

Maharashtra Board
Class X Mathematics - Algebra
Board Paper – 2014

Time: 2 hours

Total Marks: 40

Note:- (1) All questions are compulsory.
(2) Use of calculator is not allowed.

1. Attempt any five question from the following:

5

- i. For an A.P. $t_3 = 8$ and $t_4 = 12$, find the common difference d .
- ii. $(x + 5)(x - 2) = 0$, find the roots of this quadratic equation.
- iii. The following data shows the number of students using different modes of transport:

Modes of Transport	Number of Students
Bicycle	140
Bus	100
Walk	70
Train	40
Car	10

From this table, find the central angle (θ) for the Mode of Transport 'Bus'.

- iv. 'A coin is tossed'. Write the sample space 'S'.
- v. If $\sum f_i x_i = 75$ and $\sum f_i = 15$, then find the mean \bar{x} .
- vi. Write the following quadratic equation in a standard form:
 $3x^2 = 10x + 7$.

2. Attempt any four subquestions from the following:**8**

- i. State whether the following sequence is an AP or not:
1, 3, 6, 10.....
- ii. Solve the following quadratic equation by factorization method:
 $9x^2 - 25 = 0$
- iii. If the point (3, 2) lies on the graph of the equation $5x + ay = 19$, then find a.
- iv. If $12x + 13y = 29$ and $13x + 12y = 21$, find $x + y$.
- v. A die is thrown. Write the sample space (S) and number of sample points $n(S)$ and also write event A of getting even number on the upper surface and write $n(A)$.
- vi. For a certain frequency distribution, the value of mean is 20 and mode is 11. Find the value of median.

3. Attempt any three of the following subquestions:**9**

- i. Solve the equation by using the formula method.
 $3y^2 + 7y + 4 = 0$
- ii. Solve the following simultaneous equations by using Cramers's rule:
 $3x - y = 7$
 $x + 4y = 11$
- iii. Two coins are tossed simultaneously. Write the sample space 'S' and the number of sample points $n(S)$. Write the following events using set notation and mention the number of elements in each of them:
(a) A is the event of getting at least one head.
(b) B is the event of getting exactly one head.
- iv. The following table gives the frequency distribution of trees planted by different Housing Societies in a particular locality:

No. of Trees	No. of Housing Societies
10-15	2
15-20	7
20-25	9
25-30	8
30-35	6
35-40	4

Find the mean number of trees planted by Housing Societies by using 'Assumed Means Method'

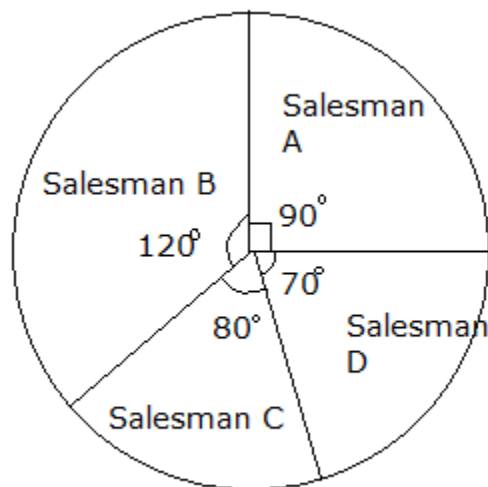
v. Represent the following data by Histogram:

Price of Sugar per kg (in Rs.)	Number of Weeks
18-20	4
20-22	8
22-24	22
24-26	12
26-28	8
28-30	6

4. Attempt any two sub-questions from the following:

8

- A farmer borrows Rs.1,000 and agrees to repay with a total interest of Rs. 140 in 12 installments, each installment being less than the preceding installment by Rs. 10. What should be his first installment?
- There are three boys and two girls. A committee of two is to be formed. Find the probability of events that the committee contains:
 - At least one girl.
 - One boy and one girl
 - Only boys.
- The sales of salesmen in a week are given in the pie diagram. Study the diagram and answer the following questions. If the total sale due to salesmen A is Rs. 18,000, then



- a) Find the total sale.
- b) Find the sale of each salesman.
- c) Find the salesman with the highest sale.
- d) Find the difference between the highest sale and the lowest sale.

5. Attempt any two of the following subquestions:

10

- i. If m times m th term of an A.P. is equal to n times its n th term, then show that $(m + n)$ th term of the A.P. is zero.
- ii. The product of four consecutive natural numbers, which are multiples of five, is Rs. 15,000. Find those natural numbers.
- iii. Draw the graphs representing the equations $4x + 3y = 24$ and $3y = 4x + 24$ on the same graph paper. Write the co-ordinates of the point of intersection of these lines and find the area of triangle formed by these lines and the X-axis.

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Note: - (1) All questions are compulsory.
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1.

i) Given:

$$t_3=8 \text{ and } t_4=12$$

$$\text{Since, } t_n=a+(n-1)d$$

$$t_3=a+(3-1)d$$

$$8=a+2d \dots (i)$$

$$t_4=a+(4-1)d$$

$$12=a+3d \dots (ii)$$

Subtracting (i) from (ii), we get

$$d=4$$

The common difference is 4.

$$ii) (x+5)(x-2)=0$$

$$x+5=0 \text{ and } x-2=0$$

$$x=-5 \text{ and } x=2$$

-5 and 2 are the roots of the quadratic equation $(x+5)(x-2)=0$.

$$iii) \text{ Total number of students} = 140 + 100 + 70 + 40 + 10 = 360$$

Central angle (θ) for the mode of Transport 'Bus' =

$$\frac{\text{Number of students using Bus}}{\text{Total number of students}} \times 360^\circ = \frac{100}{360} \times 360^\circ$$
$$= 100^\circ$$

$$\text{Central angle } (\theta) = 100^\circ$$

$$iv) \text{ Sample Space } S = \{H, T\}$$

$$n(S) = 2$$

$$v) \text{ Mean } = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{75}{15} = 5$$

$$\text{Mean} = \bar{x} = 5$$

$$vi) 3x^2 - 10x - 7 = 0 \text{ is in the standard form.}$$

2.

i) The given sequence is 1, 3, 6, 10,

Here $t_1 = 1$, $t_2 = 3$, $t_3 = 6$, $t_4 = 10$

Then,

$$t_2 - t_1 = 3 - 1 = 2$$

$$t_3 - t_2 = 6 - 3 = 3$$

$$t_4 - t_3 = 10 - 6 = 4$$

$$t_2 - t_1 \neq t_3 - t_2 \neq t_4 - t_3$$

Since the difference between two consecutive terms is not constant.

Therefore the given sequence is not an A.P.

ii) $9x^2 - 25 = 0$

$$(3x)^2 - (5)^2 = 0$$

$$(3x-5)(3x+5) = 0 \dots\dots [a^2 - b^2 = (a-b)(a+b)]$$

$$(3x-5) = 0 \text{ or } (3x+5) = 0$$

$$3x = 5 \text{ or } 3x = -5$$

$$x = \frac{5}{3} \text{ or } x = -\frac{5}{3}$$

$\therefore \left\{ \frac{5}{3}, -\frac{5}{3} \right\}$ is the solution set of the given equation.

iii) Given:

$$5x + ay = 19$$

$$(x, y) = (3, 2)$$

The point (x, y) lies on the graph of the equation; hence it satisfies the equation.

Substitute $x = 3$ and $y = 2$ in the given equation,

We get

$$5(3) + a(2) = 19$$

$$\Rightarrow 15 + 2a = 19$$

$$\Rightarrow 2a = 19 - 15$$

$$\Rightarrow 2a = 4$$

$$\Rightarrow a = \frac{4}{2} = 2$$

Therefore the value of a is 2.

iv) The given equations are

$$12x + 13y = 29 \dots (i)$$

$$13x + 12y = 21 \dots (ii)$$

Add (i) and (ii), we get

$$12x + 13y = 29$$

$$\underline{13x + 12y = 21}$$

$$25x + 25y = 50$$

$$25(x+y) = 50$$

$$x + y = 2$$

v) The sample space (S) is

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$\text{No. of sample points} = n(S) = 6$$

Let A be the event of getting an even number.

$$A = \{2, 4, 6\}$$

$$\Rightarrow n(A) = 3$$

vi) The inter-relation between the measures of central tendency is given by

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

$$20 - 11 = 3(20 - \text{Median})$$

$$9 = 3(20 - \text{Median})$$

$$\frac{9}{3} = 20 - \text{Median}$$

$$3 = 20 - \text{Median}$$

$$\text{Median} = 20 - 3$$

$$\text{Median} = 17$$

3.

i) The given quadratic equation is $3y^2 + 7y + 4 = 0$.

Comparing the given equation with $ax^2 + bx + c = 0$ we get,

$a = 3$, $b = 7$ and $c = 4$.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(7) \pm \sqrt{(7)^2 - 4(3)(4)}}{2(3)}$$

$$y = \frac{-(7) \pm \sqrt{49 - 48}}{6}$$

$$y = \frac{-(7) \pm \sqrt{1}}{6}$$

$$y = \frac{-(7) \pm 1}{6}$$

$$y = \frac{-(7)+1}{6} \quad \text{or} \quad y = \frac{-(7)-1}{6}$$

$$y = \frac{-6}{6} = -1 \quad \text{or} \quad y = \frac{-8}{6} = \frac{-4}{3}$$

$$y = -1 \quad \text{or} \quad y = -\frac{4}{3}$$

Therefore -1 and $-\frac{4}{3}$ are the roots of given equation.

ii) The given equations are

$$3x - y = 7 \dots (i)$$

$$x + 4y = 11 \dots (ii)$$

Equation (i) and (ii) are in standard form.

$$D = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = (3 \times 4) - (-1 \times 1) = 12 + 1 = 13 \neq 0$$

$$D_x = \begin{vmatrix} 7 & -1 \\ 11 & 4 \end{vmatrix} = (7 \times 4) - (-1 \times 11) = 28 + 11 = 39$$

$$D_y = \begin{vmatrix} 3 & 7 \\ 1 & 11 \end{vmatrix} = (3 \times 11) - (7 \times 1) = 33 - 7 = 26$$

By Cramer's rule, we get

$$x = \frac{D_x}{D} \quad \text{and} \quad y = \frac{D_y}{D}$$

$$x = \frac{39}{13} \quad \text{and} \quad y = \frac{26}{13}$$

$$x = 3 \quad \text{and} \quad y = 2$$

$$(x, y) = (3, 2)$$

$x = 3$ and $y = 2$ is the solution to the given equation.

iii)

The sample space (S) is

$S = \{TT, HT, TH, HH\}$

$n(s) = 4$

a) Let A be the event of getting at least one head.

$A = \{HT, HH, TH\}$

$n(A) = 3$

b) Let B be the event of getting exactly one head.

$B = \{HT, TH\}$

$n(B) = 2$

iv) By Assumed Mean Method

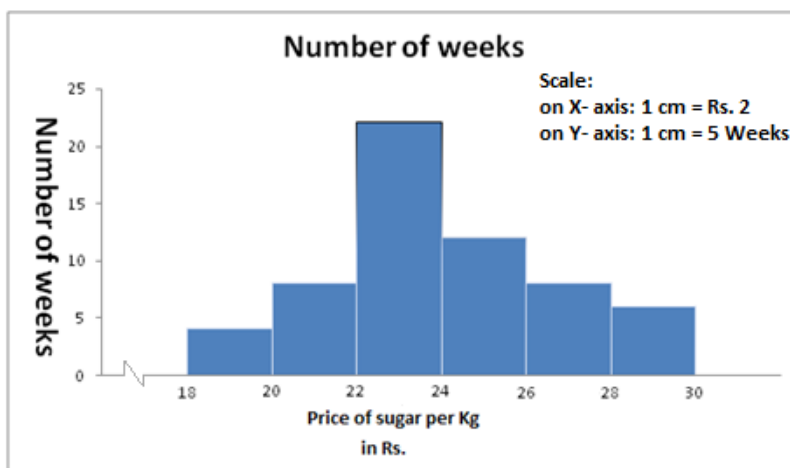
No. of trees	Class mark x_i	$d_i = x_i - A$	No of housing societies(f_i)	$f_i d_i$
10-15	12.5	-10	2	-20
15-20	17.5	-5	7	-35
20-25	22.5 $\rightarrow A$	0	9	0
25-30	27.5	5	8	40
30-35	32.5	10	6	60
35-40	37.5	15	4	60
Total	-	-	$\sum f_i = 36$	$\sum f_i d_i = 105$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{105}{36} = 2.916$$

$$\text{Mean} = \bar{x} = A + \bar{d} = 22.5 + 2.916 = 25.42$$

Therefore the mean number of trees planted by Housing Societies is 25.42.

v)



4.

- i) As each installment being less than the preceding installment by Rs. 10 the installments are in A.P.

$$S_{12} = 1000 + 140 = 1140$$

$$n = 12, d = -10$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2a + (12-1)(-10)]$$

$$1140 = 6[2a + (11)(-10)]$$

$$1140 = 6[2a - 110]$$

$$\frac{1140}{6} = [2a - 110]$$

$$190 = 2a - 110$$

$$2a = 190 + 110$$

$$2a = 300$$

$$a = \frac{300}{2}$$

$$a = 150$$

The first installment = Rs. 150.

- ii) Let the three boys be b_1, b_2, b_3 and the three girls be g_1 and g_2 .

The Sample space(S) is

$$S = \{ b_1 b_2, b_3 b_1, b_2 b_3, b_1 g_1, b_1 g_2, b_2 g_1, b_2 g_2, b_3 g_1, b_3 g_2, g_1 g_2 \}$$

$$\Rightarrow n(S)=10$$

- a) Let B be event that the committee contains only one girls.

$$B = \{ b_1 g_1, b_1 g_2, b_2 g_1, b_2 g_2, b_3 g_1, b_3 g_2, g_1 g_2 \}$$

$$\Rightarrow n(B)=7$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{10}$$

- b) Let C be the event that the committee contains one boy and one girl.

$$C = \{ b_1 g_1, b_1 g_2, b_2 g_1, b_2 g_2, b_3 g_1, b_3 g_2 \}$$

$$\Rightarrow n(C)=6$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{10}$$

- c) Let D be the event that the committee contains only boys.

$$D = \{ b_1 b_2, b_3 b_1, b_2 b_3 \}$$

$$\Rightarrow n(D)=3$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{3}{10}$$

iii) Given: Sales of salesman A = Rs. 18000

a) Sales of salesman A = Rs. 18000

$$\text{Sales of salesman A} = \frac{\text{Central angle}}{360^\circ} \times \text{Total sales}$$

$$18000 = \frac{90}{360} \times \text{Total sales}$$

$$\text{Total sales} = 18000 \times 4 = \text{Rs. } 72000$$

b) Sales of salesman B = $\frac{\text{Central angle}}{360^\circ} \times \text{Total sales}$

$$\text{Sales of salesman B} = \frac{120}{360} \times 72000$$

$$\text{Sales of salesman B} = \text{Rs. } 24000$$

$$\text{Sales of salesman C} = \frac{\text{Central angle}}{360^\circ} \times \text{Total sales}$$

$$\text{Sales of salesman C} = \frac{80}{360} \times 72000$$

$$\text{Sales of salesman C} = \text{Rs. } 16000$$

$$\text{Sales of salesman D} = \frac{\text{Central angle}}{360^\circ} \times \text{Total sales}$$

$$\text{Sales of salesman D} = \frac{70}{360} \times 72000$$

$$\text{Sales of salesman D} = \text{Rs. } 14000$$

c) Salesman B is the salesman with the highest sale.

d) Difference between the highest sale and the lowest sale

$$= \text{Sales of salesman B} - \text{Sales of salesman D}$$

$$= \text{Rs. } 24000 - \text{Rs. } 14000$$

$$= \text{Rs. } 10000$$

5.

i) Given

$$t_m = [a + (m-1)d]$$

$$t_n = [a + (n-1)d]$$

$$m(t_m) = n(t_n)$$

$$m[a + (m-1)d] = n[a + (n-1)d]$$

$$\Rightarrow m[a + md - d] = n[a + nd - d]$$

$$\Rightarrow am + m^2d - md = an + n^2d - nd$$

$$\Rightarrow am + m^2d - md - an - n^2d + nd = 0$$

$$\Rightarrow am - an + m^2d - n^2d - md + nd = 0$$

$$\Rightarrow a(m-n) + d(m^2 - n^2) - d(m-n) = 0$$

$$\Rightarrow (m-n)[a + d(m+n) - d] = 0$$

... [Divide by (m-n)]

$$\Rightarrow [a + d(m+n) - d] = 0$$

$$\Rightarrow a + (m+n-1)d = 0$$

$$\Rightarrow t_{m+n} = 0$$

- ii) Let $5x$, $5(x+1)$, $5(x+2)$ and $5(x+3)$ be four consecutive natural numbers, which are multiple of fives.

It is given that product of these consecutive numbers is 15000.

$$5x \times 5(x+1) \times 5(x+2) \times 5(x+3) = 15000$$

$$625 \times x(x+1)(x+2)(x+3) = 15000$$

$$x(x+1)(x+2)(x+3) = \frac{15000}{625}$$

$$x(x+1)(x+2)(x+3) = 24$$

When $x = 1$

Then,

$$x(x+1)(x+2)(x+3)$$

$$1(1+1)(1+2)(1+3)$$

$$1(2)(3)(4) = 24$$

Hence, the four consecutive natural numbers are

$$5x = 5 \times 1 = 5$$

$$5(x+1) = 5 \times (1+1) = 5 \times 2 = 10$$

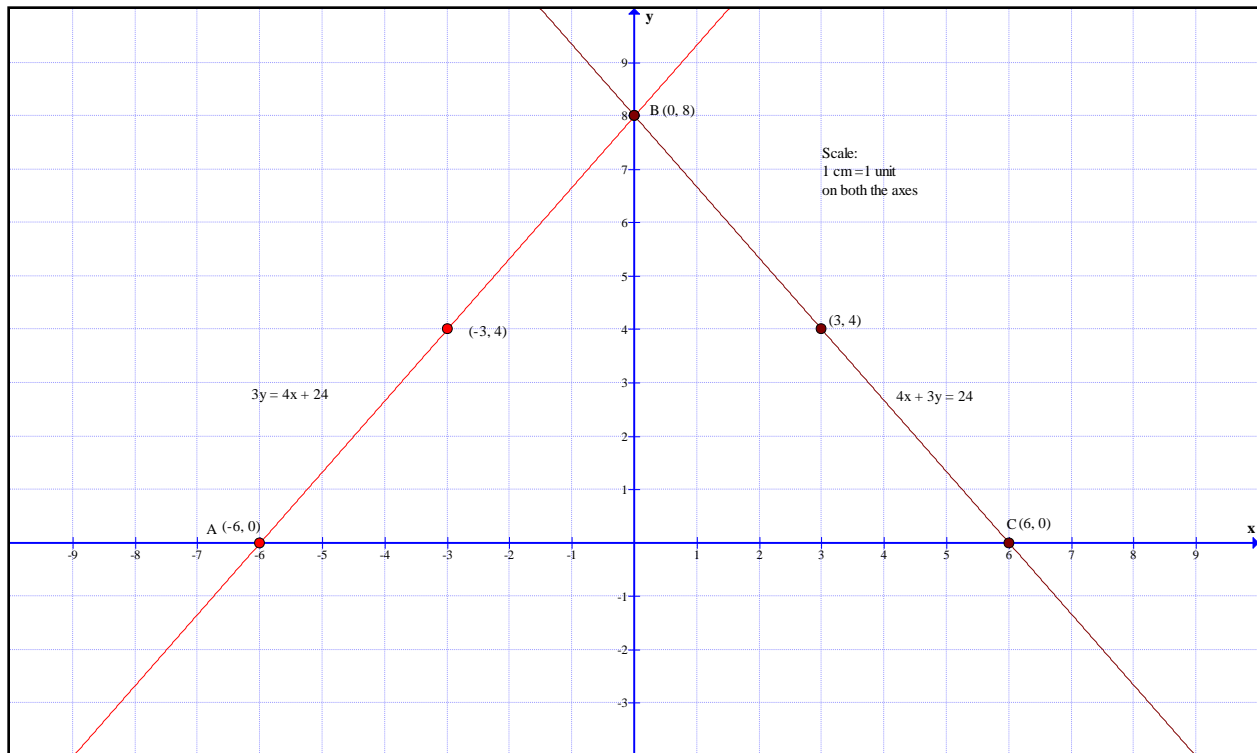
$$5(x+2) = 5 \times (1+2) = 5 \times 3 = 15$$

$$5(x+3) = 5 \times (1+3) = 5 \times 4 = 20$$

Therefore four consecutive natural numbers are 5, 10, 15 and 20.

iii) The given simultaneous equations are $4x + 3y = 24$ and $4x - 3y = -24$.

$4x + 3y = 24$... (i) $y = \frac{24 - 4x}{3}$				$4x - 3y = -24$... (ii) $y = \frac{4x + 24}{3}$			
x	0	3	6	x	0	-3	-6
y	8	4	0	y	8	4	0
(x,y)	(0, 8)	(3, 4)	(6, 0)	(x,y)	(2, 8)	(-3, 4)	(-6, 0)



From the graph A (-6, 0), B(0, 8) C (6, 0) and AC = 12 units.

Height = h = 8 units

Base = b = 12 units

Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$A(\Delta ABC) = \frac{1}{2} \times 12 \times 8$

$A(\Delta ABC) = 48$ sq units