Short Answer Questions-I (PYQ)

[2 Mark]

Q.1. Write A^{-1} for $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

Ans.

For elementary row operations, we write

$$A = IA \quad \Rightarrow \quad \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.A$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.A \qquad [\text{Applying } R_1 \leftrightarrow R_2]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}A \qquad [\text{Applying } R_2 \to R_2 - 2R_1]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix}A. \qquad [\text{Applying } R_1 \to R_1 + 3R_2]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}A \qquad [\text{Applying } R_2 \to (-1)R_2]$$

$$\Rightarrow \quad I = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}A \quad \Rightarrow \quad A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

[**Note :** B is called inverse of A, if $AB = BA = I$]

Q.2. **If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write A^{-1} in terms of A.**

Ans.

$$|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix}$$

$= -4 - 15 = -19 \neq 0$

Now, $C_{11} = -2$, $C_{12} = -5$, $C_{21} = -3$ and $C_{22} = 2$

$$\text{adj } A = \begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19}A$$

Q.3. **If $A = \begin{vmatrix} 3 & 10 \\ 2 & 7 \end{vmatrix}$, then write A^{-1}.**

Ans.

Given: $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 7 & -2 \\ -10 & 3 \end{bmatrix}^T = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

Also $|A| = 21 - 20 = 1 \neq 0$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{1}\begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

Short Answer Questions-I (OIQ)

[2 Mark]

Q.1. If A is invertible matrix of 3 × 3 and $|A| = 7$, then find $|A^{-1}|$.

Ans.

$\because \quad A^{-1} = \frac{1}{|A|} . \text{adj } A$

$|A^{-1}| = \left|\frac{1}{|A|} \text{adj } A\right| = \left(\frac{1}{|A|}\right)^3 . |\text{adj } A|$ $\quad [\because |KA| = K^n . |A|$, where n is order of $A]$

$= \frac{1}{|A|^3} . |A|^{3-1} = \frac{1}{|A|} = \frac{1}{7}$

Q.2. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ **then find the value of λ for which A^{-1} exists.**

Ans.

For existence of A^{-1}

$|A| \neq 0 \quad \Rightarrow \quad \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix} \neq 0$

$\Rightarrow \quad 2(6-5) - \lambda(0-5) + (-3)(0-2) \neq 0$ $\quad$ [Expanding along R_1]

$\Rightarrow \quad 2 + 5\lambda + 6 \neq 0 \quad \Rightarrow 5\lambda \neq -8 \quad \Rightarrow \quad \lambda \neq -\frac{8}{5}$

Hence, λ can have any value other than $-\frac{8}{5}$.

Q.3. Write the inverse of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

Ans.

Let $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$|A| = \cos^2\theta + \sin^2\theta = 1 \neq 0$

Since, $|A| \neq 0$

Now $\quad A^{-1} = \frac{\text{Adj } A}{|A|}$

$= \frac{1}{1}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Q.4. If $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ **and** $B = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$ **satisfying *AB* = *I*, then what is the inverse of *A*?**

Ans.

Given, $AB = I$

$\Rightarrow \quad A^{-1}(AB) = A^{-1}I \qquad \Rightarrow (A^{-1}A)B = A^{-1}$

$\Rightarrow \quad IB = A^{-1} \qquad \Rightarrow B = A^{-1}$

$\therefore \quad A^{-1} = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$

Q.5. For what value of *x*, the following matrix is singular?

$\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$

Ans.

For matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ being singular

$$\begin{vmatrix} 5-x & x+1 \\ 2 & 4 \end{vmatrix} = 0$$

$\Rightarrow \quad 20 - 4x - 2x - 2 = 0 \qquad \Rightarrow 18 - 6x = 0 \qquad \Rightarrow 6x = 18$

$\Rightarrow \quad x = \frac{18}{6} = 3$

Q.6. If $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$**, find *A* + *A'*, where *A'* is transpose of *A*.**

Ans.

$\because \quad A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} \quad \Rightarrow \quad A' = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$

$\therefore \quad A + A' = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$

$= 6\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$