PRACTICE SET -3

1. Let $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 1 \end{cases}$. Then for all x, $f\{g(x)\}$ is equal to:

   a. x b. 1 c. $f(x)$ d. $g(x)$

2. If $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots \text{to } \infty}}}$ then

   a. x is an irrational number b. $2 < x < 3$
   c. $x = 3$ d. None of these

3. If $z + z^{-1} = 1$, then $z^{100} + z^{-100}$ is equal to

   a. i b. – I c. 1 d. – 1

4. The inverse of $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$ is

   a. $\frac{-1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ b. $\frac{-1}{8}\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$
   c. $\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ d. $\frac{1}{8}\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

5. The sum of 100 terms of the series $.9 + .09 + .009 \ldots\ldots$ will be

   a. $1 - \left(\frac{1}{10}\right)^{100}$ b. $1 + \left(\frac{1}{10}\right)^{100}$
   c. $1 - \left(\frac{1}{10}\right)^{106}$ d. $1 + \left(\frac{1}{10}\right)^{100}$

6. If n is an integer greater than 1, then $a - {}^nC_1(a-1) + {}^nC_2(a-2) + \ldots + (-1)^n (a-n) =$

   a. a b. 0
   c. a^2 d. 2^n

7. The sum of $\frac{2}{1!} + \frac{6}{2!} + \frac{12}{3!} + \frac{20}{4!} + \ldots$ is

   a. $\frac{3e}{2}$ b. e c. 2e d. 3e

8. If $\pi < \alpha < \frac{3\pi}{2}$, then $\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} + \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} =$

   a. $\frac{2}{\sin\alpha}$ b. $-\frac{2}{\sin\alpha}$
   c. $\frac{1}{\sin\alpha}$ d. $-\frac{1}{\sin\alpha}$

9. Find real part of $\cos^{-1}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)$

   a. $\pi/3$ b. $\pi/4$
   c. $\log\left(\frac{\sqrt{3}-1}{2}\right)$ d. None of these

10. If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{when } x \neq 0 \\ 2, & \text{when } x = 0 \end{cases}$ then

   a. $\lim_{x \to 0+} f(x) \neq 2$ b. $\lim_{x \to 0-} f(x) = 0$
   c. $f(x)$ is continuous at $x = 0$ d. None of these

11. If $x^y = e^{x-y}$, then $\frac{dy}{dx} =$

   a. $\log x[\log(ex)]^{-2}$ b. $\log x.[\log(ex)]^2$
   c. $\log x.(\log x)^2$ d. None of these

12. 20 is divided into two parts so that product of cube of one quantity and square of the other quantity is maximum. The parts are

   a. 10, 10 b. 16, 4
   c. 8, 12 d. 12, 8

13. $\int \frac{1}{\sqrt{x}} \tan^4 \sqrt{x} \sec^2 \sqrt{x}\, dx =$

   a. $2\tan^5\sqrt{x} + c$ b. $\frac{1}{5}\tan^5\sqrt{x} + c$
   c. $\frac{2}{5}\tan^5\sqrt{x} + c$ d. None of these

14. The area bounded by the straight lines $x = 0$, $x = 2$ and the curves $y = 2^x$, $y = 2x - x^2$ is

   a. $\frac{4}{3} - \frac{1}{\log 2}$ b. $\frac{3}{\log 2} + \frac{4}{3}$
   c. $\frac{4}{\log 2} - 1$ d. $\frac{3}{\log 2} - \frac{4}{3}$

15. The solution of $\frac{dy}{dx} = e^x(\sin x + \cos x)$ is

   a. $y = e^x(\sin x - \cos x) + c$
   b. $y = e^x(\cos x - \sin x) + c$
   c. $y = e^x \sin x + c$
   d. $y = e^x \cos x + c$

16. The product of the perpendiculars drawn from the points $(\pm\sqrt{a^2-b^2},0)$ on the line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$, is

a. a^2 b. b^2

c. a^2+b^2 d. a^2-b^2

17. Area of the circle in which a chord of length $\sqrt{2}$ makes an angle $\frac{\pi}{2}$ at the centre is

a. $\frac{\pi}{2}$ b. 2π

c. π d. $\frac{\pi}{4}$

18. The focal chord to $y^2=16x$ is tangent to $(x-6)^2+y^2=2$, then the possible values of the slope of this chord, are

a. $\{-1,1\}$ b. $\{-2,2\}$

c. $\{-2,1/2\}$ d. $\{2,-1/2\}$

19. If $\vec{a},\vec{b},\vec{c}$ are non-zero vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, then which statement is true

a. $\vec{b}=\vec{c}$ b. $\vec{a}\perp(\vec{b}-\vec{c})$

c. $\vec{b}=\vec{c}$ or $\vec{a}\perp(\vec{b}-\vec{c})$ d. None of these

20. The point of intersection of the lines $\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1},\frac{x+3}{-36}=\frac{y-3}{2}=\frac{z-6}{4}$ is

a. $\left(21,\frac{5}{3},\frac{10}{3}\right)$ b. $(2,10,4)$

c. $(-3,3,6)$ d. $(5,7,-2)$

21. If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is

a. 324 b. 341

c. 359 d. None of these

22. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently, equals

a. $\frac{1}{2}$ b. $\frac{7}{15}$

c. $\frac{2}{15}$ d. $\frac{1}{3}$

23. A house of height 100 metres subtends a right angle at the window of an opposite house. If the height of the window be 64 metres, then the distance between the two houses is

a. 48 m b. 36 m c. 54 m d. 72 m

24. If ${}^{12}P_r=11880$, then $\sum_{i=1}^{\lambda}{}^{\lambda}C_i$ must be (where $\lambda=r+3$) = ?

a. 127 b. 227

c. 327 d. 627

25. ABCD is a rectangular field. A vertical lamppost of height 12m stands at the corner A. If the angle of elevation of its top from B is 60° and from C is 45°, then the area of the field is

a. $48\sqrt{2}$ sq.m b. $48\sqrt{3}$ sq.m

c. 48 sq.m d. $12\sqrt{2}$ sq.m

26. The following integral $\int_{\pi/4}^{\pi/2}(2\operatorname{cosec} x)^{17}dx$ is equal to

a. $\int_0^{\log(1+\sqrt{2})}2(e^u+e^{-u})^{16}du$ b. $\int_0^{\log(1+\sqrt{2})}(e^u+e^{-u})^{17}du$

c. $\int_0^{\log(1+\sqrt{2})}(e^u-e^{-u})^{17}du$ d. $\int_0^{\log(1+\sqrt{2})}2(e^u-e^{-u})^{16}du$

27. $\int\frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx$ is equal to

a. $\frac{\sqrt{2x^4-2x^2+1}}{x^2}+c$ b. $\frac{\sqrt{2x^4-2x^2+1}}{x^3}+c$

c. $\frac{\sqrt{2x^4-2x^2+1}}{x}+c$ d. $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+c$

28. If y (x) satisfies the differential equation $y'-y\tan x=2x\sec x$ and y(0) = 0, then.

a. $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$ b. $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$

c. $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$ d. $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

29. Given an isosceles triangle, whose one angle is 120° and radius of its incircle $=\sqrt{3}$. Then the area of the triangle in sq. units is

a. $7+12\sqrt{3}$ b. $12-7\sqrt{3}$

c. $12+7\sqrt{3}$ d. 4π

30. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point

a. $\left(-\frac{3}{2}, 0\right)$ b. $\left(-\frac{5}{2}, 2\right)$ c. $\left(-\frac{3}{2}, \frac{5}{2}\right)$ d. (–4, 0)

31. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x -axis at Q. If M is the mid -point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points

a. $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ b. $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$

c. $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ d. $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

32. Let $f:(-1,1) \to IR$ be such that $f(\cos 4\theta) = \frac{2}{2-\sec^2\theta}$ for $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{4},\frac{\pi}{2}\right)$, then the values of $f\left(\frac{1}{3}\right)$ is(are)

a. $1-\sqrt{\frac{3}{2}}$ b. $1+\sqrt{\frac{3}{2}}$ c. $1-\sqrt{\frac{2}{3}}$ d. $1+\sqrt{\frac{2}{3}}$

33. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to

a. $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ b. $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$

c. $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$ d. $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)}$

34. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as

a. $\sin A\cos A + 1$ b. sec A cosec A + 1

c. tan A + cot A d. sec A + cosec A

35. In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are)

a. 16 b. 18 c. 24 d. 22

36. For $x \in (0,\pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

a. infinitely many solutions b. three solutions

c. one solution d. no solution

37. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$, where $x \in R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals:

a. $\frac{1}{6}$ b. $\frac{1}{3}$ c. $\frac{1}{4}$ d. $\frac{1}{12}$

38. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is:

a. 3 b. 5

c. 7 d. 9

39. Let $S = \left\{x \in (-\pi, \pi) : x \ne 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \text{cosec } x + 2(\tan x - \cot x) = 0$ in the set S is equal to

a. $-\frac{7\pi}{9}$ b. $-\frac{2\pi}{9}$ c. 0 d. $\frac{5\pi}{9}$

40. Match the statements/expressions in Column I with the open intervals in Column II

Column I	Column II
(A) Interval contained in the domain of definition of non -zero solutions of the differential equation $(x-3)^2 y' + y = 0$	1. $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
(B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)\,dx$	2. $\left(0,\frac{\pi}{2}\right)$
(C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	3. $\left(\frac{\pi}{8},\frac{5\pi}{4}\right)$
(D) Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing	4. $\left(0,\frac{\pi}{8}\right)$
	5. $(-\pi,\pi)$

a. A→1,2,4; B→1,2,3,5; C→2,3; D→2,3

b. A→1,3; B→2,1; C→2; D→1,2,3,5

c. A→1,2,4; B→1,5; C→1,2,3,5; D→4

d. A→1,2; B→1,4; C→2,3; D→3,4

41. Match the statements in Column I with the properties Column II.

Column I	Column II
(A) Two intersecting circles	1. have a common tangent
(B) Two mutually external circles	2. have a common normal
(C) Two circles, one strictly inside the other	3. do not have a common tangent
(D) Two branches of a hyperbola	4. do not have a common normal

a. A→1,2; B→1,2; C→2,3; D→2,3
b. A→1,3; B→2,1; C→2; D→3
c. A→1,2,3; B→2; C→ 3; D→4
d. A→1,2; B→1,4; C→2,3; D→3,4

42. Match the statements given in Column I with the interval/union of intervals given in Column II

Column I	Column II
(A) The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number}, \lvert z\rvert = 1, z \neq \pm 1\right\}$ is	1. $(-\infty, -1) \cup (1, \infty)$
(B) The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	2. $(-\infty, 0) \cup (0, \infty)$
(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta) : 0 \le \theta < \frac{\pi}{2}\right\}$ is	3. $[2, \infty)$
(D) If $f(x) = x^{3/2}(3x-10),\ x \ge 0$, then f(x) is increasing in	4. $(-\infty, -1] \cup [1, \infty)$
	5. $(-\infty, 0] \cup [2, \infty)$

a. A→4; B→2, 4; C→4; D→2 b. A→4; B→5; C→3; D→3
c. A→2; B→1, 4; C→3; D→4 d. A→1; B→2; C→5; D→4

43. Match the statements in Column-I with those in Column-II.
Note: Here z takes the values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z

Column I	Column II
(A) The set of points z satisfying $\lvert z - i\lvert z\rvert\rvert = \lvert z + i\lvert z\rvert\rvert$ is contained in or equal to	1. an ellipse with eccentricity $\frac{4}{5}$
(B) The set of points z satisfying $\lvert z+4\rvert + \lvert z-4\rvert = 10$ is contained in or equal to	2. the set of points z satisfying Im $z = 0$
(C) If $\lvert\omega\rvert = 2$, then the set of points $z = \omega - 1/\omega$ contained in or equal to	3. the set of points z satisfying $\lvert\text{Im } z\rvert \le 1$
(D) If $\lvert\omega\rvert = 1$, then the set of points $z = \omega + 1/\omega$ is contained in or equal to	4. the set of points z satisfying $\lvert\text{Re } z\rvert \le 1$
	5. the set of points z satisfying $\lvert z\rvert \le 3$

a. A→3; B→1, 4; C→1,5; D→2,5
b. A→4; B→5; C→1,5; D→3
c. A→2; B→2, 5, C→3; D→4
d. A→1; B→2; C→5; D→4

44. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals

a. $-\theta$ b. $\frac{\pi}{2} - \theta$
c. θ d. $\pi - \theta$

45. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = A A^T, then $5a + b$ is equal to:

a. –1 b. 5
c. 4 d. 13

46. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is :

a. $\sqrt{2}+\sqrt{3}$ b. $3+\sqrt{2}$
c. $2-\sqrt{3}$ d. $2+\sqrt{3}$

47. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of coefficients of all the terms in this expansion, is:

a. 64 b. 2187
c. 243 d. 729

48. Let T_n be the number of all possible triangles formed by joining vertices of a n-sides regular polygon. If $T_{n+1} - T_n = 1$ then the value of n is

a. 7 b. 5
c. 10 d. 8

49. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P(computer turns out to be defective given that it is produced in plant T_1) $= 10$ P (computer turns out to be defective given that it is produced in Plant T_2), where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected

and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

a. $\frac{36}{73}$ b. $\frac{47}{79}$

c. $\frac{78}{93}$ d. $\frac{75}{83}$

50. If $5(\tan^2 x - 2\cos^2 x) = 2\cos 2x + 9$, then the value of cos 4x is

a. $\frac{2}{9}$ b. $-\frac{7}{9}$

c. $-\frac{3}{5}$ d. $\frac{1}{3}$

Answers and Solutions

1. (b) $f(g(x)) = \begin{cases} -1 & g(x) < 0 \\ 0 & g(x) = 0 \\ 1 & g(x) > 0 \end{cases}$

Since $g(x) \ge 1 > 0$

Hence $g(g(x)) = 1$

2. (c) $x = \sqrt{6+x}$,

$x > 0 \Rightarrow x^2 = 6 + x, x > 0$

$\Rightarrow x^2 - x - 6 = 0, x > 0$

$\Rightarrow x = 3, x > 0.$

3. (d) $z + z^{-1} = 1$

$\Rightarrow z^2 - z + 1 = 0$

$\Rightarrow z = -\omega$ or $-\omega^2$

For $z = -\omega$, $z^{100} + z^{-100} = (-\omega)^{100} + (-\omega)^{-100}$

$= \omega + \frac{1}{\omega} = \omega + \omega^2 = -1$

For $z = -\omega^2$, $z^{100} + z^{-100} = (-\omega^2)^{100} + (-\omega^2)^{-100}$

$= \omega^{200} + \frac{1}{\omega^{200}} = \omega^2 + \frac{1}{\omega^2} = \omega^2 + \omega$

$= -1.$

4. (a) Let the matrix of cofactors of the elements of A viz.

$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 2 & -(-4) \\ -(-3) & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

$\therefore$ adj A = transpose of the matrix of cofactors of elements of $A = \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$

$\therefore$ $A(\text{adj } A) = |A| I$.

5. (a) Series is a G.P. with $a = 0.9 = \frac{9}{10}$ and $r = \frac{1}{10} = 0.1$

$\therefore S_{100} = a\left(\frac{1-r^{100}}{1-r}\right) = \frac{9}{10}\left(\frac{1-\frac{1}{10^{100}}}{1-\frac{1}{10}}\right) = 1 - \frac{1}{10^{100}}.$

6. (b) L.H.S. $= a[C_0 - C_1 + C_2 - C_3 + ... + (-1)^n C_n]$

$+ [C_1 - 2C_2 + 3C_3 - .... + (-1)^{n-1} nC_n] = a.0 + 0 = 0$

7. (d) Let $S = \frac{2}{1!} + \frac{6}{2!} + \frac{12}{3!} + \frac{20}{4!} + \ldots$ and let

$S_1 = 2 + 6 + 12 + 20 + \ldots + T_n$

$S_1 = \quad 2 + 6 + 12 + \ldots T_{n-1} + T_n$

$0 = 2 + 4 + 6 + 8 + \ldots \text{upto } n \text{ terms } - T_n$

$T_n = 2 + 4 + 6 + 8 + \ldots\ldots \text{upto } n \text{ terms}$

$\Rightarrow T_n = \frac{n}{2}[2 \times 2 + (n-1)2]$

$= n(2 + n - 1) = n(n+1)$

$\therefore$ n^{th} term of given series

$T_n = \frac{n(n+1)}{n!}$ or $T_n = \frac{n(n+1)}{n(n-1)!}$

or $T_n = \frac{1}{(n-2)!} + \frac{2}{(n-1)!}$

Now, sum $= \sum_{n=1}^{\infty} \frac{1}{(n-2)!} + 2\sum_{n=1}^{\infty} \frac{1}{(n-1)!} = e + 2e = 3e.$

8. (b) $\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} + \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \frac{1-\cos\alpha + 1 + \cos\alpha}{\sqrt{1-\cos^2\alpha}}$

$= \frac{2}{\pm\sin\alpha} = \frac{2}{-\sin\alpha}, \left(\text{since } \pi < \alpha < \frac{3\pi}{2}\right).$

9. (b) $\because$ Expression $\cos^{-1}(\cos\theta + i\sin\theta)$

$= \sin^{-1}\sqrt{\sin\theta} - i\log(\sqrt{\sin\theta} + \sqrt{1+\sin\theta})$, where $\theta = \frac{\pi}{6}$

$\therefore$ Real part of $\cos^{-1}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = \sin^{-1}\sqrt{\frac{1}{2}} = \frac{\pi}{4}.$

10. (c) $f(0+) = f(0-) = 2$

and $f(0) = 2$

Hence $f(x)$ is continuous at $x = 0$.

11. (a) $x^y = e^{x-y}$

$\Rightarrow \quad y \log x = x - y$

$\Rightarrow \quad y = \dfrac{x}{1+\log x}$

$\Rightarrow \quad \dfrac{dy}{dx} = \log x(1+\log x)^{-2} = \log x[\log (ex)]$.

12. (d) Let $x + y = 20 \Rightarrow y = 20 - x$

and $x^3.y^2 = z \Rightarrow z = x^3 y^2$

$z = x^3(20-x)^2$

$\Rightarrow \quad z = 400x^3 + x^5 - 40x^4$

$\dfrac{dz}{dx} = 1200x^2 + 5x^4 - 160x^3$

Now $\dfrac{dz}{dx} = 0$, then $x = 12, 20$

Now $\dfrac{d^2z}{dx^2} = 2400x + 16x^3 - 480x^2$;

$\left(\dfrac{d^2z}{dx^2}\right)_{x=12} = -ve$

Hence $x = 12$ is the point of maxima

$\therefore \quad x = 12, \; y = 8.$

13. (c) $\int \dfrac{1}{\sqrt{x}} \tan^4 \sqrt{x}.\sec^2 \sqrt{x}\, dx$

Put $\tan\sqrt{x} = t \Rightarrow \dfrac{\sec^2\sqrt{x}}{2\sqrt{x}} dx = dt$,

then it reduces to

$2\int t^4 dt = \dfrac{2}{5}(\tan\sqrt{x})^5 + c = \dfrac{2}{5}\tan^5\sqrt{x} + c$

14. (d) Required area = $\int_0^2 [2^x - (2x - x^2)]\, dx$

$= \left[\dfrac{2^x}{\log 2} - x^2 + \dfrac{x^3}{3}\right]_0^2$

$= \dfrac{4}{\log 2} - 4 + \dfrac{8}{3} - \dfrac{1}{\log 2}$

$= \dfrac{3}{\log 2} - \dfrac{4}{3}.$

15. (c) Given equation $\dfrac{dy}{dx} = e^x(\sin x + \cos x)$

$\Rightarrow \quad dy = e^x(\sin x + \cos x)\, dx$

On integrating, we get $y = e^x \sin x + c$

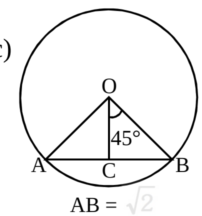
16. (b) $\left(\dfrac{b\sqrt{a^2-b^2}\cos\theta + 0 - ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}\right)\left(\dfrac{-b\sqrt{a^2-b^2}\cos\theta - ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}\right)$

$= \dfrac{-[b^2(a^2-b^2)\cos^2\theta - a^2b^2]}{(b^2\cos^2\theta + a^2\sin^2\theta)}$

$= \dfrac{b^2[a^2 - a^2\cos^2\theta + b^2\cos^2\theta]}{b^2\cos^2\theta + a^2\sin^2\theta}$

$= \dfrac{b^2[a^2\sin^2\theta + b^2\cos^2\theta]}{b^2\cos^2\theta + a^2\sin^2\theta} = b^2$

17. (c)



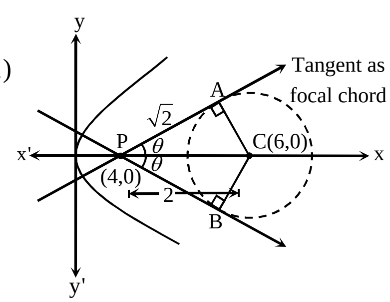
Let AB be the chord of length $\sqrt{2}$, O be centre of the circle and let OC be the perpendicular from O on AB. Then

$AC = BC = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$

In Δ OBC, $OB = BC \operatorname{cosec} 45° = \dfrac{1}{\sqrt{2}}.\sqrt{2} = 1$

$\therefore$ Area of the circle = $\pi(OB)^2 = \pi$

18. (a)



Here, the focal chord of $y^2 = 16x$ is tangent to circle $(x-6)^2 + y^2 = 2.$

$\Rightarrow$ focus of parabola as $(a,0)$ i.e., $(4,0)$

Now, tangents are drawn from $(4,0)$ to $(x-6)^2 + y^2 = 2.$

Since, PA is tangent to circle.

$\therefore \quad \tan\theta$ = slope of tangent $= \dfrac{AC}{AP} = \dfrac{\sqrt{2}}{\sqrt{2}} = 1$, or $\dfrac{BC}{BP} = -1$

$\therefore$ Slope of focal chord as tangent to circle $= \pm 1.$

19. (c) $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} \Rightarrow \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c} = 0 \Rightarrow \vec{a}\cdot(\vec{b}-\vec{c}) = 0$

$\Rightarrow$ Either $\vec{b}-\vec{c} = 0$ or $\vec{a} = 0 \Rightarrow \vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b}-\vec{c})$.

20. (a) Given lines are,

$$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1} = r_1, \text{ (say)}$$

and $\dfrac{x+3}{-36} = \dfrac{y-3}{2} = \dfrac{z-6}{4} = r_2$, (say)

$\therefore$ $x = 3r_1 + 5 = -36r_2 - 3$,

$y = -r_1 + 7 = 3 + 2r_2$

and $z = r_1 - 2 = 4r_2 + 6$

On solving, we get $x = 21,\ y = \dfrac{5}{3},\ z = \dfrac{10}{3}$

Trick: Check through options.

21. (a) Words starting from A are $5! = 120$

Words starting from I are $5! = 120$

Words starting from KA are $4! = 24$

Words starting from KI are $4! = 24$

Words starting from KN are $4! = 24$

Words starting from KRA are $3! = 6$

Words starting from KRIA are $2! = 2$

Words starting from KRIN are $2! = 2$

Words starting from KRISA are $1! = 1$

Words starting from KRISNA are $1! = 1$

Hence rank of the word KRISNA is 324.

22. (b) The number of ways of placing 3 black balls without any restriction is ${}^{10}C_3$. Since, we have total 10 places of putting 10 balls in a row. Now the number of ways in which no two black balls put together is equal to the number of ways of choosing 3 places marked '–' out of eight places.

– W – W – W – W – W – W – W –

This can be done in ${}^{8}C_3$ ways.

$\therefore$ Required probability $= \dfrac{{}^{8}C_3}{{}^{10}C_3} = \dfrac{8\times7\times6}{10\times9\times8} = \dfrac{7}{15}$

23. (a) $64\cot\theta = d$

Also $(100-64)\tan\theta = d$

or $(64)(36) = d^2$,

$\therefore$ $d = 8\times6 = 48$ m.

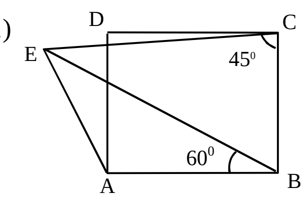
24. (a) $\because {}^{12}P_r = 12\times11\times10\times9 = {}^{12}P_4$

$\therefore$ $r = 4$

Then, $\lambda = r + 3 = 7$

$\therefore$ $\displaystyle\sum_{i=1}^{7} {}^{7}C_i = 2^7 - 1 = 127$

25. (a)



Let AE is a vertical lamppost. Given, $AE = 12$m

$\tan 45° = \dfrac{AE}{AC}$

$AC = AE = 12$m

$\tan 60° = \dfrac{AE}{AB}$

$AB = \dfrac{AE}{\sqrt{3}} = 4\sqrt{3}$

$BC = \sqrt{AC^2 - AB^2} = \sqrt{144-48} = \sqrt{96} = 4\sqrt{6}$

Area $= AB\times BC = 4\sqrt{3}\times4\sqrt{6} = 48\sqrt{2}$ sq.cm.

26. (a) $\displaystyle\int_{\pi/4}^{\pi/2} (2\operatorname{cosec} x)^{17}\,dx$

Let $e^u + e^{-u} = 2\operatorname{cosec} x$, $x = \dfrac{\pi}{4}$

$\Rightarrow$ $u = \ln(1+\sqrt{2}),\ x = \dfrac{\pi}{2}$

$\Rightarrow$ $u = 0$

$\Rightarrow$ $\operatorname{cosec} x + \cot x = e^u$ and $\operatorname{cosec} x - \cot x = e^{-u}$

$\Rightarrow$ $\cot x = \dfrac{e^u - e^{-u}}{2}$ $(e^u - e^{-u})du = -2\operatorname{cosec} x \cot x\,dx$

$\Rightarrow$ $-\displaystyle\int (e^u + e^{-u})^{17}\,\frac{(e^u - e^{-u})}{2\operatorname{cosec} x \cot x}\,du$

$= -2\displaystyle\int_{\ln(1+\sqrt{2})}^{0} (e^u + e^{-u})^{16}du$

$= \displaystyle\int_{0}^{\ln(1+\sqrt{2})} 2(e^u + e^{-u})^{16}du$

27. (d) $\int \frac{\left(\frac{1}{x^3}-\frac{1}{x^5}\right)dx}{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}$

Let $2-\frac{2}{x^2}+\frac{1}{x^4}=z$

$\Rightarrow \frac{1}{4}\int\frac{dz}{z}$

$\Rightarrow \frac{1}{2}\times\sqrt{z}+c$

$\Rightarrow \frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+c$

28. (a, d) $y'-y\tan x = 2x\sec x$

$\text{I.F.}=e^{\int \tan x\, dx}=e^{\log\cos x}=\cos x$

$\therefore\ y\cos x=\int 2x\sec x\cos x\, dx$

$\Rightarrow y\cos x = x^2+c$

$\Rightarrow y\cdot\cos x = x^2 \quad (\because y(0)=0) \Rightarrow y=x^2\sec x$

$\therefore\ y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$ and $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

29. (c) $\triangle=\frac{\sqrt{3}}{4}b^2$ . . .(i)

Also $\frac{\sin 120^\circ}{a}=\frac{\sin 30^\circ}{b}$

$\Rightarrow a=\sqrt{3}b$ and $\triangle=\sqrt{3}s$ and $s=\frac{1}{2}(a+2b)$

$\Rightarrow \triangle=\frac{\sqrt{3}}{2}(a+2b)$ . . .(ii)

From equation (i) and (ii), we get $\triangle=(12+7\sqrt{3})$

30. (d) Circle touching y-axis at (0, 2) is

$(x-0)^2+(y-2)^2+\lambda x=0$

Passes through (–1, 0)

$\therefore\ 1+4-\lambda=0 \Rightarrow \lambda=5$

$\therefore\ x^2+y^2+5x-4y+4=0$

Put y = 0

$\Rightarrow x=-1,-4$

$\therefore$ Circle passes through (–4, 0)

31. (a) Any point on the line can be taken as

$Q\equiv\{(1-3\mu),(\mu-1),(5\mu+2)\}$

$\overline{PQ}=\{-3\mu-2,\ \mu-3, 5\mu-4\}$

Now $1(-3\mu-2)-4(\mu-3)+3(5\mu-4)=0$

$\Rightarrow -3\mu-2-4\mu+12+15\mu-12=0$

$8\mu=2$

$\Rightarrow \mu=1/4$

$>\left|\frac{-3+\sqrt{3}\times 1}{2}\right|>\frac{3-\sqrt{3}}{2}$

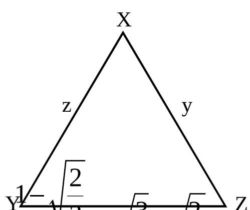
32. (a, b) $f(\cos\theta)=\frac{2}{2-\sec^2\theta}$

$f(\cos\theta)=\frac{2\cos^2\theta}{\cos 2\theta}=\frac{1+\cos 2\theta}{\cos 2\theta}$

$\cos 4\theta=\frac{1}{3}; \quad 2\cos^2 4\theta-1=\frac{1}{3}$

Or $\cos 2\theta=\pm\sqrt{\frac{2}{3}}$

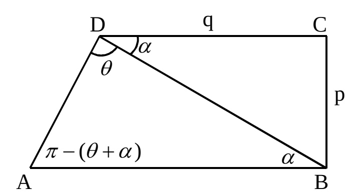
$\Rightarrow f\left(\frac{1}{3}\right)=\frac{1+\sqrt{\frac{2}{3}}}{\sqrt{\frac{2}{3}}}=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}}$



$\Rightarrow f\left(\frac{1}{3}\right)=1+\sqrt{\frac{3}{2}}$ and $f\left(\frac{1}{3}\right)=\frac{1-\sqrt{\frac{2}{3}}}{-\sqrt{\frac{2}{3}}}=\frac{\sqrt{3}-\sqrt{2}}{-\sqrt{2}}$

Or $f\left(\frac{1}{3}\right)=1-\sqrt{\frac{3}{2}}$

33. (a) $BD=\sqrt{p^2+q^2}$



$\angle ABD=\angle BDC=\alpha$

$\Rightarrow \angle DAB=\pi-(\theta+\alpha)\quad \tan\alpha=\frac{p}{q}\quad \Delta ABD$

$\frac{AB}{\sin\theta}=\frac{BD}{\sin(\pi-(\theta+\alpha))}=\frac{BD}{\sin(\theta+\alpha)}$

$\therefore\ AB=\frac{BD\sin\theta}{\sin(\theta+\alpha)}=\frac{BD^2\sin\theta}{BD\sin(\theta+\alpha)}$

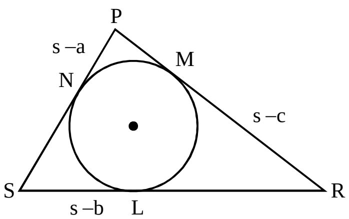
$=\frac{BD^2\sin\theta}{BD\sin\theta\cos\alpha+BD\cos\theta\sin\alpha}=\frac{(p^2+q^2)\sin\theta}{q\sin\theta+p\cos\theta}$

34. (b) Exp. $=\frac{\tan^2 A}{\tan A-1}+\frac{1}{\tan A-\tan^2 A}$

$=\frac{1}{\tan A-1}\left[\tan^2 A-\frac{1}{\tan A}\right]$

$$= \frac{\tan^2 A + \tan A + 1}{\tan A} = \tan A + \cot A + 1 = \sec A \operatorname{cosec} A + 1$$

35. (b, d)



Let $s - a = 2k - 2,\ s - b = 2k,\ s - c = 2k + 2,\ k \ne 1,\ k > 1$

Adding we get, $s = 6k$

So, $a = 4k + 2,\ b = 4k,\ c = 4k - 2$

Now, $\cos P = \frac{1}{3}$

So, sides are 22, 20, 18

36. (d) $\sin x + 2\sin 2x - \sin 3x = 3$

$\sin x + 4\sin x\cos x - 3\sin x + 4\sin^3 x = 3$

$\sin x[-2 + 4\cos x + 4(1 - \cos^2 x)] = 3$

$\sin x[2 - (4\cos^2 x - 4\cos x + 1) + 1] = 3$

$\sin x[3 - (2\cos x - 1)^2] = 3$

$\Rightarrow \sin x = 1$ and $2\cos x - 1 = 0$

$\Rightarrow x = \frac{\pi}{2}$ and $x = \frac{\pi}{3}$

Which is not possible at same time. Hence, no solution.

37. (d) $f_k = \frac{1}{4}(\sin^k x + \cos^k x)$ $\qquad f_6(x) = \frac{1}{6}(\sin^6 x + \cos^6 x)$

$f_4(x) = \frac{1}{4}(\sin^4 x + \cos^4 x)$ $\qquad f_6 K = \frac{1}{6}\left[1 - \frac{3}{4}\sin^2 2x\right]$

$f_4(x) = \frac{1}{4}\left[1 = \frac{\sin^2 2x}{2}\right]$

$f_4(x) - f_6(x) = \left[\frac{1}{4} - \frac{\sin^2 2x}{8}\right] - \left[\frac{1}{6} - \frac{\sin^2 2x}{8}\right] = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

38. (c) $0 \le x < 2\pi$

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$(\cos x + \cos 4x) + (\cos 2x + \cos 3x) = 0$

$2\cos\frac{5x}{2}\cos\frac{3x}{2} + 2\cos\frac{5x}{2}\cos\frac{x}{2} = 0$

$2\cos\frac{5x}{2}\left[2\cos x\cos\frac{x}{2}\right] = 0 \quad \cos\frac{5x}{2} = 0$ or $\cos x = 0$

or $\cos\frac{x}{2} = 0 \quad x = \frac{(2n+1)\pi}{5}$

or $x = (2n+1)\frac{\pi}{2}$ or $x = (2n+1)\pi$

$x = \left\{\frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$

Number of solution is 7

39. (c) $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$

$\Rightarrow \sqrt{3}\sin x + \cos x + 2(\sin^2 x - \cos^2 x) = 0$

$\Rightarrow \sqrt{3}\sin x + \cos x - 2\cos 2x = 0$

$\Rightarrow \sin\left(x + \frac{\pi}{3}\right) = \cos 2x$

$\Rightarrow \cos\left(\frac{\pi}{3} - x\right) = \cos 2x$

$\Rightarrow 2x = 2n\pi \pm \left(\frac{\pi}{3} - x\right)$

$\Rightarrow x = \frac{2n\pi}{3} + \frac{\pi}{9}$ or $x = 2n\pi - \frac{\pi}{3}$

$\Rightarrow -100° - 60° + 20° + 140° = 0$

40. (c) A → 1,2,4; B → 1,5; C → 1,2,3,5; D → 4

(A) $(x-3)^2\frac{dy}{dx} + y = 0 \Rightarrow \int\frac{dx}{(x-3)^2} = -\int\frac{dy}{y}$

$\Rightarrow \frac{1}{x-3} = \ln|y| + c$ so domain is 3 – {3}

(B) Put $x = t + 3$

$\int_{-2}^{2}(t+2)(t+1)t(t-1)(t-2)dt =$

$\int_{-2}^{2}t(t^2-1)(t^2-4)dt = 0$ (being odd function)

(C) $f(x) = \frac{5}{4} - \left(\sin x - \frac{1}{2}\right)^2$

Maximum value occurs when $\sin x = \frac{1}{2}$

(D) $f'(x) > 0$ if $\cos x > \sin x$

41. (a) A → 1,2; B → 1,2; C → 2,3; D → 2,3

(A) When two circles are intersecting they have a common normal and common tangent.

(B) Two mutually external circles have a common normal and common tangent.

(C) When one circle lies inside of other then, they have a common normal but no common tangent.

(D) Two branches of a hyperbola have a common normal but no common tangent.

42. (b) A → 4; B → 5; C → 3; D → 3

(A) $z = \frac{2i(x+iy)}{1-(x+iy)^2} = \frac{2i(x+iy)}{1-(x^2-y^2+2ixy)}$

Using $1-x^2=y^2$

$$Z=\frac{2ix-2y}{2y^2-2ixy}=-\frac{1}{y}.$$

$\because \quad -1\le y\le 1$

$\Rightarrow \quad -\frac{1}{y}\le -1$

Or $\quad -\frac{1}{y}\ge 1.$

(B) For domain

$$-1\le \frac{8.3^{x-2}}{1-3^{2(x-1)}}\le 1$$

$\Rightarrow \quad -1\le \frac{3^x-3^{x-2}}{1-3^{2x-2}}\le 1.$

Case-I: $\frac{3^x-3^{x-2}}{1-3^{2x-2}}-1\le 0$

$\Rightarrow \quad \frac{(3^x-1)(3^{x-2}-1)}{(3^{2x-2}-1)}\ge 0$

$\Rightarrow \quad x\in(-\infty,0]\cup(1,\infty).$

Case-II: $\frac{3^x-3^{x-2}}{1-3^{2x-2}}+1\ge 0$

$\Rightarrow \quad \frac{(3^{x-2}-1)(3^x+1)}{(3^x.3^{x-2}-1)}\ge 0$

$\Rightarrow \quad x\in(-\infty,1)\cup[2,\infty).$

So, $x\in(-\infty,0]\cup[2,\infty).$

(C) $R_1\to R_1+R_3$

$$f(\theta)=\begin{vmatrix}0 & 0 & 2\\ -\tan\theta & 1 & \tan\theta\\ -1 & -\tan\theta & 1\end{vmatrix}$$

$=2(\tan^2\theta+1)=2\sec^2\theta$

(D) $f'(x)=\frac{3}{2}(x)^{1/2}(3x-10)+(x)^{3/2}3$

$=\frac{15}{2}(x)^{1/2}(x-2)$

Increasing, when $x\ge 2$.

43. (a) $A\to 3;\ B\to 1,4;\ C\to 1,5;\ D\to 2,5$

(A) (3) $\left|\frac{z}{|z|}-i\right|=\left|\frac{z}{|z|}+i\right|, z\ne 0$

$\frac{z}{|z|}$ is unimodular complex number and lies on perpendicular bisector of i and $-i$

$\Rightarrow \quad \frac{z}{|z|}=\pm 1\Rightarrow z=\pm 1|z|\Rightarrow$ a is number Im(z) = 0.

(B) (1) $|z+4|+|z-4|=10$

z lies on an ellipse whose focus are (4, 0) and (−4, 0) and length of major axis is 10

$\Rightarrow \quad 2ae=8$ and $2a=10\Rightarrow e=4/5\ |Re(z)|\le 5.$

(C) (2, 5) $|w|=2$

$\Rightarrow \quad w=2(\cos\theta+i\sin\theta)$

$$x+iy=2(\cos\theta+i\sin\theta)-\frac{1}{2}(\cos\theta-i\sin\theta)$$

$$=\frac{3}{2}\cos\theta+i\frac{5}{2}\sin\theta$$

$\Rightarrow \quad \frac{x^2}{(3/2)^2}+\frac{y^2}{(5/2)^2}=1$

$$e^2=1-\frac{9/4}{25/4}=1-\frac{9}{25}=\frac{16}{25}$$

$\Rightarrow \quad e=\frac{4}{5}$

(D) (1, 5) $|w|=1$

$\Rightarrow \quad x+iy=\cos+i\sin\theta+\cos\theta-i\sin\theta$

$x+iy=2\cos\theta$

$|Re(z)|\le 1, |Im(z)|=0.$

44. (c) Let $z=\omega$

Now $\frac{1+z}{1+\bar{z}}=\frac{1+\omega}{1+\omega^2}=\frac{-\omega^2}{-\omega}=\omega$

$\therefore \quad \arg\frac{1+z}{1+z}=\arg\omega=\theta$ (put $z=\cos\theta+i\sin\theta$)

45. (b) $|A|I=AA^T$

$$\Rightarrow (10a+3b)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}\begin{bmatrix}5a & 3\\ -b & 2\end{bmatrix}$$

$\Rightarrow \quad 25a^2+b^2=10a+3b\ \&\ 15a-2b=0\ \&\ 10a+3b=13$

$\Rightarrow \quad 10a+\frac{3.15a}{2}=13$

$\Rightarrow \quad 65a=2\times 13$

$\Rightarrow \quad a=\frac{2}{5}$

$\Rightarrow \quad 5a=2$

$\Rightarrow \quad 2a=6\Rightarrow b=3$

$\therefore \quad 5a+b=5$

46. (d) Let the numbers be a, ar, ar^2 is G.P.

Given $a, 2ar, ar^2$ are in A.P. the $2ar = \frac{a + ar^2}{2} (a \neq 0)$

which gives $r = 2 + \sqrt{3}$, as the G.P. is an increasing G.P.

47. (d) Theoretically the number of terms are 2N + 1 (i.e. odd). But as the number of terms being odd hence considering that number clubbing of terms is done hence the solutions follows:

Number of terms = ${}^{n+2}C_2 = 28$

$\therefore \quad n = 6$

Sum of coefficient = $3^n = 3^6 = 729$

Put $x = 1$

48. (b) ${}^{n+1}C_3 - {}^{n}C_3 = 10$

$\Rightarrow$ On solving n = 5

49. (c) Let $x = P$ (computer turns out to be defective given that it is produced in plate T_2),

$$\Rightarrow \frac{7}{100} = \frac{1}{5}(10x) + \frac{4}{5}x$$

$$\Rightarrow 7 = 200x + 80x$$

$$\Rightarrow x = \frac{7}{280}$$

P (produced in T_2 / not defective)

$$= \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \frac{\frac{4}{5(1-x)}}{\frac{1}{5}(1-10x) + \frac{4}{5}(1-x)} = \frac{\frac{4}{5}\left(\frac{273}{280}\right)}{\frac{1}{5}\left(\frac{280-70}{280}\right) + \frac{4}{5}\left(\frac{273}{280}\right)}$$

$$\Rightarrow \frac{4 \times 273}{210 + 4 \times 273} = \frac{2 \times 273}{105 + 2 \times 273} = \frac{546}{651} = \frac{78}{93}$$

50. (b) $5\left[\frac{1-t}{t} - t\right] = 2(2t - 1) + 9 \quad \{\text{Let } \cos^2 x = t\}$

$$\Rightarrow 5(1 - t - t^2) = t(4t + 7) \Rightarrow 9t^2 + 12t - 5 = 0$$

$$\Rightarrow 9t^2 + 15t - 3t - 5 = 0$$

$$\Rightarrow (3t - 1)(3t + 5) = 0$$

$$\Rightarrow t = \frac{1}{3} \text{ as } t \neq -\frac{5}{3} \quad \cos 2x = 2\left(\frac{1}{3}\right) - 1 = -\frac{1}{3}$$

$$\cos 4x = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$$

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