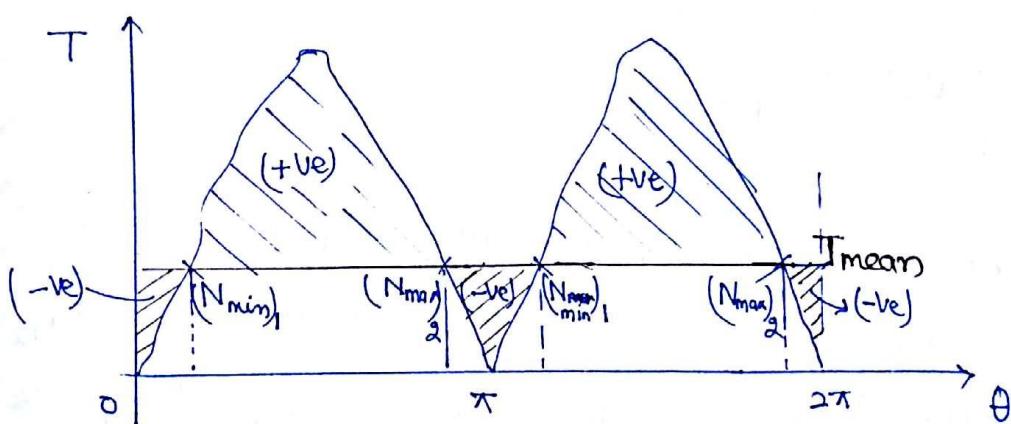


$$\text{4} \quad T_{\text{mean}} = \frac{\text{W.D./cycle}}{T_{\text{period}}}$$

~~★~~ Flywheel: it is a device which serve as a reservoir to store energy when supply of the energy is more than the requirement and release energy when requirement is more than the Supply.

Turning Moment Diagram of Double acting Steam engine:



$$T_{\text{period}} = 2\pi$$

$$(\text{WD})_{\text{cycle}} = \int_0^{2\pi} T d\theta = T_{\text{mean}} \times 2\pi$$

$$\text{★} \quad \boxed{T_{\text{mean}} = \frac{(\text{WD})_{\text{cycle}}}{2\pi}}$$

After installing Flywheel.

$$N_{\text{max}} = \text{max. of } [N_{\text{max}_1}, N_{\text{max}_2}, \dots]$$

$$N_{\text{min}} = \text{min. of } [N_{\text{min}_1}, N_{\text{min}_2}, \dots]$$

Variation of speed =  $N_{max} - N_{min}$  = Fluctuation

$C_s$  = Coefficient of fluctuation of speed

$$C_s = \frac{N_{max} - N_{min}}{N}$$

$$N = \frac{N_{max} + N_{min}}{2}$$

Coefficient of steadiness =  $m$

$$m = \frac{1}{C_s}$$

Coefficient of fluctuation of energy

$$C_E = \frac{E_{max} - E_{min}}{\text{1 cycle}}$$

Note: ~~some~~ sum of all positive areas (energy) about the mean line will be equal to sum of all negative areas below the mean line.

Fundamental equation of flywheel:—

$$E_{max} = \frac{1}{2} I \omega_{max}^2$$

$$E_{min} = \frac{1}{2} I \omega_{min}^2$$

$$\Delta E = E_{max} - E_{min} = \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2)$$

06/12/16

Variation in  $E$

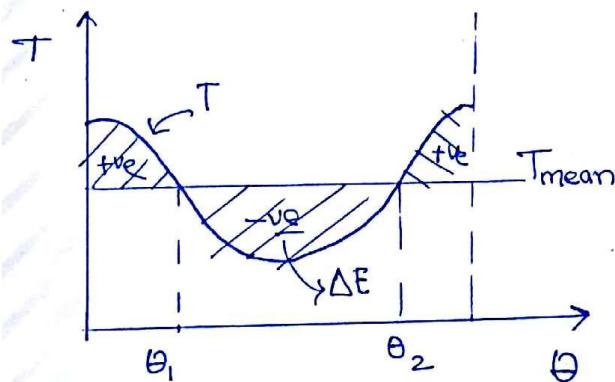
$$\Delta E = \frac{1}{2} I (w_{\max}^2 - w_{\min}^2)$$

$$\Delta E = \frac{1}{2} I \left( \frac{w_{\max}^2 - w_{\min}^2}{w} \right) (w_{\max} + w_{\min}) \times w$$

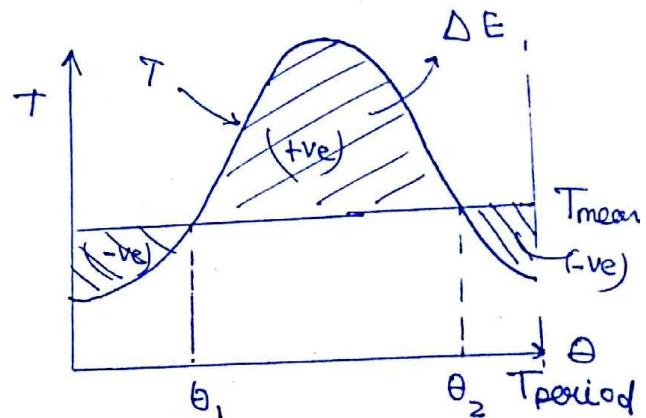
$$w = \frac{w_{\max} + w_{\min}}{2}$$

~~$\star$~~   $\Delta E = I w^2 C_s$

eq



eq



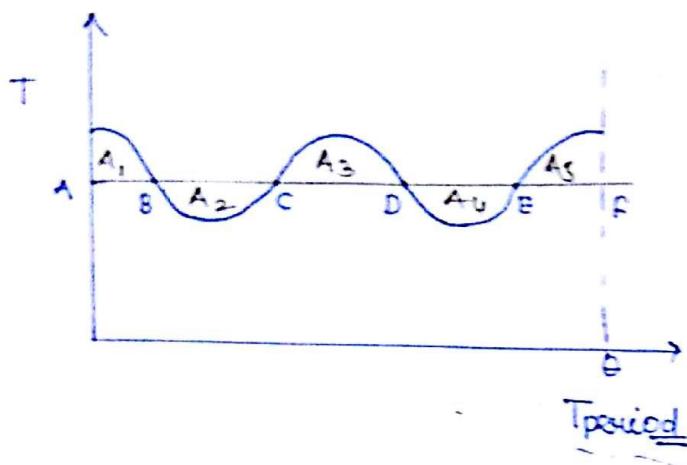
$$\Delta E = \int_{\theta_1}^{\theta_2} (T_{\text{mean}} - T) d\theta$$

$$\Delta E = \int_{\theta_1}^{\theta_2} (T - T_{\text{mean}}) d\theta$$

How to Find  $\theta_1$  &  $\theta_2$   
 $T = T_{\text{mean}}$ .

{ \* Total Area's above & below the  $T_{\text{mean}}$  line are equal. }

C



$$E_A = E$$

$$E_B = E + A_1$$

$$E_C = E + A_1 - A_2$$

$$E_D = E + A_1 - A_2 + A_3$$

$$E_E = E + A_1 - A_2 + A_3 - A_4$$

$$E_F = E + A_1 - A_2 + A_3 - A_4 + A_5$$

$$E_F = E$$

$$E_{\max} = \text{max of } [E_A, E_B, E_C, E_D, E_E, E_F]$$

$$E_{\min} = \text{min of } [E_A, E_B, E_C, E_D, E_F, E_F]$$

$$\boxed{\Delta E = E_{\max} - E_{\min}}$$

$\star \xrightarrow{\text{eq}}$   $E_{\max} = E + 70$        $E_{\min} = E - 30$

$$\Delta E = 100 \text{ unit} \begin{matrix} \rightarrow \text{cm}^2 \\ \rightarrow \text{mm}^2 \end{matrix}$$

$$\text{Scale } 1 \text{ mm}^2 = 15 \text{ Joule}$$

$$100 \text{ mm}^2 = 100 \times 15 \text{ J}$$

$A$  scale  $1 \text{ m m} = 15 \text{ Nm}$

$$1 \text{ mm} = \frac{1}{3} \text{ rad.}$$

$$100 \text{ mm}^2 = 100 \times 15 \times \frac{1}{3} \text{ (J)}$$

① A machine is coupled to a 2 stroke engine which produced a torque  $T = (800 + 180 \sin 3\theta) \text{ Nm}$  where  $\theta$  is crank angle what is the max. fluctuation in energy.

Soln

$$T = 800 + 180 \sin 3\theta$$

$$T_{\text{mean}}(2\pi/3) = \int_0^{2\pi/3} (800 + 180 \sin 3\theta) d\theta$$

$$= 800 \theta + 180 \frac{\theta \cos 3\theta}{3} \Big|_0^{2\pi/3}$$

$$= 800(2\pi/3) - 180 \left\{ \cos \cancel{\frac{2\pi}{3}} - \cos \cancel{\frac{\pi}{3}} \right\}$$

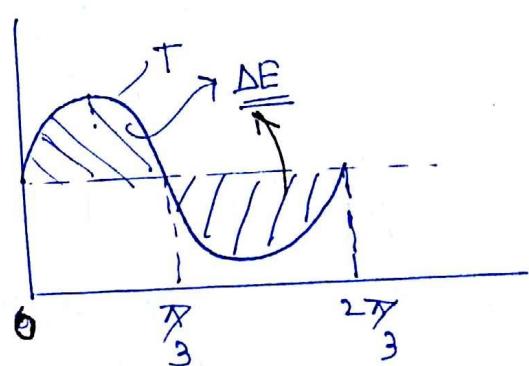
$$= \frac{1600\pi}{3} - \frac{180}{3} [+1 - 1] \quad \frac{3\theta \times 2\pi}{3}$$

$$T_{\text{mean}}(2\pi/3) = \frac{1600\pi}{3} + \cancel{\frac{360}{3}}$$

$$\Delta E = \int_0^{2\pi/3} (T - T_{\text{mean}}) d\theta \quad \boxed{T_{\text{mean}} = \underline{\underline{800 \text{ Nm}}}}$$

$$\Delta E = \int_0^{2\pi/3} 180 \sin 3\theta d\theta$$

$$\begin{aligned} \Delta E &= -\frac{180}{3} (\cos 3\theta) \Big|_0^{2\pi/3} \\ &= -\frac{180}{3} (-1-1) = 120 \underline{\underline{\text{J}}} \end{aligned}$$



Q.26

P.g.31  
WB

$$T = 10000 + 2000 \sin 2\theta - 1800 \cos 2\theta$$

$$T_{\text{mean}}(\pi) = \int_0^\pi T d\theta$$

$$\text{Ans} = T_{\text{mean}}(\pi) = \int_0^\pi (10000 + 2000 \sin 2\theta - 1800 \cos 2\theta) d\theta \\ = 10000(\pi - 0) + 2000 \left\{ \frac{\sin 2\pi}{2} - \frac{\sin 0}{2} \right\} - 1800 \left\{ \sin 2\pi - \sin 0 \right\}$$

$$T_{\text{mean}}(\pi) = 10000\pi - 2000(1-1) - 1800(0-0)$$

$$T_{\text{mean}} = 10000 \text{ Nm}$$

$$P = T_{\text{mean}} \times \omega = 10000 \times \frac{2\pi \times 250}{60}$$

$$P = 261.79 \text{ kW}$$

$$\Delta E = \int_{20.99}^{110.99} (T - T_{\text{mean}}) d\theta$$

$$= \int_{20.99}^{110.99} (10000 \sin 2\theta - 1800 \cos 2\theta) d\theta$$

$$\Delta E = \int_{20.99}^{110.99} (2000 \sin 2\theta - 1800 \cos 2\theta) d\theta$$

$$(20.99) \frac{\pi}{180}$$

$$\Delta E = 2690.72 \text{ J}$$

$$T = T_{\text{mean}}$$

$$2000 \sin 2\theta - 1800 \cos 2\theta = 0$$

$$\tan 2\theta = 0.9$$

$$2\theta = 41.98^\circ$$

$$= 180 + 41.98$$

$$= 360 + 41.98^\circ$$

$$\underline{\underline{\theta = 20.99, 110.99}}$$

$$T = 10000 + 2000 \sin 2\theta - 1800 \cos 2\theta$$

$$\frac{2\pi}{2}$$

$$\frac{2\pi}{2}$$

$$\boxed{\text{Time period} = \frac{\text{LCM of } N^2}{\text{HCF of } Ds}} \quad \frac{2\pi}{2} = \pi$$

(↑ Highest common)

$$\Rightarrow \text{eq} \quad T = 800 + 1000 \sin 3\theta - 1800 \cos 5\theta$$

$$\frac{2\pi}{3}$$

$$\pi_5$$

$$\boxed{T_{\text{period}} = \frac{2\pi}{1} = 2\pi}$$

$$P = 261.75 \text{ kW}$$

$$\zeta_s = \pm 0.25\%$$

$$\Delta E = I \omega^2 \zeta_s$$

$$\zeta_s = 0.5\%$$

$$2690.72 = I \left( \frac{2\pi \times 250}{60} \right)^2 \times 0.5 \quad \zeta_s = \frac{0.5}{100}$$

$$(ii) \quad I = 785.16 \text{ kg-m}^2$$

$$(iii) \quad (T - T_{\text{mean}})_{\theta=45^\circ} = I \alpha$$

$$(2000 \sin 2\theta - 1800 \cos 2\theta) \Big|_{\theta=45^\circ} = 785.16 \alpha$$

$$(2000 \sin 90^\circ - 1800 \cos 90^\circ) = 785.16 \alpha$$

$$\alpha = 2.54 \text{ rad/sec.}$$

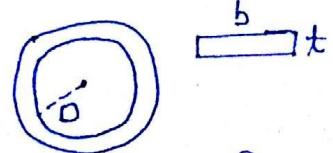
if radius of gyration given

$$I = m k^2 + t \quad \text{Given } D$$

$m = ?$

$$m = \cancel{\rho} g \times \text{Vol.} \quad b \quad t$$

$$m = g \times (\pi D b t)$$



Ring type flywheel

Q.25

P.331

$$T = 10000 + 1000 \sin \theta - 1200 \cos 2\theta$$

$$2\pi \quad \pi$$

$$T_{\text{period}} = \frac{2\pi}{1} = 2\pi$$

$$T_{\text{mean}}(2\pi) = \int_0^{2\pi} (10000 + 1000 \sin \theta - 1200 \cos 2\theta) d\theta$$

$$= \int_0^{2\pi} 10000(2\pi) + 1000 \left[ -\cos \theta \right]_0^{2\pi} - 1200 \left[ \frac{\sin 2\theta}{2} \right]_0^{2\pi}$$

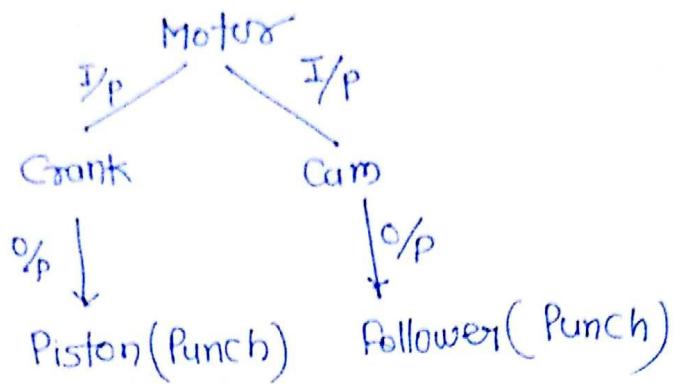
$$T_{\text{mean}}(2\pi) = 10000(2\pi) - 1000 \left[ \cos 2\pi - \cos 0 \right] - 1200 \left[ \sin \pi - \sin 0 \right]$$

$$T_{\text{mean}} = 10000 \text{ Nm}$$

$$P = T_{\text{me}} \times w = 10000 \times \left( \frac{2\pi \times 100}{60} \right)$$

$$P = 104.719 \text{ } \underline{\underline{kW}}$$

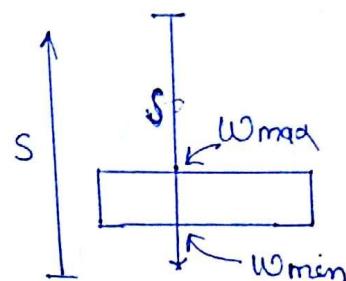
## Use of flywheel in Power Presses :-



$T_c$  - cycle time (2-strokes of punch)

$T_a$  - Actual punching time

$$T_a \ll T_c$$



$$P_{motor} = \frac{\text{Energy req./hole}}{T_c}$$

energy will be supplied  
only in cycle time

$$\Delta E = E_{req/hole} - \text{Available during punching}$$

$$\Delta E = E_{req/hole} - (P_{motor} \times T_a) = I w^2 C_s$$

$$\Delta E = \frac{1}{2} I (w_{\max}^2 - w_{\min}^2)$$

↓                      ↓  
before              after  
punching            punching

Q.28

Pg.32 hole = 720 holes/hr. =  $\frac{720}{3600} = \frac{1 \text{ hole}}{5 \text{ sec.}}$   
wB  $d = 2 \text{ mm}, t = 3 \text{ mm}$  So cycle time = 5 sec.

$$E_{\text{eq.}} = 20 \text{ J/mm}^2$$

$$T_a = \frac{1}{4} \text{ sec.}, T_c = 5 \text{ sec.}$$

$$N_{\text{max}} = 100 \text{ rpm} \quad N_{\text{min}} = 800 \text{ rpm.}$$

$$E_{\text{eq.}} = 20 \times \pi \times D \times t = 20 \times \pi \times \frac{2}{\cancel{1000}} \times \frac{3}{\cancel{1000}}$$

$$E_{\text{eq.}} = 376.99 \text{ Joule per hole.} = 120 \pi$$

$$P_{\text{Motor}} = \frac{376.99}{5} = 75.39 \text{ (W)} = 24 \pi$$

$$\Delta E = E_{\text{eq per hole}} - E_{\text{air}}$$
$$= 376.99 - 75.39 \times \frac{1}{4}$$

$$\Delta E = 120 \pi - 24 \pi \times \frac{1}{4} = 114 \pi$$

$$\Delta E = I \omega^2 S.$$

$$\Delta E = \frac{1}{2} I (\omega_{\text{max}}^2 - \omega_{\text{min}}^2) = 114 \pi$$

$$= \frac{1}{2} I \left\{ 100^2 - 80^2 \right\} \left( \frac{2\pi}{60} \right)^2 = 114 \pi$$

$$\frac{I}{g} \left( \frac{3600}{3600} \right) 4\pi^2 = 114 \pi$$

$$I = 18.14 \text{ kg-m}^2$$

$$I = m k^2$$

$$18.14 = m (0.30)^2$$

$$m = 201.59 \text{ kg}$$

Q.2]

$$P_{motor} = 1.5 \text{ kW}$$

$$P = 1500 \text{ W}$$

$$30 \text{ holes}_{\text{min}} = 1 \text{ hole}_{\frac{1}{2} \text{ sec}}$$

$$T_c = 2 \text{ sec.}$$

$$E = 1500 \times 2$$

$$E_{\text{poly}} = \underline{3000 \text{ J}}$$

$$\frac{\Theta_a}{\Theta_c} = \frac{T_a}{T_c}$$

$$\frac{30}{180} = \frac{T_a}{2}$$

$$T_a = \frac{1}{6} \text{ sec.}$$

$$W_{\text{max}} = \left( \frac{2\pi}{2} \right)$$

$$C_s = 20\% \\ = \frac{20}{100}$$

$$\Delta E = E_{\text{poly hole}} - E_{\text{other parts.}}$$

$$\Delta E = 3000 - 1500 \times \frac{1}{6} = 2750 \text{ J}$$

$$I \omega^2 C_s = 2750$$

$$I \left( \frac{2\pi}{2} \right)^2 \times \frac{20}{100} = 2750$$

$$I = 1393.16 \text{ kg-m}^2$$

### Pb-Case-1

$$T_c = 5 \text{ sec.}$$

$$S = 100 \text{ mm}$$

$$t = 5 \text{ mm}$$

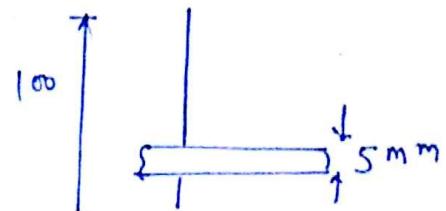
Metal sheet thick

$$200 \text{ mm} \rightarrow 5 \text{ sec.}$$

$$1 \text{ mm} \rightarrow \frac{5}{200}$$

$$5 \text{ mm} \rightarrow \frac{5}{200} \times 5 = \frac{1}{8} \text{ sec.}$$

$$T_a = \underline{\frac{1}{8}} \text{ sec.}$$



Ques In the requirement of Flywheel in punching press the cycle time is 6 sec. the stroke is 125 mm the thickness of sheet metal to be punch is to be 3 mm. the time req. for sheet metal is

Sol<sup>n</sup>.

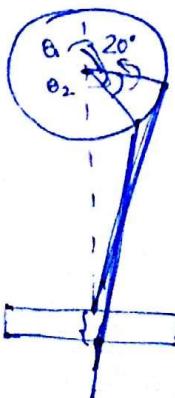
$$\frac{6}{250} \times 3 = 0.036 \times 2 \text{ sec.}$$

$$= 0.072 \text{ sec.}$$

### Pb - Case-2

$$T = 5 \text{ sec.}$$

actual punching is done in  $20^\circ$  of crank rotation.

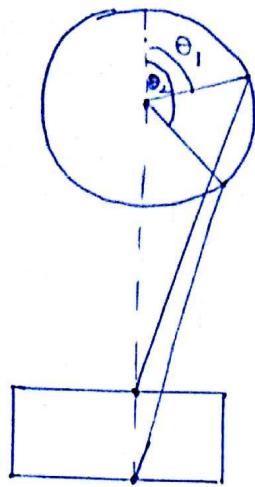


$$360^\circ \rightarrow 5 \text{ sec.}$$

$$\theta_1 - \theta_2 = 20^\circ$$

$$20^\circ \rightarrow \frac{5 \times 20}{360}$$

$$T_a = 0.277 \text{ sec.}$$



$$\theta_2 - \theta_1 = 20^\circ$$

$$W_{mean} = \left( \frac{2\pi}{5} \right) \text{ rad/sec}$$



$$W_{mean} = \frac{2\pi}{T_c} \text{ rad/sec}$$

## Design of Flywheel:-

For Rotating body

$\rho$  - density

$v$  → speed (circumferential speed)

$$\sigma_h = \rho v^2$$

↓  
hoop stress developed in flywheel  
or centrifugal stress.

$$(\sigma_{ind}) < \sigma_{per}$$

$$\rho v^2 \leq \sigma_{per}$$

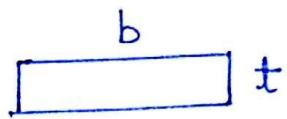
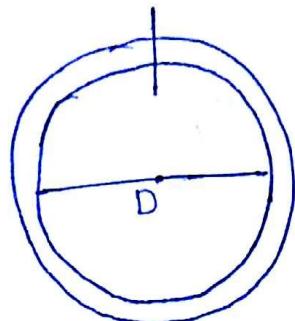
$$\text{limiting } \sigma_{ind} = \sigma_{per}$$

$$\rho v^2 = \sigma_{per}$$

$$V_{max} = \sqrt{\frac{\sigma_{per}}{\rho}}$$

$$V_{max} = \frac{\pi D N_{max}}{60}$$

### rim type flywheel:-



$$A_{x-s/c} = bt$$

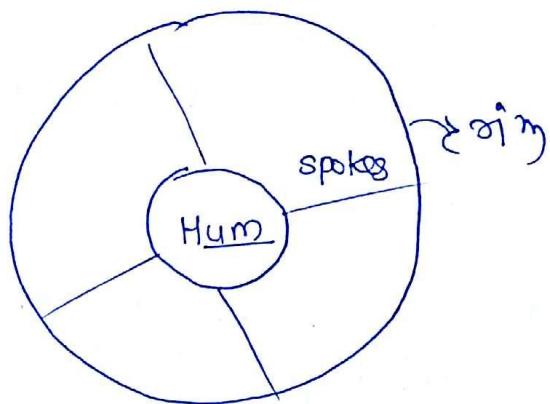
$$m = \rho \times \text{Vol.}$$

$$I = mR^2 = \frac{mD^2}{4}$$

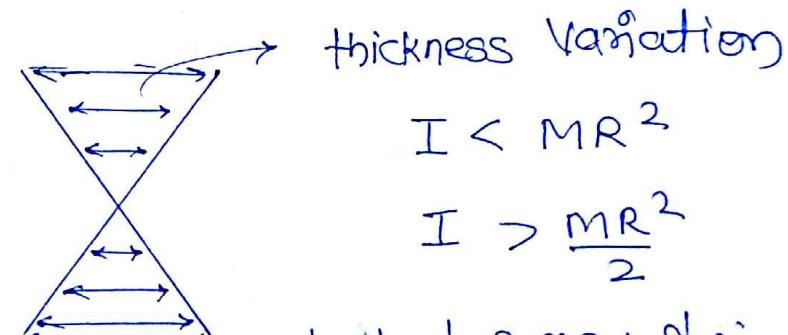
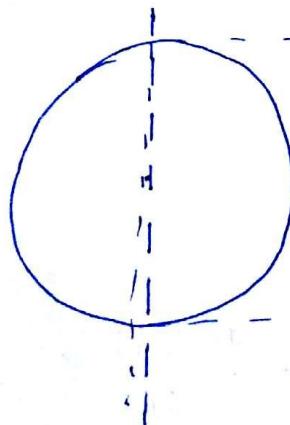
$$m = \rho \pi D (A_{x-s/c})$$

$$m = \rho \pi D b t$$

### Medium speed flywheel



### High speed flywheel :-



thickness Variation

$$I < MR^2$$

$$I > \frac{MR^2}{2}$$

$$MR^2 < I > \frac{MR^2}{2}$$

Highest energy Storing Capacity,

$$\Delta E = \frac{1}{2} I w^2$$

Note:

$$\frac{1}{2} I \omega^2 = E$$

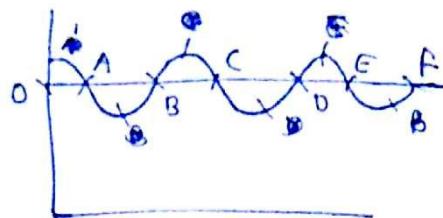
→  $\omega$  - high low - I { small Flywheel } bullet bikes

→  $\omega$  - low low - I { High Flywheel } scooter.

\* Size of a Flywheel is less for the high speed movers and vice-versa.

Ques An engine runs at a constant load at a speed of 480 rpm the crank effort diagram is drawn to a scale of  $1\text{ cm} = 2000\text{ N-m}$  torque and  $1\text{ cm} = 36^\circ$  crank angle the area of the diagram above & below the mean torque are measured in  $\text{sq. cm}$  units and are in the following order  $+1.1, -1.32, +1.53, -1.66, +1.97, -1.62$ . Design the Flywheel if the total fluctuation of speed is not to exceeds 10 rpm and the centrifugal stress in rim is not to exceeds  $5\text{ MPa}$ . You may assume that rim breadth is 2.5 time the rim thickness & 90% of MoI is due to rim.  $f = 7250\text{ kg/m}$  (material of flywheel)

Sol<sup>n</sup>



$$N = 480 \text{ rpm} \quad 1 \text{ cm} = 2000 \text{ Nm}$$

$$N_{\max} - N_{\min} = 10 \text{ rpm} \quad 1 \text{ rad} = 36' \\ = 36 \frac{\pi}{180} = \frac{\pi}{5} \text{ rad} \\ I_{\text{rim}} = 0.9 I_{\text{flywheel}} \quad S = 7250 \text{ kg/m}^2$$

$$E_0 = E$$

$$E_A = E + 1.1$$

$$E_B = E - 0.22$$

$$E_C = E + 1.31$$

$$E_D = E - 0.35$$

$$E_E = E + 1.62$$

$$E_F = E$$

$$E_{\max} = E + 1.62 \quad b = 1.25t$$

$$E_{\min} = E - 0.35$$

$$\Delta E = E + 1.62 - E - 0.35$$

$$\Delta E = 1.97 \text{ cm}^2$$

$$\Delta E = 1.97 \times 2000 \times \frac{\pi}{5} = 2475.5 \text{ J}$$

$$C_s = \frac{10}{480}$$

$$\Delta E_{\text{fly}} = I_{\text{fly}} \omega^2 C_s$$

$$2475.5 = I_{\text{fly}} \times (16\pi)^2 \times \frac{10}{480}$$

$$I_{\text{fly}} = 47.03 \text{ kg-m}^2$$

$$I_{\text{rim}} = 0.9 I_{\text{fly}} = 47.02 \times 0.9$$

$$I_{\text{rim}} = 42.327 \text{ kg m}^2 = \underline{\underline{mR^2}}$$

$$\Rightarrow gV^2 \leq 5 \times 10^6$$

$$7250 \times V^2 = 5 \times 10^6$$

$$V_{\max} = 26.2611 \text{ m/s}$$

$$\frac{\pi D N_{max}}{60} = 26.2611$$

$$\frac{m D^2}{4} = 42.3487$$

$$g \pi D (bt) \frac{D^2}{4} = 42.3487$$

$\Rightarrow \underline{N_{max}} = 485 \text{ rpm}$   
 $= 480 + 5$

$$\frac{\pi \times 485 \times D}{60} = 26.2611$$

$$D = \underline{1.034 \text{ m}}$$

$$R = \underline{0.517 \text{ m}}$$

$$\Rightarrow 7250 \times \pi \times 1.034 \times 2.5 \cancel{t} \times \cancel{t} \times \frac{(1.034)^2}{4} = 42.3487$$

$$t = \underline{0.052 \text{ ms}}$$

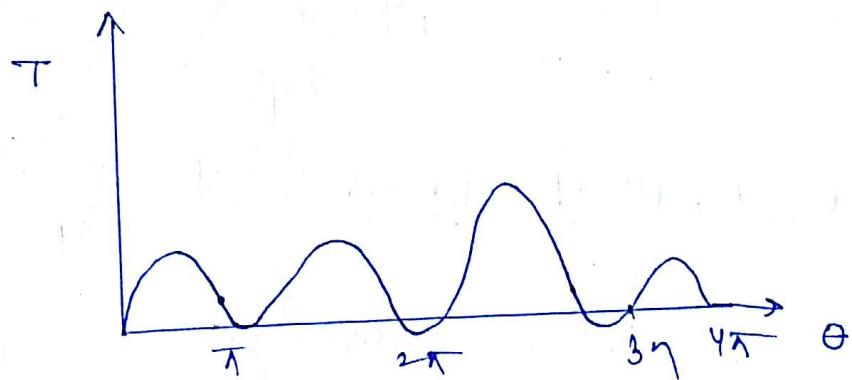
$$b = \underline{0.13 \text{ m}}$$

## Multi cylinder engine:-

Multicylinder engines are used to increase power as well as to increase the uniformity of power so that the fluctuation in engine can be decreased upto certain extent and requirement of flywheel can be suppressed.

Note:- Fixing order of an engine is decided so has to minimize unbalance forces which intern reduces the requirement of flywheel but not minimize the req. of flywheel.

### 4-5 Cylinder



### Multicylinder

