MODEL PAPER - II

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 100***

General Instructions

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and *C.* Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION A

Question number 1 to 10 carry one mark each.

1. Evaluate : $\int \frac{1}{x + x \log x} dx.$

2. Evaluate : $\int_0^1 \frac{1}{\sqrt{4x + 1}} dx.$

3. If the binary operation * defined on Q, is defined as $a * b = 2a + b - ab$, for all a, $b \in Q$, find the value of 3 * 4.

4. If $\begin{pmatrix} y + 2x & 5 \\ -x & 3 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ -2 & 3 \end{pmatrix}$, find the value of y.

5. Find a unit vector in the direction of $2\hat{i} - \hat{j} + 2\hat{k}$.

6. Find the direction cosines of the line passing through the following points:

(–2, 4, –5), (1, 2, 3)

7. If $A = [a_{ij}] = \begin{pmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{pmatrix}$ and $B = [b_{ij}] = \begin{pmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{pmatrix}$, then find $a_{22} + b_{21}$.

8. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.

9. If $A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}$, then find the value of k if $|2A| = k\,|A|$.

10. Write the principal value of $\tan^{-1}\left[\tan \frac{3\pi}{4}\right]$.

SECTION B

Question number 11 to 22 carry 4 marks each.

11. Evaluate : $\int \frac{\cos x\, dx}{(2 + \sin x)(3 + 4\sin x)}$.

OR

Evaluate : $\int x^2 \cos^{-1} x\, dx$.

12. Show that the relation R in the set of real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive, nor symmetric, nor transitive.

13. If $\log (x^2 + y^2) = 2 \tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx} = \frac{x + y}{x - y}$.

OR

If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2y}{dx^2}$.

14. Find the equation of the tangent to the curve $y = \sqrt{4x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

OR

Using differentials, find the approximate value of $f(2.01)$, where $f(x) = 4x^3 + 5x^2 + 2$.

15. Prove the following :

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right).$$

OR

Solve the following for x :

$$\cos^{-1}\left(\frac{x^2 - 1}{x^2 + 1}\right) + \tan^{-1}\left(\frac{2x}{x^2 - 1}\right) = \frac{2\pi}{3}.$$

16. Find the angle between the line $\frac{x + 1}{2} = \frac{3y + 5}{9} = \frac{3 - z}{-6}$ and the plane $10x + 2y - 11z = 3$.

17. Solve the following differential equation :

$$(x^3 + y^3)\, dy - x^2y\, dx = 0$$

18. Find the particular solution of the differential equation

$\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$, $(x \neq 0)$, given that $y = 1$ when $x = \frac{\pi}{2}$.

19. Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ba & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

20. The probability that A hits a target is $\frac{1}{3}$ and the probability that B hits it is $\frac{2}{5}$. If each one of A and B shoots at the target, what is the probability that

(i) the target is hit?

(ii) exactly one of them hits the target?

21. Find $\frac{dy}{dx}$, if $y^x + x^y = a^b$, where a, b are constants.

22. If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are vectors such that $\overrightarrow{a}.\overrightarrow{b} = \overrightarrow{a} \cdot \overrightarrow{c}$ and $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{a} \times \overrightarrow{c}, \overrightarrow{a} \neq \overrightarrow{0}$ then prove that $\overrightarrow{b} = \overrightarrow{c}$.

SECTION C

Question number 23 to 29 carry 6 marks each.

23. One kind of cake requires 200 g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as a linear programming problem and solve graphically.

24. Using integration, find the area of the region :

$$\{(x, y) : 9x^2 + y^2 \leq 36 \text{ and } 3x + y \geq 6\}$$

25. Show that the lines $\frac{x+3}{-3} - \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane containing the lines.

26. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

OR

Show that the total surface area of a closed cuboid with square base and given volume, is minimum, when it is a cube.

27. Using matrices, solve the following system of linear equations :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

28. Evaluate : $\int \frac{x^4 dx}{(x-1)(x^2+1)}$.

OR

Evaluate : $\int_1^4 [|x-1| + |x-2| + |x-4|]\, dx$.

29. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

ANSWERS

MODEL PAPER - II

SECTION A

1. $\log|(1 + \log x)| + c$

2. $\frac{1}{2}(\sqrt{5} - 1)$

3. -2

4. $y = 3$

5. $\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

6. $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

7. 5

8. $\theta = \frac{\pi}{3}$

9. $k = 4$

10. $-\frac{\pi}{4}$

SECTION B

11. $\frac{1}{5}\log\left|\frac{3 + 4\sin x}{2 + \sin x}\right| + c$

13. **OR** $\frac{1}{at}\sec^3 t$

14. $4x - 2y = 1$

14. **OR** 54.68

15. **OR** $X = 2 - \sqrt{3}$

16. $\theta = \sin^{-1}\left(\frac{8}{21}\right)$

17. $\frac{-x^3}{3y^3} + \log|y| = c$

18. $y\sin x = x + 1 - \frac{\pi}{2}$

20. (i) $\frac{3}{5}$ (ii) $\frac{7}{15}$

21. $\dfrac{dy}{dx} = \dfrac{-\left[y^{x} \log y + yx^{y-1}\right]}{xy^{x-1} + x^{y} \log x}$

SECTION C

23. Maximum number of cakes = 30 of kind one and 10 cakes of another kind.

24. $3(\pi - 2)$ square units

25. $x - 2y + z = 4$.

27. $x = 1,\ y = 2,\ z = 3$

28. $\dfrac{x^{2}}{2} + x + \dfrac{1}{2}\log|x - 1| - \dfrac{1}{4}\log|x^{2} + 1| - \dfrac{1}{2}\tan^{-1} x + c$

OR

28. $\dfrac{23}{2}$

29. Mean = 1, Variance = 0.49