Chapter - 3.1

Arithmetic Mean

Primarily, there is problem in proper understanding and comparing the data. Data are presented in form of the frequency distribution via the methods of classification and tabulation so that data can easily be understood. These methods are the primary methods of the statistical analysis. These methods do not show the important characteristic of data. These important characteristics of data are briefly represented by the measures of central tendency and for this purpose we study the statistical averages.

Each data series has such a point around which the other data are having the tendency of concentration. This value lies in about the centre of the data series. This value is a representative value of the important characteristics of data and thus known as the measure of central tendency or the average.

Accordingly to Simpson and Kaphka. "Central tendency is a such value at which other numbers are concentrated."

Measures of central tendency are also known as the statistical averages or place related measures. Statistical averages are practically very useful. With the help of these averages, the complicated and unsystematic data can be presented in the simple form. This simple form represents the population. Two or more groups can be compared. These statistical averages are considered as base in other statistical techniques and are used in the policy making process by the planners.

Characteristics of an Idle Average :

Following are the main characteristics of an idle statistical average :

1. It should be defined clearly so that its meaning can be understood easily.
2. It should be simple to understand and calculate.
3. Its value must represent all the items of the data series. If this so, the statistical average can be regarded as a representative value.
4. It must be less affected by the extreme values of the given data. Thus the minimum and maximum values must have their least effect on the value of the statistical average.
5. It must be used in the higher algebraic analysis. An idle statistical average has some special mathematical features so that it can be used in the higher algebraic analysis. For Example, if we have mean value and the frequencies of some groups, we can calculate the combined mean of these groups.

Types of Measures of Central Tendency:-

The classification of measures of central tendency can be seen as below:

Statistical averages

(a) Positional Averages

(i) Median (m)

(ii) Mode (Z)

(b) Mathematical Averages

(i) Arithmetic Mean ($\bar{X}$)

(ii) Geometric Mean (GM)

(iii) Harmonic Mean (HM)

(iv) Quadratic Mean (QM)

(c) Commercial Averages

(i) Moving Average

(ii) Progressive Average

The analysis of arithmetic mean ($\bar{X}$), median (M) and mode (Z) is the subject matter of this unit.

Arithmetic Mean :

This mean is the most familiar and important among all the mathematical averages and this is frequently used by the common people in their daily life.

The arithmetic mean is that value of the data series which is calculated by divided the sum of all items or observations by the total number of items. If the weight of 6 children is 41, 48, 47, 45, 52 and 39 kilogram, then the arithmetic mean of the weight of these 6 children is 47 kilogram. The total of weights of 6 Children is 282 kilogram which is divided by total number of students which is 6 and thus we get 47 Kilogram as mean weight.

Generally arithmetic mean is denoted by $\bar{X}$.

Type of Arithmetic Mean :

There are two types of this mean,

(1) Simple or unweighted arithmetic mean

(2) Weighted arithmetic Mean

Calculation of Simple Arithmetic Mean:

Simple arithmetic mean can be calculated in individual, discrete and continuous series separately by the direct method and the short cut method.

Calculation of $\bar{X}$ in Individual Series:

Here, $\bar{X}$ can be calculated by direct method and short-cut method.

1. Direct Method :-

In this method, the sum of all the items (ΣX) is divided by the number of items (N) and thus arithmetic mean is obtained. If X_1, X_2, ……, X_N are the N items. Here,

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_N}{N}$$

$$\bar{X} = \frac{\Sigma X}{N}$$

Example 1 : Calculate arithmetic mean from the following data:

S.No.	1	2	3	4	5	6	7	8	9	10
Marks Obt.	58	95	100	82	85	65	79	41	50	75

Solution : In this example, this sum of marks obtained (ΣX) is 730 and N =10 So,

$$\bar{X} = \frac{\Sigma X}{N}$$

$$= \frac{730}{10}$$

So, $\bar{X} = 73$

This is the average marks obtained in examination by 10 students.

2. Short-cut Method :

When the items in the data series are large, size of number is big and values are in decimals then short cut method is suitable for the calculation of $\bar{X}$. Under this method a suitable value is taken (either from the given values or from the outside) and this value is called the assumed mean (A). The value of A is then subtracted from all the given values and thus change of origin technique is followed. These

values are known as the deviations taken from assumed mean (d). All these deviated values have to be summed up (Σd) and the following formulae is used for the calculation of $\bar{X}$.

$$\bar{X} = A + \frac{\Sigma d}{N}$$

Where,

A = Assumed Mean

Σd = Sum of deviations taken from assumed mean

N = No. of items of values

Example 2 : Calculate arithmetic mean from the short cut method by considering data of example 1.

The necessary calculations required in obtaining $\bar{X}$ have been done in the following table.

S. No. of Students	Marks obtained (x)	A = 75 X-A=d
1	58	-17
2	95	20
3	100	25
4	82	7
5	85	10
6	65	-10
7	79	4
8	41	-34
9	50	-25
10	75	-25
N = 10		**Σd = -20**

Now, $\bar{X} = A + \frac{\Sigma d}{N}$

$= 75 + \frac{(-20)}{10}$

$= 75\text{-}2$

So, $\bar{X} = 73$

It can be seen that both the methods of direct and short cut give the same value of $\bar{X}$.

Calculation of $\bar{X}$ in Discrete Series :

Here, $\bar{X}$ can be calculated by both direct and short cut methods.

1. Direct Method :

In direct method, each value has to be multiplied with the respective frequency and then the multiplicated values are summed up (ΣXF). This summed up value is divided by the number of items N which is the total of all frequencies (N = ΣF). Thus,

$$\bar{X} = \frac{\Sigma XF}{N} = \frac{\Sigma XF}{\Sigma F}$$

Example 3: Calculate arithmetic mean from the below given data-series.

Variable (x)	56	60	64	68	72	76	80	84	88
Frequency (F)	10	12	16	14	10	8	17	5	4

Solution : For the calculation of $\bar{X}$, the below table has to be prepared :

Variable (x)	Frequency (F)	FX
56	10	560
60	12	720
64	16	1024
68	14	952
72	10	720
76	8	608
80	17	1360
84	5	420
88	4	352
Total	**N = 96**	**ΣFX = 6716**

Now,

$$\bar{X} = \frac{\Sigma XF}{N}$$

$$= \frac{6716}{96}$$

So, $\bar{X} = 69.96$

2. Short-cut Method :

Under short-cut method, following steps are taken to calculate $\bar{X}$:

(i) Taking assumed mean (A) from the given values of the variable.

(ii) Calculating deviations from A i.e. d = X-A.

(iii) Multiplying d with the respective frequency and getting Fd.

(iv) Finding Σfd i.e. summation of the multiplication of d with F.

(v) Applying the formula,

$$\bar{X} = A + \frac{\Sigma fd}{N}$$

Where, N = ΣF

Example 4: By using example 3, calculate X with the short cut method.

Solution : The following table is required for the computational work:

Variable (X)	Frequency (F)	A =68 d =X-A	Fd
56	10	-12	-120
60	12	-8	-96
64	16	-4	-64
68	14	0	0
72	10	4	40
76	8	8	64
80	17	12	204
84	5	16	80
88	4	20	80
Total	**N=ΣF = 96**		**Fd=188 =-280+468**

Now, $\bar{X} = A + \frac{\Sigma fd}{N}$

$$= 68 + \frac{188}{96}$$

$$= 68 + 1.96$$

So, $\bar{X} = 69.96$

Calculation of $\bar{X}$ in Continuous Series:

There are class intervals in the continuous series. The procedure of calculation of $\bar{X}$ in continuous series is similar as in discrete series. The only difference is that in continuous series mid-values are taken first of all. Thus after taking mid-values (m) the continuous series becomes just like the discrete series. In continuous series, the class intervals can be in form of exclusive series, inclusive series or the series of the unequal class-intervals. In these all cases, the X is calculated in the same way by using either direct method or by using short-cut method or by using the step deviation method.

1. Direct Method :

In this method, the mid-values are obtained which have then to be multiplied with the respective frequencies. Then sum of the multiplication of mid-values and respective frequencies is obtained. This sum has to be divided by the sum of the frequencies to get the value of $\bar{X}$. Thus,

$$\bar{X} = \frac{\Sigma Fm}{N} = \frac{\Sigma Fm}{\Sigma F}$$

Where, m is the mid-value, and

N = ΣF

2. Short-cut Method:

After getting the mid-values, the same process of calculating $\bar{X}$ is followed as done in the discrete series. The formulae to calculate $\bar{X}$ is,

$$\bar{X} = A + \frac{\Sigma Fd}{N}$$

where, d = m-A

$N = \Sigma F$

Example 5: Find out the arithmetic mean by both direct and short-cut methods in the following distribution.

Income (X) (in Rs.)	Frequency (F)
100-110	4
110-120	16
120-130	36
130-140	52
140-150	64
150-160	40
160-170	32
170-180	11

1. Direct Method :

$\overline{X} = \frac{\Sigma Fm}{N} = \frac{36465}{253}$

So, $\overline{X} = 143$

2. Short-cut Method :

$\overline{X} = A + \frac{\Sigma Fd}{N}$

$= 145 + \frac{510}{255}$

$= 145\text{-}2$

So, $\overline{X} = 143$

3. Step Deviation Method :

If the continuous series has equal class intervals and a large number of classes then the arithmetic mean should be calculate with the help of the step deviation method. By this method, the short cut method becomes easier. Under this method the deviations taken from assumed mean are divided by the common factor, which is equal in all classes and thus step deviations are found.

Following are the steps to be taken to calculate $\overline{X}$ by the step deviation method :

(i) To get mid values of all classes (m)

(ii) To take assumed mean from the mid-values (A)

(iii) To take deviations from the assumed mean d = (m-A).

(iv) To divide deviations by the common factor of class-intervals and thus to get the step deviations (d'), thus $d' = \frac{m-A}{i}$

(v) To multiply the step deviations with the respective frequencies and then to get their sum ($\Sigma Fd'$).

(vi) To use the below given formulae.

$\overline{X} = A + \frac{\Sigma Fd'}{N} \times i$

Example 6: Calculate $\overline{X}$ by using step deviation method in the data of example 5.

Solution: For the given data is exercises 5, the following table has to be prepared:

Calculation of $\overline{X}$

Income Class (Rs.) (X)	F	Mid-Value (m)	A =145 d= m-A	i=10 $d' \frac{d}{i}$	Fd'
100-110	4	105	-40	-4	-16
110-120	16	115	-30	-3	-40
120-130	36	125	-20	-2	-72
130-140	52	135	-10	-1	-52
140-150	64	145	0	0	0
150-160	40	155	10	1	40
160-170	32	165	20	2	64
170-180	11	175	30	3	33
Total	**N =255**				**Σfd'= 137-188 = -51**

Now,

$\overline{X} = A + \frac{\Sigma Fd'}{N} \times i$

$= 145 + \frac{-51}{255} \times 10$

$= 145 - \frac{-510}{255}$

= 145-2

So, $\bar{X}$ = 143

Weighted Arithmetic Mean :

All the values of the variable are given the same importance in the calculation of the simple arithmetic mean. The reality is that the different values have their different relative importance in the practical life. So it is necessary to calculate the arithmetic mean by considering the different relative importance of the different values of the variable. This relative importance of different values is assigned with the help of certain numbers. There number are known as weights. Thus the arithmetic mean calculated in this way is known as weighted arithmetic mean and denoted by $\bar{X}$w. For examples if a male labourer, a female labourer and a child labourer earn Rs. 200, 160 and 90 respectively as daily wage in a factory, then their daily average wage is Rs. 150 which is not a correct mean because here the number of labourers in each category has not been taken with account. If the number of labourers in all the above three categories is 50, 20 and 10 respectively then it is more important to calculate the arithmetic mean by considering the number of workers. This example can be explained as below:

Wage (x)	Weight (w)	XW
200	50	10,000
160	20	3200
90	10	900
	ΣW= 80	ΣXW = 14100

Now,

$$\bar{X}w = \frac{\Sigma XW}{\Sigma W}$$

$$= \frac{14100}{80}$$

So, $\bar{X}w$ = 176.125

It is clear that,

$$\bar{X}w = \frac{X_1W_1+X_2W_2+---+X_NW_N}{W_1+W_2+---+W_N}$$

$$\bar{X}w = \frac{\Sigma XW}{\Sigma W}$$

Properties of the Arithmetic Mean

Following are the properties of the arithmetic mean:

(i) Sum of deviations taken from arithmetic mean is always Zero, i.e.

$(\Sigma X - \bar{X}) = 0$

(ii) Sum of squares of deviations taken from arithmetic mean is minimum, i.e.

$\Sigma(X- \bar{X})^2$ is minimum

(iii) If any two values of $\bar{X}$, N and Σx are given, the third value can easily be obtained, i.e.

$$\bar{X} = \frac{\Sigma X}{N}, \Sigma X = \bar{X}N, N = \frac{\Sigma X}{\bar{X}}$$

(iv) If there are two or more than two groups of values of a variable and the arithmetic means and number of items of each group are given, then combined arithmetic mean can be calculated as below:

$$\bar{X}_{12} = \frac{N_1\bar{X}_1+N_2\bar{X}_2}{N_1+N_2}$$

(v) If class intervals are given in the form of exclusive and inclusive series, there is no need of any correction or adjustment to calculate the arithmetic mean.

(vi) In case of unequal class intervals there is no need of adjusting the frequencies.

(vii) If a contest number K is added or subtracted to or from all the items of data series, the arithmetic mean becomes $\bar{X}$+K or $\bar{X}$–K respectively.

(viii) If a content number K is used to divide or multiply all the values of the data series, then accordingly the arithmetic mean becomes $\bar{X}$/K or $\bar{X}$K respectively.

Miscellaneous Questions :

1. Calculate $\bar{X}$ from the below given data:

Class Interval (X)	46-55	36-45	26-35	22-25	18-21
Frequency(F)	20	36	54	32	8

Solution: In this example classes are inclusive and in the descending order and also the class-interval are not equal. There is no need of any adjustment to calculate the X as done below:

Calculation of $\bar{X}$

Class (x)	Mid-value (m)	F	A=30.5 d = M-A	Fd
46-55	50.5	20	20	400
36-45	40.5	36	10	360
26-35	30.5	54	0	0
22-25	23.5	32	-7	-224
18-21	19.5	8	-11	-88
Total				**Σfd=448**

Here,

$$\bar{X} = A + \frac{\Sigma fd}{N}$$

$$= 30.5 + \frac{448}{150}$$

$$= 30.5 + 2.99$$

So, $\bar{X} = 33.49$

2. Calculate the arithmetic mean of the following data series.

Income (Rs.)	No. of Persons
100-200	15
100-300	33
100-400	63
100-50	83
100-60	100

Solution : This is the case of cumulative frequency series which has to be converted into the normal frequency series as below:

Calculation of $\bar{X}$

Income Class (x)	F	M	A=350 d=m-a	i=100 d' =d/i	Fd'
100-200	15	150	-200	-2	-30
200-300	18	250	-100	-1	-18
300-400	30	350	0	0	0
400-500	20	450	100	1	20
500-600	17	550	200	2	34
Total	**N=100**				**ΣFd=6**

Now,

$$\bar{X} = A + \frac{\Sigma Fd}{N} \text{ xi}$$

$$= 30.5 + \frac{6}{100} \text{ x } 100$$

$$= 350 + 6$$

So, $\bar{X} = 356$

3. The average rainfall in a week (except Sunday) was 4.2cm. Due to excess rainfall on Sunday, the weekly average rainfall become 7.00cm. Find out the rainfall on Sunday.

Solution: The average rainfall in the week (except Sunday) = 4.2cm

Total rainfall in 6 days = 4.2 x 6

= 25.2 cm

In whole of the week, the average rainfall = 7cm

Total rainfall in the week

= 7 x 7= 49cm

So, rainfall on Sunday

= 49.00-25.2 = 23.8 cm

4. The mean of 50 students was 40. It was found later that marks of one student were considered

wrongly as 83 instead of the correct marks 53. Find the correct mean marks.

Solution :

Given : $\overline{X}$ = 40, N =50

Here, $\overline{X} = \frac{\Sigma X}{N}$ $\Sigma x = N\,\overline{X}$

= 50 x40 = 2000

So, total wrong marks are 2000.

Now correct ΣX= Incorrect ΣX

- Wrong Marks

+ Correct Marks

= 2000-83+53 = 1970

So, Correct $\overline{X} = \frac{Correct\ \Sigma X}{N}$

$= \frac{1970}{50}$

= 39.4

5. The average wage of 60 male workers of a company is Rs. 40 and the average wage of reaming 40 female workers (total workers in the company is 100) is Rs. 35. Find the average wage of all the workers.

Solution : Given:

N = 100, N_1 =60,

$N_2 = N-N_1$ = 100-60 =40

Here, Combined Mean (X_{12}) is : $\overline{\ }$

$\overline{X}_{12} = \frac{N_1\overline{X}_1+N_2\overline{X}_2}{N_1+N_2}$

$= \frac{60X40 + 40X35}{60+40}$

$= \frac{2400 + 1400}{100}$

$= \frac{3800}{100}$ $\overline{X}_{12}$ =Rs. 38

So, the average wage of all workers is Rs. 38.00

6. Calculate arithmetic Mean of the following data:

2, 4, 6, 8, 10, 12, 17, 21

Solution : The Arithmetic Mean is

$\overline{X} = \frac{\Sigma X}{N} = \frac{2+4+6+8+10+12+17+21}{8}$

$= \frac{80}{8}$ = 10

(i) If 2 is added in each item

(ii) If 2 is subtracted from each item

(iii) If each item is multiplied by 2

(iv) If each item is divided by 2.

Calculate arithmetic in above 4 cases Here the constant number is 2.

X	**X+2=X_1**	**X-2=X_2**	**X2= X_3**	**X/2= X_4**
2	4	0	4	1
4	6	2	8	2
6	8	4	12	3
8	10	6	16	4
10	12	8	20	5
12	14	10	24	6
17	19	15	34	8.5
21	23	19	42	10.5
Σx=80	**Σx_1=96**	**ΣX_2=64**	**ΣX_3=160**	**ΣX4=40**

(i) By adding 2:

$\overline{X}_1 = \frac{\Sigma X_1}{N} = \frac{96}{8}$ =12

So, $\overline{X}_1=\overline{X}$+2 =10+2 =12

(ii) By subtracting 2 :

$\overline{X}_2 = \frac{\Sigma X_2}{N} = \frac{64}{8}$ =8

So, $\overline{X}_2=\overline{X}$-2 =10-2 =8

(iii) By Multiplying by 2 :

$\overline{X}_3 = \frac{\Sigma X_3}{N} = \frac{160}{8}$ =20

So, $\overline{X}_3=\overline{X}$.2 =10x2 =20

(iv) By Dividing by 2 :

$\overline{X}_4 = \frac{\Sigma X_4}{N} = \frac{40}{8}$ =5

So, $\overline{X_4}$= $\overline{X}$/2 =10/2 =5

In this way the 7th property of the arithmetic mean is verified.

Uses of Arithmetic Mean:

The arithmetic mean is a simpler and easier to calculate and thus frequently used to study the economic and social problems. The concepts like average output, average cost, average income average export-import, average bonus, etc. use the arithmetic mean. This mean is an idle average even after having some demerits like that it is affected by the extreme values and gives the illusive conclusions.

Important Points :

Following are the important points related with arithmetic mean:

- The characteristics of different data can be expressed in terms of measures of central tendency.
- The measures of central tendency are the values towards with other values of data series are attracted.
- Statistical averages are useful in presenting the complicated data in the simple form, in representing the population, in comparing the groups and in formulating the future policies and plans.
- There are positional, mathematical and commercial averages studied in statistics.
- Arithmetic mean, harmonic mean and geometric mean are the mathematical averages.
- Arithmetic mean is calculated as

$$\overline{X} = \frac{\Sigma X}{N}$$

- Simple and weighted arithmetic means are the two types of arithmetic mean.
- Arithmetic mean is calculated by direct and short-cut methods in individuals, discrete and continuous series.
- In the calculation of simple arithmetic mean all the values are given the same importance while in case of weighted arithmetic mean the values are given the relative importance.
- The sum of deviations taken from arithmetic mean is always zero.

Questions for Exercise

Objective Type Questions :

1. The value which is calculated to show the property of data in brief is :

   (a) Statically Methods

   (b) Statistical Averages

   (c) Statistical Formula

   (d) Tabulation ( )

2. The objective of arithmetic mean is:

   (a) Average value of items

   (b) Equal difference of items

   (c) Mid value of items

   (d) All of the above ( )

3. Which average has the algebraic analysis ?

   (a) Arithmetic Mean

   (b) Median

   (c) Mode

   (d) All of the above ( )

4. If $\overline{X}_1$=4, $\overline{X}_2$ = 5, N_1 =10 and N_2 = 15, the value of combined mean is :

   (a) 4.5 (b) 4.6

   (c) 5 (d) 4.8 ( )

5. In a data series the sum of deviations taken from arithmetic mean is –

   (a) Maximum Sum (b) Minimum Sum

   (C) Zero Sum (d) Infinity ( )

Very Short Answer Questions:

1. Name of positional averages.
2. What is the main difference between simple arithmetic mean and weighted arithmetic mean ?
3. When is the step deviation method used to calculate the arithmetic mean ?
4. What are the first category means ?
5. Write formulae to calculate the combined arithmetic mean.

Short Answer Questions :

1. Prove with example that sum of deviations taken from arithmetic mean is always zero.
2. What will be the change in arithmetic mean of a series if a constant number is added to & subtracted from and multiplied by and divided by a constant number ?
3. Write any four properties of an idle average.
4. Explain the statistical averages.
5. What are the objectives to study the averages?

Essay Type Questions:

1. What do you mean by measures of central tendency ? Describe the properties of the idle average.
2. If the mean age of children is 11.9 years, find out the number of children in the following table for the age class (10.5-15.5).

Age (years)	No. of Children
0.5- 5.5	3
5.5-10.5	17
10.5-15.5	?
15.5-20.5	8
20.5-25.5	2

3. Calculate arithmetic mean, mode and median from the following frequency distribution.

Class	Frequency
0-5	3
5-10	4
10-15	6
15-20	12
20-25	0
25-30	14
30-35	6
35-40	5

4. Find out weighted arithmetic mean from the below given data.

Item	Expenditure (Rs.)	Weight
Food grains	940	7.5
Rent	200	2.5
Cloth	500	1.5
Fuel	250	1.0
Others	240	0.5

Answers to Objective Type Questions

(1) b (2) d (3) a (4) b (5) c

Reference Books:

1. S.C. Gupta & V.K. Kapoor: Fundamentals of Mathematical Statistics, Published by sultan chand and sons.
2. S.P.Sing: Sankhyki : Sindhant Avam Vyavhar, S. Chand
3. K.N. Nagar: Sankhyki ke Mool Tatva, Minakshi Prakashan.

Chapter - 3.2

Median

Median is a positional average. It is the mid-value of the ordered data series. After arranging the data series in either ascending or in descending order, the value which is in the middle is known as median.

Median is that value of the variable in the data series which divides the ordered series into two parts in such a way that in one part all the values are more than the median and in the second part all the values are less than the median. For example, if the daily income of 7 persons is Rs. 68, 95, 101, 118, 165, 182 and 210 respectively, here median is Rs. 118. The item of Rs. 118 is exactly in the middle of the series. There are there item (68, 95 and 101) which are less than median and another three items (165, 182, and 210) which are more than median. Generally, Median is denoted by M.

Determination of Median :

(1) Individual Series :

To calculate the median in the individual series, following steps are taken:

(i) All the given values of the variable are arranged either in ascending order or in descending order.

(ii) Following formulae is used to get the value of median-

M = Size of $(\frac{N+1}{2})$th item.

Where, N= Number of items

M = Median

It is to be noted that the above formulae gives the number of the median and not the value of median. The value of the item corresponding to this number is the median. If the number of items in the individual series is even, the serial number in the centre will not be an integer. Here, to decide the value of such serial number, the both side two integers are added and then divided by 2. The value got in this way is the median.

Example 1 : Calculate median from the following data.

31, 38, 42, 33, 35, 49, 28, 45, 39

Solution : The values have to be arranged in the ascending order as below:

S.No.	Item-Value
1	28
2	31
3	33
4	35
5	38
6	39
7	42
8	45
9	49

Here,

M= Size of $(\frac{N+1}{\Sigma})$th item

= Size of $(\frac{9+1}{2})$th item

= Size of 5^{th} item

Thus the size of 5^{th} item is 38 which is the value of Median (M).

Example 2 : Calculate the median from the following data:

15, 19, 40, 20, 18, 22, 28, 11, 15, 15, 21, 35, 30, 23, 32, 22, 11, 12.

Solution : By arranging the given data into the ascending order the following information has been tabulated.

S.No.	Item Value	S.No.	Item Value
1	11	10	21
2	11	11	22
3	12	12	22
4	15	13	23
5	15	14	28
6	15	15	30
7	18	16	32
8	19	17	35
9	20	18	40

Applying the formulae,

M= Size of $(\frac{N+1}{2})$th item

= Size of $(\frac{18+1}{2})$th item

= Size of 9.5^{th} item

$= \frac{\text{Size of 9th item + Size of 10th item}}{2}$

$= \frac{20+21}{2}$

So, M = 20.5

(2) Discrete Series :

The steps to be taken to calculate the median are:

(i) Frequencies are made cumulative.

(ii) Following formulae is used for the serial number of the median:

(iii) The cumulative frequency is used to determine the value of the number. In which cumulative frequency this number is included firstly, the value related with this is the median.

(Before the solution of the question, it has to be ensured that the data series has been arranged in an order)

Example 3: Calculate the median wage in the following distribution:

Daily Wage (Rs.)	25	10	18	26	20	30	15
No. of laborer	13	18	14	10	16	6	12

Solution : The above data series has to be arranged in the ascending order as below:

Daily wage (Rs.)	F	CF
10	18	18
15	12	30
18	14	44
20	16	60
25	13	73
26	10	83
30	6	89
Total	N=89	

The median wage is determined with the following formulae,

M= Size of $(\frac{N+1}{2})$th item

= Size of $(\frac{89+1}{2})$th item

= Size of 45^{th} item

In the above table size of 45^{th} item has to be associate with the cumulative frequency of 60 which is related with the daily wage of Rs. 20. So,

M = Rs. 20

(3) Continuous Series :

Following are the steps to be used for the calculation of median:

(i) Cumulative frequencies are calculated

(ii) The median-number is found with the help of the below given formula:

$M = \text{Size of } (\frac{N}{2})$th item

In continuous series, the median is the value of the size of $(\frac{N}{2})$th item and not of $(\frac{N+1}{2})$th item. It is so because the value of median must be same both in ascending order and in descending order. In both these situations the value of median is same when $\frac{N}{2}$ is in the centre point.

(iii) The class of the cumulative frequency along with the median number is associated firstly is selected as the median class.

(iv) After the determination of the median class, the value of median is calculated with the help of the below given formulae.

(a) When data series is in the ascending order:

$$M = L_1 + \frac{i}{f}(\frac{N}{2} - c)$$

where, M - Median

i - class interval of the median class

$(L_2 - L_1)$

f - Frequency of the median class

N - Total frequencies

c - Cumulative frequency of the class preceding to the median class.

L_1 - Lower limit of the median class.

(b) When data series is in the descending order :

$$M = L_2 + \frac{i}{f}(\frac{N}{2} - c)$$

Where,

L_1 = Lower limit of the median class

L_2 = Upper limit of the median class

Example 4 : Calculate median from the following data :

Class	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	110	125	86	45	18	12

Solution : Median is determine in the following table :

Class	**F**	**CF**
10-20	110	110
20-30	125	235
30-40	86	321
40-50	45	366
50-60	18	384
60-70	12	396
Total	**ΣF = N = 396**	

Here,

$M = \text{Size of } (\frac{N}{2})$th item

$M = \text{Size of } (\frac{396}{2})$th item

M = Size of 198th item

From the column of cumulative frequency, the value 198 is associated with CF of 235 and thus the corresponding class (20-30) is the median class in which the value of median is found.

The median is now calculated with the below formulae:

$M = L_1 + \frac{i}{f}(\frac{N}{2} - c)$

$= 20 + \frac{10}{125}(\frac{396}{2} - 110)$

$= 20 + \frac{10}{125}(198 - 110)$

$= 20 + \frac{10}{125}(88)$

$= 20 + 7.04$

$= 27.04$

So, M = 27.04

Example 5: Find the value of median from the below given data :-

Class	50-60	40-50	30-40	20-30	10-20
Frequency	4	8	15	10	7

Solution: Above data are in the descending order. So median is calculated as below:

Class	**Frequency (F)**	**Cumulative frequencies (CF)**
50-60	4	4
40-50	8	12
30-40	15	27
20-30	10	37
10-20	7	44
Total	**ΣF = 44**	

Here,

$M = \text{Size of } (\frac{N}{2})\text{th item}$

$= \text{Size of } (\frac{44}{2})\text{th item}$

$= \text{Size of } 22^{nd} \text{ item}$

Clearly, the median class is (30-40)

Now,

$M = L_2 - \frac{i}{f}(\frac{N}{2} - c)$

Here,

$L_2 = 40$, $L_1 = 30$, F = 15, N = 44, C = 12, i = 10

Putting the values is the formulae.

$M = 40 - \frac{10}{15}(\frac{44}{2} - 12)$

$M = 40 - \frac{10}{15}(22 - 12)$

$M = 40 - \frac{2}{3}(10)$

$= 40 - 6.67$

Thus, M = 33.33

Merits of Median:

Following are the merits of median.

(i) Median is a clear and well defined average.

(ii) Median is easy to calculate and understand.

(iii) As mode, median can also be determined by the inspection method.

(iv) Median is not affected by the extreme values.

(v) Median is more suitable for qualitative variables.

(vi) Median can be determined by graphical method also.

Demerits of Median :

The below given are the demerits of median:

(i) It is difficult to calculate median when the data are not arranged in ascending or descending order and when number of observation are even.

(ii) Median ignores the extreme values of the variable.

(iii) Because median lacks the algebraic qualities, so it is not useful in higher statistical methods.

(iv) Median is affected by sampling fluctuations.

Uses of Median:

Practically median is very useful because of the simple and easy method of calculation. Median is used to study the distribution of wealth and property. Social problems can also be studied with the help of median. It is very useful in measuring the attributes like health, poverty, intelligence, etc. It is proper to use median where the values are unweighted. The partition values (specially quartiles) are used in the study of dispersion and skewness.

Important Points :

- Median is a positional average.
- Median is the mid-value of the data-series after arranging it in either ascending or in descending order.
- Median is determined in individual, discrete and continuous series.
- When data series is in ascending order, the median in continuous series is determined by : $M = L_1 + \frac{i}{f}(\frac{N}{2} - c)$ and when data series is in the descending order, median is calculated by $M = L_2 - \frac{i}{f}(\frac{N}{2} - c)$
- Median is used in the analysis of wealth and property distribution, qualitative aspects, etc. Median is very important in the analysis of social problems.

Questions for Exercise

Objective Type Questions:

1. The best average for the facts which can not be assigned numerical value is:

   (a) Arithmetic Mean

   (b) Median

   (c) Mode

   (d) Harmonic Mean

2. Median in the below given data is:

   8, 11, 12, 13, 15, 18

   (a) 12.5 (b) 13 (c) 12 (d) 14

3. If Z = 18 and $\bar{X} = 20$,the median (M) is:

   (a) 29.33 (b) 19.33 (c) 18.66 (d) 9.33

Very Short Answer Questions:

1. What do you mean by median?
2. What is the formulae to calculate median when the number of items is even in the individual series ?
3. When is the best use of median?
4. Which average is more suitable for the open-ended classes ?

Short Answer Questions :

1. The median of four observations (3, 4, x and 8) is 5. What is the value of X ?
2. Why do we use $(\frac{N+1}{2})$ in discrete series and $\frac{N}{2}$ in continuous series to calculate median ?
3. It $\bar{x}=75$ and Z = 60, calculate the value of median.
4. State any four uses of median.
5. Write the formula to calculate Q_1 and Q_3.

Essay Type Questions :

1. Calculate Mode and Median from the following table :

Class	F
0-10	10
10-20	3
20-30	7
30-40	15
40-50	5

2. Critically analyse the merits and demerits of the important measures of central tendency.

Answer to objective type questions :

(1) b (2) a (3) c (4) b (5) b

Reference Books :

1. S.C. Gupta & V.K. Kapoor: Fundamentals of Mathematical Statistics, Published by Sultan Chand and Sons
2. S.P.Singh: Sankhyki : Sindhant Avam Vyavhar, S. Chand
3. K.N. Nagar: Sankhyki ke Mool Tatva, Minakshi Prakashan.

Chapter - 3.3

Mode

The English word "Mode" has been taken from the French word "La Mate " which means fashion or custom or which occurs most. The commodity, which is more fashionable or under custom, is used more by people. So mode is the item of data-series which has the maximum frequency. Mode is the value which comes maximum times in the data i.e. which has maximum frequency. It shows that point where most of the items are concentrated. This point has the maximum density or the concentration of the values so mode is a positional average. For example, a shoe manufacture wants to know about the size of the shoe which has its maximum demand in the market. If Rs. 400 per month is the model wage in a factory, it means that the maximum laburers get Rs. 400 as the monthly wage.

Determination of Mode :

Determination of mode is studied in the individual, discrete and continuous series.

1. Individual Series:

Mode is the value which comes maximum times in the data series. Following are the methods to find out the mode in the individual series.

(i) By changes individual series in the discrete series.

(ii) By changing the individual series in the continuous series, or

(iii) Finding the mode with the help of the arithmetic mean and medium.

(a) By changing Individual series in the Discrete Series:

When values in the individual series appear twice or more than twice this series should be changed into the discrete series. The values are arranged in ascending order and their frequencies are written. Now with the help of the inspection method, the maximum frequency is observed and its value is known as the mode. Mode is symbolized by 'Z'.

Example 1 : Calculate the mode in the following series. 40, 44, 46, 50, 44, 34, 38, 44,46, 42, 44

Solution:

Value (x)	34	38	40	42	44	46	50
Frequency (F)	1	1	1	1	4	2	1

It is clear by the inspection method that the value of 44 has the maximum frequency as 4. So mode is 44. Thus, Z = 44

(b) By Changing Individual Series in Continuous Series :

If any individual value does not have more than one frequency, the individual series should

be changed into continuous series. Changing individual series into the discrete series (in this situation) does not help in the determination of the mode because all the values have the same frequency. Thus individual values should be changed in the continuous series and then the class of the maximum frequency should be found which is known as the model class. After this the mode is calculated by using a formulae which is explained next.

(c) Finding Mode with the Help of Arithmetic Mean and Median:

If arithmetic mean, Median (M) and mode (Z) are calculated in the individual series, Z can be calculated with the help of their mutual relationship as explained below:

$$(\bar{X} - Z) = 3\,(\bar{X} - M)$$

Or $\quad Z = 3M - 2\bar{X}$

This formula should only be used if specially asked in the examination or if there is abnormal situation.

Example 2 : Calculate Arithmetic Mean (X), Median (M) and Mode (Z) from the following data.

4, 13, 9, 25, 17, 20, 10

Solution : By arranging above data in the ascending order –

4, 9, 10, 13, 17, 20, 25

$(\bar{X}) = \frac{\sum X}{N} = \frac{4+9+10+13+17+20+25}{N} = \frac{98}{7} = 14$

So, $\bar{X} = 14$

M= Size of $(\frac{N+1}{2})$th item

= Size of $(\frac{7+1}{2})$th item

= Size of 4th item

= 13

So, M= 13, and

$Z = 3M - 2\bar{X}$

$= (3 \times 13) - (2 \times 14)$

$= 39 - 28$

$= 11$

So, Z = 11

2. Discrete Series :

Under discrete series, there are two method to calculate the mode.

(a) Inspection Method

(b) Grouping Method

(a) Inspection Method :

When frequencies in the discrete series have the regular nature (in the beginning frequency should increase frequency must be maximum in the middle of the series and after this, frequencies must decrease continuously). In this type of series, the maximum frequency can be seen clearly. This is the inspection method of finding out the mode.

Example 3 : Calculate mode in the following series :

Weight (x) Kg.	50	52	55	58	64	70
No. of person (F)	4	10	20	11	3	2

Solution : In the above series, the frequencies are regular. So, mode can be calculated with the help of the inspection method. The maximum frequency is 20 whose corresponding value is 55 Kg. So, Mode weight is 55 Kg. Thus,

Z = 55 Kg.

(b) Grouping Method :

The grouping method is to decide the concentration point of frequencies in the distribution of the irregular frequencies.

Process of Grouping :

In the grouping method, a table has values of the variable (x) with 6 columns of frequencies as shown below :

Column	Value of X
1.	Frequencies (F)
2.	Total of 2-2 frequencies
3.	Total of 2-2 frequencies ignoring first frequency
4.	Total of 3-3 frequencies
5.	Total of 3-3 frequencies ignoring first frequencies
6.	Total of 3-3 frequencies ignoring first two frequencies

After grouping the frequencies as above, the maximum frequency in each column is selected and the analysis table is prepared. The value of the variable (x) having maximum frequency is the value of mode.

Example 4 : Find out the mode size of the collar of the shirt from the following data –

Collar Size (cm)(x)	30	31	32	33	34	35	36	37
No. of Persons (F)	2	9	3	4	8	7	8	5

Solution : Although the inspection method shows that the maximum frequency is 9, so X=31 is the mode but the frequencies are not regular, so grouping method should be used to calculate the mode so as to keep ourselves away from the doubt.

Determination of Mode by Grouping Method:

Collar Size (cm) (x)	F (I)	(II)	(III)	(IV)	(V)	(VI)	No. of maximum Frequencies/Analysis Table
30	2						0
31	9	**11**		14			I 1
32	3		12		16		0
33	4	**7**				15	I 2
34	8		12	19			III 3
35	7	**15**			23		IIII 5
36	8		15			20	III 3
37	5	**13**					I 1

According to the analysis table (last column) The maximum number is 5 whose corresponding value of X is 35, So Z = 35 cm.

3. Determination of Mode in Continuous Series:

Here, the model class in which the value of mode lies has to be determined first. The model class is determined by either the inspection method or by the grouping method (the two methods explained above). It is advised to use the grouping method to determine the model class to keep ourselves safe from the situation of doubt.

After determination of the model class, following formulae is used to calculate the value of mode. The value of mode must come within the limits of the model class.

$$Z = L_1 + \frac{F_1 - F_0}{2F_1 - F_0 - F_2} \times i$$

Where, Z – Value of mode

L_1 – Lower Limit of the model class

i – Class interval of the model class

(where i = Upper limit –Lower limit)

F_1 – Frequency of the model class

F_0 – Frequency of the class preceding to the model class.

F_2 – Frequency of the class – succeeding to the model class.

Example 5: Calculate mode from the following frequency table.

Class (x)	Frequency (F)
20-40	6
40-60	9
60-80	11
80-100	14
100-120	20
120-140	15
140-160	10
160-180	8
180-200	7

Solution : In the above example the frequencies are regular, So inspection method can be used for the determination of the model class. The maximum frequency is 20 which relates with the class of 100-120. So 100-120 is the model class in which mode lies. Applying the formula,

$$Z = L_1 + \frac{F_1 - F_0}{2F_1 - F_0 - F_2} xi$$

$$= 100 + \frac{20+4}{40-14-15} x20$$

$$= 100 + \frac{6X20}{11}$$

$$= 100 + \frac{120}{11}$$

$$= 100 + 10.9 = 110.9, \quad Z = 110.9$$

Merits of Mode:-

Following are the merits of mode:

(i) It is easy to calculate mode. Mode can be calculated by the general understanding. Mostly the mode can be calculated by the inspection method and the formula to calculate mode is not required.

(ii) It is a publically accepted query. It is frequently used in daily life.

(iii) Mode is not affected by the extreme values because the maximum frequencies are calculated generally in the middle of the series and not near the extreme values.

(iv) Mode can also be determined with the help of the graphical method.

(v) Mode is the best representative average. Mode is the value having maximum frequency which is among the given values of the variable. So mode is assumed as the most representative average.

Demerits of Mode :-

Following are the demerits of mode:

i. Mode is a unclear, uncertain and undetermined average. When all the items of the data series have same frequencies or the data series has two or more than two modes then the determination of mode becomes difficult.

ii. If mode is not determined with the help of the inspection method, the grouping method becomes more difficult.

iii. Mode ignores the extreme values which is the ignorance of the mathematical rules.

iv. The algebraic analysis of mode is not possible.

v. Mode is not an actual and representative average.

Uses of mode:

Common people can use mode in the everyday life. Mode is an useful average to use in the studies related with business, science of seasons, biology, preference of consumers, etc. Mode is frequently used specially to determine the maximum concentration of the values like average size of collar, average monthly expenditure of student, daily average number of telephone calls, number of words in the average pages of a book, average number of children to per couple, etc.

Mode can be useful in business forecasting. The forecasting regarding rainfall, wind speed, etc. is done with the help of the mode. In this way, mode is very useful in our practical life.

Miscellaneous Exercises:

Example 6 : If mode of the following series is 24, find out the missing frequency of the class(30-40).

Class	0-10	10-20	20-30	30-40	40-50
Frequency	14	2	27	?	15

Solution : Suppose unknown or missing frequency is X.

Class	**Frequency**
0-10	14
10-20	23 (F_0)
20-30	27 (F_1)
30-40	X(F_2)
40-50	14

Because mode is 24, so (20-30) is the model class. Here,

$Z= L_1+\frac{F_1-F_0}{2F_1-F_0-F_2}xi$

Given, Z= 24, L_1 =20, F_1 =27. F_0=23, F_2=X, and i =10

So, putting values in above formulae,

$24=20+\frac{27-23}{54-23-X}x10$

$24 -20 = \frac{40}{31-X}$

$4 = \frac{40}{31-X}$

$31-X = \frac{40}{4}$

31-X = 10

X =31-10 = 21 So, unknown or missing frequency (F_2) is 21.

Example 7 : Calculate arithmetic mean ($\bar{X}$) and mode (Z) from the following data:

Class Interval	**Frequency**
10-20	4
20-30	16
30-40	56
40-50	97
50-60	124
60-70	137
70-80	146
80-90	150

Solution : Cumulative frequencies given is the example which have to be changed into the class frequencies.

Calculation of $\bar{X}$: Following table shows the calculation for $\bar{X}$

Class Interval	**Frequency (F)**	**Mid-Value (m)**	**$d^1=\frac{m-a}{i}$**	Fd^1
10-20	4	15	-4	-16
20-30	12	25	-3	-36
30-40	40	35	-2	-80
40-50	41	45	-1	-41
50-60	27	55	0	0
60-70	13	65	1	13
70-80	9	75	2	18
80-90	4	85	3	12
Total	**N=1500**			**-130**

Here, A=55, $\sum$Fd'= -130, i=10 and N=150

Substituting the values in the below formulas.

$\bar{X}= A+\frac{\sum Fd'}{N}xi$

$= 55-\frac{130}{150}x10$

$= 55-\frac{26}{3}$

= 46.33

Calculation of mode (Z):

In the above example, the maximum frequency is 41 and thus the model class is (40-50) Now,

$Z = L_1 + \frac{F_1 - F_0}{2F_1 - F_0 - F_2} xi$

The relevant values are:

$L_1 = 40$, i=10, F_1=41, F_0 =40 and $F_2 = 27$

Thus,

$Z = 40 + \frac{41-40}{N82-40-27} x10$

$= 40 + \frac{1}{82-67} x10$

$= 40 + \frac{10}{15}$

$= 40 + 0.67$

$= 40.67$

So, $\bar{X} = 46.33$ and $Z = 40.67$

Example 8 : Calculate mode in the following table.

Class (x)	1-9	11-19	21-29	31-39	41-49	51-49
Frequency (F)	14	31	54	69	28	24

This is the example of inclusive series which has to be changed into the exclusive series. Because there is difference of 2 in the serial classes, so the series has to be adjusted by subtracting 1 from the lower limit and adding 1 to the upper limit of all the classes. The mode has to be calculated by using the grouping method as below:

Collar Size (cm) (x)	F (I)	(II)	(III)	(IV)	(V)	(VI)	Analysis Table
0-10	14						1(1)
10-20	31	**45**		99			III (3)
20-30	54		85		154		IIIII (6)
30-40	69	**123**				151	III (3)
			97				
40-50	28						I (1)
50-60	14	**42**		111			

It is clear that the maximum frequency is 6 which shows that model class is (20-30), so the value of mode will be in the class (20-30).

The following formula has to be used for the calculation of mode.

$Z = L_1 + \frac{F_1 - F_0}{2F_1 - F_0 - F_2} xi$

Here, L_1 =20, i =10, F_1 =54, F_0 =31, $F_2 = 69$

Now, $Z = 20 + \frac{54-31}{108-31-69} x10$

$= 20 + \frac{23}{108-100} x10$

$= 20 + \frac{230}{8}$

$= 20 + 28.8$

$= 48.8$

Because the value of Z = 48.8 which goes out the model class, so alternative formula has to be used which is,

$Z = L_1 + \frac{F_2}{F_2 + F_0} xi$

$= 20 + \frac{69}{69+31} x10$

$= 20 + \frac{690}{100}$

$= 20 + 6.90$

$= 26.90$

So, Z = 26.90

Example 9 : Find out the model value in the following example.

Central Size	F
15	5
25	9
35	13
45	21
55	20

65	15
75	8
85	3

Solution : In the example mid-values of the variable are given. In all the items the difference is of 10 each. So the limits of the classes have to be determined by $L_1 = (m-\frac{i}{2})$ and $L_2 = (m+\frac{i}{2})$ In this case grouping method has to be used as explained below:

Collar Size (cm) (x)	F (I)	(II)	(III)	(IV)	(V)	(VI)	Analysis Table	
10-20	5							-
20-30	9	**14**		27			I	1
30-40	13		22		43		II	2
40-50	21	**34**				54	IIII	5
50-60	20		41	56			IIII	5
60-70	15	**35**			43		III	3
70-80	8		23			26	I	1
80-90	3	11						-

It is clear that this is the bi-model case because both classes of (40-50) and (50-60) have the 5 as the maximum frequency. In this case the following method has to be used for the determination of mode.

Class / **Frequency**	**40-50**	**50-60**
F_0	13	21
F_1	21	20
F_2	20	15
Total	**54**	**56**

Clearly the class (50-60) has the maximum total of F_0, F_1 and F_2 as 56, so model class is (50-60) applying the formula.

$$Z = L_1+\frac{F_2}{F_2+F_0}xi$$

$$=50 + \frac{15}{21+15}x10$$

$$= 50+ \frac{150}{36}$$

$$= 50+4.17$$

$$= 54.17$$

So, Z = 54.17

It is to be noted that if the original formula is used to calculate the mode, its value comes out of the class (50-60). So alternate formula has to be used.

Important Points :-

- Generally, Mode is the item of the data series which has the maximum frequency.
- Mode is the value of maximum density or the point of concentration of values. So mode is a positional average.
- Mode is determined in individual, discrete and continuous series.
- Mode can also be calculated with the help of X and M (Z = 3 M-2X).
- There are two methods – inspection method and grouping method – to determine mode both in discrete and continuous series.
- When frequencies are not regular in data-series, grouping method should be used.
- In continuous series, the model class is firstly determined followed by the use of the formula ($Z = L1+\frac{F_2}{F_2+F_0}xi$) to calculate the mode.
- Mode is based on the assumption of the equal class intervals. If the class intervals are not same in the data series, these have to be made equal before the solution.
- Mode is a useful average in the study of business, weather science, biology, preference of consumers, etc.

Question for Exercise

Objective Type Question :

1. Which is the most uncertain average ?
   (a) Mode
   (b) Arithmetic Mean
   (c) Median
   (d) Harmonic Mean
2. That value of the item which has the maximum frequency is known as –
   (a) Arithmetic Mean
   (b) Median
   (c) Mode
   (d) All of the above
3. The proper average for the average size of the readymade garments is:
   (a) Median
   (b) Mode
   (c) Arithmetic Mean
   (d) None of the above
4. What will be the lower limit of the model class in the following data series ?

X	F
0-9	2
10-19	5
20-29	16
30-39	12
40-49	4

   (a) 19 (b) 19.5
   (c) 20 (d) 29.5
5. In which average the extreme value have the minimum effect ?
   (a) Arithmetic Mean
   (b) Geometric Mean
   (c) Median
   (d) Mode

Very Short Answer Questions:

1. 'An average Rajasthani' person puts on the shoe of number 7. To which statistical average, this statement is concerned ?
2. Write the general formulae to calculate the mode in the continuous series.
3. Define mode.
4. What are the methods to calculate the mode?
5. Write the alternative formulae of mode.

Short Answer Questions :

1. If M = 21 and X $=\overline{2}0$, Calculate Z.
2. In what circumstances the 'density test' is performed in the mode ?
3. If model class is (50-60) and F_1=40, F_0=25 and F_2= 20, then calculate mode.
4. Explain the grouping method.
5. Explain the uses of mode.

Essay Type Questions:

1. Calculate the mode from the following table by the grouping method.

Mid Value (m)	F
15	5
25	9
35	13
45	21
55	20
65	15
75	8
85	3

2. Find the mode from the following data.

Size	F
8	3
10	7
12	12
14	28
16	10
18	9
20	6

3. Calculate mode, Q_1 and Q_3 from the following data series.

Income (Rs.)	No. of Persons
100-200	15
100-300	33
100-400	63
100-500	83
100-600	100

Answers to objective type questions:

(1) a (2) c (3) b (4) b (5) d

Reference Books:

1. S.C. Gupta and V.K. Kapoor. Fundamentals of Mathematical Statistics, Published by Sultan Chand and sons.
2. S.P. Singh: Sankhyiki : Sidhant aivm Vyavhar, S. Chand.
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