4. THEORY OF FAILURE

MAXIMUM PRINCIPAL STRESS THEORY (RANKINES THEORY)

According to this theory, permanent set takes place under a state of complex stress, when the value of maximum principal stress is equal to that of yield point stress as found in a simple tensile test.

For design criterion, the maximum principal stress (σ_1) must not exceed the working stress 'σ_y' for the material

$$\boxed{\sigma_{1,2} \le \sigma_y} \quad \text{For no failure.}$$

$$\boxed{\sigma_{1,2} \le \frac{\sigma_y}{FOS}} \quad \text{For design.}$$

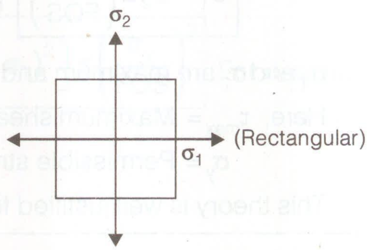
Note: For no shear failure $\tau \le 0.57\sigma_y$

Graphical representation

Remember

For brittle material which do not fail by yielding but fail by brittle fracture, this theory gives satisfactory result.

The graph is always square even for different of values of σ_1 and σ_2.



MAXIMUM PRINCIPAL STRAIN THEORY (ST. VENANT'S THEORY)

According to this theory, a ductile material begins to yield when the maximum principal strain reaches the strain at which yielding occurs in simple tension

$$\boxed{\epsilon_{1,2} \le \frac{\sigma_y}{E}} \quad \text{For no failure in uni-axial loading.}$$

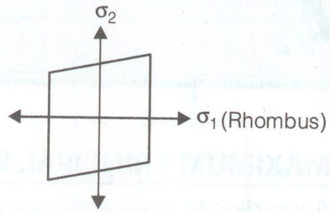
$$\boxed{\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} \le \frac{\sigma_y}{E}} \quad \text{For no failure in tri-axial loading.}$$

$$\boxed{\sigma_1 - \mu\sigma_2 - \mu\sigma_3 \le \left(\frac{\sigma_y}{FOS}\right)} \quad \text{For design. Here, } \epsilon = \text{Principal strain}$$

σ_1, σ_2 and σ_3 = Principal stresses

Graphical Representation

Remember This theory over estimate the elastic strength of ductile material.



MAXIMUM SHEAR STRESS THEORY (GUEST & TRESCA'S THEORY)

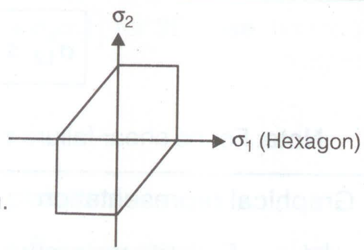
According to this theory, failure of a specimen subjected to any combination of loads when the maximum shearing stress at any point reaches the failure value equal to that developed at the yielding in an axial tensile or compressive test of the same material.

Graphical Representation

Remember

$$\tau_{max} \le \frac{\sigma_y}{2}$$ For no failure.

$$\sigma_1 - \sigma_2 \le \left(\frac{\sigma_y}{FOS}\right)$$ For design.



σ_1 and σ_2 are maximum and minimum principal stresses respectively.

Here, τ_{max} = Maximum shear stress

σ_y = Permissible stress

This theory is well justified for ductile materials.

MAXIMUM STRAIN ENERGY THEORY (HAIGH'S THEORY)

According to this theory, a body under complex stress fails when the total strain energy on the body is equal to the strain energy at elastic limit in simple tension.

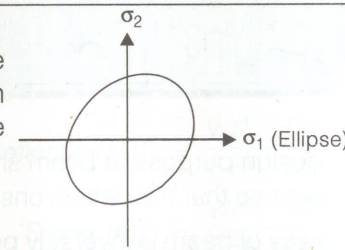
Graphical Representation

Remember

$$\left\{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right\} \le \sigma_y^2$$ for no failure.

$$\left\{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right\} \le \left(\frac{\sigma_y}{FOS}\right)^2$$ for design.

This theory does not apply to brittle material for which elastic limit stress in tension and in compression are quite different.

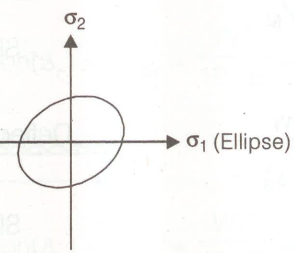


MAXIMUM SHEAR STRAIN ENERGY/DISTORTION ENERGY THEORY/MISES-HENKY THEORY

It states that inelastic action at any point in a body, under any combination of stress begins, when the strain energy of distortion per unit volume absorbed at the point is equal to the strain energy of distortion absorbed per unit volume at any point in a bar stressed to the elastic limit under the state of uniaxial stress as occurs in a simple tension/compression test.

$$\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \le \sigma_y^2$$ For no failure.

$$\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \le \left(\frac{\sigma_y}{FOS}\right)^2$$ For design.



Remember

- It can not be applied for material under hydrostatic pressure.
- All theories will give same results if loading is uniaxial.

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