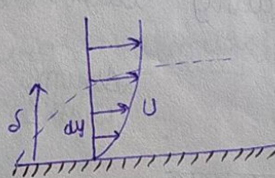
$$= \left(y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\right)_0^{\delta} = \delta - \delta + \frac{\delta}{3} = \boxed{\frac{\delta}{3}}$$

~~lecture-17~~
12/04/19

Momentum Thickness (θ)



Assume,
width = 1

In general

Effect of FIM class

1st way

(Before 6:00 PM)
Student [Fix] $\rightarrow$ Not attend class
(After 9:30 PM)
$\rightarrow$ Attend class

$$d\dot{m} = \rho(dy.1).U$$

[momemtum per sec of flowing fluid in the absence of boundary layer $= \int_0^{\delta} d\dot{m}.U_\infty$

2nd Way

Students① $\rightarrow$ Attend class (you)
Students② $\rightarrow$ Not attend class

[momentum --- in the present of boundary layer $= \int_0^{\delta} d\dot{m}.U$

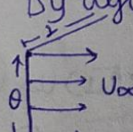
(Newton mind) [Reduction of in momentum per sec

$$= \int_0^{\delta} d\dot{m}(U_\infty - U)$$

$$= \int_0^{\delta} \rho(dy.1)U(U_\infty - U)$$

$$= \int_0^{\delta} \rho U(U_\infty - U)\,dy \quad \text{--- ①}$$

By defini.



compensation in momentum per sec $= \rho(\theta.1)U_\infty.U_\infty$ --- ②

by ① and ② eq.

$$\cancel{\rho}(\theta.1)U_\infty U_\infty = \int_0^{\delta} \cancel{\rho}.U(U_\infty - U).dy$$

$$\boxed{\theta = \int_0^{\delta} \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right)dy}$$

Momentum thickness (θ)

It is defined as the distance, measured $\perp$ to the boundary of a solid body, by which the boundary should be displace to compensate for the reduction in momentum per sec. of flowing fluid on account of boundary formation.

Kinetic energy thickness: (δ^{**})

It is defined as distance, measured $\perp$ to the boundary of solid ~~boundary~~ body by which the boundary should be displaced to compensate for the reduction in Kinetic energy per sec of ~~follo~~ flowing fluid on account of boundary layer foundation

Assume width 1

$$dm \begin{cases} \to U_\infty & \frac{1}{2}(dm\, U_\infty^2) \text{ hypo} \\ \to U & \frac{1}{2}\, dm\, U^2 \quad \text{Real} \end{cases}$$

$$dm = \rho(dy \cdot 1) \cdot U$$

K.E. per sec of flowing fluid in the absence of boundary layer $= \int_0^\delta \frac{1}{2}\, dm\, U_\infty^2$

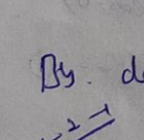
KE – " — " — " – " – $= \int_0^\delta \frac{1}{2}\, dm\, U^2$

Reduction in K.E. per sec $= \int_0^\delta \frac{1}{2}\, dm\,(U_\infty^2 - U^2)$

$$= \int_0^\delta \frac{1}{2}\rho(dy \cdot 1)(U_\infty^2 - U^2)$$

$$= \int_0^\delta \frac{1}{2}\rho U(U_\infty^2 - U^2)\, dy \quad \text{--- ①}$$

By. defini



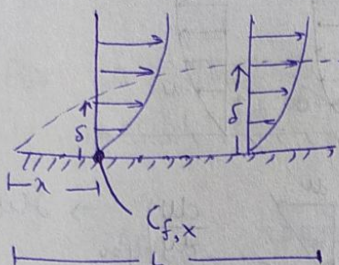
compensation in K.E per sec $= \frac{1}{2}\rho(\delta^{**} \cdot 1)U_\infty U_\infty^2$ --- ②

By eq ① and ②

$$\frac{1}{2}\cancel{\rho}(\delta^{**} \cdot 1)U_\infty \cdot U_\infty^2 = \int_0^\delta \cancel{\rho} \cdot U(U_\infty^2 - U^2)\, dy$$

$$\boxed{\delta^{**} = \int_0^\delta \frac{U}{U_\infty}\left(1 - \frac{U^2}{U_\infty^2}\right) dy}$$

In general



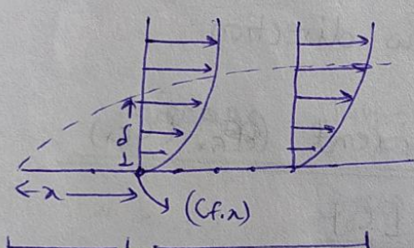
Boundary layer thickness (δ) = local

local skin-friction coefficient ($C_{f,x}$)
local

$$\boxed{C_{f,x} = \frac{\tau_{o,x}}{\frac{1}{2}\rho U_\infty^2}}$$

Avg skin friction coefficient ($\overline{C_{f,x}}$)
(Average)
(or)
Drag coefficient (C_D)

For laminar Boundry layer over a flat plate :-



For 2D steady incompressible laminar boundry layer over a flat plate.

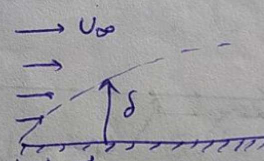
" Blasius series solution"

① Boundary layer thickness (δ)

$$\boxed{\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}}$$

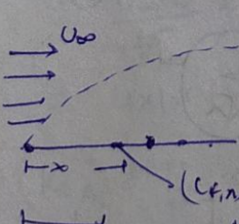
(or) this is true $\boxed{\delta \propto \sqrt{x}}$

**
confuse kar sakta hai ki ($\delta \propto x$) ~~hai~~ me (question me)



$$Re_x = \frac{\rho . U_\infty x}{\mu}$$
→ const.

② local skin friction coefficient ($C_{f,x}$)



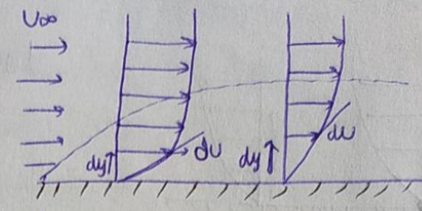
$$\boxed{C_{f,x} = \frac{0.664}{\sqrt{Re_x}}}$$

$$\boxed{C_{f,x} \propto \frac{const}{\sqrt{x}}}$$

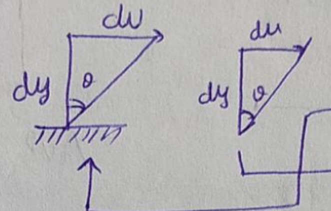
**

Shear Distribution (For laminar)

$$C_{f,x} = \frac{\tau_{0,x}}{\frac{1}{2}\rho U_\infty^2}$$



$$\frac{0.664}{\sqrt{Re_x}} = \frac{\tau_{0,x}}{\frac{1}{2}\rho U_\infty^2}$$



$\left.\frac{du}{dy}\right|_{y=0} \Rightarrow$ High

$\left.\frac{du}{dy}\right|_{y=0} \Rightarrow$ low

$$\left[\tau_{0,x} \propto \frac{1}{\sqrt{x}}\right]$$

$$\boxed{\tau_{0,x} = \frac{0.332 \cdot \rho U_\infty^2}{\sqrt{Re_x}}}$$

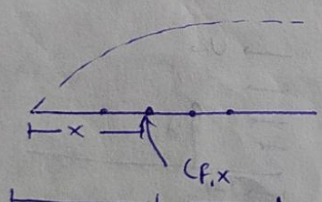
$$\boxed{\tau_0 = \mu \cdot \left.\frac{du}{dy}\right|_{y=0}}$$

$\Rightarrow$ $\left.\frac{du}{dy}\right|_{y=0}$ decreases in the flow direction

③ Average skin - friction coefficient ~~$C_{F,x}$~~ $(\overline{C_{F,x}})$

(or)

Drag coefficient $[C_D]$



$$\boxed{C_D = 2\, C_{F,x}}$$ (don't confuse this is for laminar flow)

(or)

$$\boxed{C_D = \frac{1.328}{\sqrt{Re_L}}}$$

Math →

$$C_D = \int_0^x C_{f,x}\, dx$$

$$= \frac{\int_0^x \frac{C}{\sqrt{x}} \cdot dx}{x}$$

$$= \frac{C}{x} \int_0^x \frac{1}{\sqrt{x}} \cdot dx$$

$$= \frac{C}{\cancel{x}} \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{C}{x} \frac{\sqrt{x}}{\left(\frac{1}{2}\right)}$$

$$= C_D = 2\left(\frac{C}{\sqrt{x}}\right)$$

$$\boxed{C_D = 2\, C_{F,x}}$$

For turbulent layer over a flat plate.

The most appropriate velocity profile for a turbulent boundary layer over a flat plate is based on $\left[\frac{1}{7}^{th} \text{ Power law}\right]$

Most appropriate velocity profile $\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$

Boundary layer thickness $\boxed{\frac{\delta}{x} = \frac{0.376}{Re_x^{\frac{1}{5}}}}$

Local skin-friction coefficient $\boxed{C_{f,x} = \frac{0.059}{Re_x^{\frac{1}{5}}}}$ or $C_{f,x} = \frac{C}{x^{\frac{1}{5}}}$

Average skin-friction coefficient (or) Drag coefficient (C_D) $\boxed{C_D = \frac{0.074}{Re_L^{\frac{1}{5}}}}$

Math,

$$C_D = \frac{\int_0^x C_{f,x}\,dx}{x} = \frac{\int_0^x \frac{C}{x^{\frac{1}{5}}}\,dx}{x} = \frac{C}{x}\int_0^x \frac{1}{x^{\frac{1}{5}}}\,dx$$

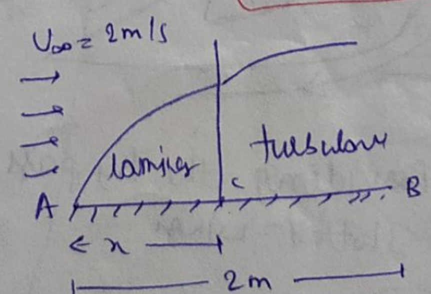
$$\cancel{C_D} = \frac{C}{x}\cdot\frac{x^{-\frac{1}{5}+1}}{-\frac{1}{5}+1} = \frac{C}{(x)^{\frac{1}{5}}}\cdot x\,\frac{1}{\left(\frac{4}{5}\right)} = \frac{5}{4}\underbrace{\left(\frac{C}{x^{\frac{1}{5}}}\right)}_{C_{f,x}}$$

$$\boxed{C_D = \frac{5}{4}C_{f,x}}$$

(Pb) $Re_{AB} = \frac{\rho\, U_\infty (2)}{\mu}$

$= \frac{U_\infty (2)}{\nu} = \frac{2\times 2}{10^{-6}}$

$= 4\times 10^6$ (Not this)



width = 2m

for avoid mistake ← put this $Re_{AC} = \frac{U_\infty \cdot x}{\nu}$

$5\times 10^5 = \frac{2x}{10^{-6}}$

$x = 0.25\,m$

$\rho_w = 998\ kg/m^3$

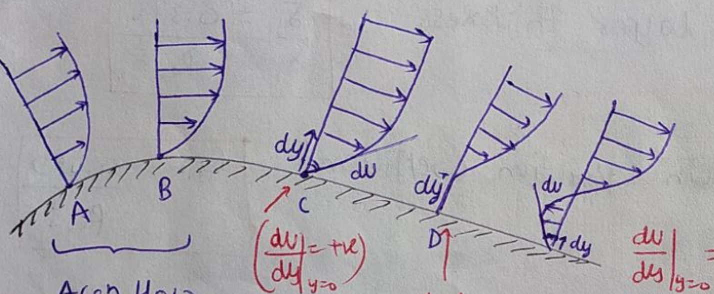
$\nu = 10^{-6}\ m^2/s$

Blasius Results

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$$

$$\frac{\delta}{0.25} = \frac{5}{\sqrt{5\times10^5}} \qquad \delta = 1.768\ mm$$

Boundary layer separation:



Acc^n flow

$\frac{dP}{dx} < 0$

(Favourable Pressure)

Deacceleration Flow

$\frac{dP}{dx} > 0$, (Adverse Pressure gradient)

$$\frac{u}{U_\infty} = \frac{y}{\delta}$$

$\frac{du}{dy}\Big|_{y=0} = (+ve)$

(Flow is attached)

$\frac{du}{dy}\Big|_{y=0} = 0$

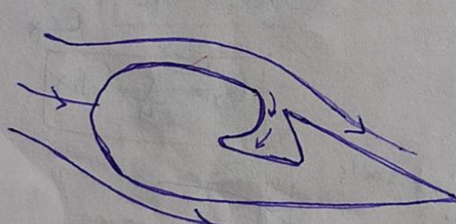
(Flow is at the verge of separation)

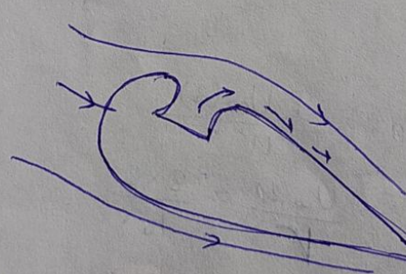
$\frac{du}{dy}\Big|_{y=0} = (-ve)$

Flow has separated

Method to control the Flow separation:

① By the set^n of suction of fluid ② By the injection of fluid





③ By Providing the by pass in slotted wings

(Pb) For laminar boundary layer over a flat plate

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Determine, δ, C_{F_x}, C_D

Soln: By Von-Karmann integral momentum eqn.

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{\partial\theta}{\partial x} \quad \text{---} (1)$$

For θ,

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy.$$

Solve then we get

$$\theta = \frac{39\delta}{280} \quad \text{--} (2)$$

(3) For $\tau_{0,x} = \mu \left.\frac{du}{dy}\right|_{y=0}$

$$= \mu \left(\frac{d}{dy}\left(U_\infty\left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\cancel{\left(\frac{y}{\delta}\right)^3}^{\,0}\right]\right)\right|_{y=0}$$

$$\tau_{0,x} = \frac{3}{2}\,\mu \cdot \frac{U_\infty}{\delta} \quad \text{--} (3)$$

By eq (1) (2) (3)

$$\frac{\overbrace{\frac{3}{2}\frac{\mu \cdot U_\infty}{\delta}}^{\tau_{ox}}}{\rho \cdot U_\infty^2} = \frac{\partial}{\partial x}\left(\frac{39\delta}{280}\right)$$

$$= \frac{3}{2}\frac{\mu \cdot \cancel{U_\infty}}{\delta\,\rho\, U_\infty^{\cancel{2}}} = \frac{\cancel{39}^{13}}{\cancel{280}_{140}} \cdot \frac{\partial\delta}{\partial x}$$

$$\frac{140}{13} \cdot \frac{\mu}{\rho \cdot U_\infty} \cdot \partial x = \delta \cdot \partial\delta$$

Int it.

$$\frac{\delta^2}{2} = \frac{140}{13} \cdot \frac{\mu}{\rho \cdot U_\infty} \cdot x + C$$

At $x = 0$, $\delta = 0$, $C = 0$

$$\frac{\delta^2}{2} = \frac{140}{13}\left(\frac{\mu}{\rho \cdot U_\infty \cdot x}\right) x^2$$

$$\frac{\delta}{x} = \sqrt{\frac{280}{13}} \times \frac{1}{\sqrt{Re_x}}$$

$$\boxed{\frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}}}$$

Re eq (3)

$$\tau_{0,x} = \frac{3}{2} \cdot \frac{\mu \cdot U_\infty}{\delta}$$

$$\left\{ \delta = \frac{4.64\, x}{\sqrt{Re_x}} \right)$$

$$\tau_{0,x} = \frac{3}{2} \cdot \frac{\mu U_\infty}{\frac{4.64 x}{\sqrt{Re_x}}}$$

$$\tau_{0,x} = \frac{3}{2 \times 4.64} \left(\frac{\mu \cdot U_\infty}{x}\right) \frac{\sqrt{Re_x} \times \rho U_\infty^2}{\rho U_\infty^2}$$

$$\tau_{0,x} = 0.323 \frac{\sqrt{Re_x}}{Re_x} \cdot \rho \cdot U_\infty^2$$

$$\boxed{\tau_{0,x} = \frac{0.323 \cdot \rho U_\infty^2}{\sqrt{Re_x}}}$$

$$\boxed{C_{f,x} = \frac{\tau_{0x}}{\frac{1}{2}\rho U_\infty^2}}$$

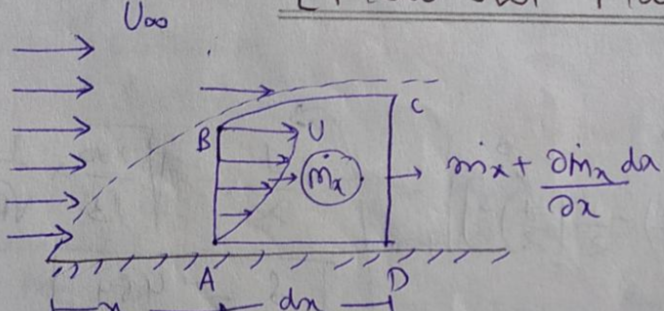
$$C_{f,x} = \frac{\frac{0.323}{\sqrt{Re_x}} \cdot \cancel{\rho} U_\infty^2}{\frac{1}{2}\cancel{\rho} U_\infty^2}$$

$$\boxed{C_{f,x} = \frac{0.646}{\sqrt{Re_x}}}$$

$$\boxed{C_D = 2 C_{f,x}}$$

Von- Karmann integral momentum eqn

[Flow over Flate Plate]



$\dot{m}_x = \int_0^\delta (\rho.(dy.1)) U$ (Assume width = 1)

Apply continuity.

$$\dot{m}_{AB} + \dot{m}_{BC} = \dot{m}_{CD}$$

$$\downarrow \quad \downarrow \quad \searrow$$

$$\dot{m}_x \quad \frac{\partial \dot{m}_x}{\partial x} dx \quad \dot{m}_x + \frac{\partial \dot{m}_x}{\partial x} dx$$

Initial momentum per sec $= (\dot{m}V)_{AB} + (\dot{m}V)_{BC}$

final momentum per sec $= (\dot{m}V)_{CD}$

$= (\dot{m}$

(Pb) $\frac{U}{V_\infty} = C_0 + C_1\left(\frac{y}{\delta}\right) + C_2\left(\frac{y}{\delta}\right)^2 + C_3\left(\frac{y}{\delta}\right)^3$ (10 marks ese) mains

Determine: C_0, C_1, C_2, C_3

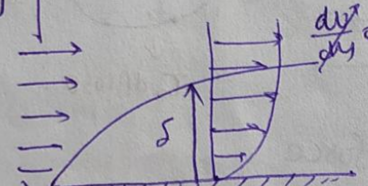
Soln (Pb) ① At $y=0$, $U=0$

$\boxed{C_0=0}$

② $y=\delta$, $U=V_\infty$

$\frac{V_\infty}{V_\infty} = \cancel{C_0}^{0} + C_1\left(\frac{\delta}{\delta}\right) + C_2\left(\frac{\delta}{\delta}\right)^2 + C_3\left(\frac{\delta}{\delta}\right)^3$ | $C_1+C_2+C_3 = 1$ --- ①

$\frac{du}{dy} = V_\infty\left[\frac{C_1}{\delta} + \frac{2C_2 y}{\delta} + \frac{3C_3 y^2}{\delta^3}\right]$ | $\frac{d^2u}{dy^2} = V_\infty\left(\frac{2C_2}{\delta^2} + \frac{6C_3 \cdot y}{\delta^3}\right)$



③ At $y=\delta$, $\frac{du}{dy}=0$

$\cancel{\frac{du}{dy}}^{0}\Big|_{y=\delta} = V_\infty\left(\frac{C_1}{\delta} + \frac{2C_2\delta}{\delta^2} + \frac{3C_3\delta^2}{\delta^3}\right)$

$C_1 + 2C_2 + 3C_3 = 0$ --- ②

④ At $y=0$, $\frac{d^2u}{dy^2}=0$

$\cancel{\frac{d^2u}{dy^2}}^{0}\Big|_{y=0} = V_\infty\left(\frac{2C_2}{\delta^2} + \frac{6C_3(0)}{\delta^3}\right)$

$C_2=0$

In general

At wall

$\tau \to$ max.

$\frac{d\tau}{dy} = 0 \left[\tau = \mu\frac{du}{dy}\right]$

$\frac{d^2u}{dy^2} = 0$

from ① and ②

$\cancel{C_1} + \cancel{C_2}^{0} + C_3 = 1$

$-\cancel{C_1} + \cancel{2C_2}^{0} + 3C_3 = 0$
$(-)\quad(-)$

$-2C_3 = 1$

$C_3 = -\frac{1}{2}$

$C_1 - \frac{1}{2} = 1$

$C_1 = \frac{3}{2}$

then, $\frac{\gamma}{\gamma-1} \cdot \frac{P}{\rho} + \frac{V^2}{2} + g \cdot z = \text{const}$

Divide by g

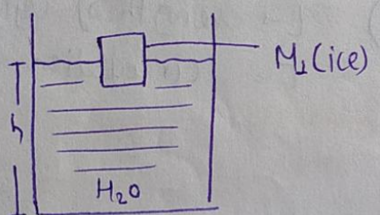
$$\boxed{\frac{\gamma}{\gamma-1} \cdot \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{const}}$$

[Fluid dynamic portion]

Ques What happen the level of water inside the beaker when ~~edges~~ ice melt

Imp

Case ①



Force analysis

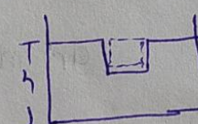
$F_B = M_1 g$

$\rho_w \forall_{displaced} \cancel{g} = M_1 \cancel{g}$

$\forall_{displaced} = \frac{M_1}{\rho_w}$

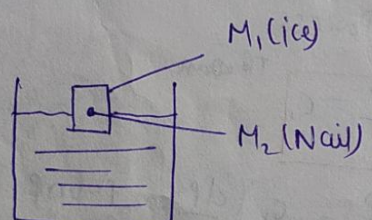
volume recovery after melting

$\forall_{recovery} = \frac{M_1}{\rho_w}$



(same level)

Case-②



Free analysis.

$F_B = (M_1 + M_2) \cdot g$

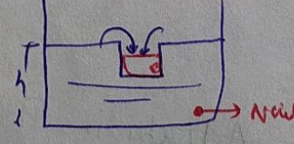
$\rho_w \forall_{displaced} \cancel{g} = (M_1 + M_2) \cancel{g}$

$\forall_{displaced} = \frac{M_1}{\rho_w} + \frac{M_2}{\rho_w}$

volume Recovery after melting

$\forall_{recovery} = \frac{M_1}{\rho_w} + \frac{M_2}{\rho_{Nail}}$ low

$\rho_{Nail} > \rho_w$



level goes down

Pipe Flow [Discussion]

9 Turbulant Flow in pipes

In general

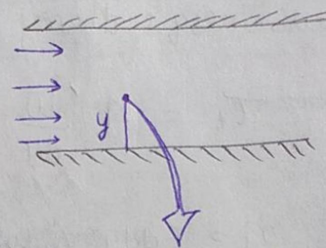
$$\text{Mean} = \frac{V+V}{2}$$

$$\text{mean} = \frac{V+V+V+\dots 10}{10}$$

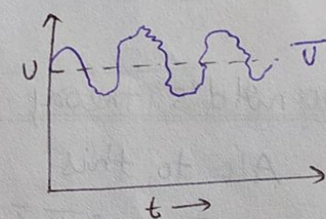
↓ more accurate

$$\text{Mean} = \frac{V+V+V+\dots 20}{20}$$

↓ almost same



$\bar{U}$ = temporal mean velocity at a distance y from the wall



$$U = \underbrace{\bar{U}}_{\text{Steady}} + \underbrace{U'}_{\text{unsteady (turbulent generate)}}$$

$$V = \bar{V} + v'$$

$$W = \bar{W} + W'$$

$$\bar{U} = \int_0^T \frac{U\,dt}{T}$$

break the flow into two part
1. Mean ($\bar{U}$)
2. turbulant fluctuations (U', V', W')

T = time taken suffiient large.

(Imp point of view)

$\overline{U'} = 0$ but $\boxed{\overline{U'}\,\overline{V_1} \neq 0}$

$\overline{V'} = 0$

$\overline{W'} = 0$

Math

$$\bar{U} = \int_0^T \frac{U\,dt}{T} \quad (U = \bar{U} + u')$$

$$\overline{U'} = \int_0^T \frac{(U - \bar{U})\,dt}{T}$$

$$\overline{U'} = \int_0^T \frac{U}{T}dt - \int_0^T \frac{\bar{U}}{T}dt$$

$$= \bar{U} - \bar{U}\frac{T}{T} = 0 \text{ (Prove)}$$

[Shear stresses in turbulant flow]

$$\boxed{\tau = \tau_v + \tau_t}$$

Due to turbulent fluctuations there is a additional shear stress b/w the layers of fluid to determine that additional shear a no. of theory were proposed.

$$\frac{1}{\sqrt{F}} = \frac{5.75}{\sqrt{8}} \log_{10} \frac{D\,\bar{U}}{2\nu}\sqrt{\frac{F}{8}} + \frac{1.75}{\sqrt{8}}$$

$$\frac{1}{\sqrt{F}} = 2.03 \log_{10} \frac{Re\sqrt{F}}{2\sqrt{8}} + \frac{1.75}{\sqrt{8}}$$

$$\frac{1}{\sqrt{F}} = 2.303 \log_{10} Re\sqrt{F} + 2.303 \log_{10} \frac{1}{2\sqrt{8}} + \frac{1.75}{\sqrt{8}}$$

$$\frac{1}{\sqrt{F}} = 2.303 \log_{10} Re\sqrt{F} - 0.9$$

By Nikuradse experi.

$$\boxed{\frac{1}{\sqrt{F}} = 2 \log_{10} Re\sqrt{F} - 0.8}$$

$$f = fn(Re)$$

By Nikuradse expe

$$\frac{1}{\sqrt{F}} = 2 \log_{10} \frac{D}{2K_s} + 1.74$$

$$\boxed{f = fn\left(\frac{K_s}{D}\right)}$$
