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In this chapter, we will discuss the concept of conditional probability of an event given that another event has occurred, which will be helpful in understanding the Baye's theorem, multiplication rule of probability and independence of events. We will also study random variable, its probability distribution; mean of probability distribution.

PROBABILITY

|TOPIC 1|

Basic of Probability and Conditional Probability

SOME BASIC DEFINITIONS

Experiment An operation which can produce some well-defined outcomes is called an experiment.

Random Experiment An experiment is called random experiment, if it satisfies the following two conditions

(i) It has more than one possible outcomes.

(ii) It is not possible to predict the outcome in advance.

e.g. When a fair coin is tossed, it may turn up a head or tail but we are not sure which one of these results will be actually obtained.

Outcomes and Sample Space A possible result of a random experiment is called its outcome. The set of all possible outcomes of a random experiment is called its sample space. It is usually denoted by S. Each element of a sample space is called a sample point or an event point.

e.g. The sample space of tossing a coin is $S = \{H, T\}$.

Trial When a random experiment is repeated under identical conditions and it does not give the same result each time but may result in anyone of the several possible outcomes, then each such action is called a trial and the outcomes are called cases. The number of times the experiment is repeated, is called the number of trials.

e.g. One toss of a coin is a trial, when the coin is tossed n times.

CHAPTER CHECKLIST

- Basic of Probability and Conditional Probability
- Multiplication Theorem on Probability and Independent Events
- Theorem of Total Probability and Baye's Theorem
- Random Variable and Its Probability Distribution

Event

A subset of the sample space associated with a random experiment is called an event.

e.g. On tossing a coin, we have sample space $S = \{H, T\}$

The event of getting a head is given by $E = \{H\}$.

Clearly, $E \subseteq S$, so $E = \{H\}$ is an event.

TYPES OF EVENTS

(i) **Impossible and Sure Events** The empty set ϕ and the sample space S describe events (as S and ϕ are also subset of S). The empty set ϕ is called an impossible event and whole sample space S is called the sure event.

e.g. When we throw, a die, then the event of getting a number greater than 6 is a impossible event and the event of getting a number less than 7 is a sure event.

(ii) **Simple Event** If an event has only one sample point of a sample space, then it is called a simple or elementary event.

e.g. Let a die is thrown, then sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

Again, let A = event of getting $3 = \{3\}$

Here, A is a simple event.

(iii) **Compound Event** If an event has more than one sample point, then it is called a compound event. **e.g.** In the experiment of tossing a coin twice, the events E = exactly one head appeared

and F = atleast one head appeared are compound events. The subsets of S associated with these events are $E = \{HT, TH\}$ and $F = \{HT, TH, HH\}$.

(iv) **Equally Likely Events** The given events are said to be equally likely, if none of them is expected to occur in preference to the other.

e.g. In throwing an unbiased die, all the six faces are equally likely to come.

(v) **Mutually Exclusive Events** Two or more events are said to be mutually exclusive, if the happening of one excludes the happening of the other i.e. if no two of them can occur together. If A and B are mutually exclusive events, then $(A \cap B) = \phi$.

e.g. In throwing a die, all the 6 faces numbered 1 to 6 are mutually exclusive, if anyone of these faces comes, the possibility of others in the same trial is ruled out.

(vi) **Exhaustive Events** A set of events are said to be exhaustive, if one of them necessarily occurs whenever the experiment is performed.

Let $E_1, E_2, \ldots, E_n$ be subsets of sample space S. Then, events $E_1, E_2, \ldots, E_n$ are exhaustive events, if $E_1 \cup E_2 \cup \ldots \cup E_n = S$.

e.g. In the experiment of throwing a die,

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let E_1 = event of getting an even number $= \{2, 4, 6\}$

and E_2 = event of getting an odd number $= \{1, 3, 5\}$

Here, $E_1 \cup E_2 = S$. So, E_1 and E_2 are exhaustive events.

(vii) **Complement of an Event** Let A be an event in a sample space S, then complement of A is the set of all sample points, which are not in A and it is denoted by A' or $\overline{A}$.

i.e. $\quad A' = \{n : n \in S, n \notin A\}$

e.g. In the random experiment of throwing a die, the sample space $S = \{1, 2, 3, 4, 5, 6\}$ and if we define the event E as getting multiple of 3, then complement of E, i.e. $\overline{E} = \{1, 2, 4, 5\}$.

PROBABILITY OF AN EVENT

If there are n elementary equally likely events associated with a random experiment and m of them are favourable to an event A, then the probability of happening or occurrence of A is denoted by $P(A)$ and defined as

$$P(A) = \frac{m}{n} = \frac{\left[\text{Number of elementary events favourable to event } A\right]}{\left[\text{Total number of elementary events to the experiment}\right]}$$

Note

(i) $0 \le P(A) \le 1$

(ii) Probability of impossible event is zero.

(iii) Probability of sure event is 1.

(iv) $P(A \cup A') = 1$

(v) $P(A \cap A') = 0$

(vi) $P(A')' = P(A)$

(vii) Sometimes, we have to select r objects from n distinct objects, then we use the formula, ${}^nC_r = \frac{n!}{r!(n-r)!}$, $0 \le r \le n$.

Coin

A coin has two sides, head and tail. If an experiment consists of more than one coin, then coins are considered as distinct, if not otherwise stated.

(i) Sample space of one coin = $\{H, T\}$

(ii) Sample space of two coins = $\{(H, T), (T, H), (H, H), (T, T)\}$

(iii) Sample space of three coins

$= \{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T), (T, T, H), (T, T, T)\}$

Die

A die has six faces marked 1, 2, 3, 4, 5 and 6. If we have more than one die, then all dice are considered as distinct, if not otherwise stated.

(i) Sample space of a die = $\{1, 2, 3, 4, 5, 6\}$

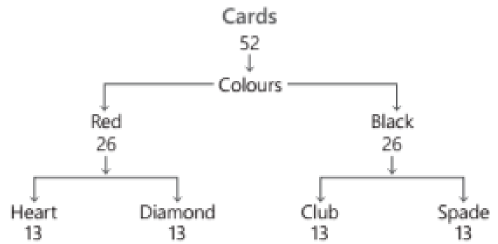
(ii) Sample space of two dice

$$= \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

Playing Cards

A pack of playing cards has 52 cards. There are 4 suits namely spade, heart, diamond and club, each having 13 cards. There are two colours, red (heart and diamond) and black (spade and club), each having 26 cards.

In 13 cards of each suit, there are 3 face cards namely king, queen and jack, so there are in all 12 face cards. 12 face cards along with 4 ace cards are together known as 16 honour cards, i.e. 4 ace, 4 king, 4 queen and 4 jack.



EXAMPLE |1| 4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamond cards?

Sol. $\because$ Total number of cards in a deck = 52 [given]

Total number of ways of selecting 4 cards out of 52 cards,

$$n(S) = {}^{52}C_4$$

Let E = event of getting 3 diamond and one another card

Then, $n(E) = {}^{13}C_3 \times {}^{39}C_1$

$\therefore$ Required probability, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{{}^{13}C_3 \times {}^{39}C_1}{{}^{52}C_4}$

Important Results on Probability

1. **Addition Theorem of Probability**

(i) For two events A and B,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are mutually exclusive events, then

$P(A \cup B) = P(A) + P(B)$

[for mutually exclusive events, $P(A \cap B) = 0$]

(ii) For three events A, B and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

If A, B and C are mutually exclusive events, then

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

[for mutually exclusive events, $P(A \cap B) = P(B \cap C) = P(C \cap A) = P(A \cap B \cap C) = 0$]

2. If A and B are two events associated with a random experiment, then

(i) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

(ii) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

(iii) $P[(A \cap \overline{B}) \cup (\overline{A} \cap B)] = P(A) + P(B) - 2P(A \cap B)$

(iv) $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$

(v) $P(\overline{A} \cup \overline{B}) = 1 - P(A \cap B)$

(vi) $P(A) = P(A \cap B) + P(A \cap \overline{B})$

(vii) $P(B) = P(A \cap B) + P(B \cap \overline{A})$

(viii) P(exactly one of A, B occurs)

$= P(A) + P(B) - 2P(A \cap B)$

$= P(A \cup B) - P(A \cap B)$

3. If A, B and C are three events, then

P(exactly one of A, B, C occurs)

$= P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(A \cap C) + 3P(A \cap B \cap C)$

4. $P(\overline{A}) = 1 - P(A)$

EXAMPLE |2| If A and B are two events such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \cap B) = 0.16$, then find $P(A \cup B)$.

Sol. Given, $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \cap B) = 0.16$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16 = 0.90 - 0.16 = 0.74$

EXAMPLE |3| There are 25 tickets bearing numbers from 1 to 25. One ticket is drawn at random. Find the probability that the number on it is a multiple of 5 or 6.

Sol. Let E = event of getting a multiple of 5

F = event of getting a multiple of 6

and $E \cap F$ = event of getting a multiple of 5 and 6, i.e. 30

$\therefore \quad P(E) = \frac{5}{25} \quad [\because E = \{5, 10, 15, 20, 25\}]$

$P(F) = \frac{4}{25} \quad [\because F = \{6, 12, 18, 24\}]$

and $P(E \cap F) = \frac{0}{25} = 0 \quad [\because E \cap F = \{\phi\}]$

$\therefore$ Required probability,

$$P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{5}{25} + \frac{4}{25} - 0 = \frac{9}{25}$$

CONDITIONAL PROBABILITY

If A and B are two events associated with the same sample space of a random experiment, then conditional probability of the event A given that B has occurred,
i.e. $P(A/B)$ is given by

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}, \text{ provided } P(B) \neq 0$$

Probability of occurrence of event B, when A has already occurred i.e. $P(B/A)$ is given by

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}, \text{ provided } P(A) \neq 0$$

Properties of Conditional Probability

Let A and B be the events of a sample space S of an experiment, then

(i) $P(S/A) = P(A/A) = 1$

(ii) If A and B are any two events of a sample space S and C is an event of S, such that $P(C) \neq 0$, then
$P\{(A \cup B)/C\} = P(A/C) + P(B/C) - P\{(A \cap B)/C\}$
In particular, if A and B are disjoint events, then
$$P\left(\frac{A \cup B}{C}\right) = P\left(\frac{A}{C}\right) + P\left(\frac{B}{C}\right)$$

(iii) $P(A'/B) = 1 - P(A/B)$, where A' is complement of A.

Note If A and B are mutually exclusive events, then $P(A/B) = 0$.

METHOD TO SOLVE PROBLEM BASED ON CONDITIONAL PROBABILITY

To solve problems based on conditional probability, we use the following steps

I. Firstly, write the sample space for given experiment and then assume the given events as A, B, which are associated with sample space.

II. Find the probability of that event A or B, which is already occurred and find $P(A \cap B)$.

III. Find the required conditional probability by using suitable formula

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \text{ or } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

Note If elementary events of the sample space are not equally likely, then probabilities $P(A \cap B)$ and $P(A)$ or $P(B)$ being calculated accordingly.

EXAMPLE |4| If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, then find

(i) $P(A \cap B)$ (ii) $P(A/B)$ (iii) $P(B/A)$. **[NCERT]**

Sol. Given, $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad \frac{7}{11} = \frac{6}{11} + \frac{5}{11} - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = \frac{6}{11} + \frac{5}{11} - \frac{7}{11}$

$\therefore \quad P(A \cap B) = \frac{4}{11}$

(ii) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{4/11}{5/11} = \frac{4}{5} \quad \left[\because P(A \cap B) = \frac{4}{11}\right]$

(iii) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{4/11}{6/11} = \frac{4}{6} = \frac{2}{3}$

EXAMPLE |5| Two dice are thrown. Find the probability of getting an even number on first die, if the outcomes on the two dice always exhibits a sum of 8.

Sol. Here, two dice are thrown. So, sample space is

$$S = \left\{\begin{matrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{matrix}\right\}$$

$\therefore \quad n(S) = 36$

Let A = event of getting an even number on first die

i.e. $$A = \left\{\begin{matrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{matrix}\right\}$$

$\therefore \quad n(A) = 18$

and B = event of getting a sum of 8
$= \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

$\therefore \quad n(B) = 5$

Here, $(A \cap B) = \{(2, 6), (4, 4), (6, 2)\}$

$\therefore \quad n(A \cap B) = 3$

Now, $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{3}{36}$

and $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{5}{36}$

Since, we have to find the conditional probability of getting an even number on first die, when the dice always exhibits a sum of B.

$\therefore$ Required probability $= P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{3/36}{5/36} = \dfrac{3}{5}$

EXAMPLE |6| Given that the two numbers appearing on throwing two dice are different. Find the probability of the events

(i) the sum of numbers on the dice is 4.

(ii) the sum of numbers on the dice is not 4. **[NCERT]**

Sol. Here, two dice are thrown, so number of outcomes in the sample space, $S = 6 \times 6 = 36$ (equally likely sample events) i.e. $n(S) = 36$.

Let E be the event of getting sum of the numbers on the dice is 4 and F be the event that numbers appearing on the two dice are different.

Then, $E = \{(1, 3), (2, 2), (3, 1)\}$

$\Rightarrow n(E) = 3$

and $F = \begin{Bmatrix} (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), \\ (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), \\ (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), \\ (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), \\ (6, 1), (6, 2), (6, 3), (6, 4), (6, 5) \end{Bmatrix}$

$\Rightarrow n(F) = 30$

Here, $E \cap F = \{(1, 3), (3, 1)\} = 2$

$\therefore \quad P(F) = \dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{30}{36} = \dfrac{5}{6}$

and $P(E \cap F) = \dfrac{2}{36} = \dfrac{1}{18}$

(i) The probability that the sum of numbers on the dice showing different number is 4,

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{1/18}{5/6} = \frac{1}{15}$$

(ii) The probability that the sum of numbers on the dice showing different numbers is not 4,

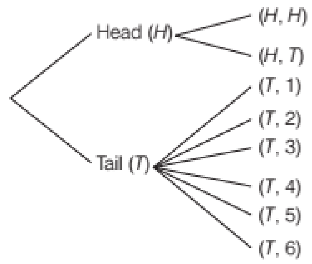
$$P\left(\frac{E'}{F}\right) = 1 - P\left(\frac{E}{F}\right) = 1 - \frac{1}{15} = \frac{14}{15}$$

EXAMPLE |7| Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that there is atleast one tail. **[NCERT; Delhi 2014C]**

Sol. The outcomes of the experiment can be represented in the following tree diagram.

The sample space S of the experiment is given as

$S = \{(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$



The probabilities of these elementary events are

$P\{(H, H)\} = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$

$P\{(H, T)\} = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$,

$P\{(T, 1)\} = \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12}$,

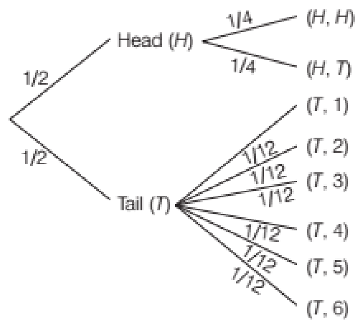
$P\{(T, 2)\} = \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12}$,

$P\{(T, 3)\} = \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12}$,

$P\{(T, 4)\} = \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12}$,

$P\{(T, 5)\} = \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12}$

and $P\{(T, 6)\} = \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12}$



Consider the following events

A = the die shows a number greater than 4

and B = there is atleast one tail.

We have, $A = \{(T, 5), (T, 6)\}$,

$B = \{(H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

and $A \cap B = \{(T, 5), (T, 6)\}$

$\therefore \quad P(B) = P\{(H, T)\} + P\{(T, 1)\} + P\{(T, 2)\} + P\{(T, 3)\} + P\{(T, 4)\} + P\{(T, 5)\} + P\{(T, 6)\}$

$\Rightarrow \quad P(B) = \frac{1}{4} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{4}$

and $P(A \cap B) = P\{(T, 5)\} + P\{(T, 6)\} = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

$\therefore$ Required probability $= P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{1/6}{3/4} = \frac{4}{18} = \frac{2}{9}$

Note Here, the elementary events are not equally likely. So, we cannot say that $P(B) = \frac{7}{8}$, $P(A \cap B) = \frac{2}{8}$.

TOPIC PRACTICE 1

OBJECTIVE TYPE QUESTIONS

1 A die is thrown once. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then, $P(A \cup B)$ is **[All India 2020]**

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1

2 From the set $\{1, 2, 3, 4, 5\}$, two numbers a and $b\,(a \neq b)$ are chosen at random. The probability that $\frac{a}{b}$ is an integer, is **[Delhi 2020]**

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$

3 Three dice are thrown simultaneously. The probability of obtaining a total score of 5 is **[Delhi 2020]**

(a) $\frac{5}{216}$ (b) $\frac{1}{6}$ (c) $\frac{1}{36}$ (d) $\frac{1}{49}$

4 A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is **[All India 2020]**

(a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

VERY SHORT ANSWER Type Questions

5 Events E and F are such that P (not E or not F) $= 0.25$. State whether E and F are mutually exclusive.

6 If $P\left(\frac{A}{B}\right) > P(A)$, then prove that $P\left(\frac{B}{A}\right) > P(B)$. **[NCERT]**

SHORT ANSWER Type I Questions

7 Three distinct numbers are chosen randomly from the first 50 natural numbers. Find the probability that all the three numbers are divisible by both 2 and 3. **[Delhi 2020]**

8 Find $[P(B/A) + P(A/B)]$, if $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$. **[Delhi 2020]**

9 The probability that atleast one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, then evaluate $P(\overline{A}) + P(\overline{B})$. **[NCERT Exemplar]**

10 If $P(\text{not } A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$. **[All India 2019]**

SHORT ANSWER Type II Questions

11 A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4. **[CBSE 2018]**

12 Given that E and F are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$. Find $P\left(\frac{E}{F}\right)$ and $P\left(\frac{F}{E}\right)$. **[NCERT]**

13 If $P(A) = 0.8$, $P(B) = 0.5$ and $P\left(\frac{B}{A}\right) = 0.4$, then find the value of $P(A \cup B)$. **[NCERT]**

14 If $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then find $P(A \cup B)' + P(A' \cup B)$. **[NCERT Exemplar]**

15 If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then find $P(A'/B') \cdot P(B'/A')$. **[NCERT Exemplar]**

16 A and B are two events such that $P(A) \neq 0$.

Find $P\left(\frac{B}{A}\right)$, if

(i) A is a subset of B. (ii) $A \cap B = \phi$. **[NCERT]**

17 A fair die is rolled. Consider the following events

$A = \{2, 4, 6\}, B = \{4, 5\}$ and $C = \{3, 4, 5, 6\}$

Find (i) $P(A \cup B / C)$ (ii) $P(A \cap B / C)$

18 Assume that each born child is equally likely to be a boy or a girl. If a family has two children, then what is the conditional probability that both are girls, given that

(i) the youngest is a girl?

(ii) atleast one is a girl? **[NCERT; Delhi 2014]**

19 A couple has 2 children. Find the probability that both are boys, if it is known that

(i) one of them is a boy.

(ii) the older child is a boy. **[Delhi 2014C]**

20 A couple has two children. Find the probability that both children are males, if it is known that atleast one of the children is male. **[NCERT]**

21 Two coins are tossed once, if E : tail appears on one coin and F : one coin shows head, then find $P\left(\frac{E}{F}\right)$. **[NCERT]**

22 A die is thrown three times. Events A and B are defined as below

A : 4 on the third throw

B : 6 on the first and 5 on the second throw.

Find the probability of A, given that B has already occurred. **[NCERT]**

23 A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared atleast once? **[NCERT]**

24 A black and a red die are rolled. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5. **[NCERT]**

25 An instructor has a question bank consisting of 300 easy true/false questions, 200 difficult true/false questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, then what is the probability that it will be an easy question, given that it is a multiple choice questions? **[NCERT]**

26 In a college, 70% students pass in Physics, 75% students pass in Mathematics and 10% students fail in both. One student is chosen at random. What is the probability that

(i) he passes in Physics and Mathematics?

(ii) he passes in Mathematics, given that he passes in Physics?

(iii) he passes in Physics, given that he passes in Mathematics?

27 In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random,

(i) find the probability that he/she reads neither Hindi nor English newspaper.

(ii) if he/she reads Hindi newspaper, then find the probability that she reads English newspaper.

(iii) if he/she reads English newspaper, then find the probability he/she reads Hindi newspaper. **[NCERT; Delhi 2011]**

28 Consider the experiment of throwing a die, if a multiple of 3 comes up, then throw the die again and if any other numbers comes, then toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'atleast one die shows 3'? **[NCERT]**

HINTS & SOLUTIONS

1. (d) We have, $A = \{4, 5, 6\}$ and $B = \{1, 2, 3, 4\}$

Now, $A \cap B = \{4\}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = \frac{6}{6} = 1$$

2. (c) We have sets of number {1, 2, 3, 4, 5}.

Sample space of choosing two numbers

$$= {}^5C_2 = \frac{5 \times 4}{1 \times 2} = 10$$

Favourable outcomes are $\left(\frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \frac{4}{2}\right) = 5$

$\therefore$ Required probability $= \frac{5}{10} = \frac{1}{2}$

3. (c) Three dice are thrown simultaneously favourable outcomes = {(1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2) (2, 1, 2) (2, 2, 1)}

= 6

Total number of outcomes $= 6^3 = 216$

$\therefore$ Required probability $= \frac{6}{216} = \frac{1}{36}$

4. (c) Let A be the event that card drawn is a spade and B be the event that card drawn is a queen. We have a total of 13 spades and 4 queen and one queen is from spade.

$$\therefore \quad P(A) = \frac{13}{52} = \frac{1}{4},$$
$$P(B) = \frac{4}{52} = \frac{1}{13}$$
and $$P(A \cap B) = \frac{1}{52}$$
$$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1/52}{1/13} = \frac{1}{4}$$

5. Given, $P(\text{not } E \text{ or not } F) = 0.25$

$\Rightarrow \quad P(E' \cup F') = P(E \cap F)' = 0.25$

$\because \quad P(E \cap F)' + P(E \cap F) = 1$

$\Rightarrow \quad P(E \cap F) = 1 - P(E \cap F)'$

$\therefore \quad P(E \cap F) = 1 - 0.25 = 0.75 \neq 0$

Hence, E and F are not mutually exclusive.

6. Given, $P\left(\frac{A}{B}\right) > P(A) \Rightarrow \frac{P(A \cap B)}{P(B)} > P(A)$

$\Rightarrow \frac{P(A \cap B)}{P(A)} > P(B) \Rightarrow P\left(\frac{B}{A}\right) > P(B)$

Hence proved.

7. Three distinct numbers are chosen from first 50 natural numbers in ${}^{50}C_3$ ways. Total numbers which is divisible by 2 and 3 from first 50 natural numbers is

$\{6, 12, 18, 24, 30, 36, 42, 48\} = 8$

$\therefore$ Required probability $= \frac{{}^8C_3}{{}^{50}C_3} = \frac{8 \times 7 \times 6}{50 \times 49 \times 48} = \frac{1}{350}$

8. We have, $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad \frac{3}{5} = \frac{3}{10} + \frac{2}{5} - P(A \cap B)$$
$$\Rightarrow \quad P(A \cap B) = \frac{3}{10} + \frac{2}{5} - \frac{3}{5} = \frac{1}{10}$$

Now $[P(B/A) + P(A/B)] = \frac{P(A \cap B)}{P(A)} + \frac{P(A \cap B)}{P(B)}$

$$= \frac{\frac{1}{10}}{3/10} + \frac{\frac{1}{10}}{2/5} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

9. We know that $A \cup B$ denotes the occurrence of atleast one of A and B and $A \cap B$ denotes the occurrence of both A and B, simultaneously.

Then, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$

$\because \; P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \quad 0.6 = P(A) + P(B) - 0.3 \Rightarrow P(A) + P(B) = 0.9$

$\Rightarrow \{[1 - P(\overline{A})] + [1 - P(\overline{B})]\} = 0.9$

$[\because P(A) = 1 - P(\overline{A}) \text{ and } P(B) = 1 - P(\overline{B})]$

$\Rightarrow P(\overline{A}) + P(\overline{B}) = 2 - 0.9 = 1.1$

10. Given, $P(A') = 0.7$, $P(B) = 0.7$ and $P\left(\frac{B}{A}\right) = 0.5$

Clearly, $P(A) = 1 - P(A') = 1 - 0.7 = 0.3$

Now, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow \quad 0.5 = \frac{P(A \cap B)}{0.3}$

$\Rightarrow \quad P(A \cap B) = 0.15$

$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} \Rightarrow P\left(\frac{A}{B}\right) = \frac{3}{14}$

11. Let us denote the numbers on black die by $B_1, B_2, \ldots, B_6$ and the numbers on red die by $R_1, R_2, \ldots, R_6$.

Then, we get the following sample space.

$$S = \left\{\begin{matrix}(B_1, R_1), (B_1, R_2), \ldots, (B_1, R_6), (B_2, R_1), (B_2, R_2), \ldots, \\ (B_2, R_6), \ldots, (B_6, R_1), (B_6, R_2), \ldots, (B_6, R_6)\end{matrix}\right\}$$

Clearly, $n(S) = 36$

Now, let A be the event that sum of number obtained on the die is 8 and B be the event that red die shows a number less than 4.

Then, $A = \{(B_2, R_6), (B_6, R_2), (B_3, R_5), (B_5, R_3), (B_4, R_4)\}$

and $B = \left\{\begin{matrix}(B_1, R_1), (B_1, R_2), (B_1, R_3), (B_2, R_1), (B_2, R_2), (B_2, R_3) \\ , \ldots, (B_6, R_1), (B_6, R_2), (B_6, R_3)\end{matrix}\right\}$

$\Rightarrow \quad A \cap B = \{(B_6, R_2), (B_5, R_3)\}$

Now, required probability,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}$$

12. Similar as Example 4 (ii) and (iii). $\left[\textbf{Ans. } \frac{2}{3} \text{ and } \frac{1}{3}\right]$

13. **Hint** $P(A \cap B) = P(A) \times P(B/A)$

and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ [**Ans.** 0.98]

14. Here, $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$

Since, $\quad P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \quad P(A \cap B) = P(A/B) \cdot P(B) = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A) = \frac{4}{5} - \frac{3}{5} + \frac{3}{10} = \frac{1}{2}$

$\therefore \quad P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{4}{5} = \frac{1}{5}$

and $P(A' \cup B) = 1 - P(A - B) = 1 - P(A \cap B')$

$= 1 - P(A) \cdot P(B') = 1 - \frac{1}{2} \cdot \frac{2}{5} = \frac{4}{5}$

$\Rightarrow \quad P(A \cup B)' + P(A' \cup B) = \frac{1}{5} + \frac{4}{5} = \frac{5}{5} = 1$

15. **Hint** $P\left(\frac{A'}{B'}\right)\times P\left(\frac{B'}{A'}\right)=\frac{[1-P(A\cup B)]^2}{[1-P(B)][1-P(A)]}$ $\left[\textbf{Ans. } \frac{25}{42}\right]$

16. **Hint** $P(A\cap B)=P(A)$ [**Ans.** (i) 1 (ii) 0]

17. Given events are

$$A=\{2,4,6\}, B=\{4,5\} \text{ and } C=\{3,4,5,6\}$$

Sample space, $S=\{1, 2, 3, 4, 5, 6\}$

Now, $A\cup B=\{2, 4, 5, 6\}, A\cap B=\{4\}$

$(A\cup B)\cap C=\{2, 4, 5, 6\}\cap\{3, 4, 5, 6\}=\{4, 5, 6\}$

and $A\cap B\cap C=\{4\}\cap\{3, 4, 5, 6\}=\{4\}$

$\therefore$ $n(S)=6, n[\{(A\cup B)\cap C\}]=3, n(A\cap B\cap C)=1$

and $n(C)=4$

(i) $P\left(\frac{A\cup B}{C}\right)=\frac{P\{(A\cup B)\cap C\}}{P(C)}=\frac{n\{(A\cup B)\cap C\}/n(S)}{n(C)/n(S)}$

$=\frac{3/6}{4/6}=\frac{3}{4}$

(ii) $P\left(\frac{A\cap B}{C}\right)=\frac{P(A\cap B\cap C)}{P(C)}=\frac{n(A\cap B\cap C)/n(S)}{n(C)/n(S)}$

$=\frac{1/6}{4/6}=\frac{1}{4}$

18. Let B represents elder child which is a boy and b represents younger child which is also a boy. Also, G represents elder child which is a girl and g represents younger child which is also a girl.

If a family has two children, then all possible cases are $S=\{Bb, Bg, Gg, Gb\}$

$\therefore$ $n(S)=4$

Let us define event A that both children are girls, then

$$A=\{Gg\} \Rightarrow n(A)=1$$

(i) Let E_1 be event that youngest child is a girl.

Then, $P\left(\frac{A}{E_1}\right)=\frac{P(A\cap E_1)}{P(E_1)}$

Now, $E_1=\{Bg, Gg\} \Rightarrow P(E_1)=\frac{2}{4}=\frac{1}{2}$

and $A\cap E_1=\{Gg\} \Rightarrow P(A\cap E_1)=\frac{1}{4}$

Thus, $P\left(\frac{A}{E_1}\right)=\frac{P(A\cap E_1)}{P(E_1)}=\frac{1/4}{1/2}=\frac{1}{2}$

$\therefore$ Required probability $=\frac{1}{2}$

(ii) Let E_2 be event that atleast one child is a girl.

Then, $E_2=\{Bg, Gg, Gb\}$

$\Rightarrow n(E_2)=3 \Rightarrow P(E_2)=\frac{3}{4}$

Now, $(A\cap E_2)=\{Gg\}\Rightarrow P(A\cap E_2)=\frac{1}{4}$

Thus, $P\left(\frac{A}{E_2}\right)=\frac{P(A\cap E_2)}{P(E_2)}=\frac{1/4}{3/4}=\frac{1}{3}$

$\therefore$ Required probability $=1/3$

19. Solve as Question 18. $\left[\textbf{Ans. } \text{(i) } \frac{1}{3} \quad \text{(ii) } \frac{1}{2}\right]$

20. **Hint** A = both children are male

and B = atleast one children is male

$\therefore$ $P(A)=\frac{1}{4}, P(B)=\frac{3}{4}, P(A\cap B)=\frac{1}{4}$

Now, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$ $\left[\textbf{Ans. } \frac{1}{3}\right]$

21. **Hint** $P(E)=\frac{2}{4}, P(F)=\frac{2}{4}, P(E\cap F)=\frac{2}{4}$

Now, $P\left(\frac{E}{F}\right)=\frac{P(E\cap F)}{P(F)}$ [**Ans.** 1]

22. **Hint** $P(B)=\frac{6}{216}, P(A\cap B)=\frac{1}{216}$

Now, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$ $\left[\textbf{Ans. } \frac{1}{6}\right]$

23. **Hint** Let E : sum of numbers is 8

and F : number 5 appears atleast once

Then, $P(E)=\frac{5}{36}, P(F)=\frac{11}{36}$ and $P(E\cap F)=\frac{2}{36}$

Required probability $=P\left(\frac{F}{E}\right)$ $\left[\textbf{Ans. } \frac{2}{5}\right]$

24. **Hint** A = sum greater than 9

and B = black die resulted 5

$\therefore$ $P(A)=\frac{6}{36}, P(B)=\frac{6}{36}, P(A\cap B)=\frac{2}{36}$,

Now, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$ $\left[\textbf{Ans. } \frac{1}{3}\right]$

25. Total number of questions,

$$n(S)=300+200+500+400=1400$$

Let E be the event that question is easy and F be the event that question is a multiple choice.

Then, $n(E)=500+300=800$

$\therefore P(E)=\frac{\text{Number of easy questions}}{\text{Total number of questions}}=\frac{n(E)}{n(S)}=\frac{800}{1400}=\frac{4}{7}$

Also, $n(F)=500+400=900$

$\therefore$ $P(F)=\frac{\text{Number of multiple choice questions}}{\text{Total number of questions}}$

$=\frac{n(F)}{n(S)}=\frac{900}{1400}=\frac{9}{14}$

and $P(E\cap F)$

$=\frac{\text{Total number of easy multiple choice questions}}{\text{Total number of questions}}$

$=\frac{500}{1400}=\frac{5}{14}$

$\therefore$ $P\left(\frac{E}{F}\right)=\frac{P(E\cap F)}{P(F)}=\frac{5/14}{9/14}=\frac{5}{9}$

26. $P(A)$ = Probability of students passed in Physics = 70/100

$P(B)$ = Probability of students passed in Mathematics = 75/100

$P(A' \cap B')$ = Probability of students fail in both = $\frac{10}{100}$

$\because P(A' \cap B') = 1 - P(A \cup B)$

$\therefore \quad P(A \cup B) = 1 - P(A' \cap B') = 1 - \frac{10}{100} = \frac{90}{100}$

(i) P(he passes in Physics and Mathematics)

$$= P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{70}{100} + \frac{75}{100} - \frac{90}{100} = \frac{55}{100} = \frac{11}{20}$$

(ii) P(he passes in Mathematics given that he passes in Physics)

$$= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{55}{100}}{\frac{70}{100}} = \frac{55}{70} = \frac{11}{14}$$

(iii) P(he passes in Physics given that he passes in Mathematics)

$$= P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{55}{100}}{\frac{75}{100}} = \frac{55}{75} = \frac{11}{15}$$

27. Let H = student reading Hindi newspaper and E = student reading English newspaper

Let $n(S) = 100$, then $n(H) = 60$, $n(E) = 40$ and $n(H \cap E) = 20$

$$\therefore P(H) = \frac{n(H)}{n(S)} = \frac{60}{100} = \frac{3}{5} \Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{40}{100} = \frac{2}{5}$$

and $P(H \cap E) = \frac{n(H \cap E)}{n(S)} = \frac{20}{100} = \frac{1}{5}$

(i) Required probability = P(student reads neither Hindi nor English newspaper)

$$= P(H' \cap E') = P(H \cup E)' = 1 - P(H \cup E)$$

$$= 1 - [P(H) + P(E) - P(H \cap E)]$$

$$= 1 - \left[\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right] = 1 - \frac{4}{5} = \frac{1}{5}$$

(ii) Required probability = P(student reads English newspaper, if he/she reads Hindi newspaper)

$$\therefore \quad P\left(\frac{E}{H}\right) = \frac{P(E \cap H)}{P(H)} = \frac{1/5}{3/5} = \frac{1}{3}$$

(iii) Required probability = P(student reads Hindi newspaper when it is given that he/she reads English newspaper)

$$\therefore \quad P\left(\frac{H}{E}\right) = \frac{P(H \cap E)}{P(E)} = \frac{1/5}{2/5} = \frac{1}{2}$$

28. Similar as Example 7. [**Ans.** 0]

|TOPIC 2|
Multiplication Theorem on Probability and Independent Events

MULTIPLICATION THEOREM

Let A and B be two events associated with a random experiment, then

$$P(A \cap B) = \begin{cases} P(A) \cdot P(B/A), \text{ where } P(A) \neq 0 \\ P(B) \cdot P(A/B), \text{ where } P(B) \neq 0 \end{cases}$$

Here, $A \cap B$ denotes the simultaneous occurrence of the events A and B. The event $A \cap B$ is also written as AB.

The above result is known as the multiplication rule of probability.

EXAMPLE |1| Find the probability of drawing a diamond card in each of the two consecutive draws from a well-shuffled pack of cards, if the card drawn is not replaced after the first draw.

Sol. Let A be the event of drawing a diamond card in the first draw and B be the event of drawing a diamond card in the second draw. Then, $P(A) = \frac{13}{52} = \frac{1}{4}$

After drawing a diamond card in first draw, 51 cards are left out of which 12 cards are diamond cards.

$\therefore \quad P(B/A)$ = Probability of drawing a diamond card in second draw when a diamond card has already been drawn in first draw

$$\Rightarrow \quad P\left(\frac{B}{A}\right) = \frac{12}{51} = \frac{4}{17}$$

Now, required probability

$$= P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right)$$

$$= \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$$

EXAMPLE |2| A bag contains 19 tickets, numbered from 1 to 19. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers.

Sol. Let A be the event of drawing an even numbered ticket in first draw and B be the event of drawing an even numbered ticket in the second draw. Then,

Required probability $= P(A \cap B) = P(A) \cdot P(B/A)$ ...(i)

Since, there are 19 tickets numbered 1 to 19 in the bag, out of which 9 are even numbered *viz.* 2, 4, 6, 8, 10, 12, 14, 16 and 18.

Therefore, $P(A) = \frac{9}{19}$

Since, the ticket drawn in the first draw is not replaced, therefore second ticket drawn is from the remaining 18 tickets, out of which 8 are even numbered.

$\therefore \quad P\left(\frac{B}{A}\right) = \frac{8}{18} = \frac{4}{9}$

Hence, required probability $= P(A \cap B)$

$= P(A) \cdot P(B/A)$ [from Eq. (i)]

$= \frac{9}{19} \times \frac{4}{9} = \frac{4}{19}$

Multiplication Rule for More than Two Events

Let E, F, G and H be four events of sample space S. Then,

(i) $P(E \cap F \cap G) = P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right)$

(ii) $P(E \cap F \cap G \cap H)$

$= P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right) \cdot P\left(\frac{H}{E \cap F \cap G}\right)$

EXAMPLE |3| Three cards are drawn successively without replacement from a pack of 52 well-shuffled cards. What is the probability that first two cards are king and the third card drawn is an ace?

Sol. There are 52 cards in a pack.

$\therefore \quad n(S) = 52$

Let A = event that the card drawn is king

and B = event that the card drawn is an ace.

Now, $P(A) = 4/52$

$P\left(\frac{A}{A}\right)$ = Probability of drawing second king when one king has already been drawn $= \frac{3}{51}$

[$\because$ remaining cards are $(52 - 1) = 51$]

$P(B/AA)$ = Probability of drawing third card to be an ace when two kings have already been drawn $= \frac{4}{50}$

Now, probability of getting first two cards are king and third card is an ace $= P(A \cap A \cap B)$

$= P(A) \cdot P\left(\frac{A}{A}\right) \cdot P\left(\frac{B}{AA}\right)$ [by multiplication theorem]

$= \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} = \frac{2}{5525}$

EXAMPLE |4| A bag contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one without replacement, then find the probability of getting all white balls.

Sol. Let A, B, C and D denote events of getting a white ball in first, second, third and fourth draw, respectively.

Then, required probability $= P(A \cap B \cap C \cap D)$

$= P(A)\, P(B/A)\, P(C/A \cap B)\, P(D/A \cap B \cap C)$ ...(i)

Now, $P(A)$ = Probability of drawing a white ball in first draw

$= \frac{5}{20} = \frac{1}{4}$

When a white ball is drawn in the first draw, there are 19 balls left in the bag, out of which 4 are white.

$\therefore \quad P(B/A) = \frac{4}{19}$

Since, the ball drawn is not replaced, therefore after drawing a white ball in second draw, there are 18 balls left in the bag, out of which 3 are white.

$\therefore \quad P(C/A \cap B) = \frac{3}{18} = \frac{1}{6}$

After drawing a white ball in third draw, there are 17 balls left in the bag, out of which 2 are white.

$\therefore \quad P(D/A \cap B \cap C) = \frac{2}{17}$

Hence, required probability $= P(A \cap B \cap C \cap D)$

$= P(A) \cdot P(B/A)\, P(C/A \cap B)\, P(D/A \cap B \cap C)$

$= \frac{1}{4} \times \frac{4}{19} \times \frac{1}{6} \times \frac{2}{17} = \frac{1}{969}$

INDEPENDENT EVENTS

Two events A and B are said to be independent, if the occurrence or non-occurrence of one event does not affect the occurrence or non-occurrence of another event. Two events E and F are said to be independent, if

$$P\left(\frac{F}{E}\right) = P(F), \text{ provided } P(E) \neq 0$$

and $$P\left(\frac{E}{F}\right) = P(E), \text{ provided } P(F) \neq 0.$$

In other words, let E and F be two events associated with the same random experiment, then E and F are said to be independent, if $P(E \cap F) = P(E) \cdot P(F)$.

Some Important Results

1. Two events E and F are said to be dependent, if they are not independent.
   i.e. $P(E \cap F) \neq P(E) \cdot P(F)$
2. Two experiments are said to be independent, if for every pair of events E and F, where E is associated with the first experiment and F with the second experiment, the probability of the simultaneous occurrence of the events E and F is the product of $P(E)$ and $P(F)$ calculated separately on the basis of two experiments.
   i.e. $P(E \cap F) = P(E) \cdot P(F)$
3. **Difference between independent events and mutually exclusive events**
   (i) Term independent is defined in terms of probability of events whereas mutually exclusive is defined in terms of subset of sample space.
   (ii) Mutually exclusive events never have an outcome common, but independent events may have common outcome. In other words, two independent events having non-zero probabilities of occurrence cannot be mutually exclusive and conversely i.e. two mutually exclusive events having non-zero probabilities of occurrence cannot be independent.
4. Three events A, B and C are said to be mutually independent, if

$$P(A \cap B) = P(A) \cdot P(B)$$
$$P(A \cap C) = P(A) \cdot P(C)$$
$$P(B \cap C) = P(B) \cdot P(C)$$
and $$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

If atleast one of the above is not true for three given events, then events are not independent.

i.e. dependent.

PROPERTIES OF INDEPENDENT EVENTS

If A and B are independent events, then

(i) A and B' are also independent events.
(ii) A' and B are also independent events.
(iii) A' and B' are also independent events.

EXAMPLE |5| An unbiased die is thrown twice. Let the event A be 'odd number on the first throw' and B be the event 'odd number on the second throw'. Check the independence of the events A and B.

Sol. On throwing a die twice, we get 36 elementary events of the experiment which are to be considered to be equally likely. Odd numbers are 1, 3 and 5.

$P(A) = P$ (odd number on the first throw)

$$= \frac{18}{36} = \frac{1}{2}$$

[∵ possible outcomes (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5) and (5, 6) i.e. 18 outcomes]

Similarly, $P(B) = P$ (odd number on the second throw)

$$= \frac{18}{36} = \frac{1}{2}$$

Also, $P(A \cap B) = P$(odd number on both throws)

$$= \frac{9}{36} = \frac{1}{4}$$

[∵ events are (1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3) and (5, 5) i.e. 9 outcomes]

Now, $P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Thus, $P(A \cap B) = P(A) \cdot P(B)$

Hence, A and B are independent events.

EXAMPLE |6| Two dice are thrown, find the probability of getting an odd number on the first die and a multiple of 3 on the other die. Also, show that both events are dependent.

Sol. Let A be the event of getting an odd number on the first die and B be the event of getting a multiple of 3 on the second die.

On throwing two dice, total outcomes = 36

Outcomes favourable to $A \cap B$, i.e. an odd number on first die and a multiple of 3 on the other die are

{(1, 3), (1, 6), (3, 3), (3, 6), (5, 3), (5, 6)}

$\therefore \quad n(A \cap B) = 6$

Then, required probability, $P(A \cap B) = \frac{6}{36} = \frac{1}{6}$

Now, $A = \{1, 3, 5\}$, $B = \{3, 6\}$

So, $n(A) = 3$, $n(B) = 2$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

and $$P(B) = \frac{2}{36} = \frac{1}{18}$$

Here, $P(A) \cdot P(B) = \frac{1}{12} \times \frac{1}{18} = \frac{1}{216} \neq P(A \cap B)$

Hence, A and B are dependent events.

Note If A and B are two dependent events, then we cannot apply $P(A \cap B) = P(A) \cdot P(B)$.

EXAMPLE |7| If each element of a second order determinant is either zero or one. What is the probability that the values of the determinants is positive? Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability 1/2. **[NCERT]**

Sol. A second order determinant has four entries which may be 0 or 1.

Total number of determinants $= 2^4 = 16$

The only positive determinants are $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$ and $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$.

Since, each entry of the above determinant can be selected with probability 1/2.

Therefore, required probability $= 3\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) = \frac{3}{16}$

EXAMPLE |8| A and B throw a pair of dice alternately. A wins the game, if he gets a total of 7 and B wins the game, if he gets a total of 10. If A starts the game, then find the probability that B wins. **[Delhi 2016]**

Sol. Here, $n(S) = 6 \times 6 = 36$

Let A = Event of getting a sum of 7 in pair of dice $= \{(1, 6), (2, 5), (3, 4), (6, 1), (5, 2), (4, 3)\}$

$\Rightarrow \quad n(A) = 6$

and B = Event of getting a sum of 10 in pair of dice $= \{(4, 6), (5, 5), (6, 4)\}$

$\Rightarrow \quad n(B) = 3$

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

and $\quad P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

$\Rightarrow \quad P(\overline{A}) = 1 - \frac{1}{6} = \frac{5}{6}$ and $P(\overline{B}) = 1 - \frac{1}{12} = \frac{11}{12}$

Now, the probability that if A start the game, then B wins

$$P(B \text{ wins}) = P(\overline{A} \cap B) + P(\overline{A} \cap \overline{B} \cap \overline{A} \cap B) + P(\overline{A} \cap \overline{B} \cap \overline{A} \cap \overline{B} \cap \overline{A} \cap B) + \dots$$

$$= P(\overline{A})\, P(B) + P(\overline{A}) P(\overline{B}) P(\overline{A}) P(B) + P(\overline{A}) P(\overline{B}) P(\overline{A}) P(\overline{B}) P(\overline{A})\, P(B) + \dots$$

[$\because$ events are independent]

$$= \frac{5}{6} \times \frac{1}{12} + \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{1}{12} + \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{1}{12} + \dots$$

$$= \frac{5}{72} + \frac{5}{72} \times \frac{55}{72} + \frac{5}{72} \times \left(\frac{55}{72}\right)^2 + \dots$$

$$= \frac{5}{72}\left[1 + \frac{55}{72} + \left(\frac{55}{72}\right)^2 + \dots\right] = \frac{5}{72}\left(\frac{1}{1 - \frac{55}{72}}\right)$$

$\left[\because \text{sum of infinite GP series is } \frac{a}{1-r}\right]$

$$= \frac{5}{72}\left(\frac{1}{17/72}\right) = \frac{5}{17}$$

TOPIC PRACTICE 2

OBJECTIVE TYPE QUESTIONS

1 A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white, is **[Delhi 2020]**

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{12}$ (d) $\frac{1}{24}$

2 Four cards are successively drawn without replacement from a deck of 52 playing cards. The probability that all the four cards are king is

(a) $\frac{1}{270721}$ (b) $\frac{1}{270722}$

(c) $\frac{1}{270724}$ (d) $\frac{1}{270725}$

3 If three mutually independent events are A, B and C, then

(a) $P(A \cap B) = P(A) \cdot P(B)$; $P(A \cap C) = P(A) \cdot P(C)$

(b) $P(B \cap C) = P(B) \cdot P(C)$

(c) $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

(d) All of the above

4 If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$ equals to **[NCERT Exemplar]**

(a) $\frac{4}{15}$ (b) $\frac{8}{45}$

(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

5 Two events A and B are said to be independent, if **[NCERT]**

(a) A and B are mutually exclusive

(b) $P(A' \cap B') = [1 - P(A)]\,[1 - P(B)]$

(c) $P(A) = P(B)$

(d) $P(A) + P(B) = 1$

VERY SHORT ANSWER Type Questions

6 Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$, find $P(A' \cap B')$. **[All India 2020]**

7 A die is tossed thrice. Find the probability of getting an even number atleast once. **[NCERT]**

SHORT ANSWER Type I Questions

8 Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black. **[NCERT]**

9 An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

10 A die marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event 'number is even' and B be the event 'number is marked red'. Find whether the events A and B are independent or not. **[Delhi2019]**

11 Prove that if E and F are independent events, then the events E and F' are also independent. **[Delhi 2017]**

12 A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events. **[All India 2017]**

13 A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'number is odd' and B be the event, 'number is green'. Are A and B independent? **[NCERT]**

SHORT ANSWER Type II Questions

14 If A and B are independent events, then prove that

(i) $P(A) = P(A \cap B) + P(A \cap \overline{B})$.

(ii) $P(A \cup B) = P(A \cap B) + P(A \cap \overline{B}) + P(\overline{A} \cap B)$.

[NCERT Exemplar]

15 Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$. Find

(i) $P(A \text{ and not } B)$ (ii) $P(A \text{ or } B)$.

16 If A and B are two independent events, then prove that the probability of occurrence of at least one of A and B is given by $1 - P(A') \cdot P(B')$. **[All India 2017C]**

17 If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, then find P (not A and not B). **[NCERT]**

18 One card is drawn at random from a well-shuffled deck of 52 cards. Events E and F are defined below

E : the card drawn is a spade

F : the card drawn is an ace

Check whether the events are dependent or independent. **[NCERT]**

19 Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

(i) both balls are red.

(ii) first ball is black and second ball is red.

[NCERT]

20 Probability of solving specific problem independently by A and B are 1/2 and 1/3, respectively. If both try to solve the problem independently, then find the probability that

(i) the problem is solved.

(ii) exactly one of them solves the problem.

[NCERT]

21 A can hit a target 4 times out of 5 times, B can hit the target 3 times out of 4 times and C can hit the target 2 times out of 3 times. They fire simultaneously. Find the probability that

(i) any two out of A, B and C will hit the target.

(ii) none of them will hit the target.

22 A and B throw a pair of dice alternately. A wins the game, if he gets a total of 6 and B wins, if she gets a total of 7. If A starts the game, then find the probability of winning the game by A in third throw of the pair of dice.

23 A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first. **[All India 2016]**

LONG ANSWER Type Questions

24 Two dice are thrown together and the total score is noted. The events E, F and G are 'a total of 4', 'a total of 9 or more' and 'a total divisible by 5', respectively. Calculate $P(E)$, $P(F)$ and $P(G)$ and decide which pairs of events, if any are independent? **[NCERT Exemplar]**

25 If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$, then find $P(A)$ and $P(B)$. **[Delhi 2015]**

HINTS & SOLUTIONS

1. (c) We have 3 white, 4 black and 2 red balls.

Total number of balls = (3 + 4 + 2) = 9

Two balls are drawn at random (without replacement).

Then, the probability that both the balls are white

$$= \frac{3}{9} \times \frac{2}{8} = \frac{6}{72} = \frac{1}{12}$$

2. (d) Let A denotes the event that the first card is king, B denotes the event that the second card is king, C denotes the event that the third card is king and D denotes the event that the fourth card is king.

Now, $P(A \cap B \cap C \cap D)$

$$= P(A) \times P(B/A) \times P(C/A \cap B) \times P(D/A \cap B \cap C)$$

$$= \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49}$$

$$= \frac{1}{13} \times \frac{1}{17} \times \frac{1}{25} \times \frac{1}{49}$$

$$= \frac{1}{270725}$$

3. (d) We know, three events A, B and C are said to be mutually independent, if

$$P(A \cap B) = P(A) \cdot P(B)$$
$$P(A \cap C) = P(A) \cdot P(C)$$
$$P(B \cap C) = P(B) \cdot P(C)$$

and $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

If any one of the above conditions is not true for three given events, we say that the events are not mutually independent.

4. (d) $P(A' \cap B') = 1 - P(A \cup B)$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[\frac{3}{5} + \frac{4}{9} - \frac{3}{5} \times \frac{4}{9}\right]$$

$[\because P(A \cap B) = P(A) \cdot P(B)]$

$$= 1 - \left[\frac{27 + 20 - 12}{45}\right]$$

$$= 1 - \frac{35}{45} = \frac{10}{45} = \frac{2}{9}$$

5. (b) A and B are independent, if $P(A \cap B) = P(A)\,P(B)$

$\therefore$ $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - P(A) - P(B) + P(A)\,P(B)$$
$$= [1 - P(A)]\,[1 - P(B)]$$

6. Given, $P(A) = 0.3$ and $P(B) = 0.6$

Now $P(A' \cap B') = P(A \cup B)' = 1 - P[A \cup B]$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - \{0.3 + 0.6 - 0.3 \times 0.6\}$$

[$\because$ A and B are independent events $\therefore P(A \cap B) = P(A)P(B)$]

$$= 1 - \{0.9 - 0.18\}$$
$$= 1 - \{0.72\} = 0.28$$

7. P (getting even number on a die) $= \frac{3}{6} = \frac{1}{2}$

$\therefore$ Required probability $= 1 -$ Probability of getting an even number in none of the throw

$$= 1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{7}{8}$$

8. There are 26 black cards in a pack of 52 cards.

Required probability $= P(E \text{ and } F)$

$$= P(E) \cdot P\left(\frac{F}{E}\right)$$

where, E : First card drawn is black
and F : Second card drawn is black.

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{26}{52} = \frac{1}{2}$$

$P\left(\frac{F}{E}\right) = \frac{25}{51}$ [after drawing 1 black card, there will be 25 black cards left]

$\therefore$ Required probability,

$$P(E \cap F) = P(E) \cdot P\left(\frac{F}{E}\right)$$

$$= \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

9. **Hint** $P(BB) = \frac{10}{15} \times \frac{9}{14}$ $\left[\textbf{Ans. } \frac{3}{7}\right]$

10. Given, a die marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed.

Clearly, $S = \{1, 2, 3, 4, 5, 6\}$

Also, we have the following events

A : the number is even

and B : the number is red

Now, two events A and B are independent, if

$P(A \cap B) = P(A) \cdot P(B)$

$\therefore$ $A = \{2, 4, 6\}$

$\Rightarrow$ $P(A) = \frac{3}{6} = \frac{1}{2}$,

$B = \{1, 2, 3\}$

$\Rightarrow$ $P(B) = \frac{3}{6} = \frac{1}{2}$

and $A \cap B =$ The number is red and even $= \{2\}$

$\Rightarrow P(A \cap B) = \frac{1}{6}$

Since, $P(A \cap B) \neq P(A) \cdot P(B)$

Therefore, A and B are not independent.

11. Given, E and F are independent events, therefore

$$P(E \cap F) = P(E)\,P(F) \quad \text{...(i)}$$

Now, we have

$P(E \cap F') + P(E \cap F) = P(E)$

$\Rightarrow$ $P(E \cap F') = P(E) - P(E \cap F)$

$\Rightarrow$ $P(E \cap F') = P(E) - P(E)P(F)$ [using Eq. (i)]

$\Rightarrow$ $P(E \cap F') = P(E)[1 - P(F)]$

$\Rightarrow$ $P(E \cap F') = P(E)P(F')$

$\therefore$ E and F' are also independent events.

Hence proved.

12. When a die is thrown, then sample space is

$S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

Also, A : number is even and B : number is red.

$\therefore \quad A = \{2, 4, 6\}, B = \{1, 2, 3\}$ and $A \cap B = \{2\}$

$\Rightarrow \; n(A) = 3, n(B) = 3$ and $n(A \cap B) = 1$

Now, $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}, P(B) = \dfrac{n(B)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}$

and $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{1}{6}$

Now, $P(A) \times P(B) = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4} \neq \dfrac{1}{6} = P(A \cap B)$

$\therefore \qquad P(A \cap B) \neq P(A) \times P(B)$

Thus, A and B are not independent events.

13. When a die is thrown, then sample space,
$S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

Also, A : Number is odd and B : Number is green

$\therefore A = \{1, 3, 5\}, B = \{4, 5, 6\}$ and $A \cap B = \{5\}$

$\Rightarrow \qquad n(A) = 3, n(B) = 3, n(A \cap B) = 1$

Now, $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}, P(B) = \dfrac{n(B)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2},$

$P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{1}{6}$

and $\quad P(A) \times P(B) = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4} \neq \dfrac{1}{6} = P(A \cap B)$

$\therefore \qquad P(A \cap B) \neq P(A) \times P(B)$

Thus, A and B are not independent events.

14. (i) **Hint** RHS $= P(A \cap B) + P(A \cap \overline{B})$

$= P(A) \cdot P(B) + P(A) \cdot P(\overline{B}) = P(A)[P(B) + P(\overline{B})$

$= P(A)[P(B) + 1 - P(B)] = P(A) =$ LHS

(ii) Similar as part (i).

15. **Hint** (i) $P(A$ and not $B) = P(A) - P(A \cap B)$

$= 0.3 - (0.3)(0.6)$

[$\because A$ and B are independent events]

(ii) $P(A$ or $B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.18$ [**Ans.** (i) 0.12 (ii) 0.72]

16. Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A) \cdot P(B) = P(A)[1 - P(B)] + 1 - P(B')$

$= P(A)P(B') - P(B') + 1$

$= 1 - P(B')[1 - P(A)] = 1 - P(A')P(B')$ **Hence proved.**

17. **Hint** $P($not A and not $B) = P(A') \times P(B')$

[$\because A$ and B are independent events]

$= [1 - P(A)][1 - P(B)] = \left(1 - \dfrac{1}{4}\right)\left(1 - \dfrac{1}{2}\right)$ $\left[\textbf{Ans. } \dfrac{3}{8}\right]$

18. **Hint** $P(E) = \dfrac{1}{4}, P(F) = \dfrac{1}{13}, P(E \cap F) = \dfrac{1}{52}$

Now, $P(E \cap F) = P(E) \times P(F)$

[**Ans.** E and F are independent events]

19. **Hint** (i) $P(RR) = \dfrac{8}{18} \times \dfrac{8}{18}$

(ii) $P(BR) = \dfrac{10}{18} \times \dfrac{8}{18}$ $\left[\textbf{Ans. } \text{(i)} \dfrac{16}{81} \text{ (ii)} \dfrac{20}{81}\right]$

20. We have, $P(A) = \dfrac{1}{2}$ and $P(B) = \dfrac{1}{3}$

$\therefore \qquad P(\overline{A}) = 1 - P(A) = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

and $\qquad P(\overline{B}) = 1 - P(B) = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

(i) P(problem is solved)

$= P$(atleast one of them will solve the problem)

$= 1 - P$(none of them solve the problem)

$= 1 - P(\overline{A}) \cdot P(\overline{B})$

[$\because A$ and B are independent events, then $\overline{A}$ and $\overline{B}$ are also independent events]

$= 1 - \dfrac{1}{2} \times \dfrac{2}{3} = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

(ii) P (exactly one of them solves the problem)

$= P(A \cap \overline{B}) + P(\overline{A} \cap B) = P(A) \cdot P(\overline{B}) + P(\overline{A}) \cdot P(B)$

[$\because A$ and B are independent events, then $A, \overline{B}$ and $\overline{A}, B$ are also independent events]

$= \dfrac{1}{2} \times \dfrac{2}{3} + \dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{2}{6} + \dfrac{1}{6} = \dfrac{3}{6} = \dfrac{1}{2}$

21. We have, $P(A) = \dfrac{4}{5}, P(B) = \dfrac{3}{4}$ and $P(C) = \dfrac{2}{3}$

Then, $\quad P(\overline{A}) = 1 - P(A) = 1 - \dfrac{4}{5} = \dfrac{1}{5},$

$P(\overline{B}) = 1 - P(B) = 1 - \dfrac{3}{4} = \dfrac{1}{4}$

and $\quad P(\overline{C}) = 1 - P(C) = 1 - \dfrac{2}{3} = \dfrac{1}{3}$

(i) P (any two of them hit the target)

$P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C)$

$= P(A) \cdot P(B) \cdot P(\overline{C}) + P(A) \cdot P(\overline{B}) \cdot P(C)$

$+ P(\overline{A}) \cdot P(B) \cdot P(C)$

[$\because A, B, C,$ are independent events]

$= \left(\dfrac{4}{5} \times \dfrac{3}{4} \times \dfrac{1}{3}\right) + \left(\dfrac{4}{5} \times \dfrac{1}{4} \times \dfrac{2}{3}\right) + \left(\dfrac{1}{5} \times \dfrac{3}{4} \times \dfrac{2}{3}\right)$

$= \dfrac{12}{60} + \dfrac{8}{60} + \dfrac{6}{60} = \dfrac{26}{60} = \dfrac{13}{30}$

(ii) P (none of them hit the target) $= P(\overline{A}) \cdot P(\overline{B}) \cdot P(\overline{C})$

[$\because A, B$ and C are independent events]

$= \dfrac{1}{5} \times \dfrac{1}{4} \times \dfrac{1}{3} = \dfrac{1}{60}$

22. **Hint** $P(A) = \dfrac{5}{36}, P(B) = \dfrac{6}{36}$

Required probability $= P(\overline{A}) \cdot P(\overline{B}) \cdot P(A)$ $\left[\textbf{Ans. } \dfrac{775}{7776}\right]$

23. Here, $n(S) = 6 \times 6 = 36$

Let E = Event of getting a total 10

$= \{(4, 6), (5, 5), (6, 4)\}$

$\Rightarrow n(E) = 3$

$\therefore$ $P(\text{getting a total of } 10) = P(E) = \dfrac{n(E)}{n(S)} = \dfrac{3}{36} = \dfrac{1}{12}$

and $P(\text{not getting a total of } 10) = P(\overline{E})$

$$= 1 - P(E) = 1 - \frac{1}{12} = \frac{11}{12}$$

Thus, $P(A \text{ getting } 10) = P(B \text{ getting } 10) = \dfrac{1}{12}$

and $P(A \text{ is not getting } 10) = P(B \text{ is not getting } 10)$

$$= \frac{11}{12}$$

Now, $P(A \text{ winning}) = P(A) + P(\overline{A} \cap \overline{B} \cap A)$

$$+ P(\overline{A} \cap \overline{B} \cap \overline{A} \cap \overline{B} \cap A) + \ldots$$

$$= P(A) + P(\overline{A})P(\overline{B})P(A) + P(\overline{A})P(\overline{B})P(\overline{A})\,P(\overline{B})P(A) + \ldots$$

$$= \frac{1}{12} + \frac{11}{12} \times \frac{11}{12} \times \frac{1}{12} + \frac{11}{12} \times \frac{11}{12} \times \frac{11}{12} \times \frac{11}{12} \times \frac{1}{12} + \ldots$$

$$= \frac{1}{12}\left[1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \ldots\right] = \frac{1}{12}\left[\frac{1}{1 - \left(\frac{11}{12}\right)^2}\right]$$

$$\left[\because \text{the sum of an infinite GP is } S_\infty = \frac{a}{1-r}\right]$$

$$= \frac{1}{12}\left[\frac{1}{\frac{144-121}{144}}\right] = \frac{12}{23}$$

Now, $P(B \text{ winning})\, 1 - P(A \text{ winning}) = 1 - \dfrac{12}{23} = \dfrac{11}{23}$

Hence, the probabilities of winning A and B are respectively, $\dfrac{12}{23}$ and $\dfrac{11}{23}$.

24. Two dice are thrown together, so number of outcomes in the sample space is $36 \Rightarrow n(S) = 36$

E = Total of 4 = $\{(2, 2), (3, 1), (1, 3)\}$

$\Rightarrow$ $n(E) = 3$

F = Total of 9 or more

$$= \left\{\begin{matrix}(3, 6), (6, 3), (4, 5), (5, 4), (4, 6), \\ (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)\end{matrix}\right\}$$

$\Rightarrow$ $n(F) = 10$

and G = Total divisible by 5

$= \{(1, 4), (4, 1), (2, 3), (3, 2), (4, 6), (6, 4), (5, 5)\}$

$\Rightarrow n(G) = 7$

Here, $(E \cap F) = \phi$ and $(E \cap G) = \phi$

Also, $(F \cap G) = \{(4, 6), (6, 4), (5, 5)\}$

$\Rightarrow$ $n(F \cap G) = 3$ and $(E \cap F \cap G) = \phi$

$\therefore$ $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{3}{36} = \dfrac{1}{12}$

$$P(F) = \frac{n(F)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

and $P(G) = \dfrac{n(G)}{n(S)} = \dfrac{7}{36}$

$$P(F \cap G) = \frac{3}{36} = \frac{1}{12}$$

and $P(F) \cdot P(G) = \dfrac{5}{18} \times \dfrac{7}{36} = \dfrac{35}{648}$

Here, we see that $P(F \cap G) \neq P(F) \cdot P(G)$

[since only F and G have common events, so only F and G are used here]

Hence, there is no pair which is independent.

25. Given, A and B are two independent events with

$$P(\overline{A} \cap B) = \frac{2}{15} \text{ and } P(A \cap \overline{B}) = \frac{1}{6}.$$

We know that, if A and B are independent events, then $\overline{A}, B$ and $A, \overline{B}$ are independent events.

Now, $P(\overline{A} \cap B) = \dfrac{2}{15} \Rightarrow P(B) \cdot P(\overline{A}) = \dfrac{2}{15}$

$\Rightarrow$ $P(B) \cdot [1 - P(A)] = \dfrac{2}{15}$

$\Rightarrow$ $P(B) - P(A) \cdot P(B) = \dfrac{2}{15}$ ...(i)

and $P(A \cap \overline{B}) = \dfrac{1}{6} \Rightarrow P(A) \cdot P(\overline{B}) = \dfrac{1}{6}$

$\Rightarrow$ $P(A) \cdot [1 - P(B)] = \dfrac{1}{6}$

$\Rightarrow$ $P(A) - P(A)P(B) = \dfrac{1}{6}$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$$P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{5-4}{30} = \frac{1}{30}$$

$\Rightarrow$ $P(A) = \dfrac{1}{30} + P(B)$ ...(iii)

Now, on substituting the value of $P(A)$ in Eq. (i), we get

$$P(B) - \left[\frac{1}{30} + P(B)\right] \cdot P(B) = \frac{2}{15}$$

Let $P(B) = x$, then

$$x - \left(\frac{1}{30} + x\right)x = \frac{2}{15}$$

$\Rightarrow$ $30x - (1 + 30x)x = 4 \Rightarrow 30x - x - 30x^2 = 4$

$\Rightarrow$ $30x^2 - 29x + 4 = 0 \Rightarrow (6x - 1)(5x - 4) = 0$

$\Rightarrow$ $x = \dfrac{1}{6}$ or $\dfrac{4}{5} \Rightarrow P(B) = \dfrac{1}{6}$ or $\dfrac{4}{5}$ $[\because x = P(B)]$

Now, if $P(B) = \dfrac{1}{6}$, then $P(A) = \dfrac{1}{5}$ [using Eq. (iii)]

and if $P(B) = \dfrac{4}{5}$, then $P(A) = \dfrac{5}{6}$

|TOPIC 3|

Theorem of Total Probability and Baye's Theorem

PARTITION OF A SAMPLE SPACE

A set of events $E_1, E_2, \ldots, E_n$ is said to represent a partition of the sample space S, if it satisfies the following conditions

(i) $E_i \cap E_j = \phi, i \neq j; i, j = 1, 2, \ldots, n$

(ii) $E_1 \cup E_2 \cup \ldots \cup E_n = S$

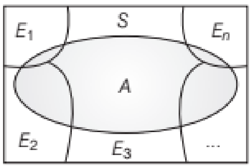
(iii) $P(E_i) > 0, \forall\, i = 1, 2, \ldots, n$

In other words, the events $E_1, E_2, \ldots, E_n$ represent a partition of the sample space S, if they are pairwise disjoint, exhaustive and have non-zero probabilities.

Note The partition of a sample space is not unique. There can be several partitions of the same sample space.

THEOREM OF TOTAL PROBABILITY

Let $\{E_1, E_2, \ldots, E_n\}$ be a partition of the sample space S and suppose that each of the events $E_1, E_2, \ldots, E_n$ has non-zero probability of occurrence.



Let A be any event associated with S, then

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + \ldots + P(E_n) \cdot P(A/E_n)$$

$$= \sum_{j=1}^{n} P(E_j) \cdot P(A/E_j)$$

EXAMPLE |1| Let bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, then find the probability of one of them being red and another black. **[Delhi 2015]**

Sol. Given, bag A = 4 black and 6 red balls

and bag B = 7 black and 3 red balls.

Let E_1 = the event that die show 1 or 2

E_2 = the event that die show 3 or 4 or 5 or 6

and E = the event that among two drawn balls, one of them is red and other is black

Now, $P(E_1) = \frac{2}{6}, P(E_2) = \frac{4}{6}$

[$\because$ total number in a die is six]

$P\left(\frac{E}{E_1}\right) = P$ (getting one red and one black from bag A)

$$= \frac{{}^4C_1 \times {}^6C_1}{{}^{10}C_2} = \frac{4 \times 6 \times 2}{10 \times 9}$$

and $P\left(\frac{E}{E_2}\right) = P$ (getting one red and one black from bag B)

$$= \frac{{}^7C_1 \times {}^3C_1}{{}^{10}C_2} = \frac{7 \times 3 \times 2}{10 \times 9}$$

$\therefore$ Total probability,

$$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$$

$$= \frac{2}{6} \cdot \left(\frac{4 \times 6 \times 2}{10 \times 9}\right) + \frac{4}{6} \cdot \left(\frac{7 \times 3 \times 2}{10 \times 9}\right)$$

$$= \frac{4 \times 6}{6 \times 10 \times 9}(4 + 7) = \frac{4 \times 6 \times 11}{6 \times 10 \times 9} = \frac{22}{45}$$

EXAMPLE |2| A bag contains $(2n + 1)$ coins. It is known that $(n - 1)$ of these coins have a head on both sides, whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n. **[All India 2016C]**

Sol. Number of coins with head on both sides = $(n-1)$

Number of fair coins = $(n+2)$

Let E_1 = Picking a coin with head on both sides

E_2 = Picking a fair coin

and A = Getting a head on tossing the coin

Then, $P(E_1) = \frac{n-1}{2n+1}, P(E_2) = \frac{n+2}{2n+1}$

$\Rightarrow P\left(\frac{A}{E_1}\right) = 1, P(A/E_2) = 1/2$

$$\therefore \quad P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$$

$$= \frac{n-1}{2n+1} \cdot 1 + \frac{n+2}{2n+1} \cdot \frac{1}{2} = \frac{3n}{2(2n+1)}$$

$$\Rightarrow \frac{3n}{2(2n+1)} = \frac{31}{42}$$

$$\Rightarrow \quad n = 31$$

BAYE'S THEOREM

If $E_1, E_2, \ldots, E_n$ are n non-empty events, which constitute a partition of sample space S, i.e.
$E_1, E_2, \ldots, E_n$ are pairwise disjoint,
$E_1 \cup E_2 \cup \ldots \cup E_n = S$ and $P(E_i) > 0, \ \forall i = 1, 2, 3, \ldots, n$.
Also, let A be any event of non-zero probability, then

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{j=1}^{n} P(E_j) \cdot P(A/E_j)}, \text{ for any } i = 1, 2, 3, \ldots, n$$

Or

Let $E_1, E_2, E_3, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is an event, which occurs together with E_i's. Then,

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{j=1}^{n} P(E_j) \cdot P(A/E_j)}$$

Here, events $E_1, E_2, \ldots, E_n$ are called **hypothesis**. The probability $P(E_i)$ is called the **priori probability** of the hypothesis E_i and the conditional probability $P(E_i/A)$ is called a **posteriori probability** of the hypothesis E_i. Baye's theorem is also called the formula for the probability of causes.

METHOD TO SOLVE PROBLEMS BASED ON BAYE'S THEOREM

To solve problems based on Baye's theorem, we use the following steps

I. Identify the events, which constitute a partition of sample space S and name them E_1, E_2, E_3 and so on.
II. Assume the other event of non-zero probability as A.
III. Find the probabilities of $E_1, E_2, E_3, \ldots$.
IV. Find the conditional probabilities of A with $E_1, E_2, E_3, \ldots$.
V. Use Baye's theorem to find required probability, i.e. $P(E_i / A)$.

EXAMPLE |3| A box contains 4 orange and 4 green balls, another box contains 3 orange and 5 green balls, one of the two box is selected at random and a ball is drawn from the box, which is found to be orange. Find the probability that the ball is drawn from the first box.

Sol. Let E_1 = event of selecting box I,
E_2 = event of selecting box II
and A = event of drawing an orange ball.

Now, $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{2}$

Now, $P(A/E_1)$ = Probability of selecting an orange ball from box I

$= \frac{4}{8} = \frac{1}{2}$

[$\because$ total balls = 8 and orange balls = 4]

and $P(A/E_2)$ = Probability of selecting an orange ball from box II

$= \frac{3}{8}$ [$\because$ total balls = 8 and orange balls = 3]

Now, $P(E_1/A)$ = Probability that drawn ball is from the first box

$$= \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{3}{8}\right)} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{3}{16}}$$

$$= \frac{1/4}{\frac{4+3}{16}} = \frac{1}{4} \times \frac{16}{7} = \frac{4}{7}$$

Hence, the probability that the ball is drawn from the first box is $\frac{4}{7}$.

Note If $P(E_1) = P(E_2) = P(E_3) = \ldots = P(E_n)$, then

$$P(E_i/E) = \frac{P(E/E_i)}{\sum_{j=1}^{n} P(E/E_j)}$$

EXAMPLE |4| There are three coins, one is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows head. What is the probability that it was the two headed coin? **[NCERT; All India 2014]**

Sol. Let us define the events as
E_1 = selecting a two headed coin,
E_2 = selecting a biased coin,
E_3 = selecting an unbiased coin,
A = head comes up.
Here, $P(E_1) = P(E_2) = P(E_3) = 1/3$

[$\because$ all coins have equal chances]

$P(A/E_1)$ = Probability that head comes up on a two headed coin

$= 1$

$P(A/E_2)$ = Probability that head comes up on a biased coin

$= 75\% = 75/100$

and $P(A/E_3)$ = Probability that head comes up on an unbiased coin

$= 1/2$

Now, by Baye's theorem, we get

$$P(E_1/A)=\frac{P(E_1)\cdot P(A/E_1)}{\left[\begin{array}{r}P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)\\+P(E_3)\cdot P(A/E_3)\end{array}\right]}$$

$$\therefore\quad P(E_1/A)=\frac{\frac{1}{3}\times 1}{\left(\frac{1}{3}\times 1\right)+\left(\frac{1}{3}\times\frac{75}{100}\right)+\left(\frac{1}{3}\times\frac{1}{2}\right)}$$

$$=\frac{1}{1+\frac{75}{100}+\frac{1}{2}}=\frac{1}{\frac{100+75+50}{100}}=\frac{100}{225}=\frac{4}{9}$$

Hence, the required probability is $4/9$.

EXAMPLE |5| In a factory which manufactures bolts, machines A, B and C manufacture respectively, 30%, 50% and 20% of the bolts. Of their outputs, 3%, 4% and 1% respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

[All India 2015]

Sol. Let E_1 = bolt is manufactured by machine A

E_2 = bolt is manufactured by machine B

E_3 = bolt is manufactured by machine C

and E = bolt is defective.

Then, we have $P(E_1)=30\%=\frac{30}{100}$, $P(E_2)=50\%=\frac{50}{100}$

and $P(E_3)=20\%=\frac{20}{100}$

Also, given that 3%, 4% and 1% bolts manufactured by machines A, B and C respectively are defective. So,

$$P\left(\frac{E}{E_1}\right)=3\%=\frac{3}{100},\quad P\left(\frac{E}{E_2}\right)=4\%=\frac{4}{100}\text{ and}$$

$$P\left(\frac{E}{E_3}\right)=1\%=\frac{1}{100}$$

Now, firstly find the probability that selected bolt which is defective is manufactured by machine B, is

$$P\left(\frac{E_2}{E}\right)=\frac{P(E_2)\cdot P\left(\frac{E}{E_2}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_3)\cdot P\left(\frac{E}{E_3}\right)}$$

$$=\frac{\frac{50}{100}\times\frac{4}{100}}{\frac{30}{100}\times\frac{3}{100}+\frac{50}{100}\times\frac{4}{100}+\frac{20}{100}\times\frac{1}{100}}$$

$$=\frac{200}{90+200+20}=\frac{200}{310}$$

$\therefore$ The probability that selected bolt which is defective is not manufactured by machine B, is

$$1-P\left(\frac{E_2}{E}\right)=1-\frac{200}{310}=\frac{110}{310}=\frac{11}{31}$$

EXAMPLE |6| Three persons A, B and C apply for a job of Manager in a private company. Chances of their selection (A, B and C) are in the ratio $1:2:4$. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

[Delhi 2016]

Sol. Let us define the following events

A = selecting person A

B = selecting person B

C = selecting person C

$$P(A)=\frac{1}{1+2+4},\quad P(B)=\frac{2}{1+2+4}\text{ and }P(C)=\frac{4}{1+2+4}$$

$$\Rightarrow\ P(A)=\frac{1}{7},\ P(B)=\frac{2}{7}\text{ and }P(C)=\frac{4}{7}$$

Let E = person introduce the changes in their profit.

Also, given $P\left(\frac{E}{A}\right)=0.8$, $P\left(\frac{E}{B}\right)=0.5$ and $P\left(\frac{E}{C}\right)=0.3$

$$\Rightarrow\quad P\left(\frac{\overline{E}}{A}\right)=1-0.8=0.2,\ P\left(\frac{\overline{E}}{B}\right)=1-0.5=0.5$$

and $P\left(\frac{\overline{E}}{C}\right)=1-0.3=0.7$

The probability that change does not take place by the appointment of C,

$$P\left(\frac{C}{\overline{E}}\right)=\frac{P(C)\cdot P\left(\frac{\overline{E}}{C}\right)}{P(A)\times P\left(\frac{\overline{E}}{A}\right)+P(B)\times P\left(\frac{\overline{E}}{B}\right)+P(C)\times P\left(\frac{\overline{E}}{C}\right)}$$

$$=\frac{\frac{4}{7}\times 0.7}{\frac{1}{7}\times 0.2+\frac{2}{7}\times 0.5+\frac{4}{7}\times 0.7}$$

$$=\frac{2.8}{0.2+1.0+2.8}=\frac{2.8}{4}=0.7$$

EXAMPLE |7| A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope, just two consecutive letter TA are visible. What is the probability that the letter came from TATANAGAR?

[NCERT Exemplar]

Sol. Let E_1 = Letter has come from CALCUTTA

E_2 = Letter has come from TATANAGAR

and E = Two consecutive letters (i.e. alphabets) TA are visible on envelope

$$\therefore\quad P(E_1)=\frac{1}{2},\ P(E_2)=\frac{1}{2},\ P\left(\frac{E}{E_1}\right)=\frac{n(E\cap E_1)}{n(E_1)}=\frac{1}{7}$$

[$\because$ 7 pairs of consecutive letters are CA, AL, LC, CU, UT, TT, TA] and $P\left(\frac{E}{E_2}\right)=\frac{n(E\cap E_2)}{n(E_2)}=\frac{2}{8}$

[$\because$ 8 pairs of consecutive letters are TA, AT, TA, AN, NA, AG, GA, AR]

$$\therefore \quad P\left(\frac{E_2}{E}\right)=\frac{P(E_2)\cdot P\left(\frac{E}{E_2}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)}$$

[using Baye's theorem]

$$=\frac{\frac{1}{2}\times\frac{2}{8}}{\frac{1}{2}\times\frac{1}{7}+\frac{1}{2}\times\frac{2}{8}}=\frac{\frac{2}{16}}{\frac{8+14}{2\times 7\times 8}}=\frac{2\times 7}{22}=\frac{7}{11}$$

Hence, the probability that the letter TA came from TATANAGAR is $\frac{7}{11}$.

TOPIC PRACTICE 3

OBJECTIVE TYPE QUESTIONS

1 The events $E_1, E_2, \ldots, E_n$ represent a partition of the sample space S, if

(a) $E_i \cap E_j = \phi, i \neq j, i, j = 1, 2, 3, \ldots, n$

(b) $E_1 \cup E_2 \cup \ldots \cup E_n = S$

(c) $P(E_i) > 0$ for all $i = 1, 2, 3, \ldots, n$

(d) All of the above

2 Let $\{E_1, E_2, \ldots, E_n\}$ be a partition of the sample space S and A be any event associated with S, then

(a) $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \cdots + P(E_n)P(A/E_n)$

(b) $P(A) = \sum_{j=1}^{n} P(E_j)P(A/E_j)$

(c) Both (a) and (b) (d) None of these

3 If $E_1, E_2, \ldots, E_n$ constitute a partition of sample space S and A is any event of non-zero probability, then $P(E_i/A)$ is equal to

(a) $\frac{P(E_i)P(A/E_i)}{\sum_{j=1}^{n} P(E_j)P(A/E_j)}$ for any $i = 1, 2, 3, \ldots, n$

(b) $\frac{P(E_i)P(E_i/A)}{\sum_{j=1}^{n} P(E_j)P(A/E_j)}$ for any $i = 1, 2, 3, \ldots, n$

(c) $\frac{P(E_i)P(E_i/A)}{P(A)}$ for any $i = 1, 2, 3, \ldots, n$

(d) None of the above

SHORT ANSWER Type II Questions

4 Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator. **[Delhi 2020]**

5 Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, then calculate the probability that it is defective. **[Foreign 2015]**

6 Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ? **[CBSE 2018]**

7 A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement) both of which are found to be red. Find the probability that the balls are drawn from the second bag. **[All India 2019]**

8 A bag contains 4 red and 4 black balls and another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the prabability that the ball is drawn from the first bag. **[NCERT]**

9 There are three coins. First is a biased that comes up tails 60% of the times, second is also a biased coin that comes up heads 75% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads. What is the probability that it was the first coin? **[Delhi 2016C]**

10 A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y. **[All India 2016]**

LONG ANSWER Type Questions

11 Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred from bag I to bag II and then ball is drawn from bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black. **[NCERT]**

12 In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answers and 1/4 be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability 1/4. What is the probability that the student knows the answer, given that he answered it correctly?

13 A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture, where the proportions are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability

(i) of a randomly chosen seed to germinate.

(ii) that it will not germinate given that the seed is of type A_3.

(iii) that it is of the type A_2 given that a randomly chosen seed does not germinate.

[NCERT Exemplar]

14 A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is 1. Find the probability that it is actually 1. **[Delhi 2014]**

15 A laboratory blood test is 99% effective in detecting a certain disease,when it is infact present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then with probability 0.005, the test will imply he has the disease). If 0.1% of the population actually has the disease, then what is the probability that a person has disease, given that his test result is positive? **[NCERT]**

16 By examining the chest X-ray, the probability that TB is detected when a person is actually suffering is 0.99. The probability of an healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffers from TB. A person is selected at random and is diagnosed to have TB. What is the probability that he actually has TB? **[NCERT Exemplar]**

17 Two groups are competing for the position on the board of directors of a corporation. The probability that the first and the second groups will win are 0.6 and 0.4, respectively. Further, if the first group wins the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 , if the second group wins. Find the probability that the new product introduce was by second group. **[NCERT]**

18 An item is manufactured by three machines A, B and C. Out of the total number of items manufactured during a specified period, 50% are manufactured on A, 30% on B and 20% on C. 2% of the items produced on A and 2% of items produced on B are defective and 3% of these produced on C are defective. All the items are stored at one godown. One item is drawn at random and is found to be defective. What is the probability that it was manufactured on machine A? **[NCERT Exemplar]**

19 A doctor is visit to a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport, are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train? **[NCERT]**

20 An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident for them are 0 .01, 0.03 and 0.15, respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver or a car driver? **[Foreign 2014]**

21 Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, then what is the probability that she threw 1, 2, 3 or 4 with the die? **[NCERT]**

22 A bag contains 3 red and 7 black balls. Two balls are selected at random one-by-one without replacement. If the second selected ball happens to be red, then what is the probability that the first selected ball is also red? **[Delhi 2014C]**

23 A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be diamonds. Find the probability of the lost card being a diamond. **[NCERT]**

24 An urn contains 4 balls. Two balls are drawn at random from the urn (without replacement) and are found to be white. What is the probability that all the balls in the urn are white? **[All India 2014C]**

HINTS & SOLUTIONS

1. (d) **Hint** By definition of partition of a sample space.

2. (c) **Hint** By the statement of theorem of total probability.

3. (a) **Hint** By the statement of Bayes theorem.

4. Let E_1 be the event that selected person is men and E_2 be the event that selected person is women, E_1 and E_2 are mutually exclusive and exhaustive event moreover $P(E_1) = P(E_2) = \frac{1}{2}$.

Let E be the event that selected person is good orator.

$\therefore \quad P(E/E_1) = \frac{5}{100} = \frac{1}{20}$ and $P(E/E_2) = \frac{25}{1000} = \frac{1}{40}$

The probability of choosing a good orator,

$$P(E) = P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)$$
$$= \frac{1}{2} \times \frac{1}{20} + \frac{1}{2} \times \frac{1}{40} = \frac{2+1}{2 \times 40} = \frac{3}{80}$$

5. Let A be the event that the picked up bulb is defective.

Let A_1, A_2 and A_3 be the events that the bulb produce by machines E_1, E_2 and E_3, respectively.

Given, $P(A_1) = 50\% = \frac{50}{100} = \frac{1}{2}$, $P(A_2) = 25\% = \frac{25}{100} = \frac{1}{4}$,

$$P(A_3) = 25\% = \frac{25}{100} = \frac{1}{4},$$
$$P\left(\frac{A}{A_1}\right) = 4\% = \frac{4}{100} = \frac{1}{25},$$
$$P\left(\frac{A}{A_2}\right) = 4\% = \frac{4}{100} = \frac{1}{25}$$

and $P\left(\frac{A}{A_3}\right) = 5\% = \frac{5}{100} = \frac{1}{20}$

By using total probability theorem,

P (getting a defective bulb)

$$= P(A_1) \cdot P\left(\frac{A}{A_1}\right) + P(A_2) \cdot P\left(\frac{A}{A_2}\right) + P(A_3) \cdot P\left(\frac{A}{A_3}\right)$$
$$= \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$
$$= \frac{1}{50} + \frac{1}{100} + \frac{1}{80} = \frac{8+4+5}{400} = \frac{17}{400} = 0.0425$$

Hence, the probability that the picked up bulb is defective, is 0.0425.

6. Let E_1 be the event that the girl gets 1 or 2

E_2 be the event that the girl gets 3, 4, 5 or 6

and A be the event that the girl gets exactly a 'tail'.

Then, $\quad P(E_1) = \frac{2}{6} = \frac{1}{3}$

and $\quad P(E_2) = \frac{4}{6} = \frac{2}{3}$

$P\left(\frac{A}{E_1}\right) = P$ (getting exactly one tail when a coin is tossed three times)

$$= \frac{3}{8}$$

$P\left(\frac{A}{E_2}\right) = P$ (getting exactly a tail when a coin is tossed once)

$$= \frac{1}{2}$$

Now, required probability

$$= P\left(\frac{E_2}{A}\right) = \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)}$$
$$= \frac{\frac{2}{3} \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{3}{8} + \frac{2}{3} \cdot \frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{8} + \frac{1}{3}} = \frac{8}{11}$$

7. Let E_1, E_2 and A denote the following events

E_1 = first bag is chosen, E_2 = second bag is chosen and A = two balls drawn at random are red.

Since, one of the bag is chosen at random.

$$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}$$

If E_1 has already occurred, i.e. first bag is chosen.

Therefore, the probability of drawing two red balls in this case $= P\left(\frac{A}{E_1}\right) = \frac{{}^5C_2}{{}^9C_2} = \frac{10}{36}$

Similarly, $P\left(\frac{A}{E_2}\right) = \frac{{}^3C_2}{{}^9C_2} = \frac{3}{36}$

We are required to find $P\left(\frac{E_2}{A}\right)$.

By Baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)}$$
$$= \frac{\frac{1}{2} \times \frac{3}{36}}{\frac{1}{2} \times \frac{10}{36} + \frac{1}{2} \times \frac{3}{36}} = \frac{\frac{3}{72}}{\frac{10}{72} + \frac{3}{72}} = \frac{\frac{3}{72}}{\frac{13}{72}} = \frac{3}{13}$$

8. Solve as Example 3. [**Ans.** $2/3$]

9. Let the events be E_1 = Choosing first coin,
E_2 = Choosing second coin, E_3 = Choosing third coin
and A = Getting head

$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Now, $P\left(\frac{A}{E_1}\right) = \frac{40}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{75}{100}$ and $P\left(\frac{A}{E_3}\right) = \frac{1}{2}$

$$\therefore P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{3}\cdot\frac{40}{100}}{\frac{1}{3}\cdot\frac{40}{100}+\frac{1}{3}\cdot\frac{75}{100}+\frac{1}{3}\cdot\frac{1}{2}} = \frac{8}{33}$$

10. Let us define the following events
E_1 : Bag X is selected
E_2 : Bag Y is selected
and E : Getting one white and one black ball in a draw of two balls.

Here, $P(E_1) = P(E_2) = \frac{1}{2}$

[$\because$ probability of selecting each bag is equal]

Now, $P\left(\frac{E}{E_1}\right)$ = Probability of drawing one white and one black ball from bag X

$$= \frac{{}^4C_1 \times {}^2C_1}{{}^6C_2} = \frac{4\times 2}{\frac{6\times 5}{2\times 1}} = \frac{16}{6\times 5} = \frac{8}{15}$$

and $P\left(\frac{E}{E_2}\right)$ = Probability of drawing one white and one black ball from bag Y

$$= \frac{{}^3C_1 \times {}^3C_1}{{}^6C_2} = \frac{3\times 3}{\frac{6\times 5}{2\times 1}} = \frac{3}{5}$$

$\therefore$ Probability that the one white and one black balls are drawn from bag Y,

$$P\left(\frac{E_2}{E}\right) = \frac{P(E_2)\cdot P\left(\frac{E}{E_2}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right)}$$

[by using Baye's theorem]

$$= \frac{\frac{1}{2}\times\frac{3}{5}}{\frac{1}{2}\times\frac{8}{15}+\frac{1}{2}\times\frac{3}{5}} = \frac{\frac{3}{5}}{\frac{8}{15}+\frac{3}{5}} = \frac{\frac{3}{5}}{\frac{8+9}{15}} = \frac{3}{5\times\frac{17}{15}} = \frac{9}{17}$$

11. Let E_1 = Red ball is transferred from bag I to bag II
and E_2 = Black ball is transferred from bag I to bag II
So, E_1 and E_2 are mutually exclusive and exhaustive events.

$\therefore \quad P(E_1) = \frac{3}{3+4} = \frac{3}{7}$ and $P(E_2) = \frac{4}{3+4} = \frac{4}{7}$

Let E be the event that the ball drawn is red.

Then, $P\left(\frac{E}{E_1}\right) = \frac{4+1}{(4+1)+5} = \frac{5}{10} = \frac{1}{2}$

and $P\left(\frac{E}{E_2}\right) = \frac{4}{4+(5+1)} = \frac{4}{10} = \frac{2}{5}$

$\therefore$ Required probability,

$$P\left(\frac{E_2}{E}\right) = \frac{P\left(\frac{E}{E_2}\right)\cdot P(E_2)}{P\left(\frac{E}{E_1}\right)\cdot P(E_1) + P\left(\frac{E}{E_2}\right)\cdot P(E_2)}$$

$$= \frac{\frac{2}{5}\times\frac{4}{7}}{\frac{1}{2}\times\frac{3}{7}+\frac{2}{5}\times\frac{4}{7}}$$

$$= \frac{\frac{8}{35}}{\frac{3}{14}+\frac{8}{35}} = \frac{\frac{8}{35}}{\frac{105+112}{14\times 35}} = \frac{8\times 14}{217} = \frac{16}{31}$$

Hence, the probability that the transferred ball is black, is $\frac{16}{31}$.

12. Let E_1 = Student knows the answer
E_2 = Student guesses the answer
and A = Student answered correctly

$\therefore \; P(E_1) = \frac{3}{4}$ and $P(E_2) = \frac{1}{4}$

Now, $P\left(\frac{A}{E_2}\right) = P$ (student answered correctly, when he guesses)

$= \frac{1}{4}$

and $P\left(\frac{A}{E_1}\right) = P$ (he answers correctly given that he knew the answer)

$= 1$

$\therefore$ Required probability,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{\left[P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)\right]}$$

[using Baye's theorem]

$$= \frac{\frac{3}{4}\times 1}{\left(\frac{3}{4}\times 1\right)+\left(\frac{1}{4}\times\frac{1}{4}\right)} = \frac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}} = \frac{\frac{3}{4}}{\frac{12+1}{16}}$$

$$= \frac{3}{4}\times\frac{16}{13} = \frac{12}{13}$$

Hence, the probability that the student knows the answer, given that he answered it correctly, is $\frac{12}{13}$.

13. We have, $A_1 : A_2 : A_3 = 4:4:2$

$\therefore \quad P(A_1) = \frac{4}{10}, P(A_2) = \frac{4}{10}$ and $P(A_3) = \frac{2}{10}$

where A_1, A_2 and A_3 denote the event of choosing flower seeds A_1, A_2 and A_3, respectively. Let E be the event that a seed germinates and $\overline{E}$ be the event that a seed does not germinate.

Then, $P\left(\frac{E}{A_1}\right) = \frac{45}{100}, P\left(\frac{E}{A_2}\right) = \frac{60}{100}, P\left(\frac{E}{A_3}\right) = \frac{35}{100}$

and $P\left(\frac{\overline{E}}{A_1}\right) = \frac{55}{100}, P\left(\frac{\overline{E}}{A_2}\right) = \frac{40}{100}, P\left(\frac{\overline{E}}{A_3}\right) = \frac{65}{100}$

(i) Probability that a randomly chosen seed to germinate,

$$P(E) = P(A_1)\cdot\left(\frac{E}{A_1}\right) + P(A_2)\cdot P\left(\frac{E}{A_2}\right) + P(A_3)\cdot P\left(\frac{E}{A_3}\right)$$

$$= \frac{4}{10}\times\frac{45}{100} + \frac{4}{10}\times\frac{60}{100} + \frac{2}{10}\times\frac{35}{100}$$

$$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{1000} = \frac{490}{1000} = 0.49$$

(ii) $P\left(\frac{\overline{E}}{A_3}\right) = 1 - P\left(\frac{E}{A_3}\right) = 1 - \frac{35}{100} = \frac{65}{100}$

(iii) $P\left(\frac{A_2}{\overline{E}}\right)$

$$= \frac{P(A_2)\cdot P\left(\frac{\overline{E}}{A_2}\right)}{P(A_1)\cdot P\left(\frac{\overline{E}}{A_1}\right) + P(A_2)\cdot P\left(\frac{\overline{E}}{A_2}\right) + P(A_3)\cdot P\left(\frac{\overline{E}}{A_3}\right)}$$

$$= \frac{\frac{4}{10}\times\frac{40}{100}}{\frac{4}{10}\times\frac{55}{100} + \frac{4}{10}\times\frac{40}{100} + \frac{2}{10}\times\frac{65}{100}}$$

$$= \frac{\frac{160}{1000}}{\frac{220}{1000} + \frac{160}{1000} + \frac{130}{1000}} = \frac{\frac{160}{1000}}{\frac{510}{1000}} = \frac{16}{51} = 0.313725 = 0.314$$

14. Let E be the event that the man reports that 1 occurs in throwing of a die, E_1 be the event that 1 occurs and E_2 be the event that 1 does not occur.

Then, $P(E_1)$ = Probability that 1 occurs $= \frac{1}{6}$

$P(E_2)$ = Probability that 1 does not occur $= \frac{5}{6}$

$P\left(\frac{E}{E_1}\right) = P$ (man reports that 1 occurs, when 1 has actually occured on the die)

$= P$ (man speaks the truth) $= \frac{3}{5}$

$P\left(\frac{E}{E_2}\right) = P$ (man reports that 1 occurs, when 1 has not occured on the die)

$= P$ (man does not speak the truth)

$= 1 - \frac{3}{5} = \frac{2}{5}$

$\therefore$ Probability that the report of the man that 1 has occurred is actually 1, is given by

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\cdot P\left(\frac{E}{E_1}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right)}$$

[by Baye's theorem]

$$= \frac{\frac{1}{6}\times\frac{3}{5}}{\frac{1}{6}\times\frac{3}{5} + \frac{5}{6}\times\frac{2}{5}} = \frac{\frac{3}{30}}{\frac{3}{30} + \frac{10}{30}} = \frac{3}{13}$$

Hence, the probability that it is actually 1 is $\frac{3}{13}$.

15. Let E_1 = Event that the person has disease

and E_2 = Event that the person is healthy.

Then, E_1 and E_2 are mutually exclusive and exhaustive events.

$\therefore \quad P(E_1) = 0.1\% = \frac{0.1}{100} = 0.001$

and $\quad P(E_2) = 1 - 0.001 = 0.999$

Let E : Event that test is positive.

$P\left(\frac{E}{E_1}\right) = P$ (result is positive, given that person has disease)

$= 99\% = \frac{99}{100} = 0.99$

$P\left(\frac{E}{E_2}\right) = P$ (result is positive, given that person does not have disease)

$= 0.5\% = \frac{0.5}{100} = 0.005$

Probability that a person has disease, given that test result is positive, is given by

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\cdot P\left(\frac{E}{E_1}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right)}$$

[by Baye's theorem]

$$= \frac{0.001\times 0.99}{0.001\times 0.99 + 0.999\times 0.005}$$

$$= \frac{0.00099}{0.00099 + 0.004995}$$

$$= \frac{0.00099}{0.005985} = \frac{990}{5985} = \frac{110}{665} = \frac{22}{133}$$

Hence, the required probability is $\frac{22}{133}$.

16. Let E_1 be the event that person is suffering from TB and E_2 be the event that person is not suffering from TB. Let E be the event that the doctor diagnoses that person has TB.

Then, $P(E_1) = \dfrac{1}{1000}$

$$P(E_2) = 1 - \frac{1}{1000} = \frac{999}{1000}$$

$P\left(\dfrac{E}{E_1}\right) = P$ (TB is detected when a person is actually Suffering)

$$= 0.99 = \frac{990}{1000}$$

and $P\left(\dfrac{E}{E_2}\right) = P$ (TB is detected when a person is not actually suffering)

$$= 0.001 = \frac{1}{1000}$$

P (A selected person has actually TB), is given by

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\cdot P\left(\frac{E}{E_1}\right)}{\left[P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right)\right]}$$

[by Baye's theorem]

$$= \frac{\left(\frac{1}{1000}\right)\left(\frac{990}{1000}\right)}{\left(\frac{1}{1000}\right)\left(\frac{990}{1000}\right) + \left(\frac{999}{1000}\right)\left(\frac{1}{1000}\right)} = \frac{\frac{990}{1000000}}{\left(\frac{990+999}{1000000}\right)}$$

$$= \frac{990}{1989} = \frac{110}{221}$$

Hence, the probability that a selected person has actually TB is $\dfrac{110}{221}$.

17. **Hint** E_1 and E_2 be the events of first group and second group wins and E be the new product is introduced.

Then, $P(E_1) = 0.6$, $P(E_2) = 0.4$, $P\left(\dfrac{E}{E_1}\right) = 0.7$

and $P\left(\dfrac{E}{E_2}\right) = 0.3$

Determine $P\left(\dfrac{E_2}{E}\right)$ by using Baye's theorem. $\left[\textbf{Ans. } \dfrac{2}{9}\right]$

18. **Hint** $P(A) = 50\%$, $P(B) = 30\%$, $P(C) = 20\%$,

$$P\left(\frac{D}{A}\right) = 2\%,\ P\left(\frac{D}{B}\right) = 2\%,\ P\left(\frac{D}{C}\right) = 3\%$$

Required probability,

$$P\left(\frac{A}{D}\right) = \frac{50\% \times 2\%}{50\% \times 2\% + 30\% \times 2\% + 20\% \times 3\%} \quad \left[\textbf{Ans. } \frac{5}{11}\right]$$

19. **Hint** E_1 = Doctor will come by train.

E_2 = Doctor will come by bus.

E_3 = Doctor will come by scooter.

E_4 = Doctor will come by other transport.

and A = Doctor will be late.

Then, $P(E_1) = \dfrac{3}{10}$, $P(E_2) = \dfrac{1}{5}$, $P(E_3) = \dfrac{1}{10}$,

$$P(E_4) = \frac{2}{5},\ P\left(\frac{A}{E_1}\right) = \frac{1}{4},\ P\left(\frac{A}{E_2}\right) = \frac{1}{3},$$

$$P\left(\frac{A}{E_3}\right) = \frac{1}{12},\ P\left(\frac{A}{E_4}\right) = 0$$

Find $P\left(\dfrac{E_1}{A}\right)$ $\left[\textbf{Ans. } \dfrac{1}{2}\right]$

20. **Hint** E_1 = Scooter driver, E_2 = Car driver,

E_3 = Truck driver and A = Meets with an accident.

$$\therefore P(E_1) = \frac{1}{6},\ P(E_2) = \frac{1}{3},\ P(E_3) = \frac{1}{2},$$

$$P\left(\frac{A}{E_1}\right) = 0.01,\ P\left(\frac{A}{E_2}\right) = 0.03,\ P\left(\frac{A}{E_3}\right) = 0.15$$

Required probability,

$$P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) = \frac{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.03}{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.03 + \frac{1}{2} \times 0.15}$$

[**Ans.** 0.019]

21. Let E_1 = Event that 5 or 6 is shown on die

and E_2 = Event that 1, 2, 3 or 4 is shown on die.

Here, $n(E_1) = 2$ and $n(E_2) = 4$

Also, $n(S) = 6$

$$\therefore \quad P(E_1) = \frac{2}{6} = \frac{1}{3} \text{ and } P(E_2) = \frac{4}{6} = \frac{2}{3}$$

Let E = The event that exactly one head show up.

$\therefore P\left(\dfrac{E}{E_1}\right) = P$ (exactly one head show up when coin is tossed thrice)

$$= P\{HTT, THT, TTH\} = \frac{3}{8}$$

[$\because$ total number of outcomes = $2^3 = 8$]

$$P\left(\frac{E}{E_2}\right) = P(\text{head shows up when coin is tossed once}) = \frac{1}{2}$$

The probability that the girl threw 1, 2, 3 or 4 with the die, if she obtained exactly one head, is given by

$$P\left(\frac{E_2}{E}\right) = \frac{P(E_2)\cdot P\left(\frac{E}{E_2}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right)}$$

[by Baye's theorem]

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{8} + \frac{1}{3}} = \frac{8}{8+3} = \frac{8}{11}$$

22. **Hint** E_1 = First ball is red

E_2 = First ball is black

and A = Second ball is red

$\therefore P(E_1) = \frac{3}{10}$ $P(E_2) = \frac{7}{10}$, $P\left(\frac{A}{E_1}\right) = \frac{2}{9}$, $P\left(\frac{A}{E_2}\right) = \frac{3}{9}$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)} \quad \left[\textbf{Ans. } \frac{2}{9}\right]$$

23. **Hint** E_1 = Missing card is diamond

E_2 = Missing card is not diamond

and A = Drawing two diamond cards from the remaining cards

$\therefore P(E_1) = \frac{1}{4}$, $P(E_2) = \frac{3}{4}$,

$P\left(\frac{A}{E_1}\right) = \frac{12}{51} \times \frac{11}{50}$,

$P\left(\frac{A}{E_2}\right) = \frac{13}{51} \times \frac{12}{50}$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)} \left[\textbf{Ans. } \frac{11}{50}\right]$$

24. **Hint** E_1 = There are two white and two other colour balls in the bag

E_2 = There are three white and one other colour balls in the bag

E_3 = There are all white balls in the bag

and A = Drawing two white balls from the bag

$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$,

$P\left(\frac{A}{E_1}\right) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}$, $\quad P\left(\frac{A}{E_2}\right) = \frac{{}^3C_2}{{}^4C_2} = \frac{1}{2}$

and $P\left(\frac{A}{E_3}\right) = \frac{{}^4C_2}{{}^4C_2} = 1$

Now, find $P\left(\frac{E_3}{A}\right)$. $\left[\textbf{Ans. } \frac{3}{5}\right]$

|TOPIC 4|

Random Variable and Its Probability Distribution

RANDOM VARIABLE

A random variable is a real valued function, whose domain is the sample space of a random experiment. Generally, it is denoted by capital letter X. Also, more than one random variables can be defined on the same sample space.

e.g. Let a coin be tossed twice.

Then, sample space, $S = \{HT, TH, HH, TT\}$

If X denotes the number of heads in two tosses, then X is a random variable and for each outcome, its value is

$$X(TT) = 0,\ X(HT) = 1,\ X(TH) = 1 \text{ and } X(HH) = 2$$

Clearly, values of X are 0, 1and 2.

Let Y denotes the number of heads minus the number of tails for each outcome of the above sample space S and for each outcome, its value is

$$Y(HH) = 2 - 0 = 2,$$
$$Y(HT) = 1 - 1 = 0,$$
$$Y(TH) = 1 - 1 = 0$$

and $\quad Y(TT) = 0 - 2 = -2$

Thus, X and Y are two different random variables defined on the same sample space S.

EXAMPLE |1| A bag contains 2 black and 1 green balls. One ball is drawn at random and then put back in the box after noting its colour. The process repeated again. Let X denotes the number of green balls recorded in the two draws, describe X.

Sol. Given a bag contains 2 black and 1 green balls. Black balls are denoted by b_1, b_2 and green ball is denoted by g_1, then the sample space for two draws is

$S = \{b_1b_1, b_1b_2, b_2b_1, b_2b_2, b_1g_1, b_2g_1, g_1b_1, g_1b_2, g_1g_1\}$

Now, X denotes the number of green balls.

Then,

$$X(b_1b_1) = X(b_1b_2) = X(b_2b_1) = X(b_2b_2) = 0,$$
$$X(b_1g_1) = X(b_2g_1) = X(g_1b_1) = X(g_1b_2) = 1$$

and $X(g_1g_1) = 2$

Hence, X is a random variable whose range is $\{0, 1, 2\}$.

Probability Distribution of a Random Variable

The system in which the values of a random variable are given along with their corresponding probability, is called probability distribution.

If X is a random variable and takes the values of $x_1, x_2, x_3, \ldots, x_n$ with respective probabilities $p_1, p_2, p_3, \ldots, p_n$.

Then, the probability distribution of X is represented by

X	x_1	x_2	x_3	...	x_n
$P(X)$	p_1	p_2	p_3	...	p_n

where $p_i > 0$ and $\sum_{i=1}^{n} p_i = 1; i = 1, 2, 3, \ldots, n$

Note If x_i is one of the possible values of a random variable X, then statement $X = x_i$ is true only at some point(s) of the sample space. Hence, the probability that X takes value x_i is always non-zero, i.e. $P(X = x_i) \neq 0$.

EXAMPLE |2| Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces.

[NCERT]

Sol. Let the number of aces is a random variable denoted by X. Here, two cards are drawn. Let A denotes an ace card and B denotes a non-ace card, then sample space for two cards is $S = \{AB, BA, AA, BB\}$.

Then, $X(AB) = 1$, $X(BA) = 1$, $X(AA) = 2$ and $X(BB) = 0$. So, X takes the values 0, 1 or 2. Since, the draws are done with replacement, two draws form independent experiments.

$$\therefore \quad P(X = 0) = P(BB) = P(B) \times P(B) = \frac{48}{52} \times \frac{48}{52} = \frac{144}{169}$$

[$\because$ in 52 cards, 4 aces and remaining 48 are non-ace cards]

$$P(X = 1) = P(AB \text{ or } BA) = P(AB) + P(BA)$$
$$= P(A) \cdot P(B) + P(B) \cdot P(A)$$
$$= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} = \frac{12}{169} + \frac{12}{169} = \frac{24}{169}$$

and $P(X = 2) = P(AA) = P(A) \cdot P(A) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

Hence, the required probability distribution is

X	0	1	2
$P(X)$	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Verification Sum of probabilities,

$$\sum_{i=1}^{n} p_i = \frac{144}{169} + \frac{24}{169} + \frac{1}{169} = \frac{144 + 24 + 1}{169} = \frac{169}{169} = 1$$

EXAMPLE |3| From a lot of 15 bulbs which include 5 defectives, a sample of 2 bulbs is drawn at random (without replacement). Find the probability distribution of the number of defective bulbs. **[Delhi 2015C]**

Sol. Total number of bulbs = 15

Number of defective bulbs = 5

$\therefore$ Good bulbs = 15 − 5 = 10

Let X be the random variable of number of defective bulbs.

$\therefore \quad X = 0, 1, 2,$

$$P(X = 0) = \frac{10}{15} \times \frac{9}{14} = \frac{9}{21}$$
$$P(X = 1) = \frac{10}{15} \times \frac{5}{14} + \frac{5}{15} \times \frac{10}{14} = \frac{10}{21}$$
$$P(X = 2) = \frac{5}{15} \times \frac{4}{14} = \frac{2}{21}$$

$\therefore$ Required probability distribution table is

X	0	1	2
$P(X)$	$\frac{9}{21}$	$\frac{10}{21}$	$\frac{2}{21}$

Mean of a Random Variable

Let X be a random variable taking values $x_1, x_2, x_3, \ldots, x_n$ with probabilities $p_1, p_2, p_3, \ldots, p_n$, respectively. Then, mean of X denoted by μ is the number $\sum_{i=1}^{n} x_i p_i$, i.e. the mean of X is the weighted average of the possible values of X, each value being weighted by its probability with which it occurs. It is also called the expectation of X, denoted by $E(X)$.

Then, $E(X) = \mu = \sum_{i=1}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$

Thus, in other words we can say that the mean or expectation of a random variable X is the sum of the products of all possible values of X by their respective probabilities.

Note

(i) Mean is a measure of location or central tendency in the sense that it roughly locates a middle or average value of the random variable.

(ii) Random variables with different probability distributions can have equal means.

METHOD OF FINDING MEAN OF A RANDOM VARIABLE

Suppose a random experiment is given to us, then to find mean and variance of a random variable associated with random experiment, we use the following steps

I. First, identify the random variable (say X) associated with given random experiment and then find all possible values $x_1, x_2, x_3, \ldots$, which X can take.

II. For each value of X obtained in step I, find the probability.

III. Write the probability distribution table for random variable X.

IV. Calculate mean by using the formula, mean $\overline{X} = \sum_{i=1}^{n} x_i \; p(x_i)$, i.e. multiply each value of x_i by the corresponding probability and then find their sum to get required mean.

EXAMPLE |4| The random variable X can take only the values 0, 1, 2, 3. Given that $P(X=0) = P(X=1) = p$ and $P(X=2) = P(X=3)$ such that $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$, find the value of p. **[Delhi 2017]**

Sol. Given, $X = 0, 1, 2, 3$ and

$P(X=0) = P(X=1) = p, P(X=2) = P(X=3)$

such that $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$

Now, $\Sigma p_i = 1 \Rightarrow p_0 + p_1 + p_2 + p_3 = 1$

$\Rightarrow p + p + x + x = 1$

[let $P(X=2) = P(X=3) = x$]

$\Rightarrow 2p + 2x = 1 \Rightarrow 2x = 1 - 2p \Rightarrow x = \frac{1-2p}{2}$

The probability distribution of X is given by

$X = x_i$	0	1	2	3
p_i	p	p	$\frac{1-2p}{2}$	$\frac{1-2p}{2}$

Now, $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$

$\Rightarrow p_0 x_0^2 + p_1 x_1^2 + p_2 x_2^2 + p_3 x_3^2$

$= 2[p_0 x_0 + p_1 x_1 + p_2 x_2 + p_3 x_3]$

$\Rightarrow p \times 0 + p \times 1^2 + \frac{1-2p}{2} \times (2)^2 + \frac{1-2p}{2} \times (3)^2$

$= 2\left[p \times 0 + p \times 1 + \frac{1-2p}{2} \times 2 + \frac{1-2p}{2} \times 3\right]$

$\Rightarrow p + (1-2p) \times 2 + (1-2p) \times \frac{9}{2}$

$= 2\left[p + (1-2p) + (1-2p) \times \frac{3}{2}\right]$

$\Rightarrow p + 2 - 4p + \frac{9}{2} - 9p = 2\left[p + 1 - 2p + \frac{3}{2} - 3p\right]$

$\Rightarrow -12p + 2 + \frac{9}{2} = 2\left[-4p + 1 + \frac{3}{2}\right]$

$\Rightarrow -12p + \frac{13}{2} = 2\left[-4p + \frac{5}{2}\right]$

$\Rightarrow -12p + 8p = 5 - \frac{13}{2}$

$\Rightarrow -4p = -\frac{3}{2} \Rightarrow 4p = \frac{3}{2}$

$\therefore p = \frac{3}{8}$

EXAMPLE |5| In a game, a man wins ₹ 5 for getting a number greater than 4 and loses ₹ 1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses. **[All India 2016]**

Sol Let X be a random variable that denotes the amount received by the man. Then, X can take values 5, 4, 3 and −3.

Now, $P(X=5) = P$ (getting a number greater than 4 in the first throw)

$= \frac{2}{6} = \frac{1}{3}$

$P(X=4) = P$ (getting a number less than or equal to 4 in the first throw and getting a number greater than 4 in the second throw)

$= \frac{4}{6} \times \frac{2}{6} = \frac{2}{9}$

$P(X=3) = P$ (getting a number less than or equal to 4 in first two throws and getting a number greater than 4 in the third throw)

$= \frac{4}{6} \times \frac{4}{6} \times \frac{2}{6} = \frac{4}{27}$

and $P(X=-3) = P$ (getting a number less than or equal to 4 in all three throws)

$= \frac{4}{6} \times \frac{4}{6} \times \frac{4}{6} = \frac{8}{27}$

Thus, the probability distribution of X is

X	5	4	3	−3
$P(X=x)$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{4}{27}$	$\frac{8}{27}$

Now, expected value of the amount

$= E(X) = \Sigma x_i p_i = 5 \cdot \frac{1}{3} + 4 \cdot \frac{2}{9} + 3 \cdot \frac{4}{27} - 3 \cdot \frac{8}{27}$

$= \frac{45 + 24 + 12 - 24}{27} = \frac{57}{27} = \frac{19}{9}$

EXAMPLE |6| Two cards are drawn simultaneously (without replacement) from a well-shuffled deck of 52 cards. Find the mean of number of red cards.

Sol. Here, we have to drawn two cards simultaneously from a deck.

Let X denotes the number of red cards (since mean and variance of red card to be calculated here), then X can take values 0 (no red card), 1 (one red card) and 2 (both red cards).

For $X = 0$, $P(X=0) = P$(getting no red card) $= \frac{^{26}C_2}{^{52}C_2}$

[∵ there are 26 red and 26 black cards in a deck]

$= \frac{\frac{26 \times 25}{2 \times 1}}{\frac{52 \times 51}{2 \times 1}} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$

For $X=1$, $P(X=1)$

$= P$(getting one red and other black card)

$$= \frac{^{26}C_1 \times {^{26}C_1}}{^{52}C_2} = \frac{26 \times 26 \times 2}{52 \times 51} = \frac{26}{51}$$

$$\text{For } X = 2,\ P(X=2) = \frac{^{26}C_2}{^{52}C_2} = \frac{\frac{26 \times 25}{2 \times 1}}{\frac{52 \times 51}{2 \times 1}} = \frac{25}{102}$$

The required probability distribution is

X	0	1	2
$P(X)$	$\frac{25}{102}$	$\frac{26}{51}$	$\frac{25}{102}$

We know that mean, $\overline{X} = \sum_{i=1}^{n} x_i\ p(x_i)$

$$\therefore \quad \overline{X} = \left(0 \times \frac{25}{102}\right) + \left(1 \times \frac{26}{51}\right) + \left(2 \times \frac{25}{102}\right)$$

$$= 0 + \frac{26}{51} + \frac{50}{102} = \frac{26}{51} + \frac{25}{51} = \frac{51}{51} = 1$$

Hence, mean = 1

EXAMPLE |7| Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6 and 7. Let X denotes the larger of the two numbers obtained. Find the mean of the probability distribution of X. **[Foreign 2015]**

Sol. The total number of pairs will be formed by taking two numbers at a time $= {^6C_1} \times {^5C_1} = 30$.

Let X denotes larger of the two numbers be obtained.

Obviously X may have values 3, 4, 5, 6 and 7.

Now, $P(X=3) = P$[getting (2,3), (3,2) out of 30 pairs]

$$= \frac{2}{30} = \frac{1}{15}$$

$P(X=4) = P$ [getting (2,4), (3,4), (4,2), (4,3) out of 30 pairs] $= \frac{4}{30} = \frac{2}{15}$

$P(X=5) = P$ [getting (2,5), (5,2), (3,5), (5,3), (4,5),(5,4) out of 30 pairs]

$$= \frac{6}{30} = \frac{3}{15}$$

$P(X=6) = P$ [getting (2,6), (6,2), (3,6), (6,3), (4,6),(6,4), (5,6), (6,5) out of 30 pairs]

$$= \frac{8}{30} = \frac{4}{15}$$

$P(X=7) = P$ [getting (2,7), (7,2), (3,7), (7,3), (4,7), (7,4), (5,7), (7,5), (6,7), (7,6) out of 30 pairs] $= \frac{10}{30} = \frac{5}{15}$

Therefore, required probability distribution is

X	3	4	5	6	7
$P(X)$	1/15	2/15	3/15	4/15	5/15

$\therefore$ Required mean $= \mu = \Sigma x_i p_i$

$$= 3 \times \frac{1}{15} + 4 \times \frac{2}{15} + 5 \times \frac{3}{15} + 6 \times \frac{4}{15} + 7 \times \frac{5}{15}$$

$$= \frac{3}{15} + \frac{8}{15} + \frac{15}{15} + \frac{24}{15} + \frac{35}{15} = \frac{85}{15} = \frac{17}{3}$$

EXAMPLE |8| An urn contains 4 white and 3 red balls. Let X be the number of red balls in a random draw of 3 balls. Find the mean of X.

Sol. When 3 balls are drawn at random, there may be no red ball, 1 red ball, 2 red balls or 3 red balls.

Let X denotes the random variable showing the number of red balls in a draw of 3 balls.

Then, X can take the values 0, 1, 2 or 3.

$P(X=0) = P$ (getting no red ball)

$= P$ (getting 3 white balls)

$$= \frac{^4C_3}{^7C_3} = \left(\frac{4 \times 3 \times 2}{3 \times 2 \times 1} \times \frac{3 \times 2 \times 1}{7 \times 6 \times 5}\right) = \frac{4}{35}$$

$P(X=1) = P$ (getting 1 red and 2 white balls)

$$= \frac{^3C_1 \times {^4C_2}}{^7C_3} = \left(\frac{3 \times 4 \times 3}{2} \times \frac{3 \times 2 \times 1}{7 \times 6 \times 5}\right) = \frac{18}{35}$$

$P(X=2) = P$ (getting 2 red and 1 white balls)

$$= \frac{^3C_2 \times {^4C_1}}{^7C_3}$$

$$= \left(\frac{3 \times 2}{2 \times 1} \times 4 \times \frac{3 \times 2 \times 1}{7 \times 6 \times 5}\right) = \frac{12}{35}$$

$$P(X=3) = P(\text{getting 3 red balls}) = \frac{^3C_3}{^7C_3} = \frac{3 \times 2 \times 1}{7 \times 6 \times 5} = \frac{1}{35}$$

Thus, the probability distribution of X is given below

$X = x_i$	0	1	2	3
p_i	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

$\therefore$ Mean, $\mu = \Sigma x_i p_i$

$$= 0 \times \frac{4}{35} + 1 \times \frac{18}{35} + 2 \times \frac{12}{35} + 3 \times \frac{1}{35} = \frac{45}{35} = \frac{9}{7}$$

TOPIC PRACTICE 4

OBJECTIVE TYPE QUESTIONS

1 Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. Then, the possible values of X are

(a) 0, 1, 3, 5 (b) 0, 1, 2, 3
(c) 0, 1, 2, 4 (d) 0, 2, 4, 6

2 The probability distribution of a discrete random variable X is given below

X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of k is **[NCERT Exemplar]**

(a) 8 (b) 16
(c) 32 (d) 48

3 For the following probability distribution.

X	−4	−3	−2	−1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

$E(X)$ is equal to **[NCERT Exemplar]**

(a) 0 (b) −1
(c) −2 (d) −1.8

4 The mean of the number obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is **[NCERT]**

(a) 1 (b) 2 (c) 5 (d) $\frac{8}{3}$

VERY SHORT ANSWER Type Questions

5 An urn contains 6 red and 3 black balls. Two balls are randomly drawn. Let X represents the number of black balls. What are the possible values of X?

6 State whether the following distribution is a probability distribution of a random variable or not.

X	3	2	1	0	−1
$P(X)$	0.3	0.2	0.4	0.1	0.05

[NCERT]

SHORT ANSWER Type II Questions

7 Find the probability distribution of X, the number of heads in a simultaneous toss of two coins. **[All India 2019]**

8 The random variable X has a probability distribution $P(X)$ of the following form, where 'k' is some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'. **[Delhi 2019]**

9 Two numbers are selected at random (without replacement) from the first five positive integers. Let X denotes the larger of the two numbers obtained. Find the mean of X. **[CBSE 2018]**

10 A biased die is such that $P(4) = \frac{1}{10}$ and other scores being equally likely. The die is tossed twice. If X is the 'number of four seen', then find the mean of the random variable X. **[NCERT Exemplar]**

11 3 defective bulbs are mixed up with 7 good ones. 3 bulbs are drawn at random. Find the probability distribution of defective bulbs.

12 A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence, find the mean of the number of tails. **[Delhi 2020]**

13 Three cards are drawn at random (without replacement) from a well-shuffled pack of 52 playing cards. Find the probability distribution of number of red cards and also find the mean of the distribution. **[Foreign 2014]**

14 Three cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of spades. Also, find the mean of the distribution. **[All India 2015]**

15 Two cards are drawn simultaneously (or successively without replacement) from a well-shuffled pack of 52 cards. Find the mean of the number of kings. **[Delhi 2019]**

16 From a lot 30 bulbs, which includes 6 defectives, 3 bulbs are drawn one-by-one at random with replacement. Find the probability distribution of number of defective bulbs. Hence, find the mean of the distribution. **[All India 2017C]**

17 A coin is biased so that the head is 2 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence, find the mean of the distribution. **[Delhi 2016C]**

18 In a meeting, 70% of the members favour a certain proposal, 30% being opposite. A member is selected at random and let $X = 0$, if he opposed and $X = 1$, if he is in favour. Find $E(X)$. **[NCERT]**

19 Suppose 10000 tickets are sold in a lottery each for ₹ 1. First prize is of ₹ 3000 and the second prize is of ₹ 2000. There are three third prizes of ₹ 500 each. If you buy one ticket, then what is your expectation? **[NCERT Exemplar]**

20 There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denotes the sum of the numbers on the two drawn cards. Find the mean of X. **[All India 2017]**

LONG ANSWER Type Questions

21 The probability distribution of a random variable X is given as under

$$P(X = x) = \begin{cases} kx^2, & x = 1, 2, 3 \\ 2kx, & x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant. Calculate

(i) $E(X)$

(ii) $P(X \geq 4)$ **[NCERT Exemplar]**

22 A random variable X has following probability distribution.

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) $P(X < 3)$

(iii) $P(X > 6)$ (iv) $P(0 < X < 3)$. **[All India 2011]**

23 Let X denotes the sum of the numbers obtained, when two fair dice are rolled. Find the mean of X. **[NCERT]**

24 There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denotes the sum of the numbers on two cards drawn. Find the mean of X. **[NCERT Exemplar]**

25 A coin is biased so that the head is 4 times as likely to occur as tail. If the coin is tossed thrice, find the probability distribution of number of tails. Hence, find the mean of the distribution. **[All India 2017C]**

26 An urn contains 3 white and 6 red balls. Four balls are drawn one-by-one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also, find mean of the distribution. **[Delhi 2016]**

27 Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find the probability distribution of the random variable X and also find the mean of the distribution. **[NCERT; All India 2014]**

28 Three numbers are selected at random (without replacement) from first six positive integers. Let X denotes the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean of the distribution. **[All India 2016]**

HINTS & SOLUTIONS

1. (d) Let M denotes the number of heads and N denotes the number of tails when a coin is tossed 6 times.

Given, X = Difference between M and $N = |M - N|$

Here, both M and N can take values 0, 1, 2, 3, 4, 5, 6 but $M + N$ is always equal to 6.

Look at the following table

M	0	1	2	3	4	5	6
N	6	5	4	3	2	1	0
X	6	4	2	0	2	4	6

Thus, we see that X takes values 0, 2, 4 and 6.

2. (c) We know that $\Sigma P(X) = 1$

$$\Rightarrow \quad \frac{5}{k} + \frac{7}{k} + \frac{9}{k} + \frac{11}{k} = 1 \Rightarrow \frac{32}{k} = 1$$

$$\therefore \quad k = 32$$

3. (d) $E(X) = \Sigma X\, P(X)$

$$= -4 \times (0.1) + (-3 \times 0.2) + (-2 \times 0.3) + (-1 \times 0.2) + (0 \times 0.2)$$

$$= -0.4 - 0.6 - 0.6 - 0.2 = -1.8$$

4. (b) Let X be the random variable representing a number on the die.

The total number of observations is six.

$P(X = 1) = \frac{3}{6} = \frac{1}{2}$, $P(X = 2) = \frac{2}{6} = \frac{1}{3}$ and $P(X = 5) = \frac{1}{6}$

Thus, required probability distribution of X is as follows:

X	1	2	5
$P(X)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$$\text{Mean } X = \Sigma X\, P(X) = 1 \times \frac{1}{2} + 2 \times \frac{1}{3} + 5 \times \frac{1}{6}$$

$$= \frac{3 + 4 + 5}{6} = \frac{12}{6} = 2$$

5. Here, two balls are drawn, so possible values of X are 0, 1and 2.

6. **Hint** $\sum_{i=1}^{n} p_i = 1.05$ [**Ans.** No]

7. When two coins are tossed, there may be 1 head, 2 heads or no head at all.
Thus, the possible values of X are 0, 1 and 2.

Now, $P(X = 0) = P$ (getting no head) $= P(TT) = \frac{1}{4}$

$P(X = 1) = P$ (getting one head) $= P(HT \text{ or } TH) = \frac{2}{4} = \frac{1}{2}$

$P(X = 2) = P$ (getting two heads) $= P(HH) = \frac{1}{4}$

Thus, the required probability distribution of X is

X	0	1	2
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

8. Given, $P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$

Making it in tabular format, we get the following

X	0	1	2	otherwise
$P(X)$	k	$2k$	$3k$	0

Since, sum of all probabilities is equal to 1.

$$\Sigma P(X = x) = 1$$
$$\Rightarrow P(X = 0) + P(X = 1) + P(X = 2) + 0 + 0 + \dots = 1$$
$$\Rightarrow k + 2k + 3k = 1$$
$$\Rightarrow 6k = 1$$
$$\Rightarrow k = \frac{1}{6}$$

9. Total number of possible outcomes

$$= {}^5P_2 = \frac{5!}{3!} = 5 \times 4 = 20$$

Here, X denotes the larger of two numbers obtained.
$\therefore$ X can take values 2, 3, 4 and 5.

Now, $P(X = 2) = P$ {getting (1, 2) or (2, 1)} $= \frac{2}{20} = \frac{1}{10}$

$P(X = 3) = P$ {getting (1, 3) or (3, 1) or (2, 3) or (3, 2)}
$= \frac{4}{20} = \frac{2}{10}$

$P(X = 4) = P$ {getting (1, 4) or (4, 1) or (2, 4) or (4, 2) or (3, 4) or (4, 3)}
$= \frac{6}{20} = \frac{3}{10}$

and $P(X = 5) = P$ {getting (1, 5) or (5, 1) or (2, 5) or (5, 2) or (3, 5) or (5, 3) or (4, 5) or (5, 4)}
$= \frac{8}{20} = \frac{4}{10}$

Thus, the probability distribution of X is

X	2	3	4	5
$P(X)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

Now, mean of $X = E(X) = \Sigma X \cdot P(X)$

$$= 2 \cdot \frac{1}{10} + 3 \cdot \frac{2}{10} + 4 \cdot \frac{3}{10} + 5 \cdot \frac{4}{10}$$
$$= \frac{1}{10}(2 + 6 + 12 + 20)$$
$$= \frac{40}{10} = 4$$

10. Since, $X =$ Number of four seen on tossing two dice, $X = 0, 1, 2$

Also, $P(4) = \frac{1}{10}$ and $P(\text{not } 4) = \frac{9}{10}$

So, $P(X = 0) = P(\text{not } 4) \cdot P(\text{not } 4) = \frac{9}{10} \cdot \frac{9}{10} = \frac{81}{100}$

$P(X = 1) = P(\text{not } 4) \cdot P(4) + P(4) \cdot P(\text{not } 4)$
$= \frac{9}{10} \cdot \frac{1}{10} + \frac{1}{10} \cdot \frac{9}{10} = \frac{18}{100}$

$P(X = 2) = P(4) \cdot P(4) = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$

Thus, we get the following table

X	0	1	2
$P(X)$	81/100	18/100	1/100

Now, mean of $X = P(X) = 0 \times \frac{81}{100} + 1 \times \frac{18}{100} + 2 \times \frac{1}{100}$
$= \frac{20}{100} = \frac{1}{5}$

11. Given, defective bulbs = 3 and good bulbs = 7

Here, total number of bulbs $= 3 + 7 = 10$
Let X denotes the number of defective bulbs.
Then, X takes the values 0, 1, 2 and 3.

Now, $P(X = 0) = P$ (getting 0 defective bulb)

$$= \frac{({}^3C_0 \times {}^7C_3)}{{}^{10}C_3} = \frac{\left(1 \times \frac{7 \times 6 \times 5}{3 \times 2 \times 1}\right)}{\left(\frac{10 \times 9 \times 8}{3 \times 2 \times 1}\right)}$$
$$= \frac{7 \times 6 \times 5}{10 \times 9 \times 8} = \frac{7}{24}$$

$P(X = 1) = P$ (getting 1 defective bulb)

$$= \frac{({}^3C_1 \times {}^7C_2)}{{}^{10}C_3}$$
$$= \frac{\left(3 \times \frac{7 \times 6}{2 \times 1}\right)}{\left(\frac{10 \times 9 \times 8}{3 \times 2 \times 1}\right)}$$
$$= \frac{3 \times 7 \times 6 \times 3}{10 \times 9 \times 8} = \frac{21}{40}$$

$P(X=2) = P$ (getting 2 defective bulbs)

$$= \frac{({}^3C_2 \times {}^7C_1)}{{}^{10}C_3} = \frac{\left(\frac{3\times 2}{2\times 1}\times 7\right)}{\left(\frac{10\times 9\times 8}{3\times 2\times 1}\right)}$$

$$= \frac{3\times 2\times 7\times 3}{10\times 9\times 8} = \frac{7}{40}$$

$P(X=3) = P$ (getting 3 defective bulbs)

$$= \frac{({}^3C_3 \times {}^7C_0)}{{}^{10}C_3} = \frac{1\times 1}{\left(\frac{10\times 9\times 8}{3\times 2\times 1}\right)}$$

$$= \frac{3\times 2\times 1}{10\times 9\times 8} = \frac{1}{120}$$

$\therefore$ Required probability distribution as follows

X	0	1	2	3
P (X)	7/24	21/40	7/40	1/120

12. Let X be the random variable which denotes the number of tails when a biased coin is tossed twice.

So, X may have values 0, 1 or 2.

Since, the coin is biased in which head is 3 times as likely to occur as a tail.

$\therefore \quad P(H) = \frac{3}{4}$ and $P(T) = \frac{1}{4}$

$$P(X=0) = P(HH) = P(H)\cdot P(H) = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$P(X=1) = P$ (one tail and one head)

$$= P(HT, TH) = P(HT) + P(TH)$$

$$= P(H)\cdot P(T) + P(T)\cdot P(H)$$

$$= \frac{3}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{3}{4} = \frac{3}{16} + \frac{3}{16} = \frac{6}{16} = \frac{3}{8}$$

$P(X=2) = P(\text{two tails}) = P(TT) = P(T)\cdot P(T)$

$$= \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

Therefore, the required probability distribution is as follows

X	0	1	2
P(X)	9/16	3/8	1/16

Now, mean $= \Sigma X\,P(X)$

$$= 0\times\frac{9}{16} + 1\times\frac{3}{8} + 2\times\frac{1}{16} = 0 + \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

13. Similar as Example 6.

Ans.

X	0	1	2	3
P(X)	2/17	13/34	13/34	2/17

, Mean = 1.5

14. Let X be a random variable that denotes the number of spades in a draw of three cards. So, X can takes values 0, 1, 2 and 3.

Now, $P(X=0) = P$ (no spade) $= \frac{39}{52}\times\frac{39}{52}\times\frac{39}{52}$

[$\because$ cards are drawn with replacement]

$$= \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$$

$P(X=1) = P$ (one spade and two non-spade)

$= P$ (I card is spade, II and III cards are non-spade) $+ P$ (I card is non-spade, II card is spade and III is non-spade) $+ P$ (I and II cards are non-spade and III card is spade)

$$= \frac{13}{52}\times\frac{39}{52}\times\frac{39}{52} + \frac{39}{52}\times\frac{13}{52}\times\frac{39}{52} + \frac{39}{52}\times\frac{39}{52}\times\frac{13}{52}$$

$$= 3\left(\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4}\right) = \frac{27}{64}$$

$P(X=2) = P$ (two spade and one non-spade)

$$= \frac{13}{52}\times\frac{13}{52}\times\frac{39}{52} + \frac{13}{52}\times\frac{39}{52}\times\frac{13}{52} + \frac{39}{52}\times\frac{13}{52}\times\frac{13}{52}$$

$$= 3\left(\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}\right) = \frac{9}{64}$$

$\therefore P(X=3) = P$ (all are spade cards)

$$= \frac{13}{52}\times\frac{13}{52}\times\frac{13}{52} = \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{64}$$

Thus, the required probability distribution is as follows

X	0	1	2	3
P(X)	27/64	27/64	9/64	1/64

Now, mean of above distribution is given by

$$E(X) = \sum_{x=0}^{3} x\cdot P(X=x)$$

$$= 0\cdot P(X=0) + 1\cdot P(X=1) + 2\cdot P(X=2) + 3\cdot P(X=3)$$

$$= 0\cdot\frac{27}{64} + 1\cdot\frac{27}{64} + 2\cdot\frac{9}{64} + 3\cdot\frac{1}{64}$$

$$= \frac{27+18+3}{64} = \frac{48}{64} = \frac{3}{4}$$

15. Let X be the number of kings obtained.

We can get 0, 1 or 2 kings.

So, value of X is 0, 1 or 2.

Total number of ways to draw 2 cards out of 52

i.e. Total ways $= {}^{52}C_2 = 1326$

$P(X=0)$

i.e. probability of getting 0 king

Number of ways to get 0 king

= Number of ways to select 2 cards out of non-king cards

= Number of ways to select 2 cards out of $(52-4)$

= 48 cards

$= {}^{48}C_2 = 1128$

$$P(X=0) = \frac{\text{Number of ways to get 0 king}}{\text{Total number of ways}} = \frac{1128}{1326}$$

$P(X=1)$ i.e. probability of getting 1 king

Number of ways to get 1 king

= Number of ways to select 1 king out of 4 king cards × number of ways to select 1 card from 48 non-king cards

$= {}^4C_1 \times {}^{48}C_1$

$= 4 \times 48 = 192$

$P(X=1) = \dfrac{\text{Number of ways to get 1 king}}{\text{Total number of ways}} = \dfrac{192}{1326}$

$P(X = 2)$, i.e. probability of geeting 2 kings

Number of ways to get 2 king

= Number of ways of selecting 2 kings out of 4 king cards

$= {}^4C_2 = 6$

$P(X = 2) = \dfrac{6}{1326}$

Now, $\mu = E(X) = \sum_{i=1}^{n} X_i p_i$

$= 0 \times \dfrac{1128}{1326} + 1 \times \dfrac{192}{1326} + 2 \times \dfrac{6}{1326}$

$= \dfrac{192+12}{1326} = \dfrac{204}{1326} = \dfrac{34}{221}$

16. **Hint** Let X denotes the number of defective bulbs drawn.

Then, P (getting defective bulbs) $= \dfrac{1}{5}$,

P (non-getting defective bulbs) $= \dfrac{4}{5}$

and $X = 0, 1, 2, 3$

Now, $P(X = 0) = \dfrac{4}{5} \times \dfrac{4}{5} \times \dfrac{4}{5} = \dfrac{64}{125}$

$P(X = 1) = 3 \times \dfrac{4}{5} \times \dfrac{4}{5} \times \dfrac{1}{5} = \dfrac{48}{125}$

$P(X = 2) = 3 \times \dfrac{4}{5} \times \dfrac{1}{5} \times \dfrac{1}{5} = \dfrac{12}{125}$

$P(X = 3) = \dfrac{1}{5} \times \dfrac{1}{5} \times \dfrac{1}{5} = \dfrac{1}{125}$

∴ Mean $= \dfrac{48}{125} + \dfrac{24}{125} + \dfrac{3}{125}$ $\left[\textbf{Ans.}\ \text{Mean} = \dfrac{3}{5}\right]$

17. Here, $P(\text{head}) = \dfrac{2}{3}$, $P(\text{tail}) = \dfrac{1}{3}$

Let X be the number of tails in two tosses.

Then, $X = 0, 1, 2$

X	0	1	2
$P(X)$	$\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$	$\frac{2}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}=\frac{4}{9}$	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
$XP(X)$	0	4/9	2/9

∴ Mean $= \Sigma XP(X)$

$= \dfrac{6}{9} = \dfrac{2}{3}$

18. **Hint**

X	0	1
$P(X)$	3/10	7/10

$\left[\textbf{Ans.}\ E(X) = \dfrac{7}{10}\right]$

19. **Hint**

X	0	500	2000	3000
$P(X)$	$\frac{9995}{10000}$	$\frac{3}{10000}$	$\frac{1}{10000}$	$\frac{1}{10000}$
P_iX_i	0	$\frac{1500}{10000}$	$\frac{2000}{10000}$	$\frac{3000}{10000}$

$\because E(X) = \Sigma XP(X)$ [**Ans.** 0.65]

20. Here, $S = \{(1, 3), (1, 5), (1, 7), (3, 1)$
$(3, 5), (3, 7), (5, 1), (5, 3), (5, 7), (7, 1), (7, 3), (7, 5)\}$

$\Rightarrow n(S) = 12$

Let random variable X denotes the sum of the numbers on two cards drawn. So, the random variables X may have values 4, 6, 8, 10 and 12.

At $X = 4$, $P(X) = \dfrac{2}{12} = \dfrac{1}{6}$,

At $X = 6$, $P(X) = \dfrac{2}{12} = \dfrac{1}{6}$

At $X = 8$, $P(X) = \dfrac{4}{12} = \dfrac{1}{3}$,

At $X = 10$, $P(X) = \dfrac{2}{12} = \dfrac{1}{6}$

At $X = 12$, $P(X) = \dfrac{2}{12} = \dfrac{1}{6}$

Therefore, the required probability distribution is as follows

X	4	6	8	10	12
$P(X)$	1/6	1/6	1/3	1/6	1/6

∴ Mean, $E(X) = \Sigma X\, P(X)$

$= 4 \times \dfrac{1}{6} + 6 \times \dfrac{1}{6} + 8 \times \dfrac{1}{3} + 10 \times \dfrac{1}{6} + 12 \times \dfrac{1}{6}$

$= \dfrac{2}{3} + 1 + \dfrac{8}{3} + \dfrac{5}{3} + 2 = \dfrac{2+8+5}{3} + 3 = \dfrac{15}{3} + 3$

$= 5 + 3 = 8$

21. The probability distribution table for given function is

X	1	2	3	4	5	6	otherwise
$P(X)$	k	$4k$	$9k$	$8k$	$10k$	$12k$	0

We know that $\Sigma P_i = 1$

∴ $k + 4k + 9k + 8k + 10k + 12k + 0 = 1$

$\Rightarrow 44k = 1 \Rightarrow k = \dfrac{1}{44}$

(i) $E(X) = \Sigma X \cdot P(X)$

$= 1 \times k + 2 \times 4k + 3 \times 9k + 4 \times 8k + 5 \times 10k + 6 \times 12k + 0$

$= k + 8k + 27k + 32k + 50k + 72k$

$= 190k$

$= 190 \times \frac{1}{44} = \frac{95}{22}$ $\left[\because k = \frac{1}{44}\right]$

(ii) $P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$

$= 30k = 30 \times \frac{1}{44} = 0.68$

22. **Hint** Use $\Sigma P(X) = 1$

[**Ans.** (i) 1/10 (ii) 3/10 (iii) 17/100 (iv) 3/10]

23. Let X denotes the sum of the numbers obtained when two fair dice are rolled. So, X may have values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12.

[as 1 cannot be the sum of two numbers on fair dice]

$P(X = 2) = P\{(1, 1)\} = \frac{1}{36}$,

$P(X = 3) = P\{(1, 2), (2, 1)\} = \frac{2}{36}$,

$P(X = 4) = P\{(1, 3), (2, 2), (3, 1)\} = \frac{3}{36}$,

$P(X = 5) = P\{(1, 4), (2, 3), (3, 2), (4, 1)\} = \frac{4}{36}$,

$P(X = 6) = P\{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\} = \frac{5}{36}$,

$P(X = 7) = P\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\} = \frac{6}{36}$,

$P(X = 8) = P\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\} = \frac{5}{36}$,

$P(X = 9) = P\{(3, 6), (4, 5), (5, 4), (6, 3)\} = \frac{4}{36}$,

$P(X = 10) = P\{(4, 6), (5, 5), (6, 4)\} = \frac{3}{36}$,

$P(X = 11) = P\{(5, 6), (6, 5)\} = \frac{2}{36}$ and

$P(X = 12) = P\{(6, 6)\} = \frac{1}{36}$

The probability distribution of X is represented by

X	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Mean, $X = \Sigma X\, P(X)$

$$= \frac{\left[\begin{array}{l} 2 \times 1 + 3 \times 2 + 4 \times 3 + 5 \times 4 + 6 \times 5 + 7 \times 6 \\ + 8 \times 5 + 9 \times 4 + 10 \times 3 + 11 \times 2 + 12 \times 1 \end{array}\right]}{36}$$

$$= \frac{[2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12]}{36}$$

$$= \frac{252}{36} = 7$$

24. Here, $S = \left\{\begin{array}{l} (1,2), (2,1), (1,3), (3,1), (2,3), (3,2), \\ (1,4), (4,1), (1,5), (5,1), (2,4), (4,2), (2,5), \\ (5,2), (3,4), (4,3), (3,5), (5,3), (5,4), (4,5) \end{array}\right\}$

$\Rightarrow n(S) = 20$

Let random variable X denotes the sum of the numbers on two cards drawn. So, the random variables X may have values 3, 4, 5, 6, 7, 8 and 9.

At $X = 3$, $P(X) = \frac{2}{20} = \frac{1}{10}$,

At $X = 4$, $P(X) = \frac{2}{20} = \frac{1}{10}$

At $X = 5$, $P(X) = \frac{4}{20} = \frac{1}{5}$,

At $X = 6$, $P(X) = \frac{4}{20} = \frac{1}{5}$

At $X = 7$, $P(X) = \frac{4}{20} = \frac{1}{5}$,

At $X = 8$, $P(X) = \frac{2}{20} = \frac{1}{10}$

At $X = 9$, $P(X) = \frac{2}{20} = \frac{1}{10}$

Therefore, the required probability distribution is as follows

X	3	4	5	6	7	8	9
$P(X)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

$\therefore$ Mean, $E(X) = \Sigma X\, P(X)$

$$= 3 \times \frac{1}{10} + 4 \times \frac{1}{10} + 5 \times \frac{1}{5} + 6 \times \frac{1}{5} + 7 \times \frac{1}{5} + 8 \times \frac{1}{10} + 9 \times \frac{1}{10}$$

$$= \frac{3}{10} + \frac{4}{10} + \frac{5}{5} + \frac{6}{5} + \frac{7}{5} + \frac{8}{10} + \frac{9}{10}$$

$$= \frac{3 + 4 + 10 + 12 + 14 + 8 + 9}{10}$$

$$= \frac{60}{10} = 6$$

25. **Hint** P (head) = $4P$ (tail) $\Rightarrow P(H) = \frac{4}{5}$, $P(T) = \frac{1}{5}$

X (Number of tails)	0	1	2	3
$P(X)$	$\frac{64}{125}$	$\frac{48}{125}$	$\frac{12}{125}$	$\frac{1}{125}$
$XP(X)$	0	$\frac{48}{125}$	$\frac{24}{125}$	$\frac{3}{125}$

$\therefore$ Mean $= \Sigma XP(X) = \frac{75}{125}$

$\left[\textbf{Ans.} \text{ Mean} = \frac{3}{5}\right]$

26. Consider, p = probability of getting a red ball $= \frac{6}{9} = \frac{2}{3}$

and q = probability of getting a white ball $= \frac{3}{9} = \frac{1}{3}$

Let X denotes the number of red balls in four draws.

Then, X can take values 0, 1, 2, 3 and 4.

Now, $P(X=0) = P$ (no red balls) $= P$ (all black balls)

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{81}$$

$P(X=1) = P$ (one red and three blacks balls)

$$= \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}$$

$$+ \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = 4 \times \frac{2}{81} = \frac{8}{81}$$

$P(X=2) = P$ (two red and two black balls)

$$= 6 \times \frac{4}{81} = \frac{24}{81}$$

$P(X=3) = P$ (three red and one black ball)

$$= 4 \times \frac{8}{81} = \frac{32}{81}$$

$P(X=4) = P$ (all red balls) $= 1 \times \frac{16}{81} = \frac{16}{81}$

Hence, the probability distribution is shown below

X	0	1	2	3	4
$P(X)$	$\frac{1}{81}$	$\frac{8}{81}$	$\frac{24}{81}$	$\frac{32}{81}$	$\frac{16}{81}$

Now, mean of distribution, $E(X) = \Sigma X \cdot P(X)$

$$= 0 \cdot \frac{1}{81} + 1 \cdot \frac{8}{81} + 2 \cdot \frac{24}{81} + 3 \cdot \frac{32}{81} + 4 \cdot \frac{16}{81}$$

$$= \frac{1}{81}[0 + 8 + 48 + 96 + 64]$$

$$= \frac{216}{81} = \frac{8}{3}$$

27. Similar as Example 7.

Ans.

X	2	3	4	5	6
$P(X)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

28. Given, X denotes the largest of the three positive integers.

So, the random variable X may have values 3, 4, 5 or 6.

$P(X=3) = P$ (getting 3 and two numbers less than 3)

$$= \frac{{}^1C_1 \times {}^2C_2}{{}^6C_3} = \frac{1}{20}$$

$P(X=4) = P$ (getting 4 and two numbers less than 4)

$$= \frac{{}^1C_1 \times {}^3C_2}{{}^6C_3} = \frac{3}{20}$$

$P(X=5) = P$ (getting 5 and two numbers less than 5)

$$= \frac{{}^1C_1 \times {}^4C_2}{{}^6C_3} = \frac{6}{20}$$

$P(X=6) = P$ (getting 6 and two numbers less than 6)

$$= \frac{{}^1C_1 \times {}^5C_2}{{}^6C_3} = \frac{10}{20}$$

$\therefore$ Probability distribution is shown below

X	3	4	5	6
$P(X)$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$

Now, mean of distribution, $E(X) = \sum X \cdot P(X)$

$$= 3 \times \frac{1}{20} + 4 \times \frac{3}{20} + 5 \times \frac{6}{20} + 6 \times \frac{10}{20}$$

$$= \frac{3 + 12 + 30 + 60}{20} = \frac{105}{20} = \frac{21}{4}$$

SUMMARY

- **Probability of an Event** If there are n elementary equally likely events associated with a random experiment and m of them are favourable to an event A, then the probability of happening or occurrence of A is denoted by $P(A)$ and is defined as $P(A) = \frac{m}{n}$.
- **Conditional Probability** If A and B are two events associated with the same sample space of a random experiment, then conditional probability of the event A given that B has occurred, i.e. $P(A/B)$ is given by

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \text{ where } P(B) \neq 0$$

- **Multiplication Theorem**
  (i) Let A and B be two events associated with a random experiment, then

$$P(A \cap B) = \begin{cases} P(A) \cdot P(B/A), \text{ where } P(A) \neq 0 \\ P(B) \cdot P(A/B), \text{ where } P(B) \neq 0 \end{cases}$$

  Here, $A \cap B$ denotes the simultaneous occurrence of the events A and B.
  (ii) Let E, F and G be three events of sample space S.
  Then, $P(E \cap F \cap G) = P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right)$
- **Theorem of Total Probability** Let $\{E_1, E_2, \ldots, E_n\}$ be a partition of the sample space S and suppose that each of the events $E_1, E_2, \ldots, E_n$ has non-zero probability of occurrence.
  Let A be any event associated with S, then

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + \ldots + P(E_n) \cdot P(A/E_n) = \sum_{j=i}^{n} P(E_j) \cdot P(A/E_j)$$

- **Baye's Theorem** If $E_1, E_2, \ldots, E_n$ are n non-empty events, which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint, $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and $P(E_i) > 0, \forall\, i = 1, 2, \ldots, n$.
  Also, let A be any event of non-zero probability, then

$$P(E_i / A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}, \text{ for any } i = 1, 2, 3, \ldots, n$$

  Here, events $E_1, E_2, \ldots, E_n$ are called **hypothesis**. The probability $P(E_i)$ is called the **priori probability** of the hypothesis E_i and the conditional probability $P(E_i / A)$ is called a **posteriori probability** of the hypothesis E_i.
- **Random Variable** A random variable is a real valued function, whose domain is the sample space of a random experiment. Generally, it is denoted by capital letter X.
- **Probability Distribution of a Random Variable** If X is a random variable and takes the values $x_1, x_2, x_3, \ldots, x_n$ with respective probabilities $p_1, p_2, p_3, \ldots, p_n$. Then, the probability distribution of X is represented by

X	x_1	x_2	x_3	...	x_n
P(X)	p_1	p_2	p_3	...	p_n

  where, $p_i > 0$ and $\sum_{i=1}^{n} p_i = 1; i = 1, 2, 3, \ldots, n$
- **Mean** of X denoted by μ is the number $\sum_{i=1}^{n} x_i p_i$, it is also called the **expectation** of X, denoted by $E(X)$.

CHAPTER PRACTICE

OBJECTIVE TYPE QUESTIONS

1 If A and B are events such that $P\left(\frac{A}{B}\right) = P\left(\frac{B}{A}\right)$, then

(a) $A \subset B$ but $A \neq B$ (b) $A = B$
(c) $A \cap B = \phi$ (d) $P(A) = P(B)$

2 If A and B are two events such that $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P(A)$ equals to **[NCERT Exemplar]**

(a) $\frac{3}{10}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$

3 If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct? **[NCERT]**

(a) $P\left(\frac{B}{A}\right) = \frac{P(A)}{P(B)}$ (b) $P\left(\frac{A}{B}\right) < P(A)$
(c) $P\left(\frac{A}{B}\right) \geq P(A)$ (d) None of these

4 A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, then probability that both are dead is **[NCERT Exemplar]**

(a) $\frac{33}{56}$ (b) $\frac{9}{64}$
(c) $\frac{1}{14}$ (d) $\frac{3}{28}$

5 The probability of obtaining an even prime number on each die when a pair of dice is rolled is **[NCERT]**

(a) zero (b) $\frac{1}{3}$
(c) $\frac{1}{12}$ (d) $\frac{1}{36}$

6 If two events are independent, then **[NCERT Exemplar]**

(a) they must be mutually exclusive
(b) the sum of their probabilities must be equal to 1
(c) Both (a) and (b) are correct
(d) None of the above is correct

7 If a die is thrown and a card is selected at random from a deck of 52 playing cards, then the probability of getting an even number on the die and a spade card is **[NCERT Exemplar]**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{3}{4}$

VERY SHORT ANSWER Type Questions

8 A card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting an ace card.

9 Two dice are thrown simultaneously. Find the probability of getting a doublet.

10 Let E and F be the events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Find whether the events are independent or not.

11 Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card jack?
[CBSE Sample Paper 2021 (Term II)]

SHORT ANSWER Type I Questions

12 In class XII of a school, 40% of students study Mathematics, 30% of students study Biology and 10% of class study both Mathematics and Biology. If a student is selected at random from the class, then find the probability that he will studying Mathematics or Biology.

13 A die has two faces each with number 1, three faces each with number 2 and one face with number 3. If die is rolled once, then determine probability of not getting 3.

14 If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, then find $P\left(\frac{A'}{B}\right)$.
[NCERT Exemplar]

15 If $P(A) = 0.4$, $P(B) = 0.8$ and $P\left(\frac{B}{A}\right) = 0.6$, then find $P(A \cup B)$. **[NCERT Exemplar]**

16 A coin is tossed 4 times. Find the mean of the probability distribution of the number of tails.

17 A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls, if 2 balls are drawn at random from the bag one-by-one without replacement. **[CBSE Sample Paper 2021 (Term II)]**

SHORT ANSWER Type II Questions

18 If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P\left(\frac{A}{B}\right) = \frac{1}{4}$, then find $P(A' \cap B')$. **[NCERT Exemplar]**

19 If A and B are two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, then find $P\left(\frac{A}{B}\right) \cdot P\left(\frac{A'}{B}\right)$. **[NCERT Exemplar]**

20 If a mother, father and son line up at random for a family picture as E (son on one end) and F (father is on middle), then find $P(E/F)$. **[NCERT]**

21 12 cards, numbered 1 to 12 are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, then find the probability that it is an even number.

22 In a college, 30% students fail in Physics, 25% fail in Mathematics and 10% fail in both. One student is chosen at random. Find the probability that she fails in Physics, if she failed in Mathematics. **[NCERT Exemplar]**

23 A bag contains 5 red, 7 green and 4 white balls. 3 balls are drawn one after another without replacement. Find the probability that

(i) the balls are white, green and green, respectively.

(ii) out of 3 balls, one is white and 2 is green.

24 A bag contains 4 red and 5 black balls. Another bag contains 3 red and 6 black balls. One ball is drawn from bag I and two balls are drawn from bag II. Find the probability that out of three, two are black and one is red.

25 A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. Find the probability of drawing 2 green balls and one blue ball. **[NCERT Exemplar]**

26 Two events E and F are independent. If $P(E) = 0.3$ and $P(E \cup F) = 0.5$, then find $P\left(\frac{E}{F}\right) - P\left(\frac{F}{E}\right)$. **[NCERT Exemplar]**

27 Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p, if they are

(i) mutually exclusive.

(ii) independent.

28 In a group of students, there are 3 boys and 3 girls. 4 students are to be selected at random from the group. Find the probability that either 3 boys and 1 girl or 3 girls and 1 boy are selected.

29 A and B appear for an interview for two posts. The probability of A's selection is $\frac{1}{3}$ and that of B's selection is $\frac{2}{5}$. Find the probability that

(i) only one of them is selected.

(ii) none is selected.

30 Three persons A, B and C fire at a target in turn, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2, respectively. Find the probability that exactly two of them hit the target. **[NCERT Exemplar]**

31 Three bags contains a number of red and white balls as follows. Bag I : 3 red balls, Bag II : 2 red balls and 1 white ball and Bag III : 3 white balls. The probability that bag i will be chosen and a ball is selected from it is $\frac{i}{6}$, where $i = 1, 2, 3$. What is the probability that

(i) a red ball will be selected?

(ii) a white ball will be selected? **[NCERT Exemplar]**

32 A box contains 2 gold and 3 silver coins. Another box contains 3 gold and 3 silver coins. A box is chosen at random and coin is drawn from it. If the selected coin is a gold coin, then find the probability that it was drawn from second box.

33 There are two bags, bag I and bag II. Bag I contains 2 white and 4 red balls and bag II contains 5 white and 3 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag II. **[All India 2010]**

34 Find the probability distribution of number of doublets in three tosses of a pair of dice. **[All India 2011C; Delhi 2010C]**

35 Two cards are drawn simultaneously (without replacement) from a well-shuffled deck of 52 cards. Find the mean of number of red cards. **[All India 2012]**

36 In a die game, a player pays a stake of ₹ 1 for each throw of a die. She receives ₹ 5, if the die shows a 3; ₹ 2, if the die shows a 1 or 6 and nothing otherwise. What is the player's expected profit per throw over a long series of throws? **[NCERT Exemplar]**

LONG ANSWER Type Questions

37 Bag I contains 3 red and 4 black balls and bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag II. **[Delhi 2011, 2010]**

38 A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. **[Delhi 2011]**

39 A factory has two machines A and B. Past record shows that machine A produced 60% of items of output and machine B produced 40% of items. Further, 2% of items, produced by machine A and 1% produced by machine B were defective. All the items are put into a stockpile and then one item is chosen at random from this and this is found to be defective. What is the probability that it was produced by machine B? **[Foreign 2011]**

40 In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs, respectively. 1%, 2% and 3% of the bulbs produced, respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by machine A.

41 If a machine is correctly setup, it produces 90% acceptable items. If it is incorrectly setup, it produces only 40% acceptable items. Past experience shows that 80% of the setups are correctly done. If after a certain setup, the machine produces 2 acceptable items, then find the probability that machine is correctly setup. **[NCERT]**

42 There are three coins. One is a two tailed coin (having tail on both faces) another is a biased coin that comes up heads 60% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed and it shows tail. What is the probability that it is a two tailed coin? **[All India 2011C]**

43 Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? (assume that these are equal number of males and females) **[NCERT; Delhi 2011]**

44 A crime is committed by one of two suspects A and B initially, there is equal evidence against both of them. In further investigation at the crime scene. It is found that the guilty party had a blood type found is 20% of the population. If the suspect A does match this blood type, whereas the blood type of suspect B is unknown, then find the probability that A is the guilty party.

45 A letter is known to have come either from LONDON or CLIFTON. On the envelope just two consecutive letters ON are visible. What is the probability that the letter has come from (i) LONDON (ii) CLIFTON?

46 Suppose we have four boxes A, B, C, D containing coloured marbles as given below:

Boxes	Marble Colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble drawn from it. If the marble is red, then what is the probability that it was drawn from

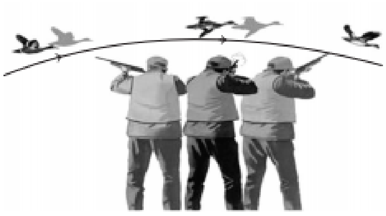
(i) box A? (ii) box B?

(iii) box C? **[NCERT]**

47 A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being of chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean of X. **[NCERT]**

CASE BASED Questions

48. A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots. **[CBSE Question Bank]**



Answer the following questions using the above information.

(i) Let A: the target is hit by A, B: the target is hit by B and C: the target is hit by C. Then, the probability that A, B and C all will hit, is

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

(ii) Referring to (i), what is the probability that B, C will hit and A will lose?

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{7}{10}$ (d) $\frac{4}{10}$

(iii) With reference to the events mentioned in (i), what is the probability that any two of A, B and C will hit?

(a) $\frac{1}{30}$ (b) $\frac{11}{30}$ (c) $\frac{17}{30}$ (d) $\frac{13}{30}$

(iv) What is the probability that none of them will hit the target?

(a) $\frac{1}{30}$ (b) $\frac{1}{60}$ (c) $\frac{1}{15}$ (d) $\frac{2}{15}$

(v) What is the probability that atleast one of A, B or C will hit the target?

(a) $\frac{59}{60}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{60}$

49. The reliability of a COVID PCR test is specified as follows

Of people having COVID, 90% of the test detects the disease but 10% goes undetected. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test and the pathologist reports him/her as COVID positive. **[CBSE Question Bank]**

Answer the following questions using the above information.

(i) What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID'?

(a) 0.001 (b) 0.1
(c) 0.8 (d) 0.9

(ii) What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID'?

(a) 0.01 (b) 0.99
(c) 0.1 (d) 0.001

(iii) What is the probability that the 'person is actually not having COVID'?

(a) 0.998 (b) 0.999
(c) 0.001 (d) 0.111

(iv) What is the probability that the 'person is actually having COVID given that 'he is tested as COVID positive'?

(a) 0.83 (b) 0.0803
(c) 0.083 (d) 0.089

(v) What is the probability that the 'person selected will be diagnosed as COVID positive'?

(a) 0.1089 (b) 0.01089
(c) 0.0189 (d) 0.189

50. In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2 and E be the events that the student knows the answer, guesses the answer and answers correctly respectively.
[CBSE Question Bank]

Answer the following questions using the above information.

(i) What is the value of $P(E_1)$?

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{3}{5}$

(ii) Value of $P(E|E_1)$ is

(a) $\frac{1}{3}$ (b) 1 (c) $\frac{2}{3}$ (d) 415

(iii) $\sum_{k=1}^{k=2} P\left(\frac{E}{E_k}\right) P(E_k)$ equals

(a) $\frac{11}{15}$ (b) $\frac{4}{15}$ (c) $\frac{1}{5}$ (d) 1

(iv) Value of $\sum_{k=1}^{k=2} P(E_k)$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{3}{5}$

(v) What is the probability that the student knows the answer given that he answered it correctly?

(a) $\frac{2}{11}$ (b) $\frac{5}{3}$ (c) $\frac{9}{11}$ (d) $\frac{13}{3}$

ANSWERS

1. (d) **2.** (c) **3.** (c) **4.** (d) **5.** (d) **6.** (d) **7.** (c) **8.** $\frac{1}{13}$

9. $\frac{1}{6}$ **10.** Not **11.** $\frac{1}{26}$ **12.** 0.6 **13.** $\frac{5}{6}$ **14.** $\frac{5}{9}$ **15.** 0.96 **16.** Mean = 2

17.

X	0	1
$P(X)$	$\frac{1}{2}$	$\frac{1}{2}$

18. $\frac{1}{4}$ **19.** $\frac{6}{25}$ **20.** 1 **21.** $\frac{5}{9}$ **22.** $\frac{2}{5}$ **23.** (i) 0.05 (ii) 0.15 **24.** 25/54 **25.** $\frac{3}{28}$ **26.** $\frac{1}{70}$

27. (i) $p = \frac{1}{10}$ (ii) $p = \frac{1}{5}$ **28.** $\frac{2}{5}$ **29.** (i) 7/15 (ii) 2/5 **30.** 0.188 **31.** (i) $\frac{7}{18}$ (ii) $\frac{11}{18}$ **32.** 5/9 **33.** 9 / 25

34.

X	0	1	2	3
$P(X)$	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

35. Mean = 1 **36.** 0.50 **37.** $\frac{35}{68}$ **38.** $\frac{3}{8}$

39. $\frac{1}{4}$ **40.** $\frac{2}{5}$ **41.** 0.953 **42.** $\frac{10}{19}$ **43.** $\frac{20}{21}$ **44.** $\frac{5}{6}$ **45.** (i) $\frac{12}{17}$ (ii) $\frac{5}{17}$ **46.** (i) $\frac{1}{15}$ (ii) $\frac{2}{5}$ (iii) $\frac{8}{15}$

47.

X	14	15	16	17	18	19	20	21
$P(X)$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Mean = 17.53

48. (i) → (c), (ii) → (a), (iii) → (d), (iv) → (b), (v) → (a)

49. (i) → (d), (ii) → (a), (iii) → (b), (iv) → (c), (v) → (b)

50. (i) → (d), (ii) → (b), (iii) → (a), (iv) → (c), (v) → (c)