CHAPTER 02

Analysis of Simple Trusses

2.1 Plane Trusses

A rigid framework made a number of bars or rods is known as plane truss. Truss is designed to resist geometrical distortion under any applied system of loading. The ends of bars or rods are joined by riveting, welding or by nuts and bolts. The bars used in trusses are slender (cross-sectional dimensions quite small compared to length) and are usually of angle section, T-section, I-section etc. Theoretically truss members carry only axial forces they are either in tension or in compression and are two force members. A truss made up of members just sufficient to keep it in equilibrium. Trusses are used in the bridge, roofing, workshops and industrial buildings, and bridge etc.

2.2 Perfect Truss

A truss which is made up of members just sufficient to keep it in equilibrium, when loaded without distortion is called perfect/stable/sufficient truss.

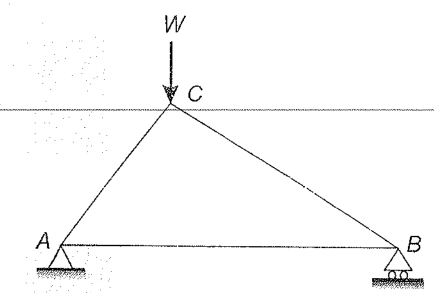


Fig. 2.1

The simplest perfect truss is a triangle, which contain three members and three joints as shown in fig. 2.1. If such a truss is loaded its shape will not be distorted. If we want to increase a joint to a triangular truss, we require two members. A truss should satisfy the Maxwell's Truss equation $m = 2j - 3$ where m is number of members and j is number of joints.

The truss which do change in shape when loaded is called an imperfect/unstable/deficient truss. For an imperfect truss $m < 2j - 3$. Such type of structure is unstable and cannot resist distortion.

A *redundant truss* is that structure which contain more number of members than required to prevent distortion of its shape when loaded with external load, for a redundant structure $m > 2j - 3$.

2.3 Type of Supports

1. **Roller Supports** : Roller support are frictionless and provide a reaction at right angles to the roller base as shown in fig. 2.2. Roller supports are frictionless and free to rotate and translate along the surface upon which the roller rests and it prevents motion only in the direction perpendicular to its base.

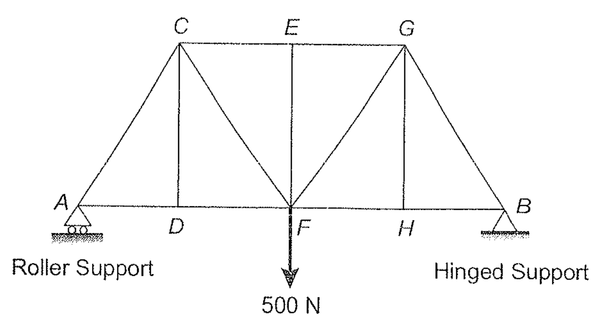


Fig. 2.2

2. Hinged Support : The hinged support is capable of resisting forces in any direction and the beam has no translatory motion in any direction, at a hinged support. This reaction can be split into two components acting in the horizontal and vertical directions (fig. 2.3) and can be determined using equation of equilibrium.
   This support cannot resist rotation of beam.

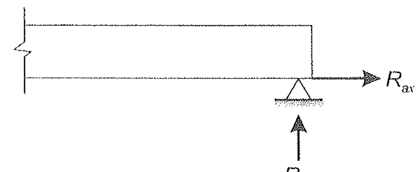


Fig. 2.3

2.4 Analysis of a Truss

The analysis of a truss involves the determining of the reaction at the supports and the stresses induced in the members due to applied loads. Assumptions made in the analysis of a truss.

1. All the members comprising the truss are rigid and lie in one plane if it is a plane truss.
2. The members are slender.
3. Each member is of uniform cross-section.
4. The members are subjected to pure axial forces and cannot develop moments at the ends.
5. The external loads and reactions act at the joints only.
6. The self weight of the members is neglected.
7. The forces are transmitted from one member to another through frictionless pins fitting perfectly in the members.

2.5 Method of Joints

In this method all forces acting on a joint must add to zero, i.e., the sum of all the vertical forces as well as the horizontal forces acting on the joint is equated to zero. The steps involved are :

1. A FBD of the entire truss is drawn and the reaction at the supports is determined using three equilibrium equation. That is sum of horizontal force is zero, sum of vertical force is zero, and moment acting at any point in truss is zero.
2. Each joint is treated separately as a free body. A certain direction of forces acting on the joint is assumed and the magnitude of forces is determined by applying the conditions of equilibrium. If the magnitude of particular force comes out positive, the assumption is of its direction is correct. However if the magnitude of the force comes out negative, the actual direction of force is reversed, i.e., opposite to assumed.
3. Start is made from a joint where there are not more than two members in which the forces are unknown, and the process is repeated from one joint to another until all the unknowns have been determined.

If the members pulls the joint to which it is connected then it is subjected to tensile force where as if the member pushes the joint to which it is connected then it is subjected to compression force. A member under tension is called a *tie*, and a member under compression is called a strut.

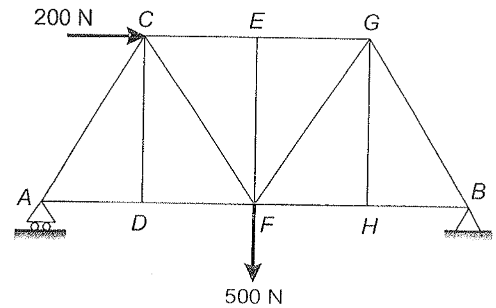
2.6 Method of Sections

The method of sections is easy and convenient when the forces in a few members of the truss is to be determined. The various steps involved are:

1. The truss is split into two parts by passing an imaginary section.
2. The imaginary section has to be such that it does not cut more than three members in which the forces are to be determined.

3. The conditions of equilibrium are applied for the one part of the truss and the unknown force in the member is determined.
4. Nature of the force in any member is determined similar to method of joints.

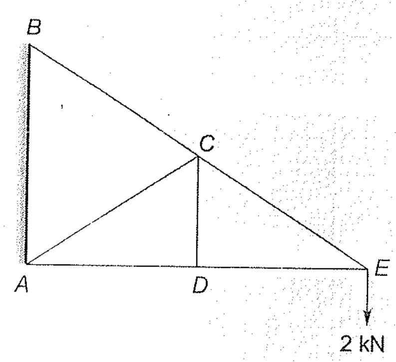
Example 2.1 For the truss shown in fig. determine the zero force member.



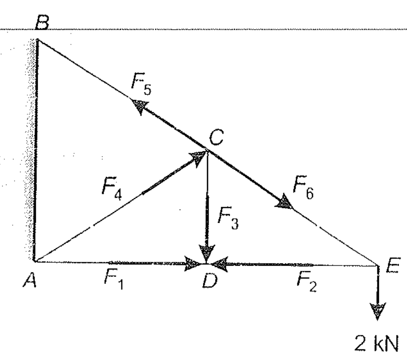
Solution:

Consider joint D for the truss shown in fig. We assume force F_1 in member DC, F_2 in member DA and F_3 in member DF. This joint is unloaded and the forces F_2 and F_3 are collinear so F_1 must be zero. Similarly forces in the members EF and HG are zero.

Example 2.2 For the truss shown in fig. determine the zero force member.



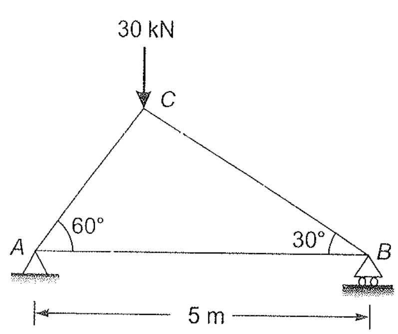
Solution:



Considering joint D of the cantilever truss shown in fig. Since the joint is unloaded and the forces F_1 and F_2 are collinear so F_3 must be zero. At joint C the forces acting are F_3, F_4, F_5 and F_6 but $F_3 = 0$. Thus at this joint acting forces are F_4, F_5 and F_6 only. This joint is unloaded and out of these three forces, two forces namely F_5 and F_6 are collinear. Thus the third force F_4 is zero.

Thus the members AC and DC will not be subjected to any load.

Example 2.3 Determine the forces in all the members of truss with the loading and support system shown in fig.



Solution:

At end B roller support provides a reaction R_b at right angles to the roller base. At end A due to hinged support, reaction can have two component acting in the horizontal and vertical direction. Since the load of 30 kN acts vertically downward, the horizontal component of reaction at A is zero and there will be only vertical reaction R_a.

Now consider the free-body diagram of the whole truss shown in fig. (a)

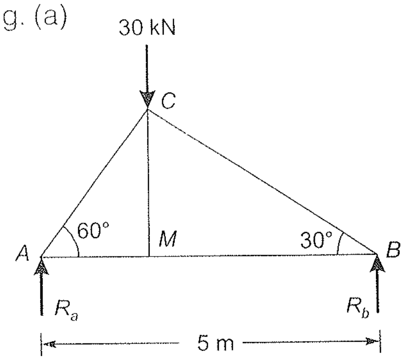


Fig. (a)

The triangle ABC is a right angled triangle with angle $ACB = 90°$

Thus
$$AC = AB\cos 60° = 5 \times 0.5 = 2.5$$

Distance of line of action of 30 kN force from A,
$$AM = AC\cos 60° = 2.5 \times 0.5 = 1.25 \text{ m}$$

Taking moments about A, we get
$$R_b \times 5 = 30 \times 1.25$$
$$R_b = \frac{30 \times 1.25}{5} = 7.5 \text{ kN}$$

Since
$$R_a + R_b = 30 \text{ kN}$$
Thus
$$R_a = 30 - 7.5 = 22.5 \text{ kN}$$

Joint A:

Consider the free body diagram of joint A with the direction of forces assumed as shown in fig. (b).

Resolving forces in vertical direction we get

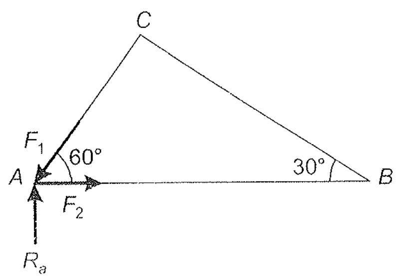


Fig. (b)

$$R_a = F_1 \sin 60°$$
Thus,
$$F_1 = \frac{R}{\sin 60°} = \frac{22.5}{0.866} = 25.97 \text{ kN (compressive)}$$

Resolving force in horizontal direction we get
$$F_2 = F_1 \cos 60° = 25.97 \times 0.5 = 12.99 \text{ kN (tensile)}$$

The force F_1 is acting towards the pin which means that the member AC is in compression.

The force F_2 is acting away from the pin which means that the member AB is in tension.

Joint B:

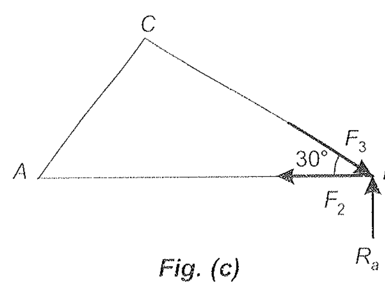


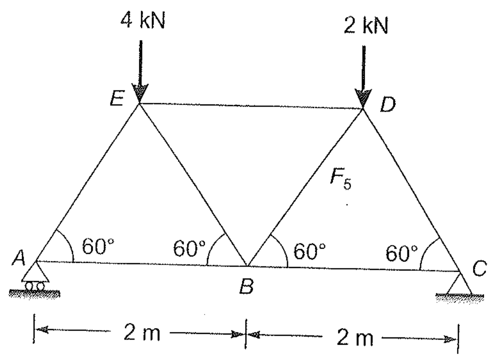
Fig. (c)

Consider the free body diagram of joint B as shown in fig. (c). The force F_2 in member AB has already been calculated above and found to be tensile. Hence force F_2 will pull the joint B and will be directed away from it as shown in fig. (c).

Resolving forces in horizontal direction
$$F_2 = F_3 \cos 30°$$
or
$$F_3 = \frac{F_2}{\cos 30°} = \frac{12.99}{0.866} = 15 \text{ kN (compressive)}$$

The force F_3 is acting towards the joint B which means that the member BC is in compression.

Example 2.4 Determine the reactions and the forces in each member of a truss supporting two loads as shown in fig.



Solution:

The reaction at the hinged support can have components acting in the horizontal and vertical directions. Since the applied loads are only in vertical direction, so reaction will be only in vertical direction only.

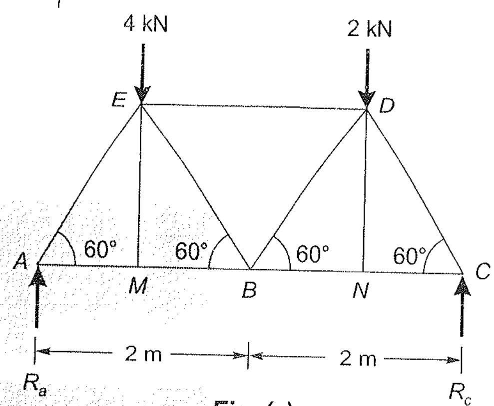


Fig. (a)

Now consider the *FBD* of the whole truss shown in fig. (a). Each of the triangular (element) is an equilateral triangle of each side equal to 2 m. Then

Distance of line of action of 4 kN force from *A*,

$$AM = AE \cos 60° = 2 \times 0.5 = 1 \text{ m}$$

Distance of line of action of 2 kN force from *A*,

$$AN = AB + BN = AB + BD \cos 60°$$

$$= 2 + 2 \times 0.5 = 3 \text{ m}$$

Taking moments about *A*, we get

$$R_c \times 4 = 2 \times 3 + 4 \times 1 = 10$$

or

$$R_c = \frac{10}{4} = 2.5 \text{ kN}$$

Since

$$R_a + R_c = 4 + 2 = 6$$

Thus

$$R_a = 6 - 2.5 = 3.5 \text{ kN}$$

Joint A :

Consider the *FBD* of joint *A* with the direction of forces assumed as shown in fig. (b). We have started with this joint because there are only two unknown forces acting at this joint.

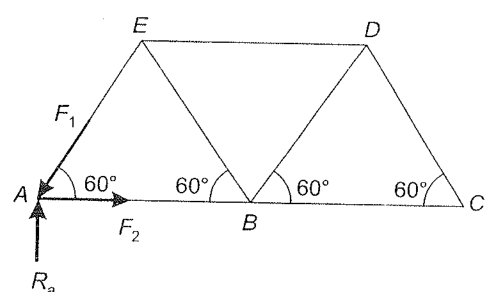


Fig. (b)

Resolving forces vertically we get

$$R_a = F_1 \sin 60°$$

or

$$F_1 = \frac{R_a}{\sin 60°}$$

$$= \frac{3.5}{0.866} = 4.04 \text{ kN (compressive)}$$

Resolving forces horizontally we get

$$F_2 = F_1 \cos 60°$$

$$= 4.04 \times 0.5 = 2.02 \text{ kN (tensile)}$$

The magnitude of forces F_1 and F_2 are both coming out to be positive and therefore assumed direction of forces are correct. The force F_1 is acting towards the pin *A* which means that member *AE* is in compression. The force F_2 is pulling the pin *A* which means that member *AB* is in tension.

Joint C : Now consider the *FBD* of joint *C* with the direction of forces assumed as shown in fig. (c). Analysis of this joint is also possible because there are only two unknown forces F_3 and F_4 acting at this joint.

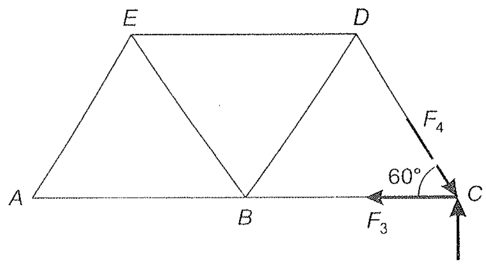


Fig. (c)

Resolving forces vertically we get

$$R_c = F_4 \sin 60°$$

or $$F_4 = \frac{R_c}{\sin 60°} = \frac{2.5}{0.866} = 2.88 \text{ kN (Comp)}$$

Resolving forces horizontally we get

$$F_3 = F_4 \cos 60° = 2.88 \times 0.5 = 1.44 \text{ kN (tensile)}$$

Joint B : Consider the *FBD* of joint *B* with the two known values F_2 in *AB* and F_3 in *BC* as shown in fig (d).

Resolving forces horizontally we get

$$F_3 + F_6 \cos 60° = F_2 + F_5 \cos 60°$$

or $$1.44 + F_6 \times 0.5 = 2.02 + F_5 \times 0.5$$

or $$(F_6 - F_5) \times 0.5 = 2.02 - 1.44 = 0.58$$

or $$F_6 - F_5 = 1.16 \qquad ...(1)$$

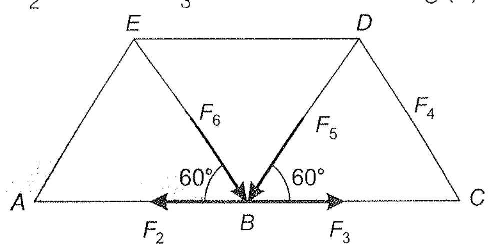


Fig. (d)

Resolving forces vertically we get

$$F_5 \sin 60° + F_6 \sin 60° = 0$$

$$F_6 + F_5 = 0 \qquad ...(2)$$

From equation (1) and (2) we get

$$F_5 = -\frac{1.16}{2} = -0.58 \text{ kN}, \; F_6 = \frac{1.16}{2} = 0.58 \text{ kN}$$

The magnitude of F_5 is coming out to be negative. So force is in opposite direction to assumed. Thus force F_5 in member *BD* is 0.58 kN tensile and force F_6 in member *BE* is 0.58 kN compressive.

Joint D: Now consider the *FBD* of joint *D* with the known values F_4 in member *DC* and F_5 in member *DB*.

Resolving forces horizontally we get:

$$F_7 + F_5 \cos 60° = F_4 \cos 60°$$

or $$F_7 = F_5 \cos 60° + F_4 \cos 60°$$

$$= 0.58 \times 0.5 + 2.88 \times 0.5$$

$$= 0.29 + 1.44 = 1.73 \text{ kN(Comp)}$$

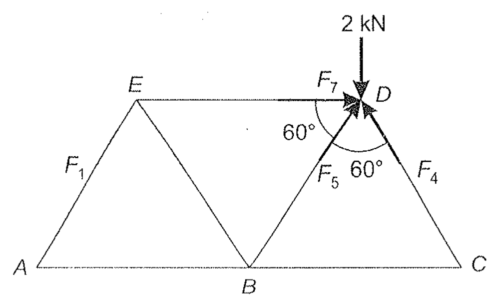
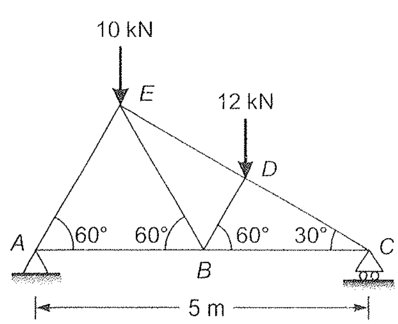


Fig. (e)

Note: There is no need to consider the equilibrium of joint *E* as the forces in all the members have been determined.

Example 2.5 Determine the forces in all the members of the truss loaded and supported as shown in fig.



Solution:

The reactions at the hinged support will be only in vertical direction as external loads are vertical.

Now consider the *FBD* of the whole truss shown in fig. (a). The triangle *AEC* is a right angled triangle with angle $AEC = 90°$.

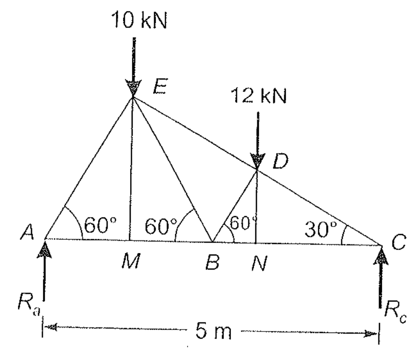


Fig. (a)

Thus

$$AE = AC\cos 60° = 5 \times 0.5 = 2.5 \text{ m}$$

$$CE = AC\sin 60° = 5 \times 0.866 = 4.33 \text{ m}$$

From the geometry of given figure it may be easily seen that ΔABE is an equilateral triangle and therefore

$$AB = AE = 2.5 \text{ m}$$

Distance of line of action of force 10 kN from joint *A* is

$$AM = AE\cos 60° = 2.5 \times 0.5 = 1.25 \text{ m}$$

The triangle *BDC* is a also right angled triangle with angle $BDC = 90°$. Therefore

$$BC = AC - AB = 5 - 2.5 = 2.5 \text{ m}$$

$$BD = BC\cos 60° = 2.5 \times 0.5 = 1.25 \text{ m}$$

Distance of line of action of force 12 kN from joint *A* is

$$AN = AB + BN = AB + BD\cos 60°$$

$$= 2.5 + 1.25 \times 0.5 = 3.125 \text{ m}$$

Taking moments about end *A*, we get

$$R_c \times 5 = 12 \times 3.125 + 10 \times 1.25 = 50$$

or

$$R_c = \frac{50}{5} = 10 \text{ kN}$$

Since $R_a + R_c = 10 + 12 = 22$

Thus $R_a = 22 - 10 = 12 \text{ kN}$

Joint A : Now consider the *FBD* of joint *A* with the direction of forces assumed as shown in fig. (b). We can start with this joint because there are only two unknown forces F_1 (in member *AE*) and F_2 (in member *AB*) acting at this joint.

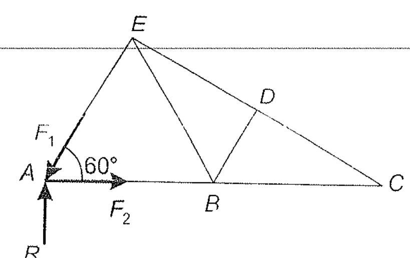


Fig. (b)

Resolving forces vertically we get

$$R_a = F_1 \sin 60°$$

or

$$F_1 = \frac{R_a}{\sin 60°} = \frac{12}{0.866} = 13.85 \text{ kN (compression)}$$

Resolving forces horizontally we get

$$F_2 = F_1\cos 60° = 13.85 \times 0.5 = 6.92 \text{ kN (tension)}$$

Joint C : Consider the *FBD* of joint *C* with the direction of forces assumed as shown in fig. (c). Analysis of this joint is also possible because there are only two unknown forces F_3 in member *BC* and F_4 in member *CD* acting at this joint.

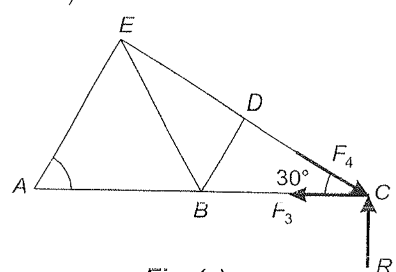


Fig. (c)

Resolving forces vertically we get

$$R_c = F_4 \sin 30°$$

or

$$F_4 = \frac{R_c}{\sin 30°} = \frac{10}{0.5} = 20 \text{ kN} \quad \text{(compression)}$$

Resolving forces horizontally we get

$$F_3 = F_4\cos 30° = 20 \times 0.866 = 17.32 \text{ kN} \quad \text{(tension)}$$

Joint B : Consider the *FBD* of joint *B* with the known values of forces F_2 in member *AB* and F_3 in member *BC* as shown in fig (d).

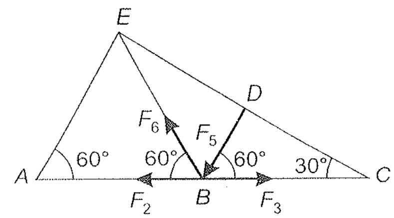


Fig. (d)

Resolving forces vertically we get

$$F_5 \sin 60° = F_6 \sin 60°$$

or $$F_5 = F_6$$

Resolving forces horizontally we get

$$F_3 = F_2 + F_5 \cos 60° + F_6 \cos 60°$$

$$17.32 = 6.92 + F_5 \times 0.5 + F_6 \times 0.5$$

or $$17.32 - 6.92 = 0.5\,F_5 + 0.5\,F_6$$

or $$F_5 = F_6 = 10.40 \text{ kN}$$

Force F_5 in member *BD* is 10.40 kN compressive and force F_6 in member *BE* is 10.40 kN tensile.

Joint D : Consider the *FBD* of joint *D* with known values of forces F_4 in member *DC* and F_5 in member *DB* a shown in fig. (e).

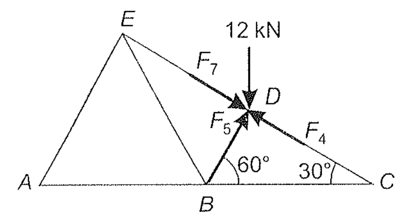


Fig. (e)

Resolving forces horizontally we get

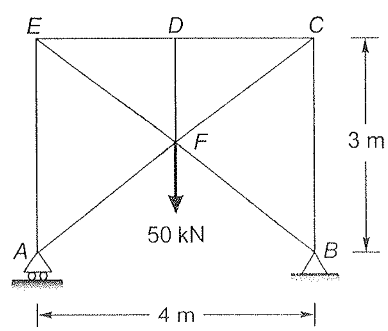
$$F_7 \cos 30° + F_5 \sin 30° = F_4 \sin 60°$$

$$F_7 \times 0.866 + 10.4 \times 0.5 = 20 \times 0.886$$

or $$F_7 = 13.995 \text{ kN} \quad \text{(Compression)}$$

Force F_7 in member *DE* is 13.995 kN compressive.

Example 2.6 A truss has been loaded and supported as shown in fig. Determine the reactions at the supports and the forces in the members of the truss.



Solution:

Both the supports will have only vertical reactions as the external load of 50 kN at the joint *F* is in vertical direction.

Further, due to symmetry $$R_a = R_b = \frac{50}{2} = 25 \text{ kN}$$

From the geometry of the truss, the angle of the inclined members with the horizontal is

$$\tan\theta = \frac{3}{4}$$

or $$\sin\theta = \frac{3}{5} \text{ and } \cos\theta = \frac{4}{5}$$

Joint A : Now consider the *FBD* of joint *A* with the direction of forces assumed as shown in fig. (a). We can start with this joint because there are only two unknown forces F_1 in member *AE* and F_2 in member *AF* acting at this joint.

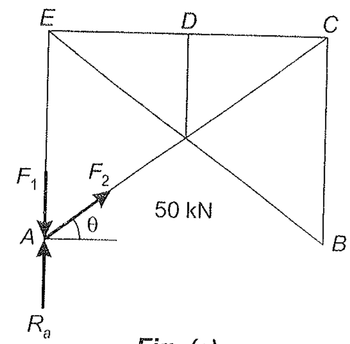


Fig. (a)

Resolving forces horizontally we get

$$F_2 \cos\theta = 0$$

or

$$F_2 = 0 \text{ (no force)}$$

Resolving forces vertically we get

$$F_1 = R_a = 25 \text{ kN (compressive)}$$

Joint E : Consider the *FBD* of joint *E* with known value of force F_1 in member *AE* as shown in fig. (b).

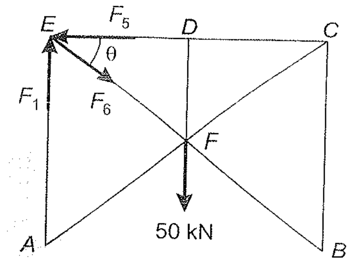


Fig. (b)

Resolving forces vertically we get

$$F_1 = F_6 \sin\theta$$

or

$$F_6 = \frac{F_1}{\sin\theta} = \frac{25}{3/5} = 41.67 \text{ kN (tensile)}$$

Resolving forces horizontally we get

$$F_5 = F_6 \cos\theta = 41.67 \times \frac{4}{5}$$

$$= 33.34 \text{ kN} \qquad \text{(Compression)}$$

Since truss is symmetrical so we get

Force in *AF* = Force in *BF* = 0

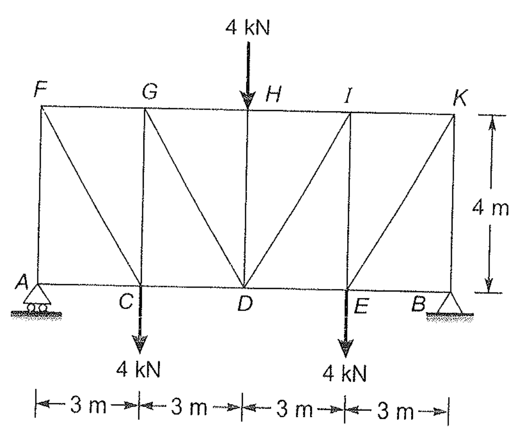
Force in *AE* = Force in *BC* = 25 kN (Compression)

Force in *FC* = force in *FE* = 41.67 kN (Tension)

Force in *CD* = force in *ED* = 33.34 kN (Compression)

There will be no force in member *DF* because F_5 and F_8 are collinear.

Example 2.7 Determine the forces in all the members of the truss loaded and supported as shows in fig.



Solution:

The reaction R_b at hinged support B will have only vertical component as all loads are acting in vertical direction only.

Due to symmetry of truss and loading

$$R_a = R_b = \frac{4+4+4}{2} = 6 \text{ kN}$$

In right angled triangle ACF,

$$CF = \sqrt{3^2 + 4^2} = 5 \text{ m}$$

or $$\sin\theta = \frac{4}{5} = 0.8 \text{ and } \cos\theta = \frac{3}{5} = 0.6$$

Here all inclined member are inclined at angle θ from horizontal.

Joint A : Now consider the *FBD* of joint *A* as shown in fig. (a).

We easily get that $F_2 = 0$ (no force)

and $F_1 = R_a = 6$ kN (compression)

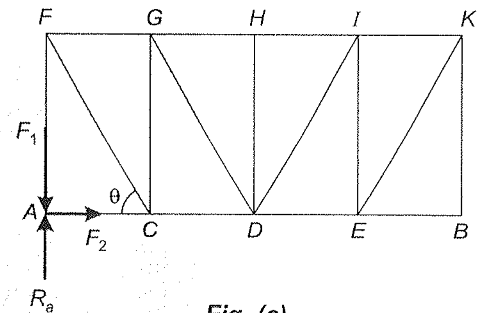


Fig. (a)

Joint F: Consider the *FBD* of joint *F* with known value of force F_1 in member *AF* as shown in fig. (b).

Resolving forces vertically we get

$$F_3 \sin\theta = F_1$$

or $$F_3 = \frac{6}{\sin\theta} = \frac{6}{0.8} = 7.5 \text{ kN (tension)}$$

Resolving forces horizontally we get

or $$F_4 = F_3 \cos\theta = 7.5 \times 0.6 = 4.5 \text{ kN (Comp)}$$

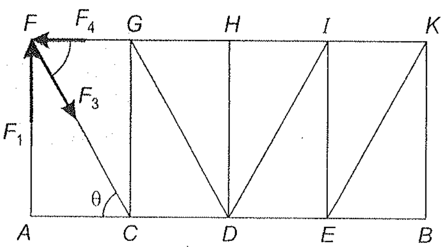


Fig. (b)

Joint C : Consider the *FBD* of joint *C* with known value of force F_3 in member *FC* and zero force in member *AC* as shown in fig. (c).

Resolving forces vertically we get

$$F_5 + F_3 \sin\theta = 4$$

or $$F_5 = 4 - F_3 \sin\theta = 4 - 7.5 \times 0.8 = -2 \text{ kN}$$

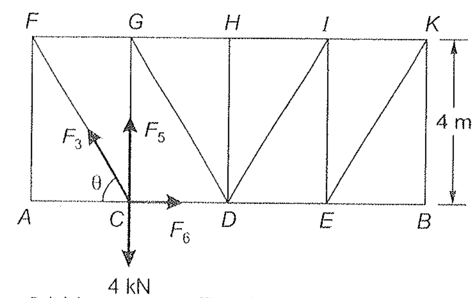


Fig. (c)

Resolving forces horizontally we get

$$F_6 = F_3 \cos\theta = 7.5 \times 0.6 = 4.5 \text{ kN} \quad \text{(tension)}$$

The negative sign with the magnitude of force F_5 shows that a wrong choice has been made while assuming its direction. Obviously the assumed direction of force in member *CG* need to be reversed. Therefore $F_5 = 2$ kN (compression) and *CG* is a compression member.

Joint G : Consider the *FBD* of joint *G* with known value of forces F_4 in member *FG* and F_5 in member *CG* as shown in fig. (d).

Resolving forces vertically we get

$$F_7 \sin\theta = F_5 = 2$$

or

$$F_7 = \frac{2}{\sin\theta} = \frac{2}{0.8} = 2.5 \text{ kN (tension)}$$

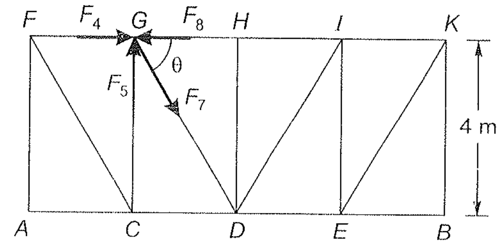


Fig. (d)

Resolving forces horizontally we get

$$F_8 = F_4 + F_7 \cos\theta$$
$$= 4.5 + 2.50 \times 0.6$$
$$= 6 \text{ kN (compression)}$$

Joint H: With reference to *FBD* of joint *H* as shown in fig. (e).

It can be easily seen that

$$F_{10} = F_8 = 6 \text{ kN (compression)}$$
$$F_9 = 4 \text{ kN (compression)}$$

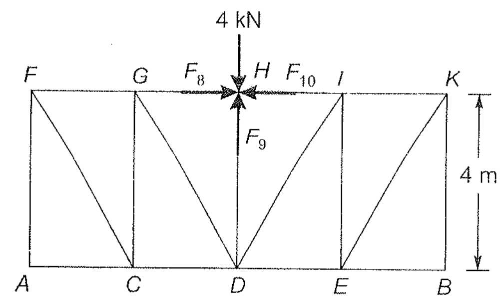
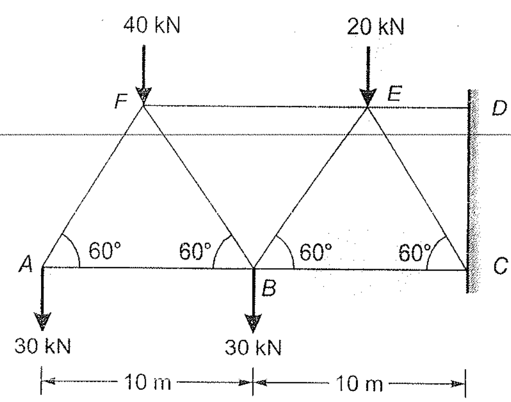


Fig. (e)

The truss is symmetrical and as such, there is no need to consider equilibrium of the joints left.

Example 2.8 Determine the forces in each member of the cantilever truss loaded as shown in fig.



Solution:

From the geometry of figure, it may be easily seen that angle of all inclined member is 60° from horizontal.

Joint A : Consider the *FBD* of joint *A* with the direction of forces assumed as shown in fig. (a).

Resolving forces vertically we get

$$F_1 \sin 60° = 30$$

or

$$F_1 = \frac{30}{\sin 60°} = \frac{30}{0.866}$$
$$= 34.64 \text{ kN (tension)}$$

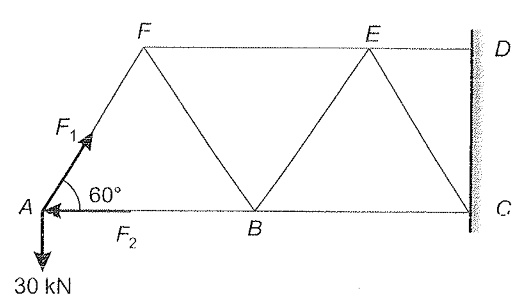


Fig. (a)

Resolving forces horizontally we get

$$F_2 = F_1 \cos 60° = 34.64 \times 0.5 = 17.32 \text{ kN} \quad \text{(compression)}$$

Joint F : Consider the *FBD* of joint *F* with known value of forces F_1 in member *AF* as shown in fig. (b).

Resolving forces vertically we get

$$F_3 \sin 60° = 40 + F_1 \sin 60°$$

$$F_3 \times 0.866 = 40 + 34.64 \times 0.866$$

or $$F_3 = 80.83 \text{ kN (Comp)}$$

Resolving forces horizontally we get

$$= \quad F_1 \cos 60° + F_3 \cos 60° \,(34.64 + 80.83) \times 0.5$$

or $$F_4 = 57.73 \text{ kN (tension)}$$

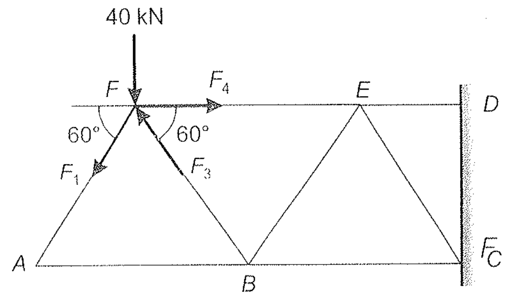


Fig. (b)

Joint B: Consider the *FBD* of joint *B* with known value of forces F_2 in member *AB* and F_3 in member *FB* as shown in figure (c)

Resolving forces vertically we get

$$F_6 \sin 60° = F_3 \sin 60° + 30$$

or $$F_6 = 115.47 \text{ kN (tensile)}$$

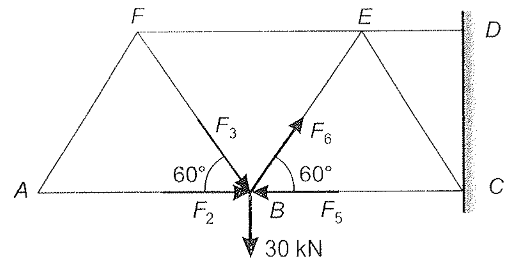


Fig. (c)

Resolving forces horizontally we get

$$F_5 = F_2 + F_3 \cos 60° + F_6 \cos 60°$$

$$F_5 = 17.32 + 80.83 \times 0.5 + 115.47 \times 0.5$$

or $$F_5 = 115.47 \text{ (compression)}$$

Joint E: Consider the *FBD* of joint *E* with known value of forces F_4 in member *EF* and F_6 in member *BE* as shown in figure (d)

Resolving forces vertically we get

$$F_7 \sin 60° = F_6 \sin 60° + 20$$

$$F_7 \times 0.866 = 115.47 \times 0.866 + 20$$

or $$F_7 = 138.56 \text{ kN (compression)}$$

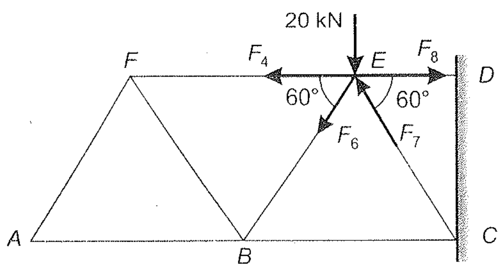


Fig. (d)

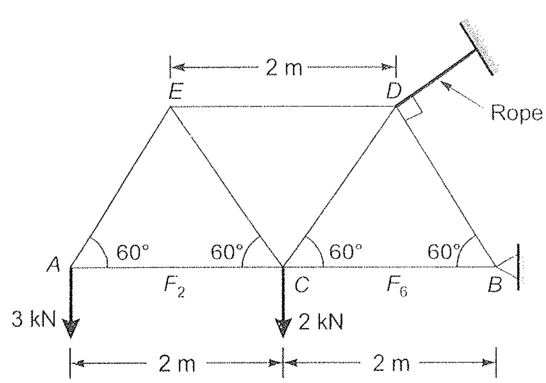
Resolving forces horizontally we get

$$F_8 = F_4 + F_6 \cos 60° + F_7 \cos 60°$$

$$= 57.73 + (115.47 + 138.56) \times 0.5$$

$$= 184.74 \text{ kN (tensile)}$$

Example 2.9 Determine the forces in each member of the truss loaded and supports as shown in fig. All members of the truss are 2 m in length.



Solution:

Let T be the tension in rope. R_{bx} and R_{by} be the reactions at the hinged support as shown in fig. (a). Consider the FBD of entire truss and take moment about point B

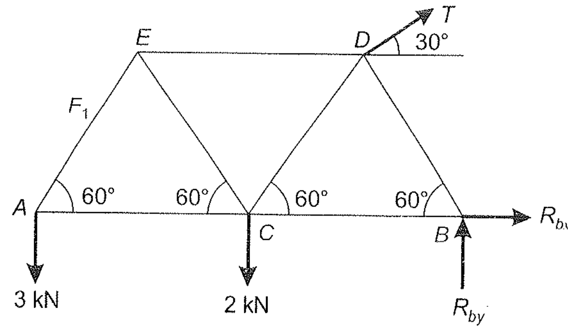


Fig. (a)

$$T \times DB = 2 \times CB + 3 \times AB$$

or $$T \times 2 = 2 \times 2 + 3 \times 4$$

or $$T = 8 \text{ kN}$$

Resolving forces horizontally we get

$$R_{bx} = T \cos 30° = 8 \times 0.866 = 6.93 \text{ kN}$$

Resolving forces vertically we get

$$R_{by} + T \sin 30° = 2 + 3 = 5$$

or $$R_{by} = 5 - 8 \times 0.5 = 1 \text{ kN}$$

Joint A : Consider FBD of joint A with the direction of forces assumed as shown in fig. (b).

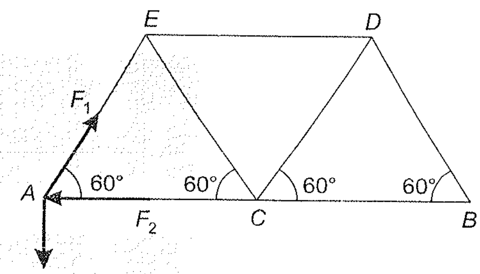


Fig. (b)

Resolving forces vertically we get

$$F_1 \sin 60° = 3$$

or $$F_1 = \frac{3}{\sin 60°} = \frac{3}{0.866}$$
$$= 3.464 \text{ kN (tensile)}$$

Resolving forces horizontally we get

$$F_2 = F_1 \cos 60°$$
$$= 3.464 \times 0.5 = 1.732 \text{ kN (compression)}$$

Joint E : Consider the FBD of joint E with known value of forces F_1 in member EA as shown in fig. (c).

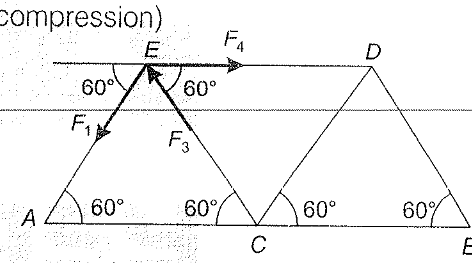


Fig. (c)

Resolving forces vertically we get

$$F_3 \sin 60° = F_1 \sin 60°$$
$$F_3 = F_1 = 3.464 \text{ kN (Comp)}$$

Resolving forces horizontally we get

$$F_4 = F_1 \cos 60° + F_3 \cos 60° = (F_1 + F_3) \cos 60°$$
$$= (3.464 + 3.464) \times 0.5 = 3.464 \text{ kN (tensile)}$$

Joint C : Consider the FBD of joint C with known value of forces F_2 in member AC and F_3 in member EC as shown in fig. (d).

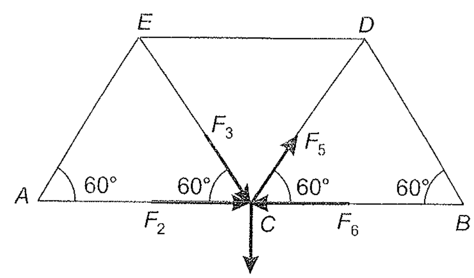


Fig. (d)

Resolving forces vertically we get

$$F_5 \sin 60° = 2 + F_3 \sin 60°$$

or $$F_5 \times 0.866 = 2 + 3.464 \times 0.866$$

or $$F_5 = \frac{2 + 3.464 \times 0.866}{0.866} = 5.77 \text{ kN (tensile)}$$

Resolving forces horizontally we get

$$F_6 = F_2 + F_3 \cos 60° + F_5 \cos 60°$$
$$= 1.732 + 3.464 \times 0.5 + 0.5 \times 5.77 = 6.35 \text{ kN} \qquad \text{(compression)}$$

Joint B : Consider the *FBD* of joint *B* with known value of forces F_6 in member *BC* as shown in fig. (e).

Resolving forces vertically we get

$$F_7 \sin 60° = R_{by} = 1$$

or
$$F_7 = \frac{1}{0.866}$$
$$= 1.155 \text{ kN (compression)}$$

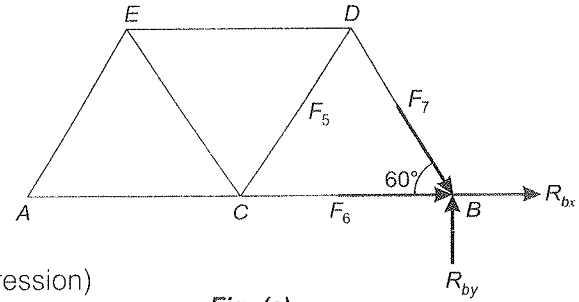
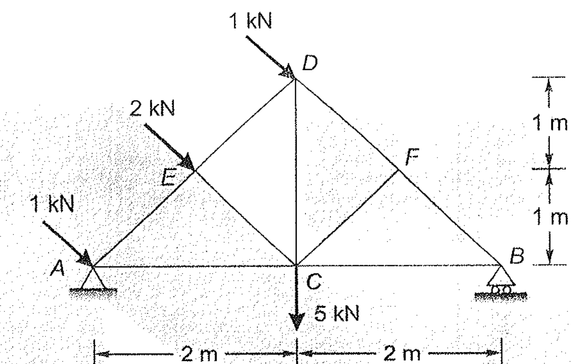


Fig. (e)

Example 2.10 For the truss loaded as shown in fig. determine the reactions at the supports and forces set up in each member of the truss.



Solution:

At end *B* roller support provides a reaction R_b at right angles to the roller base. At end *A* due to hinged support, reaction R_a can have two component acting in the horizontal and vertical direction.

From the geometry of fig. (a) we get

$$\theta = \tan^{-1}\frac{AC}{CD} = \tan^{-1}\frac{2}{2} = 45°$$

Thus $\sin\theta = \cos\theta = 0.707$

Angle of inclination of all member is 45° with horizontal.

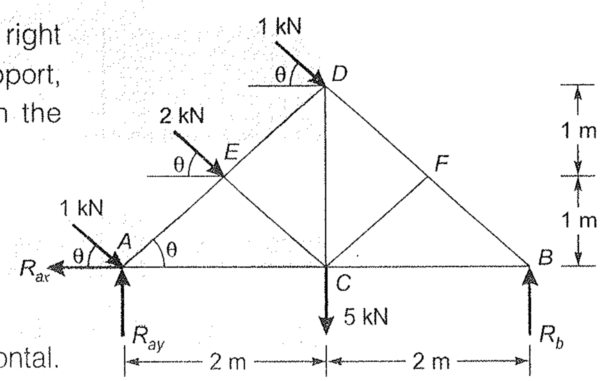


Fig. (a)

$$AD = \sqrt{AC^2 + CD^2} = \sqrt{2^2 + 2^2} = 2.828 \text{ m}$$

$$AE = \frac{AD}{2} = \frac{2.818}{2} = 1.414$$

Now consider the *FBD* of the entire truss shown in fig. (a) and taking moments about joint *A* we get

$$R_b \times AB = 2 \times AE + 1 \times AD + 5 \times AC$$

or
$$R_b \times 4 = 2 \times 1.414 + 1 \times 2.828 + 5 \times 2$$

or
$$R_b = \frac{2 \times 1.414 + 1 \times 2.828 + 10}{4} = 3.914 \text{ kN}$$

Resolving forces horizontally we get

$$R_{ax} = 1 \times \cos\theta + 2 \times \cos\theta + 1 \times \cos\theta$$

or
$$R_{ax} = (1 + 2 + 1)\cos\theta$$
$$= 4 \times 0.707 = 2.828 \text{ kN}$$

Resolving forces vertically we get

$$R_{ay} + R_b = 5 + 1 \times \sin\theta + 2 \times \sin\theta + 1 \times \sin\theta$$

or

$$R_{ay} = 5 - R_b + (1 + 2 + 1)\sin\theta$$
$$= 5 - 3.914 + 4 \times 0.707$$
$$= 3.914 \text{ kN}$$

Joint A : Consider the *FBD* of joint *A* with the direction of forces assumed as shown in fig. (b). We can start with this node because there are two unknown forces. Resolving forces vertically we get

$$1 \times \sin\theta + F_1 \sin\theta = R_{ay}$$

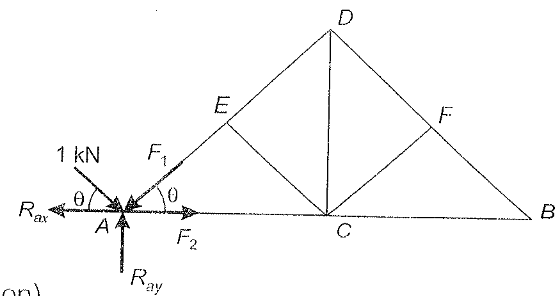


Fig. (b)

or

$$F_1 = \frac{R_{ay}}{\sin\theta} - 1 = \frac{3.914}{0.707} - 1$$
$$= 4.536 \text{ kN (compression)}$$

Resolving forces horizontally we get

$$1 \times \cos\theta + F_2 = R_{ax} + F_1\cos\theta$$

or

$$F_2 = 2.828 + 4.536 \times 0.707 - 1 \times 0.707$$
$$= 5.328 \text{ kN (tensile)}$$

Joint E: Consider the *FBD* of joint *E* with known value of force F_1 in member *AE* as shown in figure (c)

It can be easily seen that,

$$F_3 = 2 \text{ kN (compressive)}$$

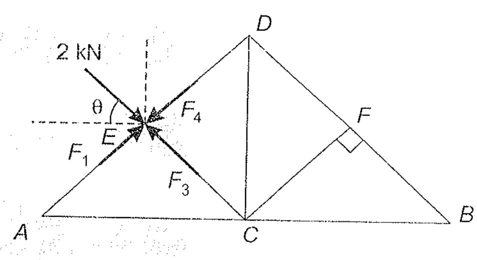


Fig. (c)

Joint D: Consider the *FBD* of joint *D* with known value of force F_4 in member *DE* as shown in figure (d)

Resolving forces horizontally we get,

$$1 \times \cos\theta + F_4 \sin\theta = F_6 \cos\theta$$

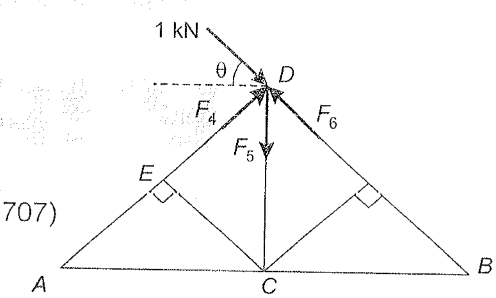


Fig. (d)

or

$$F_6 = F_4 + 1 \ (\because \sin\theta = \cos\theta = 0.707)$$

or

$$= 5.536 \text{ kN (compressive)}$$

Resolving forces vertically we get

$$1 \times \sin\theta + F_5 = F_4\cos\theta + F_6\sin\theta$$

$\Rightarrow$

$$F_5 = (F_4 + F_6 - 1) \times 0.707 = 2 \times 4.536 \times 0.707$$
$$6.414 \text{ kN (tensile)}$$

Joint F: With reference of *FBD* of joint *F* as shown in figure (e)

It can be easily seen that

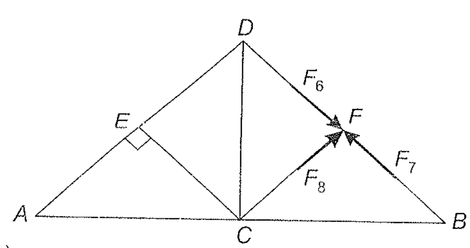


Fig. (e)

$$F_7 = F_6 = 5.536 \text{ kN (compressive)}$$

and

$$F_8 = 0$$

Joint B: Consider the *FBD* of joint *B* with known value of force F_7 in member *BC* as shown in figure (f)
Resolving forces horizontally we get

$$F_9 = F_7 \cos\theta$$
$$= 5.536 \times 0.707$$
$$= 3.914 \text{ kN (tensile)}$$

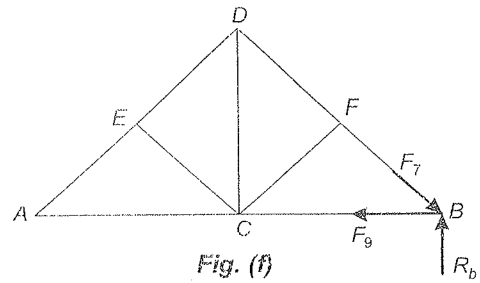
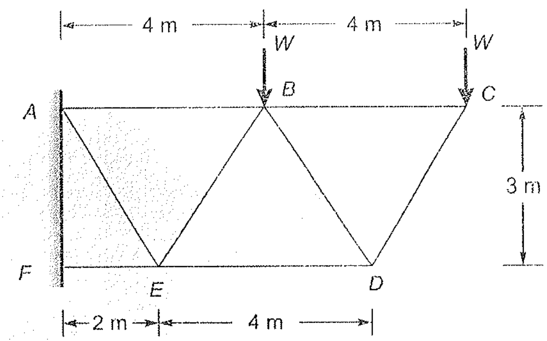


Fig. (f)

Example 2.11 Determine the value of load *W* that would produce a force of magnitude 120 kN in the *AB* member of the cantilever truss shown in fig. For this loading, find out forces in members *AE* and *FE* of the truss.



Solution:

Let a section 1 – 1 be passed in such a way that it cuts the members *AB*, *AE*, *FE* and divides the truss in two parts and assumed direction of forces is shown in fig. (a).

Consider equilibrium of right part of the truss and take moments about the point *E*,

$$W \times 2 + W \times 6 = F \times 3 = 120 \times 3$$

or

$$W = \frac{120 \times 3}{8} = 45 \text{ kN}$$

To find force F_3 in member *FE*, taking moments about the point *A*,

$$W \times 8 + W \times 4 = F_3 \times 3$$

or

$$F_3 = 4W = 4 \times 45 = 180 \text{ kN} \quad \text{(compressive)}$$

To find force F_2 in member *AE*, resolving forces vertically.

$$F_2 \sin\theta = W + W$$

or

$$F_2 = \frac{2W}{\sin\theta}$$

From the geometry of fig. (a) we get,

$$\sin\theta = \frac{AF}{AE}$$

$$= \frac{3}{\sqrt{3^3 + 2^2}} = 0.832$$

or

$$F_2 = \frac{2 \times 45}{0.832}$$

$$= 108.17 \text{ kN tensile)}$$

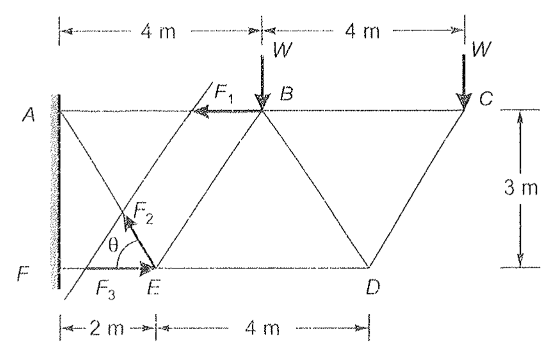
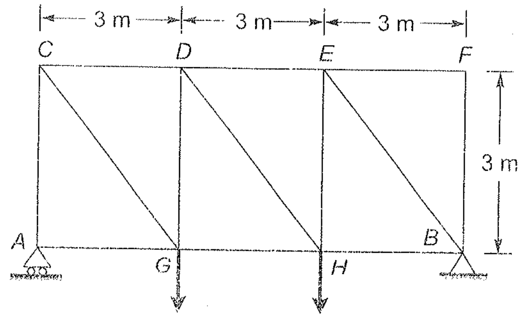


Fig. (a)

Example 2.12 For the truss loaded and supported as shown in fig., determine the forces in members *CD*, *DG* and *GH*. Use the method of sections.



Solution:

The reactions at the hinged support will be only in vertical direction as external loads are vertical.

Thus $R_a + R_b = 10 + 20 = 30$ kN

Taking moments about end *B* we get,

$$R_a \times 9 = 10 \times 6 + 20 \times 3 = 120$$

$$R_a = \frac{120}{9} = 13.33 \text{ kN}$$

Thus $R_b = 30 - 13.33 = 16.66$ kN

Let section 1-1 be passed in such a way that it cuts the members *CD*, *DG*, *GH* and divides the truss in two parts.

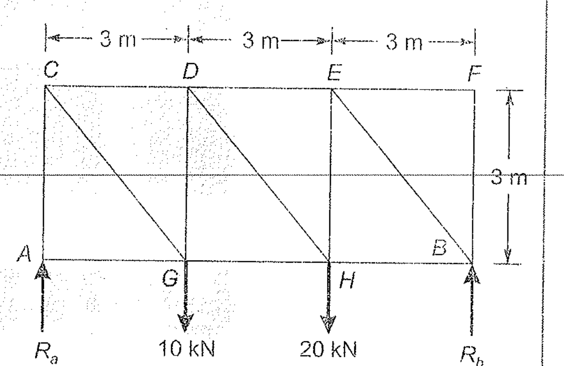


Fig. (a)

The left part of the truss (fig. (a)) is in equilibrium under the action of following forces:

(1) Reaction $R_a = 13.33$ kN

(2) 10 kN load at joint *G*

(3) Force F_1 in member *CD* (assumed compressive),

(4) Force F_2 in member *GD* (assumed tensile) and

(5) Force F_3 in member *GH* (assumed tensile).

Taking moment about point *D* we get

$$F_3 \times GD = R_a \times AG$$

or $$F_3 \times 3 = R_a \times 3$$

or $$F_3 = R_a = 13.33 \text{ kN (tensile)}$$

Taking moment about point *G* we get

$$F_1 \times AC = R_a \times AG$$

$$F_1 \times 3 = R_a \times 3$$

or $$F_1 = R_a = 13.33 \text{ kN (compression)}$$

To find F_3 in member *GH*, resolving forces horizontally

⇒ $$F_3 = F_1 = 13.33 \text{ kN (tensile)}$$

Taking moment about point *C* we get

$$10 \times AG = F_2 \times CD + F_3 \times GD$$

or $$10 \times 3 = F_2 \times 3 + F_3 \times 3$$

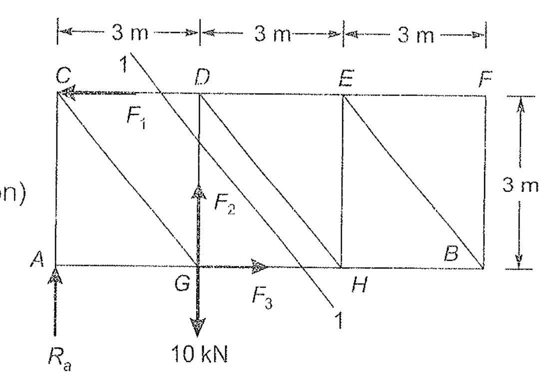
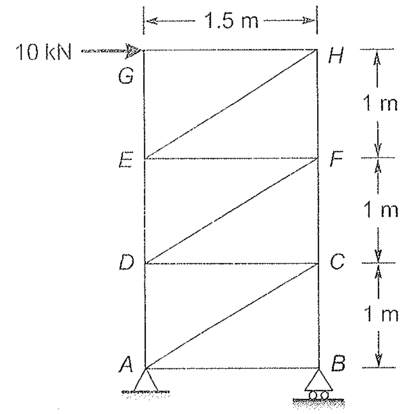


Fig. (b)

$$
\begin{aligned}
\text{or} \qquad F_2 &= 10 - F_3 \\
&= 10 - 13.33 \\
&= -3.33 \text{ kN}
\end{aligned}
$$

The negative with force F_2 indicate that it is in opposite direction as assumed. Thus force F_2 in member *GD* is 3.33 kN compressive.

Example 2.13 Using the method of sections, determine the forces in members *ED*, *DF* and *FC* of the truss shown in fig.



Solution:

Let section 1-1 be passed in such a way that it cuts the member *ED*, *DF*, *FC* and divides the truss in two parts. For considering the equilibrium of the upper part of the truss, there is no need to determine the reaction at the supports.

Let F_1, F_2 and F_3 be the forces with their directions assumed as shown in fig. (a).

From the geometry

$$\cos\theta = \frac{1.5}{\sqrt{1.5^2 + 1^2}} = \frac{1.50}{1.80} = 0.833$$

Now taking moment about point *F* we get

$$F_1 \times EF = 10 \times GF \quad \text{or } F_1 \times 1.5 = 10 \times 1$$

$$\text{or} \qquad F_1 = \frac{10}{1.5} = 6.67 \text{ kN (tensile)}$$

Resolving forces horizontally we get

$$F_2 \cos\theta = 10$$

$$\text{or} \qquad F_2 = \frac{10}{\cos\theta} = \frac{10}{0.833} = 12 \text{ kN (tensile)}$$

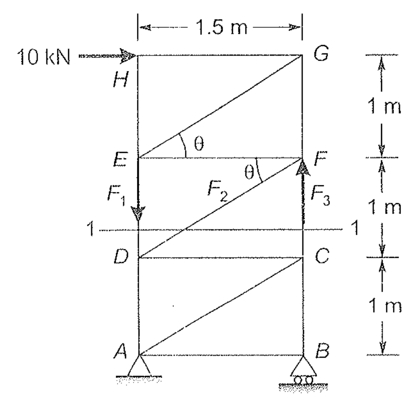


Fig. (a)

Let section 2-2 be passed in such a way that it cuts the member AD, DC and FC and divides the truss in two parts. Considering the equilibrium of the upper part of the truss, let F_3, F_4, F_5 be the forces with their directions assumed as shown in fig. (b)

Taking moment about point *D* we get

$$
\begin{aligned}
F_3 \times DC &= 10 \times HD \\
\text{or} \qquad F_3 \times 1.5 &= 10 \times 2
\end{aligned}
$$

$$\text{or} \qquad F_3 = \frac{20}{1.5} = 13.33 \text{ kN (compressive)}$$

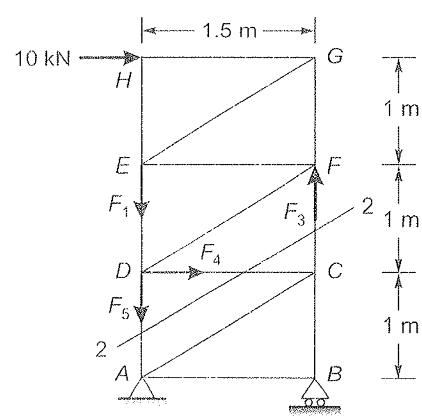


Fig. (b)

Objective Brain Teasers

Q.1 A solid sphere will be in stable equilibrium if its centre of gravity lies

(a) vertically above its geometric centre

(b) vertically below its geometric centre

(c) on the horizontal line through its centre

(d) at the centre

Q.2 Consider the following statements with regard to equilibrium

1. A body is said to be in stable equilibrium when an receiving a slight displacement, it tends to go further away from its position of rest.
2. A body is said to be in unstable equilibrium when on receiving a slight displacement, it tends to go further away from its position of rest
3. A body is in neutral equilibrium when an receiving a slight displacement, it tends to come to rest in its new position

Which of the statements given above are correct?

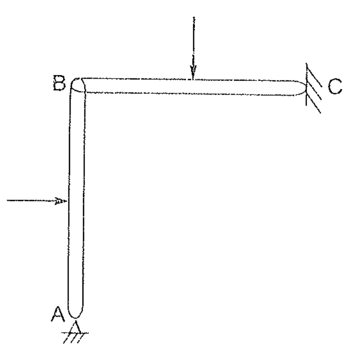
(a) 1, 2 and 3

(b) 1 and 2

(c) 2 and 3

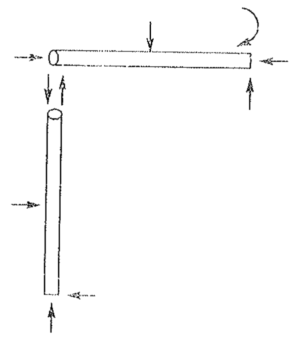
(d) 1 and 3

Q.3

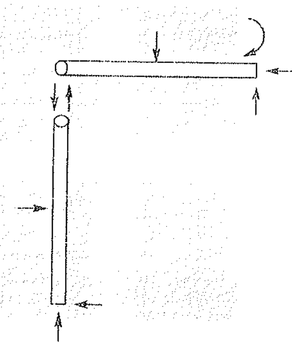


Select the correct free body diagram for the structure shown in the figure given above. The structure is hinged at *A* and *B* while *C* is fixed.

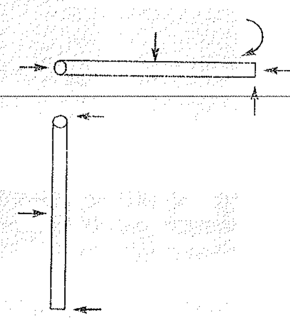
(a)



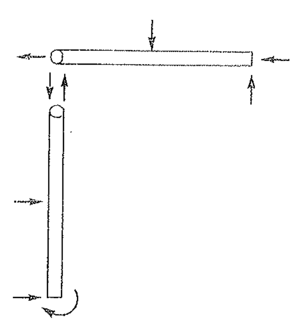
(b)



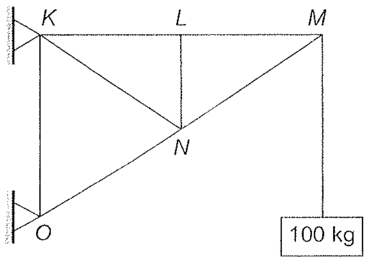
(c)



(d)

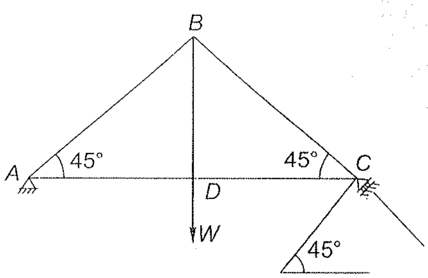


Q.4 The figure shows a pin-jointed plane truss loaded at the point *M* by hanging a mass of 100 kg. The member LN of the truss is subjected to a load of



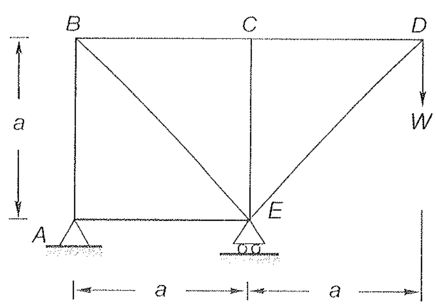
(a) 0 Newton
(b) 450 Newton in compression
(c) 981 Newton in compression
(d) 981 Newton in tension

Q.5 A truss *ABCD*, hinged at end *A* is roller supported on inclined plane at *C*. Angle of inclination of plane is 45°. Load at joint *D* is *W* as shown in figure. What is the horizontal component of reaction at *C*?

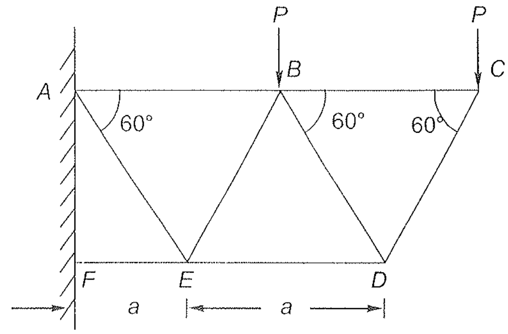


(a) Zero (b) 0.5W
(c) 0.707W (d) W

Q.6 A truss *ABCDE* is shown in figure. A load *W* acts at joint *D* as shown in figure. What is the force in member *BC*?
(a) W (tension) (b) W (comp.)
(c) $\sqrt{2}W$ (tensile) (d) None of these

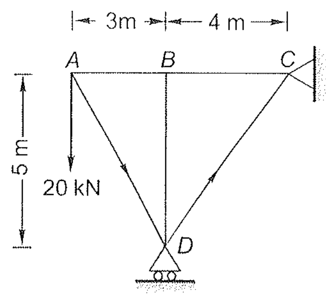


Q.7 A truss *ABCDEF*, shown in the figure carries loads *P* each at joints *B* and *C*. What is the value of *P* if force in member *AB* is 3 kN? Method of section may be used



(a) 1 kN (b) 1.299 kN
(c) 1.5 kN (d) None of these.

Q.8 A truss *ABCD* hinged at *C*, roller supported at *D* is subjected to a vertical load 20 kN at joint *A*. What is the reaction at *C* figure ?
(a) 20 kN (b) 15 kN
(c) 35 kN (d) None of these.

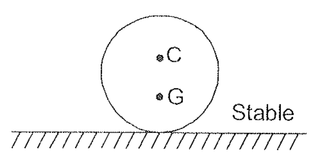


ANSWERS

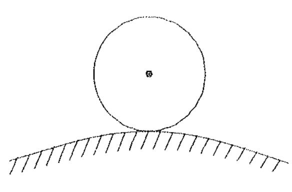
1. (b)	2. (c)	3. (a)	4. (a)	5. (b)
6. (a)	7. (b)	8. (d)		

Hints & Explanation

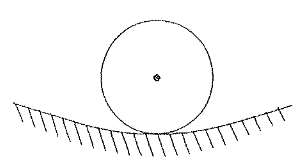
1. (b)



2. (c)

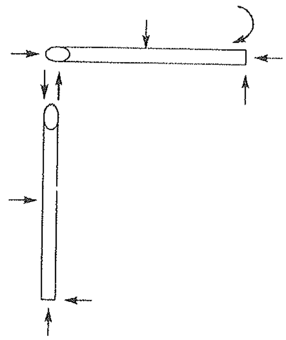


Unstable



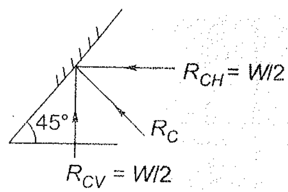
Stable equilibrium

3. (a)

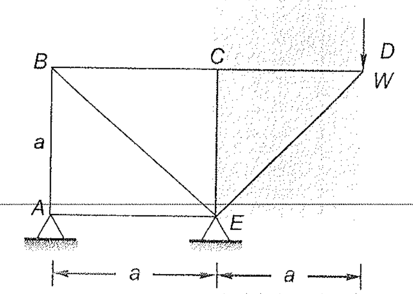


5. (b)

$$R_{CV} = \frac{W}{2} = R_{CH}.$$



6. (a)

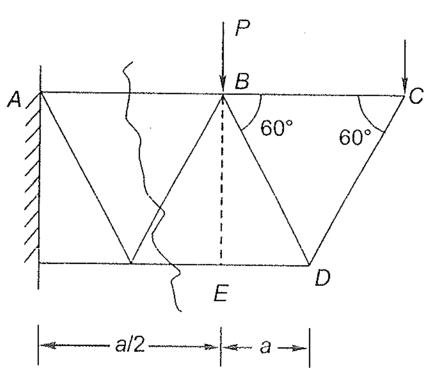


At joint D

$$F_{CD} = W \text{ (tension)}$$
$$F_{CE} = 0$$
$$F_{BC} = W \text{ (tension)}$$

7. (b)

Method of section



Moment about E

$$F_{BA} \times 0.866a = \frac{Pa}{2} + \frac{3Pa}{2}$$

$$F_{BA} = \frac{2Pa}{0.866a}$$

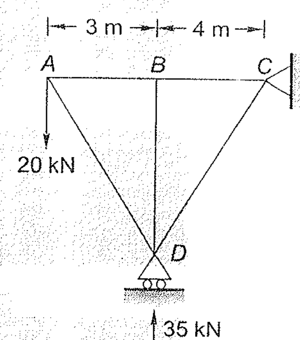
$$= 2.31P = 3 \text{ kN}$$

$$P = 1.299 \text{ kN.}$$

8. (d)

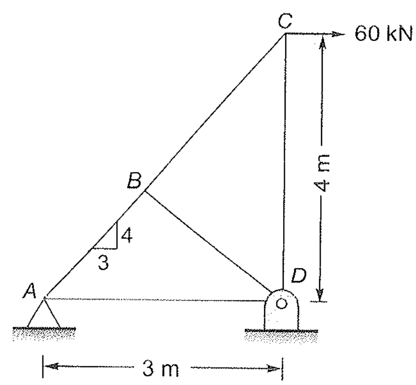
Moments about C

$$27 \times 7 = R_{DV} + 4$$
$$R_{DV} = 35 \text{ kN}$$
$$R_{CV} = 35 - 20 = 15 \text{ kN}\downarrow$$
$$R_{CH} = 0.$$



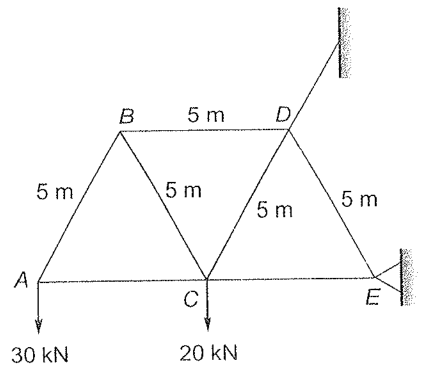
Student's Assignments

Q.1 Due to horizontal pull of 60 kN at C, what is the force induced in the member AB?



(a) 0
(b) 40 kN
(c) 80 kN
(d) 100 kN

Q.2 The magnitude of force in section BD is _____________ kN.



Q.3 The net reaction at *C* is _____________ kN.