Sample Paper-05
Mathematics
Class – XII

Time allowed: 3 hours **Maximum Marks: 100**

General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

Section A

1. Is R defined on the set A={1,2,3,4,5,6} as R={(x,y): y is divisible by x} symmetric.
2. Calculate the direction cosines of the vector $\vec{a} = 3i - 2j - 5k$.
3. What is the principal value branch of $\cos^{-1} x$?
4. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Section B

5. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right)\right]$
6. If $A = \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix}$ show that $(3A)' = 3A'$
7. Find $\frac{dy}{dx}$ if $y = \frac{\sin(ax+b)}{\cos(cx+d)}$
8. If $y = (\tan^{-1}x)^2$ show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$
9. Evaluate: $\int f'(ax+b)[f(ax+b)]^x dx$
10. Find the particular Solution of the diff. equation
    $(1+e^{2x})dy + (1+y^2)e^x$ dx = 0 given that y = 1, when x = 0
11. Show that the points $A\left(-2\hat{i}+3\hat{j}+5\hat{k}\right), B\left(\hat{i}+2\hat{j}+3\hat{k}\right) and\ C\left(7\hat{i}-\hat{k}\right)$ are collinear.
12. A – Box of oranges is inspected by examining three randomly selected oranges drown without replacement. It all the three oranges are good, the box is approved for sale, other wise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approve for sale.

Section C

13. If A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.
14. Find the intervals in which the function f given by $f(x) = \frac{4\sin x - 2x - x\cos x}{2+\cos x}$ is (i) increasing (ii) decreasing.

15. Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta.\sin\theta,\ \ y = a\sin\theta - a\theta.\cos\theta$ is at a constant distance from origin.

16. Integrate $\int \frac{dx}{\cos(x+a)\cos(x+b)}$

17. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

18. An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained.

19. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r}\times\hat{i}).(\vec{r}\times\hat{j}) + xy$.

20. Find the distance between the point (-1, -5, -10) and the pint of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane x – y +z = 5.

21. If sin $\left[\cot^{-1}(x+1)\right] = \cos(\tan^{-1}x)$, then find x.

22. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), x^2 \le$, then find $\frac{dy}{dx}$.

23. If x = a cos θ+b sin θ, y = a sin θ - b cos θ, show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

Section D

24. Let A = N x N and * be the binary operation on A defined by $(a,b)*(c,d) = (a+c, b+d)$ Show that * is commutative and associative. Find the identity element for * on A, if any.

25. Find the ratio in which the area bounded by the curves $y^2 = 12x$ *and* $x^2 = 12y$ is divided by the line x=3.

26. Find the equation of the point where the line through the points A (3, 4, 1) and B (5, 1, 6) crosses the XY plane.

27. Integrate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log(\sin x)dx.$

28. The cost of 4 kg onions, 3kg wheat and 2 kg rice is 60. The cost of 2 kg onions, 4kg wheat and 6 kg rice is 90.The cost of 6 kg onions, 2kg wheat and 3 kg rice is 70. Find the per kg cost of each of the three commodities.

29. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hour for finishing. For fabricating and finishing the maximum labour hours available are 180 and 30 respectively. The company makes a profit of rs. 8000 on each piece of model A and 12000 on each piece of model B.

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Section A

1. No. $(2,4)\in R\ but\ (4,2)\notin R$
2. $|\vec{a}|=\sqrt{(3)^2+(-2)^2+(-5)^2}=\sqrt{38}$

   $\therefore l=\frac{3}{\sqrt{38}}, m=\frac{-2}{\sqrt{38}}, n=\frac{-5}{\sqrt{38}}$
3. $[0,\pi]$
4. $2A=2\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix}$

   $4|A|=4\times(2-8)$

   $=\begin{bmatrix}2 & 4\\ 8 & 4\end{bmatrix}=4\times(-6)$

   $|2A|=8-32=-24$

   $=-24$

   Hence Prove

Section B

5. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right)\right]$

   $\tan^{-1}\left[2\cos\left(2\frac{\pi}{6}\right)\right]$

   $\tan^{-1}\left(2\cos\frac{\pi}{3}\right)$

   $\tan^{-1}\left(2.\frac{1}{2}\right)$

   $\tan^{-1}(1)$

   $\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

   $=\frac{\pi}{4}$
6. $3A=\begin{bmatrix}-3 & 15\\ 9 & 6\end{bmatrix}$

   $(3A)'=\begin{bmatrix}-3 & 9\\ 15 & 6\end{bmatrix}$

   $3A'=3\begin{bmatrix}-1 & 3\\ 5 & 2\end{bmatrix}$

   $=\begin{bmatrix}-3 & 9\\ 15 & 6\end{bmatrix}$ Hence proved

7. $y = \dfrac{\sin(ax+b)}{\cos(cx+d)}$

$$\frac{dy}{dx} = \frac{\cos(cx+d)\dfrac{d}{dx}\sin(ax+b) - \sin(ax+b)\dfrac{d}{dx}\cos(cx+d)}{\cos^2(cx+d)}$$

$$\frac{dy}{dx} = \frac{\cos(cx+d)\cos(ax+b).a + \sin(ax+b)\sin(cx+d).c}{\cos^2(cx+d)}$$

8. $y = (\tan^{-1} x)^2$ (given)

Differentiation both side w.r. to x

$$y_1 = 2\tan^{-1} x.\frac{1}{1+x^2}$$

$$(1+x^2)y_1 = 2\tan^{-1} x$$

Again differentiation both side w.r. to

$$(1+x^2)y_2 + y_1.(2x) = 2.\frac{1}{1+x^2}$$

$$(1+x^2)^2 y_2 + 2x(x^2+1)y_1 = 2$$

9. Put $f(ax+b) = t$

$$f'(ax+b).a\ dx = dt$$

$$= \int \frac{1}{a} t^n dt = \frac{1}{a}.\frac{t^{n+1}}{n+1} + c$$

$$= \frac{1}{a}.\frac{\left[f(ax+b)\right]^{n+1}}{n+1} + c$$

10. $\left(1+e^{2x}\right)dy = -\left(1+y^2\right)e^x$ dx

$$\int \frac{dy}{\left(1+y^2\right)} = -\int \frac{e^x}{1+e^{2x}}dx$$

$\tan^{-1}$ (y) + $\tan^{-1}$ e^x = c

when x = 0, y = 1

c = $\pi/2$

$\tan^{-1}$ y + $\tan^{-1}$ e^x = $\pi/2$

11. $\overrightarrow{AB} = 3\hat{i} - \hat{j} - 2\hat{k}$

$$\overrightarrow{BC} = 6\hat{i} - 2\hat{j} - 4\hat{k}$$

$$\overrightarrow{CA} = 9\hat{i} - 3\hat{j} - 6\hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{14}, \overrightarrow{BC} = 2\sqrt{14}$$

$$and \left|\overrightarrow{AC}\right| = 3\sqrt{14}$$

$$\left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$$

Hence points A, B, C are collinear.

12. Required probability $= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13}$

$= \frac{44}{91}$

Section C

13. $A = \begin{vmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{vmatrix}$

$\therefore A^2 = A \times A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2-+0 & 0-1+0 & 1-3+0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$

Now, $A^2 - 5A + 4I = \begin{vmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{vmatrix} - 5\begin{vmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{vmatrix} + 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$

Now given $A^2 - 5A + 4I + X = 0$

$\Rightarrow \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix} + X = 0$

$\Rightarrow X = -\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$

14. $f(x) = \frac{4\sin x - 2x - x\cos x}{2+\cos x}$

$= \frac{4\sin x - x(2+\cos x)}{2+\cos x}$

$= \frac{4\sin x}{2+\cos x} - \frac{x(2+\cos x)}{(2+\cos x)}$

$f(x) = \frac{4\sin x}{2+\cos x} - x$

$f'(x) = \frac{\cos x(4-\cos x)}{(2+\cos x)^2}$

$[\because -1 \le \cos x \le 1]$

Hence ,

$$\frac{\cos x(4-\cos x)}{(2+\cos x)^2} > 0 \forall x \in \left(0, \frac{\pi}{2}\right) and \left(\frac{3\pi}{2}, 2\pi\right)$$

$$\frac{\cos x(4-\cos x)}{(2+\cos x)^2} < 0 \forall x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

15. $\frac{dx}{d\theta} = -a\sin\theta + a(\theta.\cos\theta + \sin\theta)$

$$\frac{dx}{d\theta} = a\theta.\cos\theta$$

$$\frac{dy}{d\theta} = a\theta.\sin\theta$$

$$\frac{dy}{dx} = \tan\theta$$

Slope of normal $= \frac{-1}{\tan\theta}$

Equation of normal $y - y_1 = \frac{-1}{\frac{dy}{dx}}(x - x_1)$

$$y-(a\sin\theta - a.\theta\cos\theta) = \frac{-\cos\theta}{\sin\theta}\left[x-(a\cos\theta - a.\theta\sin\theta)\right]$$

$$x\cos\theta + y\sin\theta = a$$

length of $\perp$ from origin

$\frac{|0\cos\theta + 0\sin\theta - a|}{\sqrt{\cos^2\theta + \sin^2\theta}} = a$ Proved

16. $I = \int \frac{dx}{\cos(x+a).\cos(x+b)}$

$$= \frac{1}{\sin(a-b)}\int \frac{\sin[(x+a)-(x+b)]}{\cos(x+a).\cos(x+b)}$$

$$= \frac{1}{\sin(a-b)}\int \left[\frac{\sin(x+a).\cos(x+b) - \cos(x+a).\sin(x+b)}{\cos(x+a).\cos(x+b)}\right]$$

$$= \frac{1}{\sin(a-b)}\int \left[\tan(x+a) - \tan(x+b)\right]dx$$

$$= \frac{1}{\sin(a-b)}\left[\log\sec(x+a) - \log\sec(x+b)\right] + c$$

$$= \frac{1}{\sin(a-b)}\left[\log\frac{\sec(x+a)}{\sec(x+b)}\right] + c$$

17. Let E,F and A three events such that

E = selection of Bag A and F=selection of bag B

A= getting one red and one black ball of two

Here, p(E)=P(getting 1 or 2 in a throw of die)=$\frac{2}{6} = \frac{1}{3}$

$\therefore\ p(F)=1-\frac{1}{3}=\frac{2}{3}$

Also, P(A/E)=P (getting one red and one black if bag A is selected)= $\frac{{}^6C_1\times{}^4C_1}{{}^{10}C_2}=\frac{24}{45}$ and

P(A/F)=P(getting one red and one black if bag Black if bag B is selected)= $\frac{{}^3C_1\times 7C_1}{{}^{10}C_2}=\frac{21}{45}$

Now, by theorem of total probability,

p(A)=P(E).P(A/E)+P(F).P(A/F)

$\Rightarrow p(A)=\frac{1}{3}\times\frac{24}{45}+\frac{2}{3}\times\frac{21}{45}=\frac{8+14}{45}=\frac{22}{45}$

18. Let number of head be random variable X in four tosses of a coin .X may have values 0,1,2,3 or 4 obviously repeated tosses of a coin are Bernoulli trials and thus X has binomial distribution with n=4 and p= probability of getting head in one toss= $\frac{1}{2}$

q=probability of getting tail (not head) in one toss = 1 - $\frac{1}{2}$ = $\frac{1}{2}$

since, we know that P(X=r)= ${}^nC_rP^rq^{n-r}$, $\quad r=0,1,2.........n$

therefore,

P(X=0) = ${}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{4-0}=1\times1\times\left(\frac{1}{2}\right)^4=\frac{1}{16}$

$P(X=1)={}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{4-1}=4\times\left(\frac{1}{2}\right)^1\times\left(\frac{1}{2}\right)^4=\frac{4}{16}=\frac{1}{4}$

$P(X=2)={}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-2}=6\times\left(\frac{1}{2}\right)^2\times\left(\frac{1}{2}\right)^2=\frac{6}{16}=\frac{3}{8}$

$P(X=3)={}^4C_3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-3}=4\times\left(\frac{1}{2}\right)^3\times\left(\frac{1}{2}\right)^1=\frac{4}{16}=\frac{1}{4}$

$P(X=4)={}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{4-4}=1\times\left(\frac{1}{2}\right)^4\times\left(\frac{1}{2}\right)^0=\frac{1}{16}$

Now required probability distribution of X is

x	0	1	2	3	4
4P(x)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Required mean = $\mu=\sum x_ip_i$

= $0\times\frac{1}{16}+1\times\frac{1}{4}+2\times\frac{3}{8}+3\times\frac{1}{4}+4\times\frac{1}{16}=\frac{1}{4}+\frac{3}{4}+\frac{3}{4}+\frac{1}{4}=\frac{8}{4}=2$

variance = $\sigma^2_x=\sum x_ip_i-\left(\sum x_ip_i\right)^2=\sum X^2{}_ip_i-\mu^2$

$=\left(0\times\frac{1}{16}+1^2\times\frac{1}{4}+2^2\times\frac{3}{8}+3^2\times\frac{1}{4}+4^2\times\frac{1}{16}\right)-2^2$

$=\frac{1}{4}+\frac{3}{4}+\frac{9}{4}+1-4$

$=\frac{1}{4}+\frac{3}{4}+\frac{9}{4}-3$

$=\frac{1+6+9-12}{4}=\frac{4}{4}=1$

19. Here Now,

$(\vec{r}\times\hat{i})(\vec{r}\times\hat{j})+xy=\{(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{i}\}.\{(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{j}\}+xy$

$=(-y\hat{k}+z\hat{j}).(x\hat{k}-z\hat{i})+xy=(0\hat{i}+z\hat{j}-y\hat{k})\cdot(-z\hat{i}+0\hat{j}+x\hat{k})+xy$

$=0+0-xy+xy=0$

20. Let P (α,β,γ) be the point of intersection of the given line (i) and plane (ii)

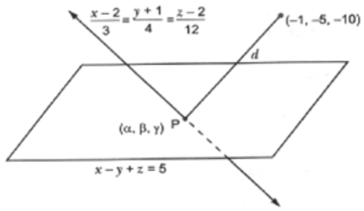
$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ …….. (i)

and x – y + z = 5 ………. (ii)

since ,point P (α,β,γ) lies on line (i)(therefore it satisfy(i)

$\Rightarrow \frac{\alpha-2}{3}=\frac{\beta+1}{4}=\frac{\gamma-2}{12}=\lambda$

$\Rightarrow \alpha=3\lambda+2;\beta=4\lambda-1;\gamma=12\lambda+2$



Also point P (α,β,γ) lie on plane (ii)

$\Rightarrow \alpha-\beta+\gamma=5$ …… (iii)

putting the value of α,β,γ in (iii) we get

$\Rightarrow 3\lambda+2-4\lambda+1+12\lambda+2=5$

$\Rightarrow 11\lambda+5=5 \Rightarrow \lambda=0$

$\Rightarrow \alpha=2;\quad \beta=-1;\quad \gamma=2$

hence the coordinate of the point of intersect ion p is (-2,-1,2)

therefore ,required distance = d = $\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$\sqrt{9+16+144}=\sqrt{169}=13\text{units}$

21. Here sin[$\cot^{-1}(x+1)$]= $\cos(\tan^{-1}x)$

let $\cot^{-1}(x+1)=\theta$

$\Rightarrow \cot\theta=x+1$

$\Rightarrow \quad \text{cosec}\,\theta=\sqrt{1+\cot^2\theta}=\sqrt{1+(x+1)^2}=\sqrt{x^2+2x+2}$

$\Rightarrow \sin\theta=\frac{1}{\sqrt{x^2+2x+2}} \qquad \Rightarrow \theta=\sin^{-1}\left(\frac{1}{\sqrt{x^2+2x+2}}\right)$

$\Rightarrow \cot^{-1}(x+1)=\sin^{-1}\left(\frac{1}{\sqrt{x^2+2x+2}}\right)$

again $\quad \tan^{-1}x=\alpha \Rightarrow \tan\alpha=x$

$\therefore \sec\alpha=\sqrt{1+\tan^2\alpha}=\sqrt{1+x^2}$

$\Rightarrow \cos\alpha=\frac{1}{\sqrt{1+x^2}} \qquad \Rightarrow \qquad \alpha=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

$\Rightarrow \quad \tan^{-1}=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

now equation (i) becomes

$$\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{x^2+2x+2}}\right)\right)=\cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)$$

$$\frac{1}{\sqrt{x^2+2x+2}}=\frac{1}{\sqrt{1+x^2}}\Rightarrow\sqrt{x^2+2x+2}=\sqrt{1+x^2}$$

$$x^2+2x+2=1+x^2 \quad\Rightarrow\quad 2x+2=1$$

$$\Rightarrow x=-\frac{1}{2}$$

22. $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

$$=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\times\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$$

$$=\tan^{-1}\left(\frac{2+2\sqrt{1-x^4}}{1+x^2-1+x^2}\right)=\tan^{-1}\left(\frac{2+2\sqrt{1-x^4}}{2x^2}\right)$$

$$=\tan^{-1}\left(\frac{1+\sqrt{1-x^4}}{x^2}\right)$$

Let $x^2=\sin\theta\Rightarrow\sin^{-1}(x^2)=\theta$

putting the value of x^2, we get

$$=\tan^{-1}\left\{\frac{1+\sqrt{1-\sin^2\theta}}{\sin\theta}\right\}$$

$$=\tan^{-1}\left\{\frac{1+\cos\theta}{\sin\theta}\right\}=\tan^{-1}\left\{\frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\}$$

$$=\tan^{-1}\left\{\cot\frac{\theta}{2}\right\}=\tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right\}$$

$$=\frac{\pi}{2}-\frac{\theta}{2}=\frac{\pi}{2}-\sin^{-1}x^2$$

$$\left(\begin{array}{l}\because 0\le x^2\le 1\\ \Rightarrow \sin 0<\sin\theta<\sin\frac{\pi}{2}\\ \Rightarrow 0<\theta<\frac{\pi}{2}\Rightarrow 0<\frac{\theta}{2}<\frac{\pi}{2}\\ \Rightarrow 0<-\frac{\theta}{2}<-\frac{\pi}{4}\\ \Rightarrow \frac{\pi}{2}>\frac{\pi}{2}-\frac{\theta}{2}>\frac{\pi}{2}-\frac{\pi}{4}\\ \Rightarrow \frac{\pi}{2}>\left(\frac{\pi}{2}-\frac{\theta}{2}\right)>\frac{\pi}{4}\\ \left(\frac{\pi}{2}-\frac{\theta}{2}\right)\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)\subset\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\end{array}\right)$$

differentiating both sides with respect to x, we get

$$\Rightarrow\ \frac{dy}{dx}=-\frac{2x}{2\sqrt{1-x^4}}$$

$$\Rightarrow\ \frac{dy}{dx}=-\frac{x}{\sqrt{1-x^4}}$$

23. Given x= a $\cos\theta$+b$\sin\theta$

$$\Rightarrow\ \frac{dx}{d\theta}=-a\sin\theta+b\cos\theta$$

Also, $y=a\sin\theta-b\cos\theta$

$$\Rightarrow\ \frac{dy}{d\theta}=a\cos\theta+b\sin\theta$$

$$\therefore \quad \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\cos\theta+b\sin\theta}{-a\sin\theta+b\cos\theta}$$

$$\frac{dy}{dx}=-\frac{x}{y}\Rightarrow\frac{d^2y}{dx^2}=-\left(\frac{y-x.\frac{dy}{dx}}{y^2}\right)$$

$$\Rightarrow y^2\frac{d^2y}{dx^2}=-y+x\frac{dy}{dx} \qquad \Rightarrow \qquad y^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$$

Section D

24. A = N x N and * is a binary operation defined on A.

$(a,b)*(c,d)=(a+c,b+d)=(c+a,d+b)=(c,d)*(a,b)$

$\therefore$ The operation is commutative

Again, $\left[(a,b)*(c,d)\right]*(e,f)=(a+c,b+d)*(e,f)=(a+c+e,b+d+f)$

And $(a,b)\left[(c,d)*(e,f)\right]=(a,b)*(c+e,e+f)=(a+c+e,b+d+f)$

Here, $\left[(a,b)*(c,d)\right]*(e,f)=(a,b)\left[(c,d)*(e,f)\right]$

$\therefore$ The operation is associative.

Let identity function be (e,f), then $(a,b)*(e,f)=(a+e,b+f)$

For identity function $a=a+e$

$\Rightarrow e=0$

And for $b+f=b$

$\Rightarrow f=0$

As $0\neq$ N, therefore, identity-element does not exist.

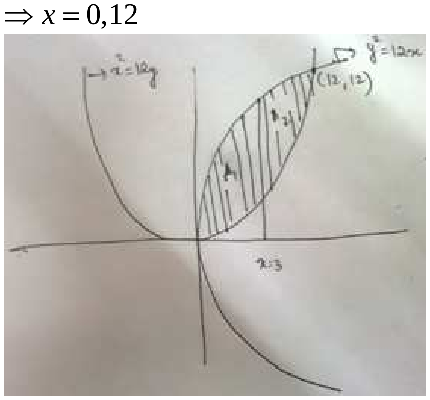
25. The point of intersection of the curves $y^2=12x$, $x^2=12y$:

$y=\frac{x^2}{12}\Rightarrow y^2=\frac{x^4}{144}$

$\Rightarrow 12x=\frac{x^4}{144}$

$\Rightarrow x(x^3-1728)=0$

$\Rightarrow x=0,12$



The shaded area is the required area.

$$A_1 = \int_0^3 (y_2 - y_1)\,dx$$
$$= \int_0^3 \left(\sqrt{12x} - \frac{x^2}{12}\right)dx$$
$$= \sqrt{12}\,\frac{x^{3/2}}{3/2}\Big|_0^3 - \frac{x^3}{36}\Big|_0^3 = \frac{45}{4}$$
$$A_2 = \int_3^{12} (y_2 - y_1)\,dx$$
$$= \int_3^{12} \left(\sqrt{12x} - \frac{x^2}{12}\right)dx$$
$$= \sqrt{12}\,\frac{x^{3/2}}{3/2}\Big|_3^{12} - \frac{x^3}{36}\Big|_3^{12} = \frac{147}{4}$$

Thus, ratio of the areas is 45:147=15:49

26. The vector equation of the line through the point A and B is

$$\vec{r} = 3\hat{i} + 4\hat{j} + k + \lambda[(5-3)\hat{i} + (1-4)\hat{j} + (6-1)\hat{k}]$$
$$\vec{r} = 3\hat{i} + 4\hat{j} + \hat{k} + \lambda(2\hat{i} - 3\hat{j} + 5\hat{k})....(i)$$

Let P be the point where the line AB crosses the XY plane. Then the position vector $\vec{r}$ of the point P is the form

$$x\hat{i} + y\hat{j}$$
$$x\hat{i} + y\hat{i} = (3+2\lambda)\hat{i} + (4-3\lambda)\hat{j} + (1+5\lambda)\hat{k}$$
$$x = 3 + 2\lambda \,|\, y = 4 - 3\lambda$$
$$x = 13/5,\ y = 23/5$$
$$req.\ point\ is \left(\frac{13}{5}, \frac{23}{5}, 0\right)$$

27. $I = \int_{\pi/4}^{\pi/2} \cos 2x \log(\sin x)dx.$

$$= \log(\sin x).\frac{\sin 2x}{2}\Big|_{\pi/4}^{\pi/2} - \int_{\pi/4}^{\pi/2} \frac{1}{\sin x}\cos x.\frac{\sin 2x}{2}dx$$
$$= 0 - \log\left(\frac{1}{\sqrt{2}}\right)\frac{1}{2} - \int_{\pi/4}^{\pi/2} \frac{1}{\cancel{\sin x}}\cos x.\frac{\cancel{2}\cancel{\sin x}\cos x}{\cancel{2}}dx$$
$$= -\frac{1}{2}\log\left(\frac{1}{\sqrt{2}}\right) - \int_{\pi/4}^{\pi/2} \frac{1+\cos 2x}{2}dx = -\frac{1}{2}\log\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{8} + \frac{1}{4}$$

28. Let x be the cost of 1 kg onions, y be the cost of 1 kg wheat, z be the cost of 1 kg rice.
Thus we get the following equations:

4x+3y+2z=60
2x+4y+6z=90
6x+2y+2z=70

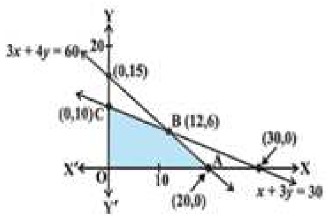
$$Let\ A=\begin{bmatrix}4 & 3 & 2\\ 2 & 4 & 6\\ 6 & 2 & 3\end{bmatrix}, b=\begin{bmatrix}60\\ 90\\ 70\end{bmatrix}$$

$$|A|=50\neq 0, A^{-1}=\frac{1}{50}\begin{bmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{bmatrix}, X=A^{-1}b=\frac{1}{50}\begin{bmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{bmatrix}\begin{bmatrix}60\\ 90\\ 70\end{bmatrix}=\begin{bmatrix}5\\ 8\\ 8\end{bmatrix}$$

$\therefore x=5,\ y=8,\ z=8$

Thus , per kg cost of onions, wheat and rice is Rs.5,Rs.8, Rs.8 respectively.

29. Suppose x is the number of pieces of model A and y is the number of pieces of model B.
Then, Profit Z= 8000x + 12000y
The mathematical formulation of the problem is as follows:
Max Z= 8000x+12000y
s.t $9x+12y\leq 180($ *fabricating constra* int)
$3x+4y\leq 60$
$x+3y\leq 30($ *finishing constra* int)
$x\geq 0, y\geq 0$



We graph the above inequalities. The feasible region is as shown in the figure. The corner points are O,A,B and C. The co-ordinates of the corner points are (0,0),(20,0), (12,6),(0,10).

Corner Point	Z=8000x +12000y
(0,0)	0
(20,0)	16000
(12,6)	**16800**
(0,10)	12000

Thus profit is maximized by producing 12 units of A and 6 units of B and maximum profit is 16800.