CBSE Test Paper 05

CH-12 Three Dimensional Geometry

1. The plane XOZ divides the join of (1 , -1 , 5) and (2 , 3 ,4) in the ratio k : 1 , then the value of k is

   a. 3

   b. 1/ 3

   c. - 3

   d. - 1/ 3

2. The coordinates of the foot of perpendicular from the point A (1, 1 , 1) , on the line joining the points B (1 , 4 , 6) and C (5 , 4 , 4) are

   a. (-3 , 0 , 4)

   b. (- 4 , 5 , 3)

   c. (3 , 4 ,5)

   d. (4 , 5, 3)

3. The points (1 , 1 , 0) , (0 , 1 , 1) , (1 , 0 , 1) , and (2/3 , 2/3 , 2/3) are

   a. none of these

   b. coplanar

   c. non coplanar

   d. the vertices of a parallelogram

4. The points A (0 , 2 , 0) , B ($\sqrt{3}$, 1 , 0) , and C($\frac{1}{\sqrt{3}}, 1, \frac{2\sqrt{2}}{\sqrt{3}}$) are the vertices of

   a. a scalene triangle

b. none of these

c. an equilateral triangle

d. an isosceles triangle

5. The lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} and \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5} are$

   a. none of these

   b. parallel

   c. intersecting

   d. skew

6. A tetrahedron has vertices at O (0 , 0 ,0), A (1 , 2, 1) , B (2 , 1 , 3) and C (- 1 , 1 , 2) , then the angle between the faces OAB and ABC will be

   a. $cos^{-1}\left(\frac{19}{35}\right)$

   b. 90^0

   c. $cos^{-1}\left(\frac{17}{31}\right)$

   d. 30^0

7. Fill in the blanks:

   The equation of z-axis, are ________.

8. Fill in the blanks:

   If the distance between the point (a, 2, 1) and (1, -1, 1) is 5, then a = ________.

9. Fill in the blanks:

   The distance of the point P(2, 3, 5) from the xy-plane is ________.

10. Find the ratio in which the line segment joining the points (2, 4, -3) and (-3, 5, 4) divided by the XY-plane.

11. Find the octant in which the points (-3, 1, 2) and. (-3, 1, -2) lie.

12. Prove by using distance formula that the points P (1,2,3), Q (-1, -1, -1) and R (3,5,7) are collinear.

13. Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled isosceles triangle.

14. Find the equation of the set of points which are equidistant from the points B(1, 2, 3) and C(3, 2, -1).

15. Find the distance between the following pairs of points:
    (i) (2, 3, 5) and (4, 3, 1)
    (ii) (-3, 7, 2) and (2, 4, -1)
    (iii) (-1, 3, -4) and (1, -3, 4)
    (iv) (2, -1, 3) and (-2, 1, 3)

Solution

1. (b) 1/ 3

   Explanation: since XOZ plane is given y coordinate must be zero

   k x 3 +1 x(-1) / k+1 =0

   3k -1 = 0

   k=1/3

2. (c) (3 , 4 ,5)

   Explanation: Equation of line BC

   $\frac{x-1}{4}=\frac{y-4}{0}=\frac{z-6}{-2}=\lambda$

   hence $x=4\lambda+1, y=4\ and\ z=-2\lambda+6$

   Let M (x,y,z) be any point on line BC

   Since AM Perpendicular to BC

   drs of $AM=(4\lambda+1-1,0,-2\lambda+6-1)4(4\lambda)+0-2(-2\lambda+5)=0$

   $20\lambda=10$

   $\lambda=1/2$

   Hence M (x, y, z) = (4*1/2+1,4, -2*1/2+6)

   M = (3,4,5)

3. (b) coplanar

   Explanation:
   $\vec{a}=1\hat{i}+1\hat{j}+0\hat{k}),\vec{b}=(0\hat{i}+1\hat{j}+1\hat{k}),\vec{c}=((\hat{i}+0\hat{j}+1\hat{k}),\vec{d}=(2/3\hat{i}+2/3\hat{j}+2$
   $\overrightarrow{AB}=-1\hat{i}+0\hat{j}+1\hat{k},\overrightarrow{AC}=-0\hat{i}+-1\hat{j}+1\hat{k},\overrightarrow{AD}=-1/3\hat{i}+-1/3\hat{j}+2/3\hat{k}$

Consider $\begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ -1/3 & -1/3 & 2/3 \end{vmatrix} = -1(-1/3)+1(-1/3)$

=0

Hence points are coplanar

4. (c) an equilateral triangle

   Explanation: To check which type of triangle these points form we need to check the distance between the two points

   AB=$\sqrt{(\sqrt{3})^2+1+0}=2$

   BC = $\sqrt{(\frac{1}{\sqrt{3}}-\sqrt{3})^2+(\frac{2\sqrt{2}}{\sqrt{3}})^2}=2$,

   similarly AC=2 by distance formula between two points

   Since all sides are same so the triangle is equilateral

5. (c) intersecting

   Explanation:

   here (a1,b1,c1)=(2,3,4)

   here (a2,b2,c2)=(3,4,5)

   Consider $\begin{vmatrix} x2-x1 & y2-y1 & z2-z1 \\ a1 & b1 & c1 \\ a2 & b2 & c2 \end{vmatrix}$

   We get

   $\begin{vmatrix} 0 & 0 & 0 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$

   Since the shortest distance is zero hence the lines are intersecting

6. (a) $cos^{-1}\left(\frac{19}{35}\right)$

   Explanation:

let n1 and n2 be the normals to the plane OAB and ABC

$$\vec{n}_1 = \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

n1=(5,-1,-3)

$$\vec{n}_2 = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$

n2=(1,-5,-3)

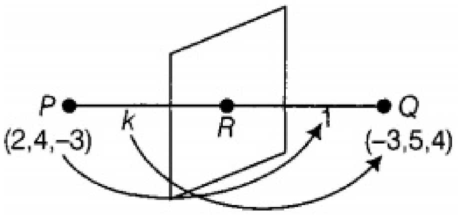
$$cos\theta = \frac{\vec{n}_1 . \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

$$= \frac{5+5+9}{\sqrt{35}.\sqrt{35}}$$

$$= \frac{19}{35}$$

$$\theta = cos^{-1}(\tfrac{19}{35})$$

7. x = 0, y = 0
8. a = 5 or -3
9. 5
10. Suppose, the line segment joining the points P (2, 4, -3) and Q (-3, 5, 4) is divided by the XY-plane at a point R in the ratio k: 1. Then, the coordinates of R are
$\left(\frac{-3k+2}{k+1}, \frac{5k+4}{k+1}, \frac{4k-3}{k+1}\right)$
$\left[\because \text{ coordinates of internal division, } \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right)\right]$
Since, R lies on XY-plane, so its z-coordinate must be zero i.e., z = 0



$\therefore \quad \frac{4k-3}{k+1} = 0$

$\Rightarrow$ 4k - 3 = 0

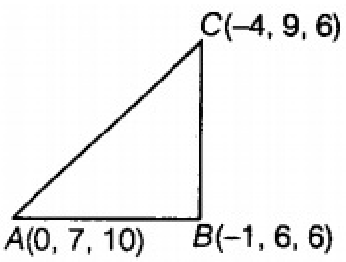
$\Rightarrow \quad k = \frac{3}{4}$

So, the required ratio is $\frac{3}{4} : 1$ or 3:4 internally.

11. The point (-3, 1, 2) named as (X'OYZ) and thus lies in II octant and the point (-3, 1, - 2) named as (X'YZ') and thus lies in VI octant.

12. Using distance formula, we obtain
PQ = $\sqrt{(-1-1)^2+(-1-2)^2+(-1-3)^2}$ = $\sqrt{4+9+16}$ = $\sqrt{29}$
QR = $\sqrt{(3+1)^2+(5+1)^2+(7+1)^2}$ = $\sqrt{16+36+64}$ = $\sqrt{116}$ = 2 $\sqrt{29}$
and, PR = $\sqrt{(3-1)^2+(5-2)^2+(7-3)^2}$ = $\sqrt{4+9+16}$ = $\sqrt{29}$
Clearly, QR = PQ + PR. Therefore, points Q, P, R are collinear and P lies between Q and R.

13. Let A (0, 7, 10), B (-1, 6,6) and C (-4, 9, 6) be the given points. We have,



Now, $AB = \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}$ [$\because$ distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$]

$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}$

$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$

and $AC = \sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}$

$= \sqrt{16+4+16}$

$\therefore \quad AC = \sqrt{36} = 6$...... (i)

Now, AB^2 + BC^2 $= (3\sqrt{2})^2 + (3\sqrt{2})^2$ = 18 + 18 = 36

$\therefore$ AB^2 + BC^2 = AC^2 [from Eq. (i)]

Also, AB = BC $= 3\sqrt{2}$

Hence, ABC is a right isosceles triangle.

14. Let A(x, y, z) be any point which is equidistant from points B(1, 2, 3) and C(3, 2, -1)

Then

$AB = \sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2}$

$AC = \sqrt{(x-3)^2 + (y-2)^2 + (z+1)^2}$

It is given that AB = AC

$\therefore \sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2} = \sqrt{(x-3)^2 + (y-2)^2 + (z+1)^2}$

$\Rightarrow$ $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = (x - 3)^2 + (y - 2)^2 + (z + 1)^2$

$\Rightarrow$ $x^2 + 1 - 2x + z^2 + 9 - 6z = x^2 + 9 - 6x + z^2 + 1 + 2z$

$\Rightarrow$ - 2x - 6z + 10 = -6x + 2z + 10

$\Rightarrow$ - 2x - 6z + 6x - 2z = 0

$\Rightarrow$ 4x - 8z = 0

$\Rightarrow$ x - 2z = 0

15. Let A(2, 3, 5) and B(4, 3, 1) be two points. Then

$AB = \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2} = \sqrt{4+0+16} = \sqrt{20} = 2\sqrt{5}$ units

(ii) Let A(-3, 7, 2) and B(2, 4, -1) be two points. Then

$AB = \sqrt{[2-(-3)]^2 + (4-7)^2 + (-1-2)^2}$

$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2} = \sqrt{25+9+9} = \sqrt{43}$ units

(iii) Let A(-1, 3, -4) and B(1, -3, 4) be two points. Then

$AB = \sqrt{[1-(-1)]^2 + (-3-3)^2 + [4-(-4)]^2}$

$= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26}$

(iv) Let A(2, -1, 3) and B(-2, 1, 3) be two points. Then

$AB = \sqrt{(-2-2)^2 + [1-(-1)]^2 + (3-3)^2}$

$= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$

$= \sqrt{16+4+0} = \sqrt{20} = 2\sqrt{5}$ units