Jai Ganesha

FLUID MECHANICS

(20 hour)
Fluid Statics
- Study of the fluid when it is in static condition

(10 hrs)
Fluid Kinematic
- Study of the motion of fluid without considering cause of it

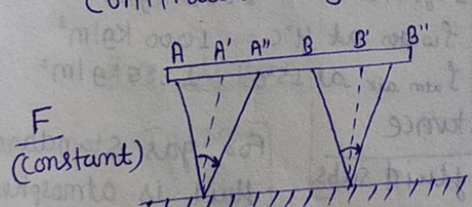
(40 hrs)
Fluid dynamic
- Study of motion of fluid with considering the cause of motion

In general -
1. solid
2. liquid } Fluid
3. Gases }

- **Fluids** Fluids are the substances that deforms conti - nously under the application of tangential force no matter How small it is.

Fluid

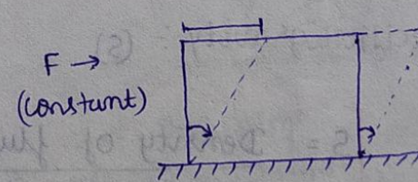
continaus deformation



Deforms permanantly no matter How small is applied tangential force

Solid

Fixed deformation



Deforms Permanently Only when the applied force is more than elastic limit otherwise elastic they regain their original Position when applied force is removed.

② the rate of deformation is important. | In solid, the deformation is important.

Fluid of continuum :-

A macroscopic ~~Macheo~~ system when the inter molecular distance are neglible as compare to dimension of system we can assume that adjacent to 1 molecule to another molecule without any space therefore entire fluid mass system can be treated as continaus distribution of mass and such continaus mass of fluid is known as continuum (It considered statistical avg effect of molecules)

For O_2
$T = 20°C$
$P = 1$ atm
→ 3×10^6 molecule
$V = 1 mm^3$

Fluid property :-

① **Density** (ρ) :- It is defined as the mass of a fluid substance per unit volume.

$$\boxed{\rho = \frac{M}{V}}$$

In M.K.S = Kg/m^3
In C.G.S. = gm/cm^3
$1\ gm/cm^3 = 1000\ Kg/m^3$
ρ_{water} at $4°C = 1000\ Kg/m^3$
$\rho_{atm\ air}$ at $15°C = 1.225\ Kg/m^3$

② **Specific gravity** :- (S)

$$\boxed{S = \frac{\text{Density of fluid substance}}{\text{Density of standard fluid subs.}}}$$

For gas standard fluid is atmospheric air as well as Hydrogen gas

For std. fluid
For liquid ⇒ H_2O at $4°C$
For gases ⇒ Air

Ex.
$S_{Hg} = 13.6$
$\frac{\rho_{Hg}}{\rho_w} = 13.6$
$\frac{\rho_{Hg}}{10^3} = 13.6$
$[\rho_{Hg} = 13.6 \times 10^3\ kg/m^3]$

③ **Specific volume** :-

Reciprocal of density $= \frac{1}{\rho} = \frac{V}{m}$

unit $\frac{m^3}{Kg}$

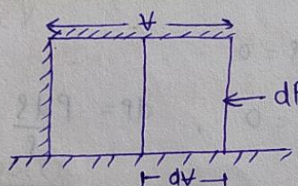
③ Weight density or specific wt. - It is defined as the weight of fluid substance per unit volume (γ)

$$\frac{m}{V} g = \rho g \quad / \quad \frac{wt}{V} = \boxed{\frac{mg}{V} = \rho g} \quad \text{(N/m}^2\text{) unit}$$

④ compressibility :- (β) It is defined as the reciprocal of bulk modulus of ellasticity (K) of fluids.

$$\boxed{\beta = \frac{1}{K}}$$

K ⇒ Bulk modulus of ellasticity



$$K = \frac{\text{Stress}}{\text{Strain}} = \boxed{\frac{dP}{-\frac{d\forall}{\forall}}}$$

$\beta = 0$, fluid is incompressible
$\beta \neq 0$ fluid is compressible

In general,

$m = \rho \forall$

Different. it

$\int dm = \rho d\forall + \forall d\rho$

$dm = 0, \quad 0 = \rho d\forall + \forall d\rho$

$-\rho d\forall = \forall d\rho$

$$\boxed{-\frac{d\forall}{\forall} = \frac{d\rho}{\rho}}$$

$$K = \rho \cdot \frac{dP}{d\rho} \quad **$$

$$\boxed{\beta = \frac{1}{\rho}\frac{d\rho}{dP}}$$

If $\frac{d\rho}{dP} = 0$ then $\boxed{\beta = 0}$

If $\frac{d\rho}{dP} \neq 0$ then $\boxed{\beta \neq 0}$

For liquids

T= 20°C

P= 1 atm , ρ_w = 998 kg/m³

P= 100 atm , ρ_w = 1003 kg/m³

% change $= \frac{1003 - 998}{998} \times 100 = 0.5$ (negligible)

• liquid is generally incompressible

For gases

$$\boxed{P = \rho R T}$$

Note : $\boxed{\text{Mach Number} = \frac{V}{C}}$ C = velocity of sound in medium

In gaseous flow is $ma \le 0.3$ flow is considered as incompressible.

⑤ <u>Isothermal compressibility of gases :-</u>

$$P = \rho.R.T$$

$$\frac{P}{\rho} = (\text{constant})$$

$$P\rho^{-1} = (\text{constant})$$

Differentiate it

$$\rho^{-1} dP - \rho^{-2} P.d\rho = 0$$

$$\rho^{-1}\left[dP - P\frac{d\rho}{\rho}\right] = 0 \quad , \quad dP = P\frac{d\rho}{\rho} \quad , \quad P = \frac{dP \times \rho}{d\rho}$$

** Sound is propagated in fluid due to compressibility of medium

Sound of speed

$$C = \sqrt{\frac{K}{\rho}}$$

$$\boxed{P = \frac{dP}{\frac{d\rho}{\rho}}} \quad , \quad \boxed{K_{iso} = P} \quad \boxed{\beta_{iso} = \frac{1}{P}}$$

⑥ <u>Adiabatic compressibility of gases :-</u>

$$\frac{P}{\rho^{\gamma}} = \text{const.} \quad \text{(or)} \quad P.\forall^{\gamma} = \text{const.} \quad [\gamma \rightarrow \text{Gamma}]$$

$$P.\rho^{-\gamma} = \text{const.}$$

Diff. it

$$\rho^{-\gamma} dP - \gamma \rho^{-\gamma-1}.P.d\rho = 0$$

$$\rho^{-\gamma}\left| dP - \gamma.P.\frac{d\rho}{\rho}\right| = 0$$

$$dP - \gamma.P.\frac{d\rho}{\rho} = 0$$

$$dP = \gamma.P.\frac{d\rho}{\rho}$$

$$\frac{dP}{\frac{d\rho}{\rho}} = \gamma.P$$

$$\boxed{\gamma = \frac{C_p}{C_v}}$$

C_p → specific heat at const. Pressure

C_v → specific heat at const. volume

$$\boxed{K_{adia} = \gamma.P}$$

$$\boxed{\beta_{adia} = \frac{1}{\gamma.P}}$$

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Differentiate it

$$\rho^{-1} dP - \rho^{-2} P . d\rho = 0$$

$$\rho^{-1} \left[dP - P \frac{d\rho}{\rho} \right] = 0 \quad , \quad dP = P \frac{d\rho}{\rho} \quad , \quad P = \frac{dP \times \rho}{d\rho}$$

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$$P . \rho^{-\gamma} = \text{const.}$$

Diff. it

$$\rho^{-\gamma} dP - \gamma \rho^{-\gamma - 1} . P . d\rho = 0$$

$$\rho^{-\gamma} \left| dP - \gamma . P . \frac{d\rho}{\rho} \right| = 0$$

$$dP - \gamma . P . \frac{d\rho}{\rho} = 0$$

$$dP = \gamma . P . \frac{d\rho}{\rho}$$

$$\boxed{\gamma = \frac{C_P}{C_V}}$$

C_P → specific heat at const. pressure

C_V → specific heat at const. volume

$$\frac{dP}{\frac{d\rho}{\rho}} = \gamma . P$$

$$\boxed{K_{adia} = \gamma . P}$$

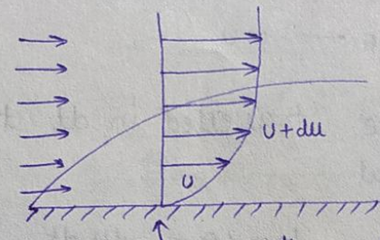
$$\boxed{\beta_{adia} = \frac{1}{\gamma . P}}$$

In general

Flow Physics

- **Flow over a Flat Plate :-**

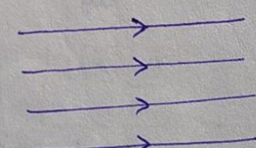
When a real fluid flow over a solid body the fluid Particles at the surface of body flow with the same velocity as that the surface to satisfy no slip cond^n so,



Relative velocity of velocity fluid Particles and at th surface of the body (solid) is zero

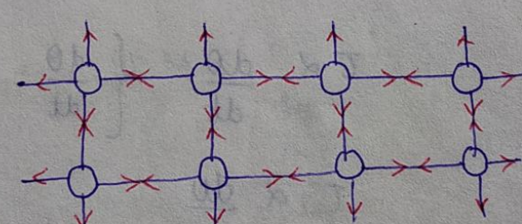
- **Viscosity**

When the two adjacent layer of fluid are in relative motion then they resist the motion of each other such a fundamental property of fluid is known as $\boxed{\text{viscosity}}$



- **Cause of viscosity**

For liquid



$\boxed{\mu_{water} >>> \mu_{air}}$

$\mu \rightarrow$ measurement of internal resistance

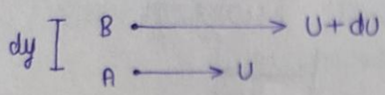
Intermolecular force of attra
$\Rightarrow$ Cohesive force

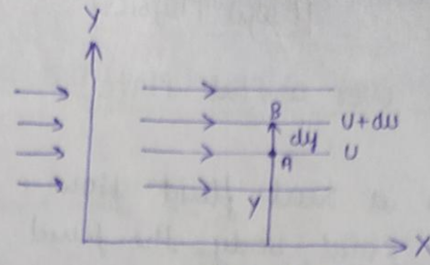
For gases

cohesion $\Rightarrow$ almost Nil **

[Random motion of molecule]

(A) Angular deformation :-

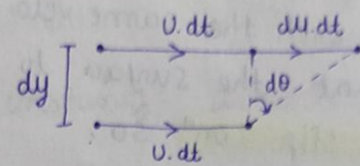




Flow direction

Distance travelled in dt time interval

$$\tan d\theta = \frac{dU.dt}{dy}$$



$$\left\{\begin{array}{l} d\theta = \text{very very small} \\ \tan d\theta \approx d\theta \end{array}\right\}$$

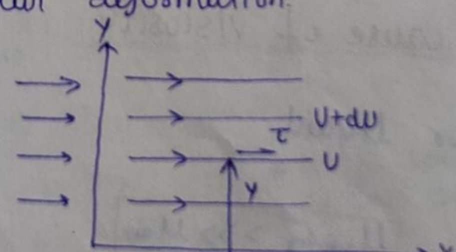
$$d\theta = \frac{dU}{dy}.dt \quad , \quad \boxed{\frac{d\theta}{dt} = \frac{dU}{dy}}$$

[Rate of angular deformation]

(B) Newton law of viscosity :-

A/c to this the shear stress on a layer at a distance Y from the wall is directly proportional to the rate of angular deformation.

$$\tau \propto \frac{d\theta}{dt} \quad \left\{\frac{d\theta}{dt} = \frac{dU}{dy}\right\}$$



Flow dir^n

$$\tau \propto \frac{dU}{dy}$$

$$\boxed{\tau = \mu \times \frac{dU}{dy}}$$

Slope

$Y = mc$

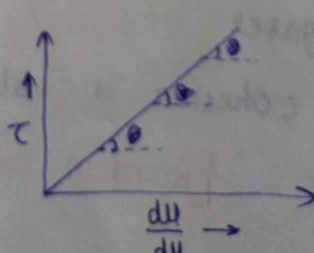
μ = coeff. of viscosity

or

Absolute viscosity

or

Dynamic viscosity



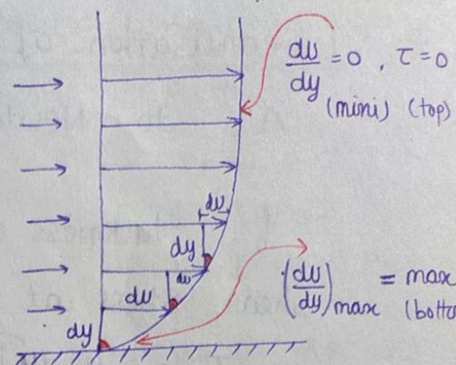
Slope of curve $= \mu$

EX- H_2O, Air, Hg, Petrol, Kerosene.

$\tau = \mu \frac{du}{dy} \to 0$

$\tau = 0$

The value of $\frac{du}{dy}$ is maximum at the wall and gradually decreases in the transverse dirⁿ to the flow.



© Dynamic viscosity (μ)

In S.I.

$$\tau = \mu \frac{du}{dy}$$

$$\frac{N}{m^2} = \mu \times \frac{m}{s} \times \frac{1}{m} \quad \text{then} \quad \boxed{\mu \Rightarrow \frac{N.s}{m^2}} \; ***$$

$$\mu = kg \times \frac{m}{s^2} \times \frac{1}{m^2} \times s \quad \text{then} \quad \boxed{\mu = \frac{kg}{m\text{-}s}} \quad [ML^{-1}T^{-1}]$$

In C.G.S.

$$1 \text{ Poise} = \frac{1 \text{ gm}}{cm\text{-}s}$$

$$\boxed{1 \text{ Poise} = \frac{1}{10} \frac{kg}{m\text{-}s}}$$

ⓓ Kinematic viscosity (ν)

$$\boxed{\nu = \frac{\mu}{\rho}}$$

In M.K.S

$$\nu = \frac{\frac{kg}{m\text{-}s}}{\frac{kg}{m^3}} = m^2/s$$

$$\boxed{\nu = m^2/s}$$

In C.G.S

$$1 \text{ stoke} = \frac{1 \, cm^2}{s}$$

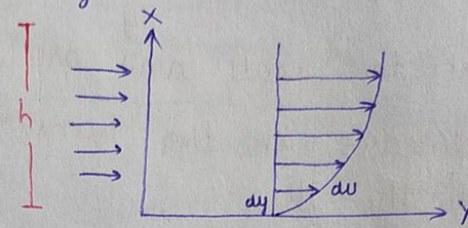
$$\boxed{1 \text{ stoke} = \frac{1}{10^4} \frac{m^2}{s}}$$

Linearlisation of Newton's law of viscosity →

A/c to Newton's law of viscosity

$$\tau = \mu \cdot \frac{du}{dy}$$

h = thickness of flow



Flow direction

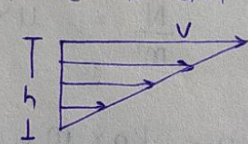
Shear Stess at wall

$$\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0}$$

du hai

$$\left.\frac{du}{dy}\right|_{y=0} = \text{Slope of tangent at } y=0$$

If $h \Rightarrow$ very-very small then any velocity Profile can be treated as straight



$$\left.\frac{du}{dy}\right|_{y=0} = \frac{V}{h}$$

$$\boxed{\tau_0 = \mu \cdot \frac{V}{h}}$$

Relative velocity

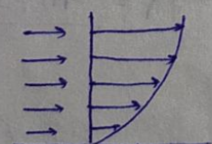
very-very small gap

In general.

$$\tau = \mu \frac{du}{dy}$$

calculate the shear at wall

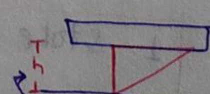
Given $U = f^n(y)$



$$\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0}$$

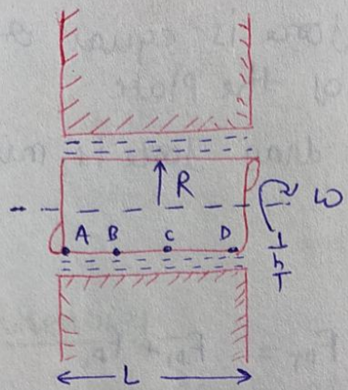
If $U = f^n(y)$ is nt given

Ex.



Very very small gap then it became straight line

Pb-①



$V = \omega R$

$\tau = \mu \cdot \frac{V}{h}$

$\omega_A = \omega_B = \omega_C = \omega_D$

$V_A = V_B = V_C = V_D$

Power consumed ?

Soln $P = T \cdot \omega$

$= (F \cdot R) \cdot \omega$

$= (\tau \cdot A) \cdot R \cdot \omega$

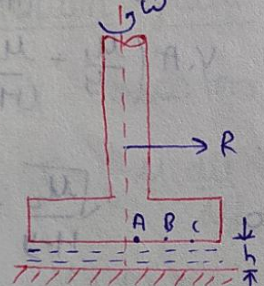
$= \mu \cdot \frac{V}{h} (2\pi R L) \cdot R \cdot \omega$

$= \mu \cdot \frac{V}{h} (2\pi R^2 \omega L)$

$= \frac{2\pi \mu \cdot \omega^2 R^3 \cdot L}{h}$

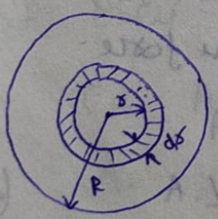
Pb - ②

***



$\omega_A = \omega_B = \omega_C$

$V_A \neq V_B \neq V_C$



Determine torque required

Soln $dT = dF \cdot r$

$= (\tau \cdot dA) \cdot r$

$= \left(\frac{\mu \cdot V}{h}\right)(2\pi r\, dr) \cdot r$

$= \mu \left(\frac{\omega \cdot r}{h}\right)(2\pi r^2 dr)$

$= \frac{2\pi \mu \omega r^2 dr}{h}$

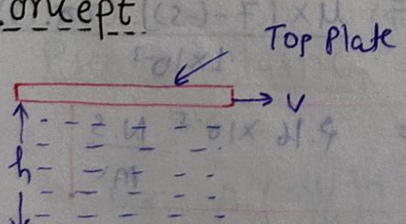
integrate....

$T = \frac{2\pi \mu \cdot \omega}{h} \int_0^R r^3 dr$ **

$= \cancel{A R^4}$ $\boxed{\frac{\pi \cdot \mu \cdot \omega R^4}{2h}}$

direct formula based

Concept



$h \Rightarrow$ very-2 small

(Plate move through the water surface)

Determine the dirⁿ of shear force on top and bottom plate