A

Total No. of Questions - 24

Total No. of Printed Pages - 4

Regd. No.

Part - III

MATHEMATICS, Paper - II (A)

(Algebra and Probability)

(English Version)

Time : 3 Hours ***Max. Marks : 75***

Note : This question paper consists of **three** Sections *A, B* and *C.*

SECTION A

10 × 2 = 20

I. **Very short answer** type questions.

i) Answer **all** questions.

ii) Each question carries **two** marks.

1. Find the real and imaginary parts of the complex number $\frac{a+ib}{a-ib}$.

2. Represent the complex number $2+3i$ in argand plane.

3. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that

$$\frac{1}{2+\omega}+\frac{1}{1+2\omega}=\frac{1}{1+\omega}.$$

4. Find the values of m, if the equation $x^2-15-m(2x-8)=0$ have equal roots.

5. Find the polynomial equation whose roots are the reciprocals of the roots of $x^4 - 3x^3 + 7x^2 + 5x - 2 = 0$.

6. If ${}^{n}P_7 = 42 \cdot {}^{n}P_5$, then find n.

7. Find the number of ways of selecting 4 boys and 3 girls from a group of 8 boys and 5 girls.

8. If the coefficients of $(2r+4)^{th}$ term and $(3r+4)^{th}$ term in the expansion of $(1+x)^{21}$ are equal, then find r.

9. Find the mean deviation about the median for the following data : 4, 6, 9, 3, 10, 13, 2

10. The mean and variance of a binomial distribution are 4 and 3 respectively. Fix the distribution and find $P(X \geq 1)$.

SECTION B

5 × 4 = 20

II. **Short answer** type questions.

i) Attempt **any five** questions.

ii) Each question carries **four** marks.

11. If $x + iy = \dfrac{1}{1 + Cos\,\theta + i\,Sin\,\theta}$, then show that $4x^2 - 1 = 0$.

12. Solve $2x^4 + x^3 - 11x^2 + x + 2 = 0$.

13. Find the sum of all 4-digit numbers that can be formed using the digits 0, 2, 4, 7, 8 without repetition.

14. Find the number of ways of forming a committee of 5 members out of 6 Indians and 5 Americans so that always the Indians will be in majority in the committee.

15. Resolve $\dfrac{x^2 - x + 1}{(x+1)(x-1)^2}$ into partial fractions.

16. If A, B are two events with $P(A \cup B) = 0.65$, $P(A \cap B) = 0.15$, then find the value of $P(A^C) + P(B^C)$.

17. Find the probability of drawing an ace or a spade from a well-shuffled pack of 52 playing cards.

SECTION C

5 × 7 = 35

III. **Long answer** type questions.

i) Attempt **any five** questions.

ii) Each question carries **seven** marks.

18. If $Cos\,\alpha + Cos\,\beta + Cos\,\gamma = 0 = Sin\,\alpha + Sin\,\beta + Sin\,\gamma$, prove that

$$Cos^2\alpha + Cos^2\beta + Cos^2\gamma = \frac{3}{2} = Sin^2\alpha + Sin^2\beta + Sin^2\gamma.$$

19. Solve $3x^3 - 26x^2 + 52x - 24 = 0$, given that the roots are in geometric progression.

20. For $r = 0, 1, 2, \ldots\ldots\ldots, n$, prove that

$$C_0 \cdot C_r + C_1 \cdot C_{r+1} + C_2 \cdot C_{r+2} + \ldots\ldots + C_{n-r} \cdot C_n = {}^{2n}C_{(n+r)}$$

and hence deduce that

i) $C_0^{\,2} + C_1^{\,2} + C_2^{\,2} + \ldots\ldots + C_n^{\,2} = {}^{2n}C_n$

ii) $C_0 \cdot C_1 + C_1 \cdot C_2 + C_2 \cdot C_3 + \ldots\ldots + C_{n-1} \cdot C_n = {}^{2n}C_{n+1}$

21. Find the sum of the infinite series $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+.........$

22. Find the variance and standard deviation of the following frequency distribution :

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

23. If A, B, C are three independent events of an experiment such that, $P\left(A \cap B^C \cap C^C\right)=\frac{1}{4}$, $P\left(A^C \cap B \cap C^C\right)=\frac{1}{8}$, $P\left(A^C \cap B^C \cap C^C\right)=\frac{1}{4}$, then find $P(A), P(B), P(C)$.

24. The range of a random variable X is $\{0, 1, 2\}$. Given that $P(X=0)=3c^3, P(X=1)=4c-10c^2, P(X=2)=5c-1$

(i) Find the value of c.

(ii) $P(X<1)$, $P(1<X\leq 2)$ and $P(0<X\leq 3)$.

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