DAY THIRTY FOUR

Statistics

Learning & Revision for the Day

- Measures of Central Tendency
- Measure of Dispersion

Measures of Central Tendency

A value which describes the characteristics of entire data, is called an **average** or a **central value**. Generally, an average lies in the central part of the data and therefore such values are called the **measure of central tendency**.

There are five measures of central tendency, which are given below :

1. Mean (Arithmetic Mean)

The sum of all the observations divided by the number of observations, is called **mean** and it is denoted by $\bar{x}$. The most stable measure of central tendency is mean.

- If $x_1, x_2, \dots, x_n$ be the n observations, then mean is given by

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum x_i}{n}$$

or $\bar{x} = A + \frac{\sum_{i=1}^{n} d_i}{n}$, where $d_i = x_i - A$ and A is assumed mean.

- If corresponding frequencies of n observations are $f_1, f_2, \dots, f_n$, then

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \dots + f_nx_n}{f_1 + f_2 + \dots + f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \quad \text{or} \quad \bar{x} = A + \frac{\sum_{i=1}^{n} f_i d_i}{N},$$

where $d_i = x_i - A$ and $N = \sum_{i=1}^{n} f_i$

- If corresponding weights are $w_1, w_2, \dots, w_n$, then **weighted mean**, $\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i}$.
- If $\bar{x}_1, \bar{x}_2, \dots, \bar{x}_k$ be the means of k sets of observations of size $n_1, n_2, \dots, n_k$ respectively, then their **combined mean**,

$$\bar{x}_{1k} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + \dots + n_k\bar{x}_k}{n_1 + n_2 + \dots + n_k}$$

Here, $\bar{x}_1$ = Mean of first set of observations

n_1 = Number of observations in first set

$\bar{x}_2$ = Mean of second set of observations
n_2 = Number of observations in second set and so on.

NOTE
- The sum of the deviations of the individual values from AM is always zero, i.e. $\Sigma(x_i - \bar{x}) = 0$.
- The sum of squares of deviations of the individual values is least when taken from AM i.e. $\Sigma(x_i - \bar{x})^2$ is least.

2. Geometric Mean (GM)

If $x_1, x_2, \ldots x_n$ are n observations, then nth root of the product of all observations is called **geometric mean.**

- If $x_1, x_2, \ldots, x_n$ are n non-zero positive observations, then
$$GM = (x_1 \cdot x_2 \ldots x_n)^{1/n} = \text{antilog}\left(\frac{1}{n}\sum_{i=1}^{n} \log x_i\right)$$
- If corresponding frequencies of each observation are $f_1, f_2, \ldots, f_n$, then
$$GM = [x_1^{f_1} \cdot x_2^{f_2} \ldots x_n^{f_n}]^{\frac{1}{N}} = \text{antilog}\left[\frac{1}{N}\sum_{i=1}^{n} f_i \log x_i\right]$$
where, $N = \sum_{i=1}^{n} f_i$

3. Harmonic Mean (HM)

The harmonic mean of any set of non-zero observations, is the reciprocal of the arithmetic mean of the reciprocals of the observations.

- The harmonic mean of n items $x_1, x_2, \ldots, x_n$ is defined as
$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}} = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}} = \left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}\right]^{-1}$$
- If corresponding frequencies of each observation are $f_1, f_2, \ldots, f_n$, then $HM = \left[\frac{1}{N}\sum_{i=1}^{n}\frac{f_i}{x_i}\right]^{-1}$
- Relation between AM, GM and HM $(GM)^2 = (AM) \cdot (HM)$

4. Median

If the observations are arranged in ascending or descending order, then the value of the middle observation is defined as the **median**.

- Let $x_1, x_2, \ldots, x_n$ be n observations, arranged in ascending or descending order, then

If n is odd, then Median $= \frac{n+1}{2}$ th observation

If n is even, then Median $= \frac{\left[\left(\frac{n}{2}\right)\text{th} + \left(\frac{n}{2}+1\right)\text{th}\right]\text{observation}}{2}$

- If in a continuous distribution the total frequency is N, then the class whose cumulative frequency is either equal to $N/2$ or just greater than $N/2$ is called **median class** and in that case,
$$\text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h,$$
where, l = Lower limit of median class
f = Frequency of median class
h = Size of median class
c = Cumulative frequency of class preceding the median class

5. Mode

Mode is the observation which has maximum frequency.

- If $x_1, x_2, \ldots, x_n$ are the n observations and corresponding frequencies are $f_1, f_2, \ldots, f_n$, then the observation of maximum frequency is a modal value.
- In a continuous distribution the interval which has maximum frequency called **modal class** and in that case,
$$\text{mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h,$$
where, l = Lower limit of modal class
f_1 = Frequency of modal class
f_0 = Frequency of the class preceding the modal class
f_2 = Frequency of the class succeeding the modal class
h = Class size
- Relation between Mean, Median and Mode
Mode = 3 Median − 2 Mean (Emperical formula)
- It is not necessary that a distribution has unique mode.

Measure of Dispersion

A measure of dispersion is designed to state the extent to which the individual observations vary from their average.

The commonly used measures of dispersion are :

Range

The difference between the maximum and the minimum observations is called **range**.

i.e. Range $= L - S$.
where, L = Maximum observation
and S = Minimum observation

Coefficient of range $= \frac{L-S}{L+S}$

Mean Deviation (MD)

The mean, of the absolute deviations of the values of the variable from a measure of their average is called **Mean Deviation** (MD).

- If $x_1, x_2, \ldots, x_n$ are n observations, then $MD = \frac{\sum_{i=1}^{n}|x_i - z|}{n}$
where, z = mean or mode or median

- If corresponding frequencies of each observation are $f_1, f_2, \ldots, f_n$, then

$$\text{MD} = \frac{\sum_{i=1}^{n} f_i |x_i - z|}{N}, \text{ where } N = \sum_{i=1}^{n} f_i$$

- Coefficient of MD $= \dfrac{\text{Mean deviation}}{\text{Corresponding average}}$
- Mean deviation is least when deviations are taken from median.

Standard Deviation (SD)

The square root of the arithmetic mean of the squares of deviations of the observations from their arithmetic mean is called **standard deviation** and it is denoted by σ.

- If $x_1, x_2, \ldots, x_n$ are n observations, then

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2}$$

- If corresponding frequencies of each observation are $f_1, f_2, \ldots, f_n$, then

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{N}} = \sqrt{\frac{1}{N}\left(\sum_{i=1}^{n} f_i x_i^2\right) - \left(\frac{1}{N}\sum_{i=1}^{n} f_i x_i\right)^2},$$

where $N = \sum_{i=1}^{n} f_i$

Variance

The square of SD is called **variance** and it is denoted by σ^2.

- Coefficient of variation $= \dfrac{\sigma}{\bar{x}} \times 100\%$

- Two different series having n_1 and n_2 observations and whose corresponding means and variances are $\bar{x}_1, \bar{x}_2$ and σ_1^2, σ_2^2. Then, their **combined variance,**

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2},$$

where $d_1 = (\bar{x}_1 - \bar{x}_{12}),\ d_2 = (\bar{x}_2 - \bar{x}_{12})$

and $\bar{x}_{12} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

NOTE
- Standard deviation is always less than range.
- Standard deviation of n natural numbers is $\sigma = \left[\dfrac{1}{12}(n^2-1)\right]^{1/2}$.
- Mean deviation $= \dfrac{4}{5}\sigma$.

Effect of average and dispersion on change of origin and scale

	Change of origin	Change of scale
Mean	Dependent	Dependent
Median	Dependent	Dependent
Mode	Dependent	Dependent
Standard deviation	Independent	Dependent
Variance	Independent	Dependent

- If $\bar{x}$, m_e, m_o and σ represent the mean, median, mode and standard deviation, respectively, of $x_1, x_2, \ldots, x_n$. Then,
  (i) Mean of $ax_1 + b, ax_2 + b, \ldots, ax_n + b$, is $a\bar{x} + b$
  (ii) Median of $ax_1 + b, ax_2 + b, \ldots, ax_n + b$, is $am_e + b$
  (iii) Mode of $ax_1 + b, ax_2 + b, \ldots, ax_n + b$, is $am_o + b$
  (iv) Standard deviation of $ax_1 + b, ax_2 + b, \ldots, ax_n + b$, is $|a| \cdot \sigma$.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 The mean of a data set consisting of 20 observations is 40. If one observation 53 was wrongly recorded as 33, then the correct mean will be → **JEE Mains 2013**
(a) 41 (b) 49 (c) 40.5 (d) 42.5

2 If the variance of 1, 2, 3, 4, 5, ..., 10 is $\dfrac{99}{12}$, then the standard deviation of 3, 6, 9, 12, ..., 30 is
(a) $\dfrac{297}{4}$ (b) $\dfrac{3}{2}\sqrt{33}$ (c) $\dfrac{3}{2}\sqrt{99}$ (d) $\sqrt{\dfrac{99}{12}}$

3 The mean of n terms is $\bar{x}$. If the first term is increased by 1, second by 2 and so on, then the new mean is
(a) $\bar{x} + n$ (b) $\bar{x} + \dfrac{n}{2}$
(c) $\bar{x} + \dfrac{n+1}{2}$ (d) None of these

4 The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data is → **JEE Mains 2015**
(a) 16.8 (b) 16.0 (c) 15.8 (d) 14.0

5 If the sum of deviation of a set of values $x_1, x_2, \ldots x_n$ measure from 59 is 20 and from 54 is 70, then sample size (n) and the sample mean is
(a) 10, 61 (b) 10, 55.67 (c) 6, 55.67 (d) 6, 44

6 The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is
(a) 40% (b) 20% (c) 80% (d) 60%

7 The weighted mean of first n natural numbers whose weights are equal to the squares of corresponding numbers is

(a) $\frac{n+1}{2}$ (b) $\frac{3n(n+1)}{2(2n+1)}$ (c) $\frac{(n+1)(2n+1)}{6}$ (d) $\frac{n(n+1)}{2}$

8 A distribution consists of three components with frequencies 20, 25 and 30 having means 25, 10 and 15 respectively. The mean of the combined distribution is

(a) 14 (b) 16 (c) 17.5 (d) 20

9 A car completes the first half of its journey with a velocity v_1 and the rest half with a velocity v_2. Then the average velocity of the car for the whole journey is

(a) $\frac{v_1+v_2}{2}$ (b) $\sqrt{v_1 v_2}$ (c) $\frac{2v_1 v_2}{v_1+v_2}$ (d) None of these

10 The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set is

(a) increased by 2
(b) decreased by 2
(c) two times the original median
(d) remains the same as that of original set

11 The median of 19 observations of a group is 30. If two observations with values 8 and 32 are further included, then the median of the new group of 21 observations will be

(a) 28 (b) 30 (c) 32 (d) 34

12 If a variable takes the discrete values

$\alpha+4, \alpha-\frac{7}{2}, \alpha-\frac{5}{2}, \alpha-3, \alpha-2, \alpha+\frac{1}{2}, \alpha-\frac{1}{2}, \alpha+5$

(where, $\alpha > 0$), then the median is

(a) $\alpha-\frac{5}{4}$ (b) $\alpha-\frac{1}{2}$ (c) $\alpha-2$ (d) $\alpha+\frac{5}{4}$

13 Median of ${}^{2n}C_0, {}^{2n}C_1, {}^{2n}C_2, {}^{2n}C_3, \ldots, {}^{2n}C_n$ (where, n is even) is

(a) ${}^{2n}C_{\frac{n}{2}}$ (b) ${}^{2n}C_{\frac{n+1}{2}}$

(c) ${}^{2n}C_{\frac{n-1}{2}}$ (d) None of these

14 If the median and the range of four numbers $\{x, y, 2x+y, x-y\}$, where $0 < y < x < 2y$, are 10 and 28 respectively, then the mean of the numbers is **➔ JEE Mains 2013**

(a) 18 (b) 10 (c) 5 (d) 14

15 Find the mean deviation from the median of the following data. **➔ NCERT Exemplar**

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	5	3	6	2

(a) 7.08 (b) 7 (c) 7.1 (d) 7.05

16 If the mean deviations about the median of the numbers $a, 2a, \ldots, 5a$ is 50, then $|a|$ is equal to

(a) 3 (b) 4 (c) 5 (d) 2

17 If the mean deviation of number $1, 1+d, 1+2d, \ldots, 1+100d$ from their mean is 255, then d is equal to

(a) 10.0 (b) 20.0 (c) 10.1 (d) 20.2

18 Consider any set of 201 observations $x_1, x_2, \ldots, x_{200}, x_{201}$. It is given that $x_1 < x_2 < \ldots < x_{200} < x_{201}$. Then, the mean deviation of this set of observations about a point k is minimum when k is equal to

(a) x_{110} (b) x_1 (c) x_{101} (d) x_{201}

19 If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true? **➔ JEE Mains 2016**

(a) $3a^2-26a+55=0$ (b) $3a^2-32a+84=0$
(c) $3a^2-34a+91=0$ (d) $3a^2-23a+44=0$

20 Mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation. **➔ NCERT Exemplar**

(a) 10.20 (b) 10.24 (c) 10.29 (d) 10.27

21 Coefficient of variation of two distributions are 50 and 60 and their arithmetic means are 30 and 25, respectively. Difference of their standard deviation is **➔ NCERT Exemplar**

(a) 0 (b) 1 (c) 1.5 (d) 2.5

22 If MD is 12, the value of SD will be

(a) 15 (b) 12 (c) 24 (d) None of these

23 If SD of variate x is σ_x, then the SD of $\frac{ax+b}{p}, \forall a, b, p \in R$ is

(a) $\left|\frac{a}{p}\right|\sigma_x$ (b) $\left|\frac{p}{a}\right|\sigma_x$ (c) $\frac{p}{a}\sigma_x$ (d) $\frac{a}{p}\sigma_x$

24 If the standard deviation of the observations $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ is $\sqrt{10}$. The standard deviation of the observations 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25 will be

(a) $\sqrt{10}+20$ (b) $\sqrt{10}+10$
(c) $\sqrt{10}$ (d) None of these

25 A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 g and a standard deviation of 2 g. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 g. The correct mean and standard deviation (in gram) of fishes are respectively **➔ AIEEE 2011**

(a) 28, 4 (b) 32, 2 (c) 32, 4 (d) 28, 2

26 The variance of first 50 even natural numbers is **➔ JEE Mains 2014**

(a) $\frac{833}{4}$ (b) 833 (c) 437 (d) $\frac{437}{4}$

27 Mean of 5 observations is 7. If four of these observations are 6, 7, 8, 10 and one is missing, then the variance of all the five observations is **➔ JEE Mains 2013**

(a) 4 (b) 6 (c) 8 (d) 2

28 The mean of the numbers *a, b,* 8, 5, 10 is 6 and the variance is 6.80. Then, which one of the following gives possible values of *a* and *b*?

(a) $a = 3, b = 4$ (b) $a = 0, b = 7$
(c) $a = 5, b = 2$ (d) $a = 1, b = 6$

29 In an experiment with 15 observations on x, the following results were available $\Sigma x^2 = 2830$, $\Sigma x = 170$. One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then, the corrected variance is

(a) 78.0 (b) 188.66 (c) 177.33 (d) 8.33

30 For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(a) $\frac{5}{2}$ (b) $\frac{11}{2}$ (c) 6 (d) $\frac{13}{2}$

31 If a variable x takes values x_i such that $a \le x_i \le b$, for $i = 1, 2, \ldots, n$, then

(a) $a^2 \le \text{Var}(x) \le b^2$ (b) $a \le \text{Var}(x) \le b$
(c) $\frac{a^2}{4} \le \text{Var}(x)$ (d) $(b - a)^2 \ge \text{Var}(x)$

32 All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given? ➔ **JEE Mains 2013**

(a) Mean (b) Median (c) Mode (d) Variance

33 Let $x_1, x_2, \ldots, x_n$ be n observations. Let $w_i = l \cdot x_i + k$ for $i = 1, 2, \ldots, n$, where l and k are constants. If the mean of x_i's is 48 and their standard deviation is 12, the mean of w_i's is 55 and standard deviation of w_i's is 15. The values of l and k should be ➔ **NCERT Exemplar**

(a) $l = 1.25, k = -5$ (b) $l = -1.25, k = 5$
(c) $l = 2.5, k = -5$ (d) $l = 2.5, k = 5$

34 If n is a natural number, then

Statement I The mean of the squares of first n natural number is $\frac{(n+1)(2n+1)}{6}$.

Statement II $\Sigma n = \frac{n(n+1)}{2}$

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

35 Let $x_1, x_2, \ldots, x_n$ be n observations and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance.

Statement I Variance of $2x_1, 2x_2, \ldots, 2x_n$ is $4\sigma^2$.
Statement II Arithmetic mean $2x_1, 2x_2, \ldots, 2x_n$ is $4\bar{x}$. ➔ **AIEEE 2012**

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 The mean age of a combined group of men and women is 25 yr. If the mean age of the group of men is 26 and that of the group of women is 21, then the percentage of men and women in the group is

(a) 40, 60 (b) 80, 20 (c) 20, 80 (d) 60, 40

2 The average of the four-digit numbers that can be formed using each of the digits 3, 5, 7 and 9 exactly once in each number, is

(a) 4444 (b) 5555 (c) 6666 (d) 7777

3 Suppose a population A has 100 observations 101, 102, ..., 200 and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations respectively, then $\frac{V_A}{V_B}$ is equal to

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) $\frac{2}{3}$ (d) 1

4 If the value observed are 1, 2, 3, ..., n each with frequency 1 and n is even, then the mean deviation from mean equals to

(a) n (b) $\frac{n}{2}$ (c) $\frac{n}{4}$ (d) None of these

5 In a class of 19 students, seven boys failed in a test. Those who passed scored 12, 15, 17, 15, 16, 15, 19, 19, 17, 18, 18 and 19 marks. The median score of the 19 students in the class is

(a) 15 (b) 16 (c) 17 (d) 18

6 If G is the geometric mean of the product of r sets of observations with geometric means $G_1, G_2, G_3, \ldots, G_r$ respectively, then G is equal to

(a) $\log G_1 + \log G_2 + \ldots + \log G_r$
(b) $G_1 \cdot G_2 \ldots G_r$
(c) $\log G_1 \cdot \log G_2 \cdot \log G_3 \ldots \log G_r$
(d) None of the above

7 In a class of 100 students, there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average marks of the girls?

(a) 73 (b) 65 (c) 68 (d) 74

8 An aeroplane flies around a squares the sides of which measure 100 mile each. The aeroplane covers at a speed of 100 m/h the first side, at 200 m/h the second side, at 300 m/h the third side and 400 m/h the fourth side. The average speed of the aeroplane around the square is

(a) 190 m/h (b) 195 m/h (c) 192 m/h (d) 200 m/h

9 The first of two samples has 100 items with mean 15 and SD = 3. If the whole group has 250 items with mean 15.6 and SD = $\sqrt{13.44}$, the SD of the second group is

(a) 4 (b) 5 (c) 6 (d) 3.52

10 If a variable takes the values $0, 1, 2, \ldots, n$ with frequencies proportional to the binomial coefficients ${}^nC_0, {}^nC_1, \ldots, {}^nC_n$, then mean of the distribution is

(a) $\frac{n}{2}$ (b) $\frac{n(n+1)}{2}$ (c) $\frac{n(n-1)}{2}$ (d) $\frac{2}{n}$

11 The marks of some students were listed out of 75. The SD of marks was found to be 9. Subsequently the marks were raised to a maximum of 100 and variance of new marks was calculated. The new variance is

(a) 81 (b) 122 (c) 144 (d) 125

12 If $x_1, x_2, \ldots, x_n$ are n observations such that $\Sigma x_i^2 = 400$ and $\Sigma x_i = 80$. Then, a possible value of n among the following is

(a) 12 (b) 9 (c) 14 (d) 16

13 In a set of $2n$ observations, half of them are equal to a and the remaining half are equal to $-a$. If the standard deviation of all the observations is 2, then the value of $|a|$ is **➜ JEE Mains 2013**

(a) 2 (b) $\sqrt{2}$ (c) 4 (d) $2\sqrt{2}$

14 If $\sum_{i=1}^{9} (x_i - 5) = 9$ and $\sum_{i=1}^{9} (x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_i, x_2, \ldots, x_9$ is **➜ JEE Mains 2018**

(a) 9 (b) 4 (c) 2 (d) 3

15 If $\bar{x}_1$ and $\bar{x}_2$ are the means of two distributions such that $\bar{x}_1 < \bar{x}_2$ and $\bar{x}$ is the mean of the combined distribution, then

(a) $\bar{x} < \bar{x}_1$ (b) $\bar{x} > \bar{x}_2$

(c) $\bar{x} = \frac{\bar{x}_1 + \bar{x}_2}{2}$ (d) $\bar{x}_1 < \bar{x} < \bar{x}_2$

ANSWERS

SESSION 1

1. (a)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (d)
11. (b)	**12.** (a)	**13.** (a)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (b)	**20.** (b)
21. (a)	**22.** (a)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (a)	**29.** (a)	**30.** (b)
31. (d)	**32.** (d)	**33.** (a)	**34.** (b)	**35.** (c)					

SESSION 2

1. (b)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (a)
11. (c)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (d)					

Hints and Explanations

SESSION 1

1 Given; $\frac{\Sigma x_{\text{incorrect}}}{20} = 40$

$\Rightarrow \quad \Sigma x_{\text{incorrect}} = 20 \times 40 = 800$

$\therefore \quad \Sigma x_{\text{correct}} = 800 - 33 + 53 = 820$

$\Rightarrow \quad \frac{\Sigma x_{\text{correct}}}{20} = \frac{820}{20}$

$\therefore$ Correct mean = 41

2 Given, $\sigma^2 = \frac{99}{12} = \frac{33}{4} \Rightarrow \sigma = \frac{\sqrt{33}}{2}$

Clearly, SD of required series

$= 3\sigma = \frac{3\sqrt{33}}{2}$

3 Let the observations be $x_1, x_2, \ldots, x_n$.

Then, $\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$

Now, when the first term is increased by 1, second term by 2 and so on, then the observations will be $(x_1 + 1), (x_2 + 2), \ldots, (x_n + n)$.

$\therefore$ New mean

$= \frac{(x_1 + 1) + (x_2 + 2) + \ldots + (x_n + n)}{n}$

$= \frac{(x_1 + x_2 + \ldots + x_n)}{n} + \frac{(1 + 2 + \ldots + n)}{n}$

$= \bar{x} + \frac{n(n+1)}{2n} = \bar{x} + \frac{n+1}{2}$

4 Given, $\frac{x_1 + x_2 + x_3 + \ldots + x_{16}}{16} = 16$

$\Rightarrow \sum_{i=1}^{16} x_i = 16 \times 16$

Sum of new observations

$= \sum_{i=1}^{18} y_i = (16 \times 16 - 16) + (3 + 4 + 5) = 252$

Number of observations = 18

$\therefore$ New mean $= \frac{\sum_{i=1}^{18} y_i}{18} = \frac{252}{18} = 14$

5 We have, $\sum_{i=1}^{n} (x_i - 59) = 20$ ...(i) and $\sum_{i=1}^{n}(x_i - 54) = 70$ ...(ii)

From Eq. (i), we get

$$\sum_{i=1}^{n} x_i - 59n = 20$$

$\Rightarrow \sum_{i=1}^{n} x_i = 20 + 59n$ ...(iii)

and similarly, from Eq. (ii), we get

$\sum_{i=1}^{n} x_i = 70 + 54n$ ...(iv)

From Eqs. (iii) and (iv), we get

$70 + 54n = 20 + 59n$

$\Rightarrow 5n = 50 \Rightarrow n = 10$

Now, from Eq. (iii), we get

$$\sum_{i=1}^{n} x_i = 610$$

$\therefore$ Sample mean = 61

6 Let the number of boys and girls be x and y.

$\therefore \quad 52x + 42y = 50(x + y)$

$\Rightarrow \quad 52x + 42y = 50x + 50y$

$\Rightarrow \quad 2x = 8y$

$\Rightarrow \quad x = 4y$

$\therefore$ Total number of students in the class

$= x + y = 4y + y = 5y$

$\therefore$ Required percentage of boys

$$= \left(\frac{4y}{5y} \times 100\right)\% = 80\%$$

7 Required mean

$$= \frac{1\cdot 1^2 + 2\cdot 2^2 + \dots + n\cdot n^2}{1^2 + 2^2 + \dots + n^2}$$

$$= \frac{1^3 + 2^3 + \dots + n^3}{1^2 + 2^2 + \dots + n^2} = \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)(2n+1)}{6}}$$

$$= \frac{n(n+1)}{4} \times \frac{6}{2n+1} = \frac{3n(n+1)}{2(2n+1)}$$

8 Here, $\bar{x}_1 = 25, \bar{x}_2 = 10, \bar{x}_3 = 15$

and $n_1 = 20, n_2 = 25, n_3 = 30$

Now, combined mean

$$x_{13} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$$

$$= \frac{500 + 250 + 450}{75} = \frac{1200}{75} = 16$$

9 Clearly, average velocity

$$= \text{HM of } (v_1, v_2) = \frac{2v_1 v_2}{v_1 + v_2}$$

10 After arranging the terms in ascending order median is the $\left(\frac{n+1}{2}\right)$th term,

i.e. 5th term.

Here, we increase largest four observations of the set which will come after 5th term.

Hence, median remains the same as that of original set.

11 Since, there are 19 observations. So, the middle term is 10th.

After including 8 and 32, i.e. 8 will come before 30 and 32 will come after 30.

Here, new median will remain 30.

12 Firstly arrange the given data in ascending order.

$$\alpha - \frac{7}{2}, \alpha - 3, \alpha - \frac{5}{2}, \alpha - 2, \alpha - \frac{1}{2}, \alpha + \frac{1}{2}, \alpha + 4, \alpha + 5$$

$\therefore$ Median $= \frac{1}{2}$[Value of 4th item + Value of 5th item]

$$= \frac{\alpha - 2 + \alpha - \frac{1}{2}}{2} = \frac{2\alpha - \frac{5}{2}}{2} = \alpha - \frac{5}{4}$$

13 Total number of terms $= n + 1$ (which is odd)

$\therefore$ Median $= \left(\frac{n+1+1}{2}\right)$th term

$= \left(\frac{n}{2} + 1\right)$th term $= {}^{2n}C_{n/2}$

14 First we arrange four numbers according to the condition

$0 < y < x < 2y$ i.e. $x - y, y, x, 2x + y$

Median $= \frac{\text{2nd term} + \text{3rd term}}{2} = 10$

$\Rightarrow \quad y + x = 20$ ...(i)

Range $= (2x + y) - (x - y) = 28$

$\Rightarrow \quad x + 2y = 28$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = 12, y = 8$

So, four numbers are 4, 8, 12, 32.

$\therefore$ Mean $= \frac{4 + 8 + 12 + 32}{4} = \frac{56}{4} = 14$

15

Class interval	Mid value x_i	f	cf	$\lvert x - M_d\rvert$	$f\lvert x - M_d\rvert$
0-6	3	4	4	11	44
6-12	9	5	9	5	25
12-18	15	3	12	1	3
18-24	21	6	18	7	42
24-30	27	2	20	13	26
Total		**20**			**140**

Now, $\frac{N}{2} = \frac{20}{2} = 10$, which lies in the interval 12-18.

$l = 12, cf = 9, f = 3$

$\therefore M_d = 12 + \frac{10 - 9}{3} \times 6$

$$\left[\because M_d = l + \frac{\frac{N}{2} - cf}{2} \times h\right]$$

$= 12 + 2 = 14$

$\therefore$ Mean deviation $= \frac{\Sigma f_i\lvert x_i - M_d\rvert}{N} = \frac{140}{20} = 7$

16 Median of $a, 2a, 3a, 4a, \dots, 50a$ is

$$\frac{25a + 26a}{2} = (25.5)a$$

Mean deviation about median

$$= \frac{\sum_{i=1}^{50}\lvert x_i - \text{median}\rvert}{n}$$

$\Rightarrow 50 = \frac{1}{50}\{2\lvert a\rvert(0.5 + 1.5 + 2.5 + \dots + 24.5)\}$

$\Rightarrow 2500 = 2\lvert a\rvert\cdot\frac{25}{2}(2 \times 0.5 + 24 \times 1)$

$= 2\lvert a\rvert\cdot\frac{25}{2}(25)$

$\therefore \quad \lvert a\rvert = 4$

17 Clearly,

$$(\bar{x}) = \frac{\text{Sum of quantities}}{n} = \frac{\frac{n}{2}(a + l)}{n}$$

$$= \frac{1}{2}[1 + 1 + 100d] = 1 + 50d$$

Now, MD $= \frac{1}{n}\Sigma\lvert x_i - \bar{x}\rvert \Rightarrow 255$

$$= \frac{1}{101}[50d + 49d + 48d + \dots + d + 0 + d + \dots + 50d]$$

$$= \frac{2d}{101}\left(\frac{50 \times 51}{2}\right)$$

$$\therefore d = \frac{255 \times 101}{50 \times 51} = 10.1$$

18 Given that, $x_1 < x_2 < x_3 < \dots < x_{201}$

Hence, median of the given observation

$= \left(\frac{201 + 1}{2}\right)$th item $= x_{101}$

Now, deviation will be minimum of taken from the median.

Hence, mean deviation will be minimum, if $k = x_{101}$.

19 We know that, if $x_1, x_2, \dots, x_n$ are n observations, then their standard deviation is given by $\sqrt{\frac{1}{n}\Sigma x_i^2 - \left(\frac{\Sigma x_i}{n}\right)^2}$

We have, $(3.5)^2 = \frac{(2^2 + 3^2 + a^2 + 11^2)}{4} - \left(\frac{2 + 3 + a + 11}{4}\right)^2$

$\Rightarrow \frac{49}{4} = \frac{4 + 9 + a^2 + 121}{4} - \left(\frac{16 + a}{4}\right)^2$

$\Rightarrow \frac{49}{4} = \frac{134 + a^2}{4} - \frac{256 + a^2 + 32a}{16}$

$\Rightarrow \frac{49}{4} = \frac{4a^2 + 536 - 256 - a^2 - 32a}{16}$

$\Rightarrow 49 \times 4 = 3a^2 - 32a + 280$

$\Rightarrow 3a^2 - 32a + 84 = 0$

20 $\therefore$ New mean,

$$\bar{x} = \frac{100 \times 40 + 3 + 27 - 30 - 70}{100}$$

$= \frac{4000-70}{100} = \frac{3930}{100} = 39.3$

$\because \Sigma x^2 = N(\sigma^2 + \bar{x}^2)$

$\therefore$ SD $= 100(100+1600) = 170000$

New $\Sigma x^2 = 170000 - (30)^2 - (70)^2 + (3)^2 + (27)^2$
$= 170000 - 900 - 4900 + 9 + 729 = 164938$

$\therefore$ New SD $= \sqrt{\frac{\text{New } \Sigma x^2}{N} - (\text{New } \bar{x})^2}$

$= \sqrt{\frac{164938}{100} - (39.3)^2}$

$= \sqrt{1649.38 - 1544.49} = \sqrt{104.89}$

$= 10.24$

21 Given, coefficient of variation, $C_1 = 50$
and coefficient of variation, $C_2 = 60$
We have, $\bar{x}_1 = 30$ and $\bar{x}_2 = 25$

$\because \quad C = \frac{\sigma}{\bar{x}} \times 100 \Rightarrow 50 = \frac{\sigma_1}{30} \times 100$

$\Rightarrow \sigma_1 = 15$ and $60 = \frac{\sigma_2}{25} \times 100$

$\Rightarrow \sigma_2 = 15$

$\therefore$ Required difference,

$\sigma_1 - \sigma_2 = 15 - 15 = 0$

22 We know that MD $= \frac{4}{5}$ SD

$\therefore$ SD $= \frac{5}{4}$ MD $= \frac{5}{4} \times 12 = 15$

23 Let $u = \frac{ax+b}{p}$, then $\bar{u} = \frac{a\bar{x}+b}{p}$

$\therefore$ SD $= \sqrt{\frac{\Sigma(u-\bar{u})^2}{\Sigma f}} = \sqrt{\frac{a^2}{p^2}\frac{\Sigma(x-\bar{x})^2}{\Sigma f}}$

$= \sqrt{\frac{a^2}{p^2}\sigma_x^2} = \left|\frac{a}{p}\right|\sigma_x$

24 The new observations are obtained by adding 20 to each previous observation. Hence, the standard deviation of new observations will be same i.e. $\sqrt{10}$.

25 Correct mean = Old mean +2

$= 30 + 2 = 32$

As standard deviation is independent of change of origin.

$\therefore$ It remains same.

$\Rightarrow$ Standard deviation = 2

26 Here, $\bar{x} = \frac{\Sigma x_i}{n}$

$= \frac{2+4+6+8+\ldots+100}{50}$

$= \frac{50 \times 51}{50} = 51$

$[\because \Sigma 2n = n(n+1)$, here $n = 50]$

Variance, $\sigma^2 = \frac{1}{n}\Sigma x_i^2 - (\bar{x})^{-2}$

$\sigma^2 = \frac{1}{50}(2^2 + 4^2 + \ldots + 100^2) - (51)^2 = 833$

27 Mean, $7 = \frac{6+7+8+10+x}{5}$

$\Rightarrow \quad x = 4$

Variance

$= \frac{(6-7)^2 + (7-7)^2 + (8-7)^2 + (10-7)^2 + (4-7)^2}{5}$

$= \frac{1^2 + 0 + 1^2 + 3^2 + 3^2}{5} = \frac{20}{5} = 4$

28 According to the given condition,

$6.80 = \frac{\left[\begin{array}{c}(6-a)^2 + (6-b)^2 + (6-8)^2 \\ + (6-5)^2 + (6-10)^2\end{array}\right]}{5}$

$\Rightarrow 34 = (6-a)^2 + (6-b)^2 + 4 + 1 + 16$

$\Rightarrow \quad (6-a)^2 + (6-b)^2 = 13 = 9 + 4$

$\Rightarrow \quad (6-a)^2 + (6-b)^2 = 3^2 + 2^2$

$\therefore \quad a = 3, b = 4$

29 Given, $n = 15$, $\Sigma x^2 = 2830$
and $\Sigma x = 170$

Since, one observation 20 was replaced by 30, then

$\Sigma x^2 = 2830 - 400 + 900 = 3330$

and $\Sigma x = 170 - 20 + 30 = 180$

$\therefore$ Variance, $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$= \frac{3330}{15} - \left(\frac{180}{15}\right)^2$

$= \frac{3330 - 2160}{15} = 78.0$

30 Here, $\sigma_1^2 = 4, \sigma_2^2 = 5, \bar{x}_1 = 2, \bar{x}_2 = 4,$
$n_1 = n_2 = 5$

Clearly, combined mean

$x_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$= \frac{1}{2}(2+4) = 3$

Combined variance,

$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}$

$= \frac{1}{2}[4 + 5 + 1 + 1] = \frac{11}{2}$

31 Since, SD < Range $\Rightarrow \sigma \le (b-a)$

$\sigma^2 \le (b-a)^2 \Rightarrow (b-a)^2 \ge \text{Var}(x)$

32 Since, variance is independent of change of origin. So, variance will not change whereas mean, median and mode will increase by 10.

33 Given, $w_i = lx_i + k$

$M(x_i) = \bar{x} = 48, \sigma(x_i) = 12$

$M(\bar{w}) = 55$ and $\sigma(w) = 15$

$M(w_i) = lM(x_i) + M(k)$

$55 = l \times 48 + k \quad \ldots(i)$

and $\sigma(w_i) = l\sigma(x_i) + \sigma(k)$

$\Rightarrow \quad 15 = l(12) + 0$

$\Rightarrow \quad l = \frac{15}{12} = 1.25$

From Eq. (i),

$55 = 1.25 \times 48 + k$

$\Rightarrow \quad k = 55 - 60$

$\therefore \quad k = -5$

34 Required mean $= \frac{\Sigma n^2}{n}$

$= \frac{n(n+1)(2n+1)}{6n} = \frac{(n+1)(2n+1)}{6}$

and Statement II is also a true Statement.

35 Clearly, variance of $2x_1, 2x_2, \ldots, 2x_n$ is $2^2 \cdot \sigma^2 = 4\sigma^2$.

and AM $= \frac{2x_1 + 2x_2 + 2x_3 + \ldots + 2x_n}{n}$

$= 2\left(\frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}\right) = 2\bar{x}$

SESSION 2

1 Let x be the number of men and y be the number of women.

Then, we have

$25 = \frac{x \cdot 26 + y \cdot 21}{x + y}$

$\left[\because \text{Combined mean } = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}\right]$

$\Rightarrow \quad 25x + 25y = 26x + 21y$

$\Rightarrow \quad x = 4y$

Now, percentage of men

$= \frac{x}{x+y} \times 100$

$= \frac{4y}{5y} \times 100 = 80\%$

and hence percentage of women

$= 20\%$

2 Number of required four-digit numbers

$= 4 \times 3 \times 2 \times 1 = 24$

and the sum of all the required four-digit numbers

$= (3+5+7+9) \times (4-1)! \times \left(\frac{10^4 - 1}{10 - 1}\right)$

$= 24 \times 6 \times \frac{9999}{9}$

Now, required average

$= \frac{24 \times 6 \times 9999}{9 \times 24} = 6666$

3 Since, variance is independent of change of origin. So, variance of observations 101, 102, 200 is same as variance of 151, 152, ..., 250.

$\therefore \quad V_A = V_B$

$\Rightarrow \quad \frac{V_A}{V_B} = 1$

4 Clearly, mean

$(\bar{x}) = \frac{1+2+3+\dots+n}{n}$

$= \frac{n(n+1)}{2n} = \frac{n+1}{2}$

Now, mean deviation from mean

$= \frac{1}{n}\Sigma|x_i - \bar{x}|$

$= \frac{1}{n}\left[\left|1-\frac{n+1}{2}\right| + \left|2-\frac{n+1}{2}\right| + \dots + \left|n-\frac{n+1}{2}\right|\right]$

$= \frac{1}{n}\left[\left|\frac{1-n}{2}\right| + \left|\frac{3-n}{2}\right| + \dots + \left|\frac{n-1}{2}\right|\right]$

$= \frac{1}{n}\left[\frac{n-1}{2} + \frac{n-3}{2} + \dots + \frac{n-3}{2} + \frac{n-1}{2}\right]$

$= \frac{1}{n}[(n-1)+(n-3)+\dots+1]\frac{1}{2}$

$\frac{1}{2n}\left[\frac{n}{2}[n-1+1]\right] = \frac{n}{4}$

5 Clearly, median score = score of $\left(\frac{19+1}{2}\right) = 10$th student.

Since, seven boys are failed out of 19, therefore according to given information there score will be less than 12.
Now, on arranging these scores in ascending order, we get score of 10th student is 15.

6 Taking X as the product of variates $X_1, X_2, \dots, X_r$, we get

$\log X = \log X_1 + \log X_2 + \log X_3 + \dots + \log X_r$

$\Rightarrow \Sigma \log X = \Sigma \log X_1 + \Sigma \log X_2 + \Sigma \log X_3 + \dots + \Sigma \log X_r$

$\Rightarrow \frac{1}{n}\Sigma \log X = \frac{1}{n}\Sigma \log X_1 + \frac{1}{n}\Sigma \log X_2 + \frac{1}{n}\Sigma \log X_3 + \dots + \frac{1}{n}\Sigma \log X_r$

$\Rightarrow \log G = \log G_1 + \log G_2 + \dots + \log G_r$

$\Rightarrow G = G_1 \cdot G_2 \cdot \dots G_r$

7 Since, total number of students = 100
and number of boys = 70

$\therefore$ Number of girls = (100 − 70) = 30

Now, the total marks of 100 students

$= 100 \times 72 = 7200$

And total marks of 70 boys

$= 70 \times 75 = 5250$

Total marks of 30 girls

$= 7200 - 5250 = 1950$

$\therefore$ Average marks of 30 girls

$= \frac{1950}{30} = 65$

8 Using the harmonic mean formula,

$\frac{1}{H} = \frac{1}{N}\sum_{i=1}^{n}\frac{f_i}{x_i} \Rightarrow H = \frac{1}{\frac{1}{N}\sum_{i=1}^{n}\frac{f_i}{x_i}}$

$\therefore$ Average speed

$= \frac{400}{100\left(\frac{1}{100}+\frac{1}{200}+\frac{1}{300}+\frac{1}{400}\right)}$

$= 192$ m / h

9 We know,

$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$,

where $d_1 = m_1 - a, d_2 = m_2 - a, a$ being the mean of the whole group.

$\therefore \quad 15.6 = \frac{100 \times 15 + 150 \times m_2}{250}$

$\Rightarrow \quad m_2 = 16$

Thus,

$13.44 = \frac{[(100 \times 9 + 150 \times \sigma^2) + 100 \times (0.6)^2 + 150 \times (0.4)^2]}{250}$

$\Rightarrow \quad \sigma = 4$

10 Here,

$N = \sum_{i=1}^{n} f_i = k({}^nC_0 + {}^nC_1 + \dots + {}^nC_n)$

$= k(1+1)^n = k2^n$ where, k is a constant of proportionality.

and $\Sigma f_i x_i = k(1 \cdot {}^nC_1 + 2 \cdot {}^nC_2 + \dots + n \cdot {}^nC_n)$

$= kn\left[1 + (n-1) + \frac{(n-1)(n-2)}{2!} + \dots + 1\right]$

$= kn2^{n-1}$

$\therefore$ Mean, $\bar{x} = \frac{1}{2^n}(n \cdot 2^{n-1}) = \frac{n}{2}$

11 Given, $\sigma = 9$

Let a student obtains x marks out of 75. Then, his marks out of 100 are $\frac{4x}{3}$. Each observation is multiply by $\frac{4}{3}$.

$\therefore$ New SD, $\sigma = \frac{4}{3} \times 9 = 12$

Hence, variance is $\sigma^2 = 144$.

12 Given that, $\Sigma x_i^2 = 400$ and $\Sigma x_i = 80$, since $\sigma^2 \geq 0$

$\Rightarrow \quad \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \geq 0$

$\Rightarrow \quad \frac{400}{n} - \frac{6400}{n^2} \geq 0$

$\therefore \quad n \geq 16$

13 Mean $= \frac{\underbrace{(a + a + \dots + n \text{ times}) + (-a - a - \dots - n \text{ times})}}{2n}$

$= 0$

$\therefore \quad \text{SD} = \sqrt{\frac{(x_i - \bar{x})^2}{2n}}$

$= \sqrt{\frac{a^2 + a^2 + \dots + 2n \text{ times}}{2n}}$

$2 = \sqrt{\frac{(a)^2\, 2n}{2n}} = |a|$

14 Since, standard deviation is remain unchanged, if observations are added or subtracted by a fixed number

We have, $\sum_{i=1}^{9}(x_1 - 5) = 9$

and $\quad \sum_{i=1}^{9}(x_1 - 5)^2 = 45$

$SD = \sqrt{\frac{\sum_{i=1}^{9}(x_1 - 5)^2}{9} - \left(\frac{\sum_{i=1}^{9}(x_1 - 5)}{9}\right)^2}$

$\Rightarrow \quad SD = \sqrt{\frac{45}{9} - \left(\frac{9}{9}\right)^2}$

$\Rightarrow \quad SD = \sqrt{5-1} = \sqrt{4} = 2$

15 Let n_1 and n_2 be the number of observations in two distributions having means $\bar{x}_1$ and $\bar{x}_2$ respectively. Then,

$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

Now consider,

$\bar{x} - \bar{x}_1 = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} - \bar{x}_1$

$= \frac{n_2(\bar{x}_2 - \bar{x}_1)}{n_1 + n_2} > 0 \quad [\because \bar{x}_2 > \bar{x}_1]$

$\Rightarrow \quad \bar{x} > \bar{x}_1 \qquad \dots(\text{i})$

Similarly, $\bar{x} - \bar{x}_2 = \frac{n_1(\bar{x}_1 - \bar{x}_2)}{n_1 + n_2} < 0$

$\Rightarrow \quad \bar{x} < \bar{x}_2 \qquad \dots(\text{ii})$

From Eqs. (i) and (ii), we get

$\bar{x}_1 < \bar{x} < \bar{x}_2$.