CBSE Board
Class X Mathematics
Sample Paper 3 (Standard) – Solution

Time: 3 hrs **Total Marks: 80**

Part A

Section I

1. HCF of two numbers is 27 and their LCM is 162.
Let the other number be x.
Product of two numbers = HCF × LCM = 27 × 162
$\Rightarrow$ 54x = 27 × 162
$\Rightarrow$ x = 81
$\Rightarrow$ The other number is 81.

OR

Let x = $0.\overline{8}$ … (i)
10x = $8.\overline{8}$ … (ii)
Subtracting (i) from (ii)
$\Rightarrow$ 9x = 8
$\Rightarrow$ x = $\frac{8}{9}$

2. Mean = 27, median = 33
Mode = 3median – 2mean
$\Rightarrow$Mode = 3 × 33 – 2 × 27
$\Rightarrow$Mode = 45

3. $\sqrt{\frac{1+\sin A}{1-\sin A}}$

$$= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \frac{1+\sin A}{\cos A} \qquad \because \sin^2 A + \cos^2 A = 1$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

4. Given: $\tan\theta = \frac{a}{b}$

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1 + \tan\theta}{1 - \tan\theta} == \frac{1 + \frac{a}{b}}{1 - \frac{a}{b}} = \frac{b + a}{b - a}$$

5. The distance between the points A(4, p) and B(1, 0) is 5.

$\Rightarrow$ AB = 5

$\Rightarrow AB^2 = 25$

$\Rightarrow (4 - 1)^2 + p^2 = 25$

$\Rightarrow 9 + p^2 = 25$

$\Rightarrow p^2 = 16$

$\Rightarrow p = \pm 4$

OR

The point on y-axis, below x-axis, at a distance of 4 units from x-axis is A(0, –4).

6. In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$

$\Rightarrow \Delta ABC \sim \Delta DEF$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AB)^2}{(DE)^2} = \frac{5^2}{7^2} = \frac{25}{49}$$

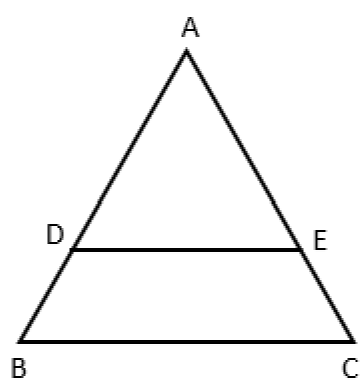
OR

DE || BC

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{4}{7} = \frac{6}{EC}$$

$\Rightarrow$ EC = 10.5 cm



7. For an event E, the value of P(E) + P(not E) = 1.

8. From the diagram,

In Δ PTO,
$PT^2 + TO^2 = PO^2$
$24^2 + 7^2 = PO^2$
$PO^2 = 576 + 49$
$PO^2 = 625$
PO = 25 cm

9. First n even natural numbers are 2, 4, 6,....2n
$a = 2, a_n = 2n$

$S_n = \frac{n}{2}(2 + 2n) = n(n+1)$

OR

$\sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

$\Rightarrow 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots$

$\Rightarrow a = 2\sqrt{2}, d = \sqrt{2}, n = 4$

$\Rightarrow a_3 = 4\sqrt{2}$

$\Rightarrow a_4 = a_3 + d = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$

10. $7x^2 - 12x + 18 = 0$
$\Rightarrow a = 7, b = -12, c = 18$
Let α, β be the roots of the equation

$\alpha + \beta = \frac{-b}{a} = \frac{12}{7}$

$\alpha\beta = \frac{c}{a} = \frac{18}{7}$

$\Rightarrow \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{12}{7}}{\frac{18}{7}} = \frac{12}{18} = \frac{2}{3}$

11. $3x + 5y = 0$ and $kx + 10y = 0$
Condition for the system of equations to have non-zero solutions,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$a_1 = 3, a_2 = k, b_1 = 5, b_2 = 10$

$\Rightarrow \frac{3}{k} = \frac{5}{10} \Rightarrow k = 6$

12. The volume of the given figure is $\frac{1}{3}\pi h\left(r^2 + R^2 + rR\right)$.

13. Let α and β are the roots of the given quadratic equation.

According to the question, $\alpha = 2$ ...(i)

Comparing $x^2 + 3x + k$ with $ax^2 + bx + c$, we get $a = 1, b = 3, c = k$

$\alpha + \beta = -3$ and $\alpha\beta = k$

From (i) $2 + \beta = -3 \Rightarrow \beta = -5$

$\alpha\beta = k \Rightarrow k = -10$

14. Product of zeroes of Cubic Polynomial = $\frac{-d}{a} = \frac{-9}{2}$

$\Rightarrow$Product of two zeroes × third zero = $\frac{-9}{2}$

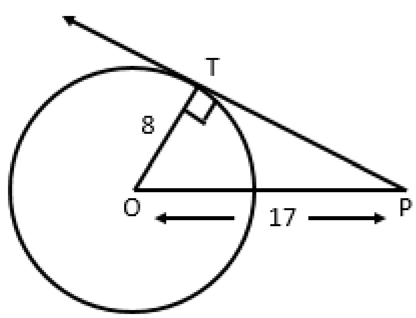
$\Rightarrow$ third zero = $\frac{-9}{2 \times 3} = \frac{-3}{2}$

15. The word is MOBILE.

The vowels in the word are O, I, E and the total number of words are 6.

The required probability = 3/6 = ½

16. In ΔOTP,



$OP^2 = OT^2 + TP^2$ $\quad \because$ OT is perpendicular to the tangent

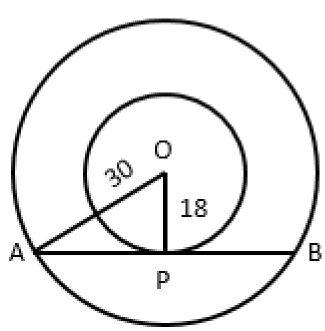
$\Rightarrow TP^2 = OP^2 - OT^2$

$\Rightarrow TP^2 = 17^2 - 8^2$

$\Rightarrow TP^2 = 289 - 64 = 225$

$\Rightarrow TP = 15$ cm

OR



Let O be the centre of the concentric circles of radii 30 cm and 18 cm respectively.
Let AB be a chord of the larger circle touching the smaller circle at P.
So, OP is perpendicular to AB as AP is tangent to the smaller circle at P.
Then AP = PB since OP is perpendicular to AB.
Using Pythagoras theorem in triangle OPA,
$\Rightarrow OA^2 = OP^2 + AP^2$
$\Rightarrow 30^2 = 18^2 + AP^2$
$\Rightarrow AP^2 = 576$
$\Rightarrow AP = 24$ cm
$\Rightarrow AB = 2AP = 48$ cm
Hence, the length of the chord is 48 cm.

Section II

17.

(a) Angles $\angle LKM$ and $\angle JKL$ are called as Linear Pair of angles.
(b) $m\angle LKM + m\angle JKL = 180°$…… Linear Pair
$\Longrightarrow 2x - 15 + m\angle LKM = 180°$
$\Longrightarrow m\angle LKM = 195° - 2x$
(c) In $\triangle LKM$,
$m\angle LKM + m\angle LMK + m\angle KLM = 180°$ …angle sum property of a triangle
$\Longrightarrow 195° - 2x + 50 + x = 180°$
$\Longrightarrow x = 65° = m\angle KLM$
(d) $m\angle LKM = 195° - 2x = 195 - 2(65) = 195 - 130 = 65°$
In $\triangle LKM$, $m\angle LKM = m\angle KLM = 65°$
$\Longrightarrow \triangle LKM$ is an isosceles triangle.
(e) $m\angle LKJ = 2x - 15 = 2(65) - 15 = 130 - 15 = 115°$

18.

(a) The coordinates of CAVE of DEATH is (5, 3).
(b) The coordinates of THREE PALMS is (6, 4).
(c) The coordinates FOUR CROSS CLIFF and CAVE of DEATH are (2, 3) and (5, 3) respectively.
Distance between them = $\sqrt{(5-2)^2 + (3-3)^2} = \sqrt{9} = 3$ units
(d) The distance of SKULL ROCK from x – axis is 5 units.
(e) The mid – point of CAVE of DEATH and THREE PALMS
$= \left(\frac{5+6}{2}, \frac{3+4}{2}\right) = (5.5, 3.5)$

19.

(a) $x^2 - 18x + 81 = x^2 - 9x - 9x + 81 = x(x - 9) - 9(x - 9) = 0$

$\Rightarrow (x - 9)(x - 9) = 0$

$\Rightarrow x = 9$ or $x = 9$

(b) Zeroes of a polynomial can be expressed graphically. Number of zeroes of a polynomial is equal to the number of points where the graph of polynomial **Intersects x – axis.**

Number of zeroes of the given polynomial(refer image) is equal to 3.

(c) The degree of a Quadratic Polynomial is 2.

(d) A highway underpass is parabolic in shape and a parabola is the graph that results from $p(x) = ax^2 + bx + c$ which has two zeroes (as it is a quadratic polynomial).

Product of zeroes = $6 \times 3 = 18$ and sum of the zeroes = $6 + 3 = 9$

x^2 – (sum of zeroes)x + product of zeroes

$= x^2 - 9x + 18$

(e) $f(m) = m^2 = m^2 + 0m + 0$ is a Quadratic Polynomial.

The number of zeroes that f(x) can have is 2.

20.

(a)

Time (in sec)	No. of students(f)	X	fx
20 – 40	7	30	210
40 – 60	10	50	500
60 – 80	15	70	1050
80 – 100	5	90	450
100 – 120	3	110	330
	$\Sigma f = 40$		$\Sigma fx = 2540$

Mean time taken by a student to finish the race = 2540/40 = 63.5 seconds

(b) The modal class is 60 – 80 as it has the highest frequency i.e 15.

Lower limit of the modal class = 60

(c) Mean, Median and Mode are measures of central tendency.

(d)

Time (in sec)	No. of students(f)	cf
20 – 40	7	7
40 – 60	10	17
60 – 80	15	32
80 – 100	5	37
100 – 120	3	40
	N= $\Sigma f = 40$	

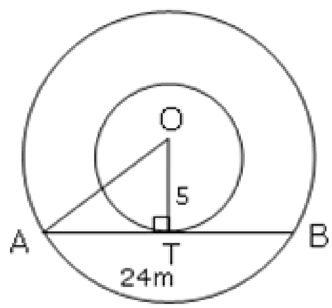
Here $N/2 = 40/2 = 20$, Median Class = 60 – 80, Modal Class = 60 – 80
Sum of upper limits of median class and modal class = 80 + 80 = 160

(e) Number of students who finished the race within 1 min = 7 + 10 = 17

Part B

Section III

21. Let O be the centre of concentric circles.
AB be the chord of larger circle and OT be the radius of smaller circle.



So $OT \perp AB$ since tangent is $\perp$ to radius at its point of contact.
AT = TB = 12 m
(Since perpendicular from centre to the chord bisects it)
So, in ΔOAT,
$OA^2 = OT^2 + AT^2$
$OA^2 = 5^2 + 12^2 = 169$
$\Rightarrow OA = 13$ cm
Thus, the radius of the larger circle is 13 cm.

22. Let n be the required number of spheres.
Since, the spheres are melted to form a cylinder.
So, the volume of all the n spheres will be equal to the volume of the cylinder.

$$n \times \frac{4}{3} \times \pi \times 3 \times 3 \times 3 = \pi \times 2 \times 2 \times 45$$

$\therefore n = 5$
Thus, the required number of spheres which are melted to form the cylinder is 5.

OR

Let x cm be the edge of the new cube.
Volume of the new cube = Sum of the volumes of three cubes
$x^3 = 3^3 + 4^3 + 5^3$
$x^3 = 27 + 64 + 125$
$x^3 = 216$
$x = 6$.
Edge of the new cube is 6 cm long.

23. Given sides of a triangle are 9 cm, 18 cm, and 16 cm.

Consider, $9^2 + 16^2 = 81 + 256 = 337 \neq 18^2$.

Hence, these sides cannot form right triangle.

OR

The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

Let a be the area of smaller triangle and A be the area of the larger triangle.

$$\frac{a}{A} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$\Rightarrow \frac{48}{A} = \frac{4}{9}$ $\qquad$ $\because$ Area of smaller triangle = 48 cm^2

$\Rightarrow$ A = 108 cm^2

24. There are 10 ribs in an umbrella.

The area between two consecutive ribs subtends an angle of $\frac{360°}{10} = 36°$ at the centre of the assumed flat circle.

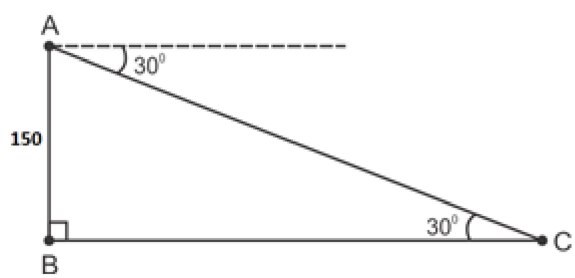
Area between two consecutive ribs of circle $= \frac{36°}{360°} \times \pi r^2$

$$= \frac{36°}{360°} \times \frac{22}{7} \times (40)^2$$

$$= \frac{1}{10} \times \frac{22}{7} \times 40 \times 40$$

$$= 502.86 \text{ cm}^2$$

25. Let AB be the tower and BC be distance between tower and car.



In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{150}{BC}$$

$$\therefore BC = 150\sqrt{3} \text{ m}$$

Hence, distance between the tower and car is $150\sqrt{3}$ m.

26. According to the question,

$n(S) = 20$

1. Let A be an event of getting a number divisible by 2 and 3.

$\therefore A = \{6, 12, 18\}$

$\therefore n(A) = 3$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{20}$

2. Let B be an event of getting a prime number.

$\therefore B = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$\therefore n(B) = 8$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{8}{20} = \frac{2}{5}$

Section IV

27. $(a - b)x^2 + (b - c)x + (c - a) = 0$

The given equation will have equal roots, if

$(b - c)^2 - 4(a - b)(c - a) = 0$

$b^2 + c^2 - 2bc - 4(ac - bc - a^2 + ab) = 0$

$b^2 + c^2 + 4a^2 + 2bc - 4ab - 4ac = 0$

$(b + c - 2a)^2 = 0$

$b + c - 2a = 0$

$b + c = 2a$

28. $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots\tan 89^\circ$

$= \tan(90^\circ - 89^\circ) \tan(90^\circ - 88^\circ) \tan(90^\circ - 87^\circ)\ldots\ldots\tan 87^\circ \tan 88^\circ \tan 89^\circ$

$= \cot 89^\circ \cot 88^\circ \cot 87^\circ\ldots\ldots\tan 87^\circ \tan 88^\circ \tan 89^\circ$

$= \cot 89^\circ \tan 89^\circ \cot 88^\circ \tan 88^\circ \cot 87^\circ \tan 87^\circ\ldots\cot 44^\circ \tan 44^\circ \tan 45^\circ$

$= 1 \times 1 \times 1 \ldots \times 1 = 1$

$\Rightarrow \tan 1^\circ \tan 2^\circ \tan 3^\circ\ldots\ldots\tan 89^\circ = 1$

29. Let $6 + \sqrt{3}$ be rational and equal to $\frac{a}{b}$.

Then, $\frac{6 + \sqrt{3}}{1} = \frac{a}{b}$, where a and b are co primes, $b \neq 0$

$\therefore \sqrt{3} = \frac{a}{b} - 6 = \frac{a - 6b}{b}$

Here a and b are integers.

So, $\frac{a - 6b}{b}$ is rational.

Therefore, $\sqrt{3}$ is rational.

This is a contradiction as $\sqrt{3}$ is irrational.
Hence, our assumption is wrong.
Thus, 6 + $\sqrt{3}$ is an irrational number.

OR

Let us assume, on the contrary that $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b (b ≠ 0) such that $\sqrt{5} = \frac{a}{b}$

Where a and b are co-prime integers.

$\sqrt{5} = \frac{a}{b} \Rightarrow a = \sqrt{5}b \Rightarrow a^2 = 5b^2$

Therefore, a^2 is divisible by 5 then a is also divisible by 5.
So a = 5k, for some integer k.

$\text{Now}, a^2 = (5k)^2 = 5(5k^2) = 5b^2$

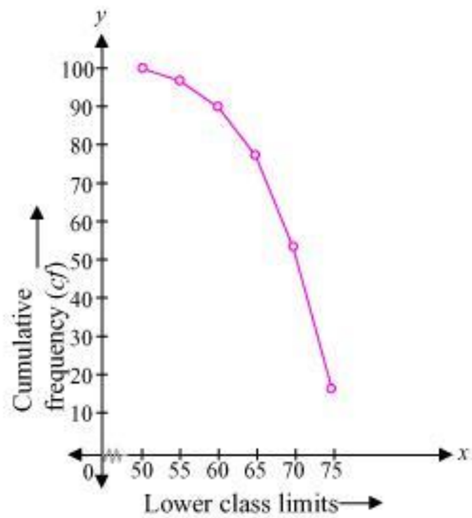
$\Rightarrow b^2 = 5k^2$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.
This implies that a and b have 5 as a common factor.
And this is a contradiction to the fact that a and b are co-prime.
So our assumption that $\sqrt{5}$ is rational is wrong.
Hence, $\sqrt{5}$ cannot be a rational number. Therefore, $\sqrt{5}$ is irrational.

30. We can obtain cumulative frequency distribution of more than type as following:

Production yield (lower class limits)	Cumulative frequency
More than or equal to 50	100
More than or equal to 55	100 - 2 = 98
More than or equal to 60	98 - 8 = 90
More than or equal to 65	90 - 12 = 78
More than or equal to 70	78 - 24 = 54
More than or equal to 75	54 - 38 = 16

Now, taking lower class limits on x-axis and their respective cumulative frequencies on y-axis, we can obtain the ogive as follows:



31.

$$\text{L.H.S.} = \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = \frac{\left(\sqrt{\sec\theta - 1}\right)^2 + \left(\sqrt{\sec\theta + 1}\right)^2}{\left(\sqrt{\sec\theta + 1}\right)\left(\sqrt{\sec\theta - 1}\right)}$$

$$= \frac{\sec\theta - 1 + \sec\theta + 1}{\sqrt{\sec^2\theta - 1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta}$$

$$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta = \text{R.H.S.}$$

OR

$$\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$$

$$= \frac{\sin(90^\circ\text{-}70^\circ)}{\sin 20^\circ} + \frac{\sin(90^\circ\text{-}59^\circ)}{\sin 31^\circ} - 8\left(\frac{1}{2}\right)^2$$

$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 8 \times \frac{1}{4} = 1 + 1 - 2 = 0$$

Since, $\cos\theta = \sin(90^\circ - \theta)$

32. Let a − d, a and a + d be three terms in A.P.

According to the question,

$a - d + a + a + d = 3$

$3a = 3 \Rightarrow a = 1$

$(a - d)(a)(a + d) = -8$

$a(a^2 - d^2) = -8$

Putting the value of a = 1, we get,

$1 - d^2 = -8 \Longrightarrow d^2 = 9$ or $d = \pm 3$

Thus, the required three terms are −2, 1, 4 or 4, 1, −2

33.

CI	50-60	60-70	70-80	80-90	90-100	100-110	Total
f_i	5	3	4	p	2	13	27 + p
x_i	55	65	75	85	95	105	
$f_i x_i$	275	195	300	85p	190	1365	2325+ 85p

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\Rightarrow 86 = \frac{2325 + 85p}{27 + p}$$

$$\Rightarrow 86p + 2322 = 2325 + 85p$$

$$\Rightarrow p = 3$$

Section V

34. Steps of construction :

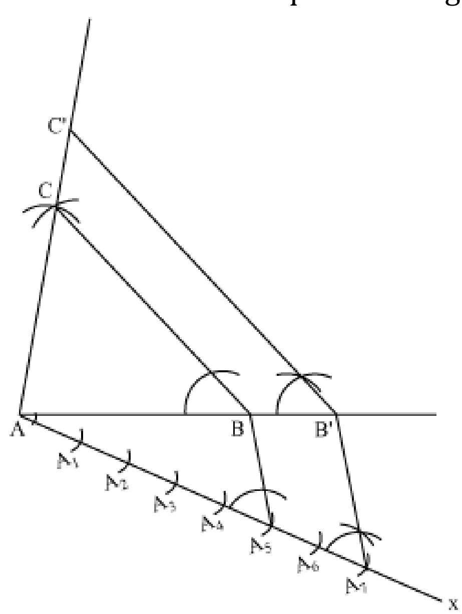
i. Draw a line segment AB of 5 cm. Taking A and B as centres, draw two arcs of 6 cm and 7 cm radius respectively. Let these arcs intersect each other at point C. ΔABC is the required triangle having lengths of sides as 5 cm, 6 cm and 7 cm respectively.

ii. Draw a ray AX making acute angle with the line AB on opposite side of vertex C.

iii. Locate 7 points A_1, A_2, A_3, A_4, A_5, A_6, A_7 (as 7 is greater between 5 and 7) on line AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

iv. Join BA_5 and draw a line through A_7 parallel to BA_5 to intersect extended line segment AB at point B'.

v. Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. ΔAB'C' is the required triangle.

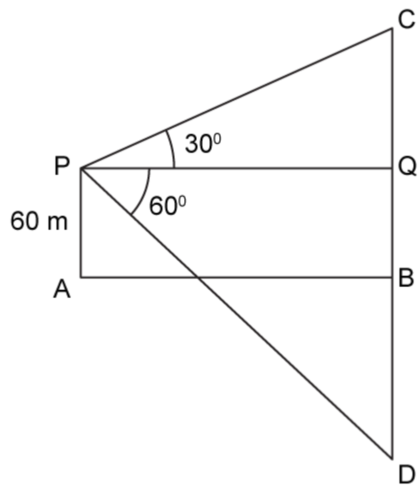


35. Let C be the cloud and D be its reflection. Let the height of the cloud be H metres.

BC = BD = H

BQ = AP = 60 m.

Therefore CQ = H − 60 and DQ = H + 60



In ΔCQP,

$$\frac{PQ}{CQ} = \cot 30°$$

$$\Rightarrow \frac{PQ}{H-60} = \sqrt{3}$$

$\Rightarrow$ PQ = $(H - 60)\sqrt{3}$ m ....(i)

In ΔDQP,

$$\frac{PQ}{DQ} = \cot 60°$$

$$\Rightarrow \frac{PQ}{H+60} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow PQ = \frac{(H+60)}{\sqrt{3}} \quad ....(ii)$$

From (i) and (ii),

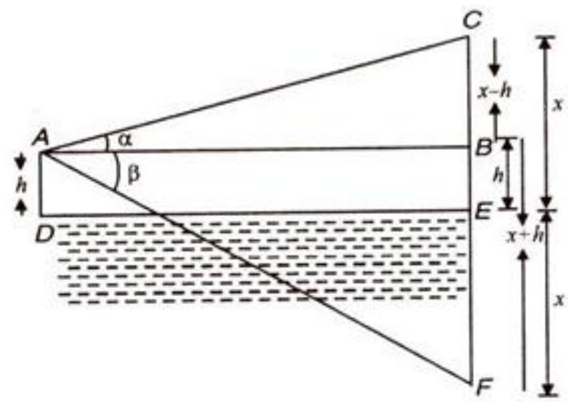
$$(H-60)\sqrt{3} = \frac{(H+60)}{\sqrt{3}}$$

$\Rightarrow$ 3H − 180 = H + 60

$\Rightarrow$ H = 120

Thus, the height of the cloud is 120 m.

OR



Let height of the cloud above water level be = x

Then, BC = x – h and BF = x + h

$\frac{BC}{AB} = \tan\alpha$

$\Rightarrow \quad x - h = AB \tan\alpha \quad \ldots(i)$

and $\quad x + h = AB \tan\beta \quad \ldots(ii)$

Dividing (i) by (ii), $\frac{x-h}{x+h} = \frac{\tan\alpha}{\tan\beta}$,

use Componendo – Dividendo to get x

$$\frac{x - h + x + h}{x - h - x - h} = \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta}$$

$$\frac{2x}{-2h} = \frac{(\tan\beta + \tan\alpha)}{-(\tan\beta - \tan\alpha)} \text{ or } x = h\frac{(\tan\beta + \tan\alpha)}{(\tan\beta - \tan\alpha)}$$

36. Diameter of graphite = 1 mm = 0.1 cm

Therefore, radius of graphite = $\frac{0.1}{2}$ = 0.05 cm

Length of pencil = 10 cm

Volume of graphite = $\pi r^2 h = \frac{22}{7} \times (.05)^2 \times 10 = 0.0785 \text{ cm}^3$

Therefore, weight of graphite = volume × density = 0.0785 × 2.3 = 0.180 gm

Diameter of the pencil = 0.7 cm

Therefore, radius of the pencil = 0.35 cm

Therefore, volume of the pencil = $\pi R^2 h = \frac{22}{7} \times (0.35)^2 \times 10 = 3.85 \text{ cm}^3$

Therefore, volume of wood = Volume of pencil – Volume of graphite

= $(3.85 – 0.0785) \text{ cm}^3$

= 3.771 cm^3

Weight of wood = Volume × density = 3.771 × 0.6 = 2.2626 gm