be selected?

ii. a white ball is selected?

OR

There are three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and take out a coin. If the coin is of gold, then what is the probability that the other coin in box is also of gold?

37. A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of ΔABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

OR

Reduce the equation $\vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k})$ + 6 = 0 to the normal form and hence find the length of the perpendicular from the origin to the plane.

38. A manufacturer produces nuts and bolts. It takes 1 hours of work on machine A and 3 hours on machine B to produce a package of nuts and bolts. He earns a profit of Rs 17.50 per package on nuts and Rs 7.00 per package on bolts. How many package of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day?

OR

Minimize $z = 3x + 5y$ subject to the constraints $x + 2y \leq 2000$, $x + y \leq 1500$, $y \leq 600$, $x \geq 0$ and $y \geq 0$

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Solution

Part - A Section - I

1. Let $x_1, x_2 \in R$ be such that $f(x_1) = f(x_2)$

   $\Rightarrow x_1^3 = x_2^3$

   $\Rightarrow x_1 = x_2$

   Therefore, f is one-one function, hence $f(x) = x^3$ is injective.

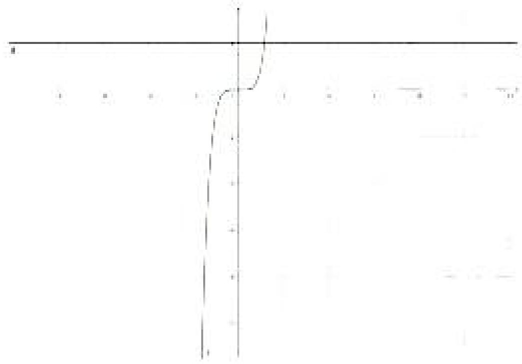
   OR

   To show: $f: R \rightarrow R : : f(x) = x^5$ is one - one and onto.

   Proof:

   $f(x) = x^5$

   $\Rightarrow y = x^5$

   Since the lines do not cut the curve in 2 equal valued points of y, therefore, the function f(x) is one - one.

   The range of $f(x) = (-\infty, \infty) = R$(Codomain)

   $\therefore$ f(x) is onto

   Hence, showed $f: R \rightarrow R : f(x) = x^5$ is one - one and onto.

2. Given set {1,2,3}

   Relation R = {(1, 2), (2, 1)}.

   We know that for relation R to be transitive,

   $(x, y) \in R$ and $(y, z) \in R \implies (x, z) \in R$

Here, $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$

Hence, Relation R is not transitive.

OR

Given that R = {(x, y) : x and y live in the same locality}

Clearly, $(x,x) \in R$ as x and x live in the same locality.

$\Rightarrow$ R is reflexive.

Now, if $(x,y) \in R$, then x and y live in the same locality.

$\Rightarrow$ y and x live in the same locality.

$\Rightarrow (y,x) \in R$

$\Rightarrow$ R is symmetric.

Further, let $(x,y), (y,z) \in R$

$\Rightarrow$ x and y live in the same locality and y and z live in the same locality.

$\Rightarrow$ x and z live in the same locality

$\Rightarrow (x,z) \in R$

$\Rightarrow$ R is transitive.

Therefore, R is reflexive, symmetric and transitive.

3. Given $R = \{(a, b) : 2a + 3b = 30, \forall (a,b) \in N\}$

Now according to the question $(a,b) \equiv 2a + 3b = 30$:

$\Rightarrow R = \{(3, 8),(6, 6),(9, 4),(12, 2)\}$

NOTE: 0 is a whole number that's why its not considered in this set,beacause 0 is not a natural number

4. We have $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

Now, $A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$\therefore (A^T)^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} = A$

Hence proved

5. Here, $a_{ij} = \frac{(i+j)^2}{2}$

$\therefore a_{11} = \frac{(1+1)^2}{2} = 2, a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}, a_{13} = \frac{(1+3)^2}{2} = 8$

$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}, a_{22} = \frac{(2+2)^2}{2} = 8, a_{23} = \frac{(2+3)^2}{2} = \frac{25}{2}$

Therefore, A = $\begin{bmatrix} 2 & \frac{9}{2} & 8 \\ \frac{9}{2} & 8 & \frac{25}{2} \end{bmatrix}$

OR

Order of matrix = Number of rows x Number of columns.

Thus order = (1×1)

6. Since we know

A(adj(A)) = |A|.I

2 I = |A|.I (Given A(adj A) = 2 I)

|A| = 2

Also, $|\text{adj } A| = |A|^{n-1}$

$= (2)^{3-1}$

$= (2)^2$

= 4

|adj A| = 4

7. We have I = $\int \sin^3 x \cos x dx$

Put $\sin x = t$

$\Rightarrow \quad \cos x dx = dt$

$\therefore \ I = \int \sin^3 x \cos x dx$

$\Rightarrow \ I = \int t^3 dt$

$= \frac{t^4}{4} + c \Rightarrow \ I = \frac{\sin^4 x}{4} + c$

OR

According to the question, $I = \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

Let, $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad \frac{1}{\sqrt{x}} dx = 2dt$

$I = \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

$= 2 \int \cos t dt$

$= 2 \sin t + C$

$= 2 \sin \sqrt{x} + C$

8. we have, $\int_0^a f(x)dx = \frac{a^2}{2} + \frac{a}{2} sin\, a + \frac{\pi}{2} cos\, a$
Differentiating w.r.t a, we get,
f(a) = a + $\frac{1}{2}(sin\, a + acos\, a) - \frac{\pi}{2} sin\, a$
put a = $\frac{\pi}{2}$, $f(\frac{\pi}{2}) = \frac{\pi}{2} + \frac{1}{2} - \frac{\pi}{2} = \frac{1}{2}$

9. Given differential equation may be written as (3x + 5y) $\frac{dy}{dx}$ - $4x^2$ = 0
In the given equation, the highest-order derivative is $\frac{dy}{dx}$ and its power is 1
∴ Its order = 1 and degree = 1.

OR

The given equation is $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right)$ = 0
In this equation, the highest-order derivative is $\frac{d^2y}{dx^2}$, so its order is 2
It has a term cos $\left(\frac{dy}{dx}\right)$, so its degree is not defined.

10. The given equation of curve is,
y = x^2 .... (i)
∴ Slope $= \frac{dy}{dx} = 2x$ .... (ii)
By the given condition, we have,
Slope = tan 45^o = 1 ..... (iii)
From (ii) and (iii), we have
2x = 1
$\Rightarrow \quad x = \frac{1}{2}$
∴ From (i)
$y = \frac{1}{4}$
Thus, the required point is
$\left(\frac{1}{2}, \frac{1}{4}\right)$

11. Domain of function is R.
f(x) = x^2
f'(x) = 2x
for x > 0 f'(x) > 0 i.e. increasing
for x < 0 f'(x) < 0 i.e. decreasing
hence it is neither increasing nor decreasing in R

12. Here, we have to find the sum of vectors. Given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$,

$\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

Sum of the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is given by the respective sum of components of the vectors.

Thus,

$$\vec{a} + \vec{b} + \vec{c} = (\hat{i} - 2\hat{j} + \hat{k}) + (-2\hat{i} + 4\hat{j} + 5\hat{k}) + (\hat{i} - 6\hat{j} - 7\hat{k})$$
$$= -4\hat{j} - \hat{k}$$

13. Given equation of plane is $\overrightarrow{r}.[(5-2t)\hat{i} + (3-t)\hat{j} + (25+t)\hat{k}] = 15$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) \cdot \left[(5-2t)\hat{i} + (3-t)\hat{j} + (25+t)\hat{k}\right] = 15$$

$(5 - 2t)x + (3 - t)y + (25 + t)z = 15$

14. Let $\vec{a}$ and $\vec{b}$ be the position vector of the points A(-1, 0, 2) and B(3, 4, 6)

Then, the required equation is,

$$\vec{r} = \vec{a} + \lambda\left(\vec{b} - \vec{a}\right)$$
$$= \left(-\hat{i} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 6\hat{k}\right)$$

15. Here,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

This product in the LHS is possible only because of the order of the first matrix is 3x3 and that of the second one is 3x1 which will result in the order of the product matrix as 3x1 which will be equal to the RHS.

$$\Rightarrow \begin{bmatrix} x \\ -y \\ -z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ x = 1, y = 0 and z = -1.

16. As 10 coins are tossed simultaneously, therefore the total number of outcomes are 2^{10} = 1024.

the favourable outcomes of getting at least 4 heads will be

${}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} = 848$

Therefore, the required probability

$$= \frac{\text{The favourable outcomes}}{\text{The total number of outcomes}}$$
$$= \frac{848}{1024}$$
$$= \frac{53}{64}$$

Section - II

17. i. (a) ₹ 2
ii. (c) ₹17
iii. (a) ₹7
iv. (a) ₹20
v. (a) ₹22

18. i. (a) 30 km
ii. (b) 20 km/hr
iii. (c) 15 km
iv. (a) $tan^{-1}(\frac{3}{4})$
v. (d) 20 km

Part - B Section - III

19. $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$
$= \tan^{-1}\tan\left(\pi + \frac{\pi}{8}\right)$
$= \tan^{-1}\left(\tan\left(\frac{\pi}{8}\right)\right)$
$= \frac{\pi}{8}$

20. The given points are collinear. Hence area of triangle is zero i,e
$\Leftrightarrow \begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0$
$\Leftrightarrow \begin{vmatrix} 5 & 5 & 1 \\ k-5 & -4 & 0 \\ 5 & 2 & 0 \end{vmatrix} = 0 \; [R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1]$
$\Leftrightarrow 1 \cdot [2(k-5) + 20] = 0$
$\Leftrightarrow 2k + 10 = 0 \Leftrightarrow k = -5$
Hence, k = -5.

OR

We know that
A (adj A) = |A| I_n = (adj A) A
$\Rightarrow \left(\frac{1}{|A|}A\right)$ (adj A) = I_n = (adj A) $\left(\frac{1}{|A|}A\right)$ [$\because$ |A| $\neq$ 0]
$\Rightarrow$ (adj A)$^{-1}$ = $\frac{1}{|A|}$ A

21. Given: $(x^2 - 5x + 8)(x^3 + 7x + 9)$

Let $y = (x^2 - 5x + 8)(x^3 + 7x + 9)$

$\Longrightarrow$ $y = (x^2 - 5x + 8)(x^3 + 7x + 9)$

$\Longrightarrow$ $y = x^5 + 7x^3 + 9x^2 - 5x^4 - 35x^2 - 45x + 8x^3 + 56x + 72$

$\Longrightarrow$ $y = x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72$

Now, differentiate both sides with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^5\right) - \frac{d}{dx}\left(5x^4\right) + \frac{d}{dx}\left(15x^3\right) - \frac{d}{dx}\left(26x^2\right) + \frac{d}{dx}(11x) + \frac{d}{dx}(72)$$

$$\frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11$$

22. Given: $f(x) = x\sqrt{1-x}$

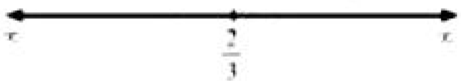
$\Rightarrow f'(x) = \sqrt{1-x} - \frac{x}{2\sqrt{1-x}} = \frac{2-3x}{2\sqrt{1-x}}$

To find the local maxima or minima of a function, we must have

$f'(x) = 0$

$\Rightarrow \frac{2-3x}{2\sqrt{1-x}} = 0$

$\Rightarrow x = \frac{2}{3}$



Since, f'(x) changes from positive to negative when x increases through $\frac{2}{3}, x = \frac{2}{3}$ is a point of maxima.

Thus, the local maximum value of f(x) at x = $\frac{2}{3}$ is given by $\frac{2}{3}\sqrt{1-\frac{2}{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}$.

23. We have,

$\int_0^a \sqrt{x}\,dx = \frac{2}{3}\left[x^{3/2}\right]_0^a = \frac{2}{3}a^{3/2}$ ....(i)

Let $I = \int_0^{\pi/2} \sin^3 x dx$, then

$$I = \int_0^{\pi/2} \frac{3\sin x - \sin 3x}{4} dx = \frac{1}{4}\int_0^{\pi/2}(3\sin x - \sin 3x)dx = \frac{1}{4}\left[-3\cos x + \frac{1}{3}\cos 3x\right]_0^{\pi/2}$$

$$\Rightarrow \quad I = \frac{1}{4}\left[\left(-3\cos\frac{\pi}{2} + \frac{1}{3}\cos\frac{3\pi}{2}\right) - \left(-3 + \frac{1}{3}\right)\right]$$

$$= \frac{1}{4}\left[0 - \left(-3 + \frac{1}{3}\right)\right] = \frac{1}{4}\left[3 - \frac{1}{3}\right] = \frac{2}{3} \ldots\ldots (ii)$$

It is given that $\int_0^a \sqrt{x}dx = 2a\int_0^{\pi/2} \sin^3 x dx$

$\Rightarrow \frac{2}{3}a^{3/2} = 2a\left(\frac{2}{3}\right)$

$\Rightarrow a^{3/2} = 2a \Rightarrow a^3 = 4a^2 \Rightarrow a^2(a-4) = 0$

$\Rightarrow a = 0, 4$ [Using(i) and (ii)]

When a = 4, we get

$\int_a^{a+1} xdx = \int_4^5 xdx = \left[\frac{x^2}{2}\right]_4^5 = \frac{25}{2} - \frac{16}{2} = \frac{9}{2}$

When a = 0, we get

$\int_a^{a+1} xdx = \int_0^1 xdx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$

Hence, $\int_a^{a+1} xdx = \frac{9}{2}\ or, \frac{1}{2}$

OR

Let I = $\int x^2 \cos x\, dx$

Now taking x^2 as the first function and cos x as the second function, we get

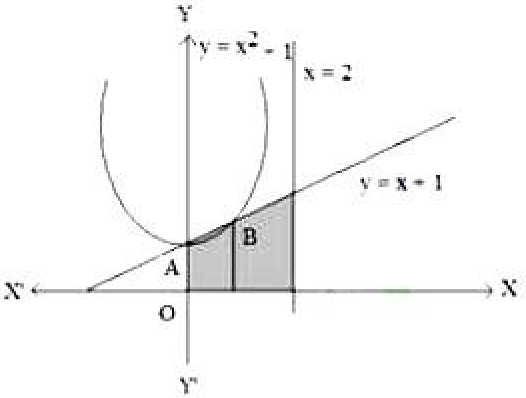
I = $x^2 \int \cos x\, dx - \int\left(\frac{d}{dx}x^2 \int \cos x dx\right) dx$

= $x^2 \sin x - \int 2x \sin x\, dx$

= $x^2 \sin x - 2 [x \int \sin x - \int \left\{\frac{d}{dx}(x) \int \sin x\, dx\right\} dx]$

= $x^2 \sin x - 2 [- x \cos x + \int \cos x\, dx]$

= $x^2 \sin x + 2x \cos x - 2 \sin x + C$

24. 

y = x^2 + 1

y = x + 1

x = 2

Area $= \int_0^1 (x^2+1)\,dx + \int_1^2 (x+1)\,dx$

$= \left[\left(\frac{x^3}{3} + x\right)\right]_0^1 + \left[\left(\frac{x^2}{2} + x\right)\right]_1^2$

$= \left[\left(\frac{1}{3} + 1\right) - 0\right] + \left[(2+2) - \left(\frac{1}{2} + 1\right)\right]$

$= \frac{23}{6}$ sq units.

25. We have, x $\cos^2$ y dx = y $\cos^2$ x dy

$\Rightarrow \frac{x}{\cos^2 x} dx = \frac{y}{\cos^2 y} dy$

$\Rightarrow$ x $\sec^2$ x dx = y $\sec^2$ y dy

Integrating both sides, we get

$\int x.\sec^2 x dx = \int y.\sec^2 y dy$ ...[using integration by parts]

$\Rightarrow x\int \sec^2 x dx - \int\left\{\frac{d}{dx}(x)\int \sec^2 x dx\right\}dx$

$= y\int \sec^2 y dy - \int\left\{\frac{d}{dy}(y)\int \sec^2 y dy\right\}dy$

$\Rightarrow$ x tan x - $\int$ tan x dx = y tan y - $\int$ tan y dy

$\Rightarrow$ x tan x - log |sec x| = y tan y - log |sec y| + C

$\Rightarrow$ x tan x - y tan y = log |sec x| - log |sec y| + C is the required solution.

26. Given: Equation of the curve $y = x^3 - 11x + 5$ ....(i)

Equation of the tangent y = x - 11....(ii)

$\Rightarrow x - y - 11 = 0$

From equation (i),

$\frac{dy}{dx} = 3x^2 - 11$

= Slope of the tangent at (x, y)

But from eq. (ii), the slope of the tangent $= \frac{-a}{b} = -\frac{-1}{-1} = 1$

$\therefore\ 3x^2 - 11 = 1$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

From equation (i), when x = 2, y = 8 - 22 + 5 = -9

And when x = -2, y = -8 + 22 + 5 = 19

Since (-2, 19) does not satisfy equation (ii), therefore the required point is (2, -9)

27. Suppose A, B and C are the points which are represented by;

$\vec{A} = -2i + 3j$; $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{C} = 7\hat{i} - \hat{k}$

Coordinates of these points are A(-2, 3, 0); B(1, 2, 3) and C(7, 0, -1).

So, $\overrightarrow{AB}$ = (1 + 2, 2 - 3, 3 - 0) = (3, -1, 3)

$\overrightarrow{AC}$ = (7 + 2, 0 - 3, -1 - 0) = (9, - 3, -1)

$\begin{vmatrix} 3 & -1 & 3 \\ 9 & -3 & -1 \end{vmatrix}$ = 3(1 + 9) + 1(-3 - 27) + 3(-9 + 9) = 0

Therefore, points A, B and C are collinear.

28. P(A ∩ B) = P(A) P(B) [because A and B are independent events]

$\frac{1}{6} = P(A)P(B)$

$\Rightarrow P(A) = \frac{1}{6P(B)}$ ...(i)

$P(\bar{A} \cap \bar{B})$ = [1 - P(A)][1 - P(B)]

$\Rightarrow \frac{1}{3}$ = [1 - P(A)][1 - P(B)]

$\Rightarrow \frac{1}{3} = \left[1 - \frac{1}{6P(B)}\right]$ [1 - P(B)] [on using (i)]

Let P (B) = x

$\Rightarrow \left(\frac{6x-1}{6x}\right)(1 - x) = \frac{1}{3}$

$\Rightarrow (6x - 1)(1 - x) = 2x$

$\Rightarrow 6x - 6x^2 - 1 + x = 2x$

$\Rightarrow 6x^2 - 5x + 1 = 0$

$\Rightarrow$ (2x - 1)(3x - 1) = 0

$\Rightarrow x = \frac{1}{2}$ or $x = \frac{1}{3}$

If $P(B) = \frac{1}{2}$, then $P(A) = \frac{1}{3}$ If $P(B) = \frac{1}{3}$, then $P(A) = \frac{1}{2}$

OR

Let p denote the probability of getting a total of 7 in a single throw of a pair of dice.

Therefore, we have,

$p = \frac{6}{36} = \frac{1}{6}$ {∵ The sum can be 7 in any one of the ways}

(1, 6), (6,1), (2,5), (5, 2), (3, 4), (4, 3)

$q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

Let X be a random variable denoting the number of successes in 7 throws of a pair of dice. Therefore, X is a binomial variate

with parameters n= 7 and $p = 1/6$ with

$P(X = r) = {}^7C_r\left(\frac{1}{6}\right)^r\left(\frac{5}{6}\right)^{7-r}, r = 0, 1, 2, \ldots, 7$ ......(i)

Therefore, we have,

Probability of 6 success = P(X = 6) $= {}^7C_6\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^{7-6}$ [Using (i)]

$= 35(\frac{1}{6})^7$

Section - IV

29. We observe the following properties of f.

Injectivity: Let x, y be any two elements in A. Then

$x \neq y \Rightarrow x|x| \neq y|y| \Rightarrow f(x) \neq f(y)$

So, f: A $\rightarrow$B is an injective map.

Surjectivity: We have,

$$f(x) = x\,|x| = \begin{cases} x^2, \text{ if } x \geq 0 \\ -x^2, \text{ if } x < 0 \end{cases}$$

If $0 \leq x \leq 1$, then f(x) = x^2 takes all values between 0 and 1 including these two points.

Also, if $-1 \leq x < 0$, then f(x) = $-x^2$ takes all values between -1 and 0 including -1. Therefore, f(x) takes every value between -1 and 1 including -1 and 1. So, range of f is same as its co-domain.

Hence, f: A $\rightarrow$ B is an onto function.

Thus, f: A$\rightarrow$ B is both one-one and onto.

Hence, it is a bijection.

30. When $x \neq \frac{\pi}{4}$

$\tan\left(\frac{\pi}{4} - x\right)$ and cot 2x are continuous in $\left[0, \frac{\pi}{2}\right]$

Thus, the quotient function

$\frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}$ is continuous in $\left[0, \frac{\pi}{2}\right]$ for each $x \neq \frac{\pi}{4}$

So, if f(x) is continuous at $x = \frac{\pi}{4}$, then it will be everywhere continuous in $\left[0, \frac{\pi}{2}\right]$

Now,

Let us consider the point $x = \frac{\pi}{4}$

Given, $f(x) = \frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot(2x)}, x \neq \frac{\pi}{4}$

We have

(LHL at $x = \frac{\pi}{4}$) = $\lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{4} - h\right)$

$$= \lim_{h \to 0}\left(\frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{4} + h\right)}{\cot\left(\frac{\pi}{2} - 2h\right)}\right)$$

$$= \lim_{h \to 0}\left(\frac{\tan(h)}{\tan(2h)}\right)$$

$$= \lim_{h \to 0}\left(\frac{\frac{\tan(h)}{h}}{\frac{2\tan(2h)}{2h}}\right)$$

$$= \frac{1}{2}\left(\frac{\lim_{h \to 0} \frac{\tan(h)}{h}}{\lim_{h \to 0} \frac{\tan(2h)}{2h}}\right) = \frac{1}{2}$$

(RHL at $x = \frac{\pi}{4}$) =

$$\lim_{x \to \frac{\pi}{4}^+} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{4} + h\right)$$

$$= \lim_{h \to 0} \left(\frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{4} - h\right)}{\cot\left(\frac{\pi}{2} + 2h\right)} \right)$$
$$= \lim_{h \to 0} \left(\frac{\tan(-h)}{-\tan(2h)} \right)$$
$$= \lim_{h \to 0} \left(\frac{\tan(h)}{\tan(2h)} \right)$$
$$= \lim_{h \to 0} \left(\frac{\frac{\tan(h)}{h}}{\frac{2\tan(2h)}{2h}} \right)$$
$$= \frac{1}{2} \left(\frac{\lim_{h \to 0} \frac{\tan(h)}{h}}{\lim_{h \to 0} \frac{\tan(2h)}{2h}} \right) = \frac{1}{2}$$

If f(x) is continuous at $x = \frac{\pi}{4}$, then

$f\left(\frac{\pi}{4}\right) = \frac{1}{2}$

Hence, for $f\left(\frac{\pi}{4}\right) = \frac{1}{2}$, the function f(x) will be everywhere continuous in $\left[0, \frac{\pi}{2}\right]$

31. First, write the conditions for the applicability of Rolle's theorem:

For a Real valued function 'f':

i. The function 'f' needs to be continuous in the closed interval [a,b].
ii. The function 'f' needs differentiable on the open interval (a,b).
iii. f(a) = f(b)

Then there exists at least one c in the open interval (a,b) such that f'(c) = 0.

Given function is:

$\Rightarrow f(x) = \log(x^2 + 2) - \log 3$ on $[-1, 1]$

We know that logarithmic function is continuous and differentiable in its own domain.

We check the values of the function at the extremum,

$\Rightarrow f(-1) = \log((-1)^2 + 2) - \log 3$

$\Rightarrow f(-1) = \log(1 + 2) - \log 3$

$\Rightarrow f(-1) = \log 3 - \log 3$

$\Rightarrow f(-1) = 0$

$\Rightarrow f(1) = \log(1^2 + 2) - \log 3$

$\Rightarrow f(1) = \log(1 + 2) - \log 3$

$\Rightarrow f(1) = \log 3 - \log 3$

$\Rightarrow f(1) = 0$

We have got $f(-1) = f(1)$. So, there exists a c such that $c \in (-1, 1)$ such that $f'(c) = 0$.

Let's find the derivative of the function f,

$\Rightarrow f'(x) = \frac{d(\log(x^2+2) - \log 3)}{dx}$

$\Rightarrow f'(x) = \frac{1}{x^2+2}\frac{d(x^2+2)}{dx} - 0$

$\Rightarrow f'(x) = \frac{2x}{x^2+2}$

We have f'(c) = 0

$\Rightarrow \frac{2c}{c^2+2} = 0$

$\Rightarrow 2c = 0$

$\Rightarrow c = 0 \in (-1,1)$

$\therefore$ Rolle's theorem is verified as per the condition of the same stated above.

OR

ATQ , $x^x + y^x = 1$

$\Rightarrow e^{\log x^x} + e^{\log y^x} = 1$ {As $e^{\text{Log } a} = a$}

$\Rightarrow e^{x \log x} + e^{x \log y} = 1$

Differentiating with respect to x using chain rule,

$\frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{x\log y}\right) = \frac{d}{dx}(1)$

$\Rightarrow e^{x\log x}\frac{d}{dx}(x\log x) + e^{x\log y}\frac{d}{dx}(x\log y) = 0$

$\Rightarrow e^{x\log x}\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] + e^{\log y^x}\left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = 0$

$\Rightarrow x^x\left[x\left(\frac{1}{x}\right) + \log x(1)\right] + y^x\left[x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y(1)\right] = 0$

$\Rightarrow x^x[1 + \log x] + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$

$\Rightarrow y^x \times \frac{x}{y}\frac{dy}{dx} = -[x^x(1 + \log x) + y^x \log y]$

$\Rightarrow \left(xy^{x-1}\right)\frac{dy}{dx} = -[x^x(1 + \log x) + y^x \log y]$

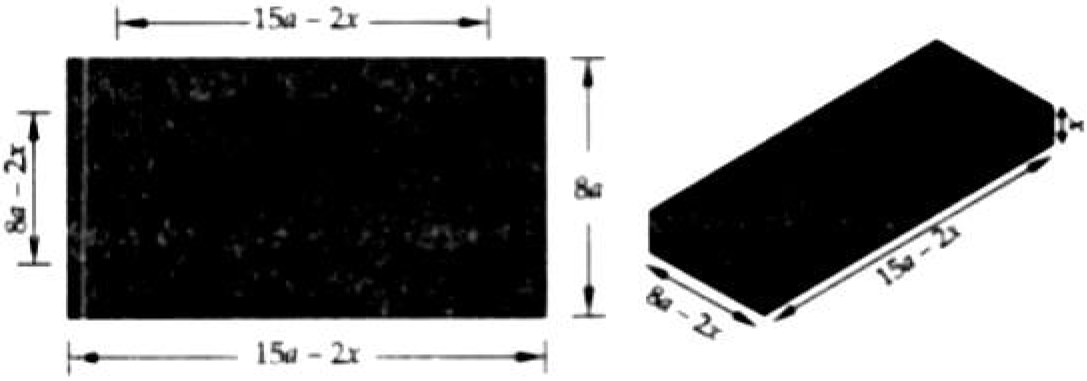
$\Rightarrow \frac{dy}{dx} = -\left[\frac{x^x(1+\log x)+y^x\log y}{xy^{x-1}}\right]$

LHS=RHS

Hence Proved.

32. Suppose the sides of rectangular sheet be 8a and 15a units respectively. Let the length of each side of the squares of same size removed from each comer of the sheet be x units. Then, the dimensions of the open box, formed by folding up the flaps,are: Length = 15a - 2x, breadth = 8a - 2x, height = x

Suppose V be the volume of the box formed. Then,



V = (15a - 2x) (8a - 2x) x

$\Rightarrow$ V = $120a^2$ x - $46ax^2$ + $4x^3$ diff w.r.t x

$\Rightarrow \frac{dV}{dx} = 120a^2 - 92ax + 12x^2$ and $\frac{d^2V}{dx^2} = -92a + 24x$

The critical points V are given by $\frac{dV}{dx} = 0$.

$\because \frac{dV}{dx} = 0$

$\Rightarrow 120a^2 = -92ax + 12x^2 \Rightarrow 30a^2 - 23ax + 3x^2 = 0 \Rightarrow (5a - 3x)(6a - x) = 0 \Rightarrow x = 6a, x = \frac{5a}{3}$

But x = 6a is not possible as for x = 6a breadth = 8a - 12a = -4a, which is not possible, thus, $\frac{5a}{3}$.

When $x = \frac{5a}{3}$, $\frac{d^2V}{dx^2} = -92a + 40a = -52a < 0$, Therefore, V is maximum when $x = \frac{5a}{3}$.

It is given that total area of four squares removed from each corner of the sheet is 100 sq. units.

$\because 4x^2 = 100 \Rightarrow x^2 = 25 \Rightarrow \frac{25a^2}{9} = 25 \Rightarrow a^2 = 9 \Rightarrow a = 3$

Therefore, the dimensions of the sheet are 15a = 45 and 8a = 24.

33. We have,

$I = \int_1^2 e^{2x}\left(\frac{1}{x} - \frac{1}{2x^2}\right) dx$

$I = \int_1^2 \frac{1}{x} \cdot e^{2x} - \int_1^2 \frac{1}{2x^2} \cdot e^{2x} dx$

$\Rightarrow I = I_1 - I_2$

Now, $I_1 = \int_1^2 \frac{1}{x} e^{2x}$ (By parts we have)

$\Rightarrow I_1 = [\frac{1}{x}]_1^2 \cdot \int_1^2 e^{2x} dx - \int_1^2 -\frac{1}{x^2} \frac{e^{2x}}{2} dx$

$\Rightarrow I_1 = \left[\frac{1}{x} \cdot \frac{e^{2x}}{2}\right]_1^2 + \int_1^2 \frac{1}{2x^2} e^{2x} dx$

$\Rightarrow I_1 = \left[\frac{1}{2x} e^{2x}\right]_1^2 + I_2$

As, $I = I_1 - I_2$

$\Rightarrow I = \left[\frac{1}{2x} e^{2x}\right]_1^2 - I_2 + I_2$

$\Rightarrow$ I = $\left[\frac{1}{2x}e^{2x}\right]_1^2$ = $\frac{1}{2}\left[\frac{1}{2}e^4 - e^2\right]$

$\Rightarrow$ I = $\frac{1}{4}$ e^2 (e^2 - 1)

34. According to the question, $A_n = \int_0^{\frac{\pi}{4}} (tan\ x)^n dx$

For 0 < x < $\frac{\pi}{4}$, 0 < $tan\ x$ <1

we have $0 < (tan\ x)^{n+1} < (tan\ x)^n$ for each n $\in N$

$\Rightarrow\ \int_0^{\frac{\pi}{4}} (tan\ x)^{n+1} dx < \int_0^{\frac{\pi}{4}} (tan\ x)^n dx$

$\Rightarrow\ A_{n+1} < A_n$

For n>2,

$A_n + A_{n+2} = \int_0^{\frac{\pi}{4}} [(tan\ x)^n + (tan\ x)^{n+2}] dx$

$= \int_0^{\frac{\pi}{4}} (tan\ x)^n (1 + tan^2\ x) dx$

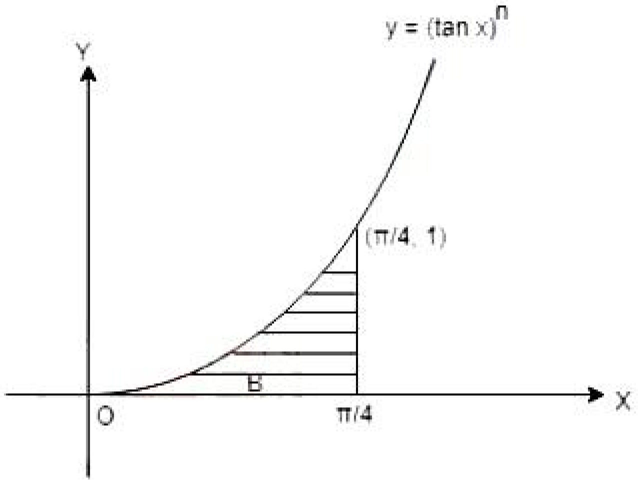
$(1 + tan^2\ x = sec^2 x)$

$= \int_0^{\frac{\pi}{4}} (tan\ x)^n sec^2\ x dx$

$(\int (tan\ x)^n sec^2\ x dx = \left[\frac{(tan\ x)^{n+1}}{n+1}\right])$

$= \left[\frac{(tan\ x)^{n+1}}{n+1}\right]_0^{\frac{\pi}{4}}$

= $\frac{1}{n+1}$



$\therefore$ If A_n is the area bounded by the curve y = $(tan\ x)^n$ and the lines x = 0, y = 0 and $x = \frac{\pi}{4}$. Then, for n > 2, $A_n + A_{n+2} = \frac{1}{n+1}$

OR

The slope of tangent to a curve y=f(x) at (x, f(x)) is 2x+1.

Therefore, we have,

$\frac{dy}{dx} = 2x + 1$

on integrating both sides, we get

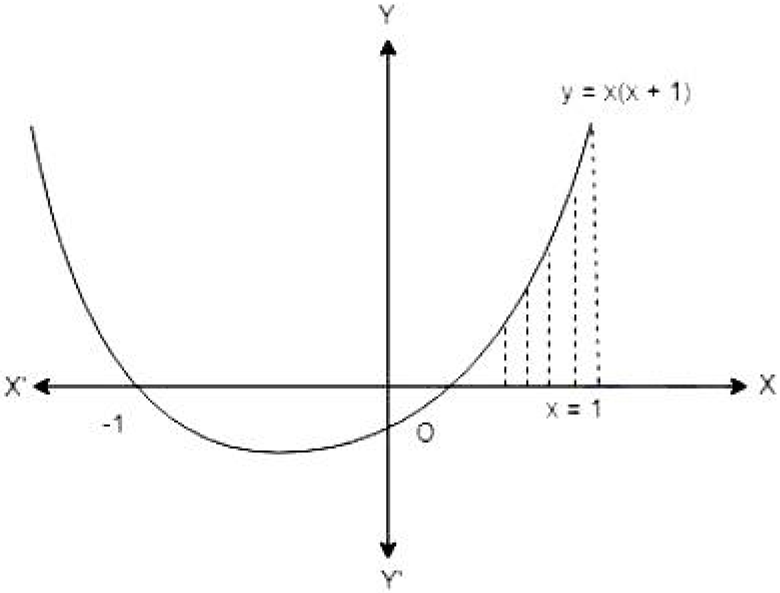
$\int dy = \int (2x + 1)dx$

y = x^2 + x + c which passes through (1, 2)

$\therefore$ 2 = 1 + 1 + c

c = 0

$\therefore$ y = x^2 + x



Thus, the required area bounded by X-axis, the curve and x = 1

= $\int_0^1 (x^2 + x)dx$

= $\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1$

= $\frac{1}{3} + \frac{1}{2}$

$= \frac{5}{6}$ sq.units

35. The given differential equation is,

$x^2dy + y(x + y)\,dx = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y(x+y)}{x^2} = -\left(\frac{y}{x} + \frac{y^2}{x^2}\right)$

$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$

$\Rightarrow$ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = -\left(\frac{vx}{x} + \frac{(vx)^2}{x^2}\right)$

$\Rightarrow x\frac{dv}{dx}$ = -v-v^2-v = -2v -v^2

$\Rightarrow \frac{dv}{2v+v^2} = -\frac{dx}{x}$

Integrating both the sides we get:

$\int \frac{dv}{2v+v^2} = -\int \frac{dx}{x} + \log|c|$

$\Rightarrow \int \frac{dv}{1+2v+v^2-1}$ = -ln |x| + ln|c|

$\Rightarrow \int \frac{dv}{(v+1)^2-1^2}$ + ln|x| = ln|c|

$\Rightarrow \frac{1}{2}\ln\left|\frac{v+1-1}{v+1+1}\right|$ + ln|x| = ln|c|

$\Rightarrow \ln\left|\frac{v+1-1}{v+1+1}\right|$ + 2 ln|x| = 2 ln|c|

Re-substituting the value of y = vx we get

$\Rightarrow \ln\left|\frac{\frac{y}{x}}{\frac{y}{x}+2}\right|$ + $\ln x^2$ = ln$|c|^2$

$\Rightarrow \ln\left|\frac{y}{y+2x}\right|$ + $\ln x^2$ = ln$|c|^2$

$\Rightarrow x^2y = c^2(y+2x)$, which is the required solution.

Section - V

36. Bag I: 3 red balls and 0 white ball.

Bag II: 2 red balls and 1 white ball.

Bag III: 0 red ball and 3 white balls.

Let E_1, $E_2$and E_3 be the events that bag I, bag II and bag III is selected and a ball is chosen from it.

$P(E_1) = \frac{1}{6}, P(E_2) = \frac{2}{6}$ and $P(E_3) = \frac{3}{6}$

i. Let E be the event that a red ball is selected. Then, Probability that red ball will be selected.

$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot \left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right)$
$= \frac{1}{6} \cdot \frac{3}{3} + \frac{2}{6} \cdot \frac{2}{3} + \frac{3}{6} \cdot 0$
$= \frac{1}{6} + \frac{2}{9} + 0$
$= \frac{3+4}{18} = \frac{7}{18}$

ii. Let F be the event that a white ball is selected.

$\therefore P(F) = P(E_1) \cdot P\left(\frac{F}{E_1}\right) + P(E_2) \cdot P\left(\frac{F}{E_2}\right) + P(E_3) \cdot P\left(\frac{F}{E_3}\right)$
$= \frac{1}{6} \cdot 0 + \frac{2}{6} \cdot \frac{1}{3} + \frac{3}{6} \cdot 1 = \frac{1}{9} + \frac{3}{6} = \frac{11}{18}$

Note: P(F) = 1 - P(E) $= 1 - \frac{7}{18} = \frac{11}{18}$ [since, we know that P(E) + P(F) = 1]

OR

There are three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and take out a coin. If the coin is of gold, then we have to find the probability that the other coin in box is also of gold.

Let us define the events as

E_1: Box I is selected

E_2: Box II is selected

E_3 · Box III is selected

A: The drawn coin is a gold coin

Since events E_1, E_2 and E_3 are mutually exclusive and exhaustive events.

$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Now, $P(A/E_1)$

= Probability that a gold coin is drawn from box I

$= \frac{2}{2} = 1$ [$\because$ box I contain both gold coins]

$P(A/E_2) =$ Probability that a gold coin is drawn from box II=0[$\because$ box II has both silver coins]

and $P(A/E_3)$ = Probability that a gold coin is drawn from box III $= \frac{1}{2}$

[$\because$ box III contains 1 gold and 1 silver coin]

The probability that other coin in box is also of gold = The probability that the drawing gold coin from bag I

$= P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{[P(E_1)P(A/E_1) + P(E_2)P(A/E_2)]}$

[by Baye' s theorem]

$$= \frac{\frac{1}{3} \times 1}{\left(\frac{1}{3} \times 1\right) + \left(\frac{1}{3} \times 0\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)}$$

$$= \frac{1}{1 + 0 + \frac{1}{2}} = \frac{1}{3/2} = \frac{2}{3}$$

Hence, the required probability is $\frac{2}{3}$.

37. Suppose the equation of the variable plane is

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$....(i)

Above plane (i) meets the X-axis, Y-axis and Z-axis at the point A(a, 0, 0), B (0, b, 0) and C(0, 0, c) respectively and let (α, β, γ) be the coordinates of the centroid of ΔABC.

$\Rightarrow \alpha=\frac{a+0+0}{3}, \beta=\frac{0+b+0}{3}$

and $\gamma=\frac{0+0+c}{3}$

$\Rightarrow \quad \alpha=\frac{a}{3}, \beta=\frac{b}{3}$ and $\gamma=\frac{c}{3}$

$\Rightarrow \quad a=3\alpha, b=3\beta$ and $c=3\gamma$ .....(ii)

$\therefore$ 3p = length of the perpendicular from (0, 0, 0) to the plane (i)

$\Rightarrow 3p=\frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$

$\Rightarrow \quad 3p=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$

$\Rightarrow \quad \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{9p^2}$

$\Rightarrow \quad \frac{1}{9\alpha^2}+\frac{1}{9\beta^2}+\frac{1}{9\gamma^2}=\frac{1}{9p^2}$ [using Equation (ii)]

$\Rightarrow \quad \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{1}{p^2}$

The locus of the centroid is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

OR

Given equation of plane is,

$\vec{r}\cdot(\hat{i}-2\hat{j}+2k)$ + 6 = 0

$\vec{r}\cdot(\hat{i}-2\hat{j}+2k)$ = -6

Multiplying both the sides by (- 1),

$\vec{r}\cdot(-\hat{i}+2\hat{j}-2\hat{k})$ = 6

$\vec{r}\cdot\vec{n}$ = 6 ...(i)

Here, $\vec{n}=-\hat{i}+2\hat{j}-2\hat{k}$

$|\vec{n}|=\sqrt{(-1)^2+(2)^2+(-2)^2}$

$=\sqrt{1+4+4}$

$=\sqrt{9}$

= 3

Dividing equation (i) by $|\vec{n}|$ = 3 both the sides,

$\vec{r}\cdot\frac{\vec{n}}{|\vec{n}|}=\frac{6}{|\vec{n}|}$

$\vec{r} \cdot \frac{1}{3}(-\hat{i} + 2\hat{j} - 2\hat{k}) = \frac{6}{3}$

$\vec{r}\left(-\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = 2$ ...(ii)

We know that, equation of a plane at distance d from origin and normal to unit vector $\hat{n}$ is

$\vec{r} \cdot \hat{n} = d$ ...(iii)

Comparing equation (ii) and (iii),

d = 2

$\hat{n} = -\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$

So, normal form of equation of plane is,

$\vec{r} \cdot \left(-\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = 2$

Length of perpendicular from origin to plane

d = 2 unit.

38.

Let the manufacture produce x nuts and y bolts

$Z = 17.50x + 7y$

$x + 3y \leq 12$

$3x + y \leq 12$

$x, y \geq 0$

Solving, $x + 3y = 12$ and $3x + y = 12$,we get,

$x = 3, y = 3$

Maximum profit

Z = Rs 17.50(3)+7(3) = Rs 73.50

OR

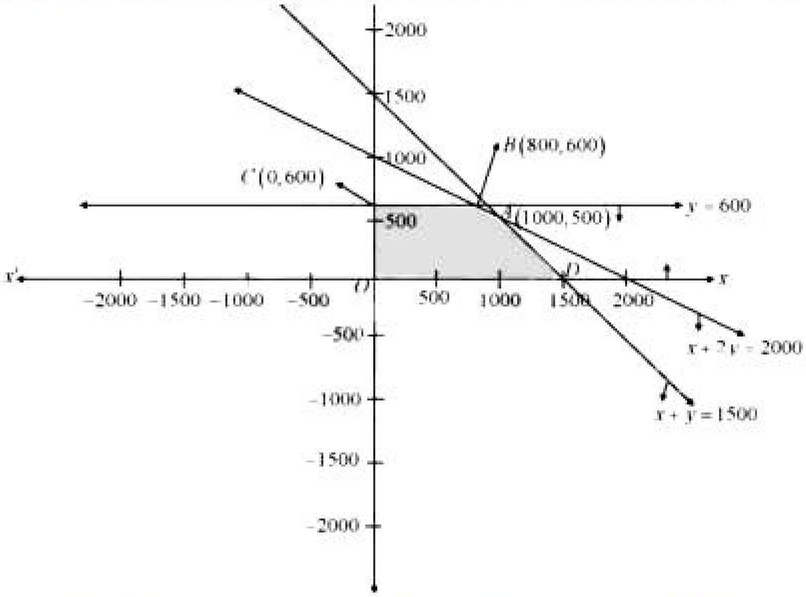
Given,

Objective function, Maximize $z = 3x + 5y$

subject to the constraints

$x + 2y \leq 2000$, $x + y \leq 1500$, $y \leq 600$, $x \geq 0$ and $y \geq 0$

Now, draw the line $x + 2y = 2000$, $x + y = 1500$, and $y = 600$



and shaded region is the feasible region satisfied by above inequalities. Here, the feasible region is bounded.

Now,

Corner points (x, y)	Z = 3x + 5y
(0, 0)	0
(1500, 0)	3.1500 + 5.0 = 4500
(1000, 500)	3.1000 + 5.500 = 5500
(0, 500)	0 + 500.5 = 2500

Hence the maximum value of z is 5500, which occurs at A(1000, 500).