

MATHEMATICS

Textbook for Class XI



राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

ISBN 81-7450-486-9

First Edition

February 2006 Phalguna 1927

Reprinted

October 2006 Kartika 1928

November 2007 Kartika 1929

December 2008 Pausa 1930

December 2009 Agrahayana 1931

January 2011 Pausa 1932

February 2012 Magha 1933

December 2012 Pausa 1934

November 2013 Kartika 1935

December 2014 Pausa 1936

May 2016 Vaishakha 1938

December 2016 Pausa 1938

December 2017 Agrahayana 1939

PD 400T BS

© **National Council of Educational
Research and Training, 2006**

₹ 180.00

Printed on 80 GSM paper with NCERT
watermark

Published at the Publication Division by
the Secretary, National Council of
Educational Research and Training, Sri
Aurobindo Marg, New Delhi 110 016 and
printed at Pankaj Printing Press, D-28,
Industrial Area, Site-A, Mathura -
281 001 (Uttar Pradesh)

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Foreword

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the Textbook Development Committee responsible for this

book. We wish to thank the Chairperson of the advisory group in Science and Mathematics, Professor J.V. Narlikar and the Chief Advisor for this book Professor P.K. Jain for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time and contribution. As an organisation committed to the systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

New Delhi
20 December 2005

Director
National Council of Educational
Research and Training

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Acknowledgements

The Council gratefully acknowledges the valuable contributions of the following participants of the Textbook Review Workshop: P. Bhaskar Kumar, *P.G.T.*, Jawahar Navodaya Vidyalaya, Ananthpur, (A.P.); Vinayak Bujade, *Lecturer*, Vidarbha Buniyadi Junior College, Sakardara Chowk Nagpur, Maharashtra; Vandita Kalra, *Lecturer*, Sarvodaya Kanya Vidyalaya Vikashpuri District Centre, New Delhi; P.L. Sachdeva, Deptt. of Mathematics, Indian Institute of Science, Bangalore, Karnataka; P.K. Tiwari, *Assistant Commissioner (Retd.)*, Kendriya Vidyalaya Sangathan; Jagdish Saran, Department of Statistics, University of Delhi; Quddus Khan, *Lecturer*, Shibli National P.G. College Azamgarh (U.P.); Sumat Kumar Jain, *Lecturer*, K.L. Jain Inter College Sasni Hathras (U.P.); R.P. Gihare, *Lecturer* (BRC), Janpad Shiksha Kendra Chicholi Distt. Betul (M.P.); Sangeeta Arora, *P.G.T.*, A.P.J. School Saket, New Delhi; P.N. Malhotra, *ADE* (Sc.), Directorate of Education, Delhi; D.R. Sharma, *P.G.T.*, J.N.V. Mungespur, Delhi; Saroj, *P.G.T.* Government Girls Sr. Secondary School, No. 1, Roop Nagar, Delhi; Manoj Kumar Thakur, *P.G.T.*, D.A.V. Public School, Rajender Nagar, Sahibabad, Ghaziabad (U.P.) and R.P. Maurya, *Reader*, DESM, NCERT, New Delhi.

Acknowledgements are due to Professor M. Chandra, *Head*, Department of Education in Science and Mathematics for her support.

The Council acknowledges the efforts of the Computer Incharge, Deepak Kapoor; Rakesh Kumar, Kamlesh Rao and Sajjad Haider Ansari, D.T.P. Operators; Kushal Pal Singh Yadav, Copy Editor and Proof Readers, Mukhtar Hussain and Kanwar Singh.

The contribution of APC–Office, administration of DESM and Publication Department is also duly acknowledged.

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