CBSE Test Paper 04

Chapter 9 Differential Equations

1. The order of the equation $\frac{d^3y}{dx^3} + x^2(\frac{d^2y}{dx^2})^3 = 0$ is
    a. 2
    b. 3
    c. 4
    d. 1
2. General solution of $sec^2 xtany\, dx + sec^2 y\, tanxdy = 0$ is
    a. $y = tanx^2 tany + C$
    b. y = tan2x tany + C
    c. y = tanx tan2y + C
    d. tanx tany = C
3. To form a differential equation from a given function
    a. Differentiate the function once and add values to arbitrary constants
    b. Differentiate the function successively as many times as the number of arbitrary constants
    c. Differentiate the function twice and eliminate the arbitrary constants
    d. Differentiate the function once and eliminate the arbitrary constants
4. Find the particular solution for $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$; y = 0 when x = 1.
    a. $\cos\left(\frac{y}{x}\right) = \log|2ex|$
    b. $\cos\left(\frac{y}{2x}\right) = \log|3ex|$
    c. $\cos\left(\frac{y}{x}\right) = \log|ex|$
    d. $\cos\left(\frac{2y}{x}\right) = \log|ex|$
5. A homogeneous equation of the form $\frac{dy}{dx} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution

a. y = vx
b. x = vy
c. v = yx
d. x = v

6. The solution of $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ is ________.
7. General solution of $\frac{dy}{dx} + y = sinx$ is ________.
8. The number of arbitrary constants in the general solution of a differential equation of order three is ________.
9. Find the differential equation representing the family of curves $V = \frac{A}{r} + B$ where A and B are arbitrary constants.
10. Verify that the function is a solution of the corresponding diff equation, $x + y = \tan^{-1}y$; $y^2y^{'} + y^2 + 1 = 0$.
11. Verify that the function is a solution of the corresponding diff eq. $x + y = \tan^{-1}y$; $y^2y^1 + y^2 + 1 = 0$.
12. Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$.
13. Form a differential equation representing the given family of curve by elimination arbitrary Constants a and b. $y = ae^{3x} + be^{-2x}$.
14. Solve the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$.
15. Form the differential equation by eliminating A and B in $Ax^2 + By^2 = 1$.
16. Find the equation of a curve passing through origin and satisfying the differential equation $\left(1 + x^2\right)\frac{dy}{dx} + 2xy = 4x^2$.
17. Solve $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0$; $y = \pi/4$, when x = 1.
18. Find the particulars solution of differential equation: $(1 + x^2)dy + 2xy\,dx = \cot x\,dx$

Solution

1. b. 3

   Explanation: Since the highest derivative term is $\frac{d^3y}{dx^3}$ hence the order is 3.

2. d. tanxtany = C

   Explanation: $\sec^2 x\tan y dx+\sec^2 y \,\text{tanx}\, dy=0$

   $\sec^2 x\tan y dx=-\sec^2 y\,\text{tanx}\, dy$

   $\int\frac{\sec^2 x}{\tan x}dx=-\int\frac{\sec^2 y}{\tan y}dy$

   $\ln\ tanx=-\ln\ tany+\ lnc$

   Since $\int\frac{f'(x)}{f(x)}dx=\ln\ f(x)+c$

   $\ln\ tanx+\ln\ tany=ln\ c$

   $ln(\ tanx+\ tany)=\ lnc$

   $tanx\ tany=c$

3. b. Differentiate the function successively as many times as the number of arbitrary constants

   Explanation: We shall differentiate the function equal to the number of arbitrary constant so that we get equations equal to arbitrary constant and then eliminate them to form a differential equation

4. c. $\cos\left(\frac{y}{x}\right)=\ \log|ex|$

   Explanation: Let y = vx $\frac{dy}{dx}=v+x\frac{dv}{dx}$

   Question becomes $v+x\frac{dv}{dx}=v-cosecv$

   $x\frac{dv}{dx}=-cosecv$

   $-\int sinvdv=\int\frac{dx}{x}$

   $cosv=logx+logc$

   $cos(\frac{y}{x})=logx+logc$

   when x = 1 and y = 0

   $cos(\frac{0}{1})=log1+logc$ { log c =1 }

   $c=e$

   $cos(\frac{y}{x})=logx+loge$

$cos(\frac{y}{x}) = log|ex|$

5. b. $x = \ \nu y$

Explanation: A homogeneous equation of the form $\frac{dy}{dx} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution x= vy.so that it becomes variable separable form and integration is then possible

6. $3y(1 + x^2) = 4x^3 + c$
7. $y = ce^{-x} + \frac{\sin\ x}{2} - \frac{\cos\ x}{2}$
8. 3
9. According to the question,the family of curves is given by,

   $V = \frac{A}{r} + B$, where A and B are arbitrary constants.

   On differentiating both sides w.r.t. r, we get

   $\frac{dV}{dr} = \frac{-A}{r^2} + 0 \Rightarrow \frac{dV}{dr} = \frac{-A}{r^2}$ ...(i)

   Now, again differentiating both sides w.r.t. r, we get

   $\frac{d^2V}{dr^2} = \frac{2A}{r^3}$

   $\Rightarrow \quad \frac{d^2V}{dr^2} = \frac{2}{r^3} \times \left(-r^2 \frac{dV}{dr}\right)$ [from Eq. (i)]

   $\Rightarrow \quad \frac{d^2V}{dr^2} = -\frac{2}{r}\frac{dV}{dr}$

   Thus, the required differential equation is

   $\frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = 0.$
10. $x + y = \tan^{-1}y$

    Differentiating w.r.t x,we get,

    $1 + y' = \frac{1}{1+y^2}y'$

    $\Rightarrow 1 + y^2 + y' + y'y^2 = y'$

    $\Rightarrow 1 + y^2 + y'y^2 = 0$

    Hence Proved.
11. $x + y = \tan^{-1}y$

    $1 + y^1 = \frac{1}{1+y^2}y^1$.

    $(1 + y^1)(1 + y^2) = y^1$

    $1 + y^2 + y^1 + y^1y^2 = y^1$

    $1 + y^2 + y^1y^2=0$

    $1 + y^2 + y^2\frac{dy}{dx} = 0$

proved.

12. The equation is of the type $\frac{dy}{dx}+Py=Q$, which is a linear differential equation.

Now I.F.= $\int \frac{1}{x}dx=e^{\log x}=x.\left(e^{\log f(x)}=f(x)\right)$

Therefore, solution of the given differential equation is

$y.x=\int xx^2dx \Rightarrow yx=\frac{x^4}{4}+c$

Hence, $y=\frac{x^3}{4}+\frac{c}{x}$

13. $y=ae^{3x}+be^{-2x}$

$\Rightarrow \frac{dy}{dx}=a3e^{3x}-2be^{-2x}$ (i)

$\Rightarrow \frac{d^2y}{dx^2}=9ae^{3x}+4be^{-2x}$ (ii)

(ii) - (i),we get,

$\frac{d^2y}{dx^2}-\frac{dy}{dx}=6(ae^{3x}+be^{-2x})$

$\Rightarrow \frac{d^2y}{dx^2}-\frac{dy}{dx}=6y$

14. $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$

$\int \frac{dy}{\sqrt{1-y^2}}=\int \frac{-dx}{\sqrt{1-x^2}}$

$\sin^{-1}(y')+\sin^{-1}(x)=c$

15. Given equation is $Ax^2 + By^2 = 1$

On differentiating both sides w.r.t. x, we get

$2Ax+2By\frac{dy}{dx}=0$

$\Rightarrow 2By\frac{dy}{dx}=-2Ax$

$\Rightarrow By\frac{dy}{dx}=-Ax \Rightarrow \frac{y}{x}.\frac{dy}{dx}=-\frac{A}{B}$

Again, differentiating w.r.t. x, we get

$\frac{y}{x}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\left(\frac{x\frac{dy}{dx}-y}{x^2}\right)=0$

$\Rightarrow \frac{y}{x}\cdot\frac{d^2y}{dx^2}+\frac{x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right)}{x^2}=0$

$\Rightarrow xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right)=0$

$\Rightarrow xyy''+x(y')^2-yy'=0$

16. Given that, $\left(1+x^2\right)\frac{dy}{dx}+2xy=4x^2$

Dividing both sides by $\left(1+x^2\right)$

$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{4x^2}{1+x^2}$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$P = \frac{2x}{1+x^2}, Q = \frac{4x^2}{1+x^2}$

$\therefore I.F = e^{\int Pdx} = e^{\int \frac{2x}{1+x^2}dx}$

Put $1 + x^2 = t \Rightarrow 2xdx = dt$

$I.F = 1 + x^2 = e^{\int \frac{dt}{t}} = e^{\log t} = e^{\log(1+x^2)} = 1 + x^2$

The general solution is

$y.\left(1 + x^2\right) = \int \frac{4x^2}{1+x^2}\left(1 + x^2\right)dx + C$

$\Rightarrow y.\left(1 + x^2\right) = \int 4x^2dx + C$

$\Rightarrow y.\left(1 + x^2\right) = 4\frac{x^3}{3} + C$ ...(i)

Since, the curve passes through origin, then substituting

x = 0 and y = 0 in Eq. (i), we get

C = 0

The required equation of curve is

$y\left(1 + x^2\right) = \frac{4x^3}{3}$

$\Rightarrow y = \frac{4x^3}{3(1+x^2)}$

17. $\left\{x\sin^2\left(\frac{y}{x}\right) - y\right\}dx + xdy = 0$

$\sin^2\left(\frac{y}{x}\right) - \frac{y}{x} + \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right)$ .....(i)

let y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

put $\frac{dy}{dx}$ in eq (i)

$v + x\frac{dv}{dx} = v - \sin^2 v$

$\int \cos ec^2 v dv = \int -\frac{dx}{x}$

-cot v = -log x + c

log x - cot v = c

$\log x - \cot\left(\frac{y}{x}\right) = c$

When x = 1, $y = \frac{\pi}{4}$

c = -1

$\log x - \cot\left(\frac{y}{x}\right) = -1$

$\log x - \cot\left(\frac{y}{x}\right) = -log\, e$

$\log ex = \cot\left(\frac{y}{x}\right)$

18. $(1+x^2)dy + 2xy\,dx = \cot x\,dx$

$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{\cot x}{1+x^2}$

Given differential equation is of the form, $\frac{dy}{dx} + py = Q$

$P = \frac{2x}{1+x^2}, Q = \frac{\cot x}{1+x^2}$

$I.F = e^{\int p\,dx}$

$= e^{\int \frac{2x}{1+x^2}}$

$= 1 + x^2$

Solution is,

$y \times (1+x^2) = \int \frac{\cot x}{1+x^2} \times \left(1+x^2\right) dx + c$

$\Rightarrow y(1+x^2) = \log(\sin x) + c$

$\Rightarrow y = \frac{\log(\sin x)}{1+x^2} + \frac{c}{1+x^2}$