# HOTS (Higher Order Thinking Skills)

**Que 1. If $\alpha, \beta, \gamma$ be Zeros of polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.**

**Sol.** P (x) = $6x^3 + 3x^2 - 5x + 1$ so a = 6, b = 3, c = – 5, d = 1

$\because$ $\alpha, \beta$ *and* $\gamma$ are zeros of the polynomial p (x).

$\therefore$ $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-3}{6} = \frac{-1}{2}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = \frac{-5}{6}$ and $\alpha\beta\gamma = \frac{-d}{a} = \frac{-1}{6}$

Now $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{-5/6}{-1/6} = 5$

**Que 2. Find the zeros of the polynomial $f(x) = x^3 - 12x^2 + 39x - 28$, if it is given that the zeros are in A.P.**

**Sol.** If $\alpha, \beta, \gamma$ are in A.P., then,

$\beta - a = \gamma - \beta \quad \Rightarrow \quad 2\beta = \alpha + \gamma$ ….(i)

$\alpha + \beta + y = \frac{-b}{a} = \frac{-(-12)}{1} = 12 \quad \Rightarrow \quad \alpha + \gamma = 12 - \beta$ ….(ii)

From (i) and (ii)

Putting the value of $\beta$ in (i), we have

8 = $\alpha + \gamma$ ….(iii)

$\alpha\beta\gamma = -\frac{d}{a} = \frac{-(-28)}{1} = 28$

$(\alpha\,\gamma)\,4 = 28 \quad or \quad \alpha\,\gamma = 7 \quad or = \frac{7}{\alpha}$ ….(iv)

Putting the value of $\gamma = \frac{7}{\alpha} in\ (iii)$, we get

$8 = \alpha + \frac{7}{\alpha} \quad \Rightarrow \quad 8a = \alpha^2 + 7$

$\Rightarrow \quad \alpha^2 - 8a + 7 = 0 \quad \Rightarrow \quad \alpha^2 - 7\alpha - 1\alpha + 7 = 0$

$\Rightarrow \quad \alpha(\alpha - 7) - 1(\alpha - 7) = 0 \quad \Rightarrow \quad (\alpha - 1)(a - 7) = 0$

$\Rightarrow$ $\alpha$ = 1 or $\alpha$ = 7

Putting $\alpha = 1$ in $(iv)$, we get

$\gamma = \frac{7}{1}$

or $\gamma = 7$

and $\beta = 4$

$\therefore$ zeros are 1, 7, 4.

Putting $\alpha = 7$ in $(iv)$, we get

$\gamma = \frac{7}{7}$

or $\gamma = 1$

and $\beta = 4$

$\therefore$ zeros are 7, 4, 1.

**Que 3. If the polynomial $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + \alpha$. Find k and $\alpha$.**

**Sol.** By division algorithm, we have Dividend = Divisor × Quotient + Remainder

⇒ Dividend – Remainder = Divisor × Quotient

⇒ Dividend – Remainder is always divisible by the divisor.

When $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $x^2 - 2x + k$ the remainder comes out to be $x + a$.

$\therefore\ f(x) - (x + a) = x^4 - 6x^3 + 16x^2 - 25x + 10 - (x + a)$

$= x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$

is exactly divisible by $x^2 - 2x + k$.

Let us now divide $x^4 - 6x^2 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$.

$x^2 - 2x + k)\ x^4 - 6x^3 + 16x^2 - 26x + 10 - a\ (\ x^2 - 4x + (8 - k)$

$x^4 - 2x^3 + kx^2$

$-\quad +\quad -$

$-4x^3 + (16 - k)x^2 - 26x + 10 - a$

$-4x^3 + 8x^2 \quad - 4kx$

$+\quad -\quad +$

$(8 - k)x^2 - (26 - 4k)x + 10 - a$

$(8 - k)x^2 - (16 - 2k)x + (8k - k^2)$

$-\quad +\quad -$

$(-10 + 2k)x + (10 - a - 8k + k^2)$

For $f(x) - (c + \alpha) = x^4 - 6x^3 + 16x^2 - 26x + 10 - \alpha$ to be exactly divisible by $x^2 - 2x + k$, we must have

$(-10 + 2k)x + (10 - \alpha - 8k + k^2) = 0$ *for all* $x$

⇒ $-10 + 2k = 0$ *and* $10 - a - 8k + k^2 = 0$

⇒ $k = 5$ *and* $10 - a - 40 + 25 = 0$

⇒ $k = 5$ *and* $a = -5$.