

CHAPTER 7

Percents

7-1. Percent of Change

The word **percent** means *hundredth* or *out of every hundred*.

To write a decimal or a fraction as a percent, multiply the decimal or the fraction by 100 and add the % sign. Convert the fraction to decimal.

To write a percent as a decimal or a fraction, multiply the percent by $\frac{1}{100}$, and drop the % sign.

Simplify the fraction.

Example 1 □ Write each decimal or fraction as a percent.

a. 0.65 b. $\frac{3}{16}$

Solution □ a. $0.65 = 0.65 \times 100\% = 65\%$

Multiply the decimal by 100 and add the % sign.

c. $\frac{3}{16} = \frac{3}{16} \times 100\% = \frac{300}{16}\% = 18.75\%$

Multiply the fraction by 100 and add the % sign. Convert the fraction to decimal.

Example 2 □ Write 175% as a decimal and a fraction.

Solution □ $175\% = 175 \times \frac{1}{100} = \frac{175}{100} = 1.75$

Multiply the amount of percent by $\frac{1}{100}$, and drop the % sign. Simplify the fraction.

$$175\% = 175 \times \frac{1}{100} = \frac{175}{100} = \frac{7}{4}$$

The percent a quantity increases or decreases from its original amount is the **percent of change**.

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

Example 3 □ a. A \$300 tablet is on sale for \$234. What is the percent of discount?

b. The population of Sunny Hills increased from 12,000 to 15,840 in ten years. What is the percent increase of the population?

Solution □ a. percent discount = $\frac{\text{amount of discount}}{\text{original amount}}$

$$= \frac{300 - 234}{300} = \frac{66}{300} = 0.22$$

There was a 22% discount.

b. percent increase = $\frac{\text{number of increase}}{\text{original number}}$

$$= \frac{15,840 - 12,000}{12,000} = \frac{3,840}{12,000} = 0.32$$

There was a 32% increase in population.

Exercises - Percent of Change

1

Which of the following is equivalent to 0.03 % of 4?

- A) 0.12
- B) 0.012
- C) 0.0012
- D) 0.00012

2

$$\frac{1}{400} =$$

- A) 0.25%
- B) 0.025%
- C) 0.0025%
- D) 0.00025%

3

The quantities x and y are positive. If x is decreased by 20 percent and y is increased by 20 percent, then the product of x and y is

- A) unchanged
- B) decreased by 4%
- C) increased by 5%
- D) decreased by 6%

4

By what percent is 4.5×10^5 greater than 9×10^4 ?

- A) 200%
- B) 400%
- C) 500%
- D) 600%

5

The temperature increased from 60°F to 72°F . What is the percent increase in temperature?

- A) 15%
- B) $\frac{50}{3}\%$
- C) 20%
- D) $\frac{70}{3}\%$

6

This year's enrollment in Mesa School District is 6,000, which is 20 percent higher than last year's. What was last year's enrollment in Mesa School District?

7

If 125% of x is 80 and x is $n\%$ of 400, what is the value of n ?

7-2. Percents and Equations

You can solve a percent problem by writing and solving an equation or a proportion. Three types of percent equations and corresponding verbal phrases are illustrated below.

	Verbal Phrase	Algebraic Expression	Equation or Proportion
1. Finding the Part	What is 15% of 72?	$n = 0.15 \times 72$ $\frac{15}{100} = \frac{n}{72}$	Write an equation. Write a proportion.
2. Finding the Percent	What percent of 20 is 6?	$\frac{n}{100} \times 20 = 6$ $\frac{n}{100} = \frac{6}{20}$	Write an equation. Write a proportion.
3. Finding the Whole	17 is 25% of what number?	$17 = 0.25 \times n$ $\frac{25}{100} = \frac{17}{n}$	Write an equation. Write a proportion.

Example 1 □ Translate each verbal phrase into an algebraic equation and a proportion. Then solve.

- What is 0.3% of 4?
- What percent of 30 is 5?
- 64 is 250% of what number?

Solution □ a. $x = 0.003 \times 4 = 0.012$

$$\frac{x}{4} = \frac{0.3}{100}$$

$$100x = 0.3 \times 4$$

$$x = 1.2 \div 100 = 0.012$$

Write an equation.

Write a proportion.

Cross products

Divide by 100 and solve.

$$\text{b. } \frac{p}{100} \times 30 = 5$$

$$p = 5 \times \frac{100}{30} = \frac{50}{3}$$

$$\frac{p}{100} = \frac{5}{30}$$

$$30p = 500$$

$$p = \frac{500}{30} = \frac{50}{3}$$

Write an equation.

Multiply both sides by $\frac{100}{30}$ and simplify.

Write a proportion.

Cross products

Divide by 30 and simplify.

$$\text{c. } 64 = 2.5 \times n$$

$$n = \frac{64}{2.5} = 25.6$$

$$\frac{250}{100} = \frac{64}{n}$$

$$250n = 6400$$

$$n = \frac{6400}{250} = 25.6$$

Write an equation. $250\% = 2.5$

Divide each side by 2.5.

Write a proportion.

Cross products

Divide each side by 250.

Exercises - Percents and Equations

1

28% of what number is 7?

2

3.6 is 240% of what number?

3

$\frac{1}{2}\%$ of 180 is what number?

4

$3\frac{1}{3}\%$ of what number is 2.5?

5

26.4 is 0.55% of what number?

6

What percent of 12 is 8?

- A) 60%
- B) $66\frac{2}{3}\%$
- C) 75%
- D) $130\frac{1}{3}\%$

7

54 is 120% of k .

Which of the following proportions could be used to solve the above expression?

- A) $\frac{100}{120} = \frac{54}{k}$
- B) $\frac{54}{100} = \frac{120}{k}$
- C) $\frac{100}{54} = \frac{120}{k}$
- D) $\frac{120}{100} = \frac{54}{k}$

8

If Kevin's monthly salary of \$4,500 is 72 percent of Paul's monthly salary, what is Paul's monthly salary?

- A) \$3,240
- B) \$5,150
- C) \$5,870
- D) \$6,250

7-3. Percent Word Problems

Mixture of Two Different Solutions

Example 1 □ How many milliliters of 65% acid solution must be added to 60 milliliters of a 40% acid solution in order to make a 50% acid solution?

Solution □ Let x = the amount of 65% acid solution added.

	Total amount \times % acid = Amount of acid		
40% solution	60	40%	0.4×60
65% solution	x	65%	$0.65x$
New solution	$60 + x$	50%	$0.5(60 + x)$

Original amount of acid + added acid = new amount of acid

$$0.4 \times 60 + 0.65x = 0.5(60 + x)$$

$$24 + 0.65x = 30 + 0.5x$$

$$0.15x = 6$$

$$x = 40$$

Therefore, 40 milliliters of 65% acid solution must be added.

Interest and Investments

Example 2 □ Bob invested \$7,500 in stocks and bonds. The stocks pay 6.5% interest a year and the bonds pay 8% interest a year. His interest income is \$528 this year. How much money was invested in stocks?

Let x = the amount invested in stocks.

Then, $7500 - x$ = the amount invested in bonds.

	Amount invested \times Rate = Interest		
Stock	x	.065	$0.065x$
Bond	$7500 - x$.08	$0.08(7500 - x)$

Interest from stocks + interest from bonds = total interest income

$$0.065x + 0.08(7500 - x) = 528$$

$$0.065x + 600 - 0.08x = 528$$

$$-0.015x = -72$$

$$x = 4800$$

Therefore, \$4,800 was invested in stocks.

Discounts and Tax

Example 3 □ The sale price of a laptop is \$505.44 after 35% discount and 8% additional tax. What was the original price of the laptop before discount and tax?

Let x = the original price of the laptop before discount and tax.

$$x - .35x = 0.65x$$

The price of laptop after 35% discount.

$$0.65x(1 + 0.08) = 0.702x$$

The price of laptop after 8% of tax.

$$0.702x = 505.44$$

$$x = 720$$

The original price of the laptop was \$720.

Exercises - Percent Word Problems

1

There are n candies in a jar. If one candy is removed, what percent of the candies are left in terms of n ?

- A) $100(1-n)\%$
- B) $100(\frac{1}{n}-1)\%$
- C) $100(n-\frac{1}{n})\%$
- D) $100(\frac{n-1}{n})\%$

2

The price of a cellphone was discounted by 25% and then discounted an additional 20%, to become \$348. What was the original price of the cellphone before it was discounted twice?

- A) \$580.00
- B) \$620.00
- C) \$650.00
- D) \$680.00

3

A chemist mixes a 40% acid solution and a 30% acid solution. How many liters of the 40% solution must be added to produce 50 liters of a solution that is 36% acid?

- A) 24
- B) 26
- C) 30
- D) 32

4

Victor invests part of his \$5,000 in a savings account that pays 4.5% annual simple interest. He invests the rest in bonds that pay 8% annual simple interest. Let s be the amount invested in savings and r be the amount invested in bonds. Victor's total income in one year from these investments is \$305.50. Which of the following systems of equations represents this relationship?

- A) $\begin{cases} 0.045s + 0.08r = 5,000 \\ s + r = 305.50 \end{cases}$
- B) $\begin{cases} 0.08s + 0.045r = 5,000 \\ s + r = 305.50 \end{cases}$
- C) $\begin{cases} s + r = 5,000 \\ 0.045s + 0.08r = 305.50 \end{cases}$
- D) $\begin{cases} s + r = 5,000 \\ 0.08s + 0.045r = 305.50 \end{cases}$

5

A sporting goods store added 50% profit cost and 8% tax to the price of a backpack, which then became \$129.60. What was the price of the backpack before adding profit and tax?

6

There are 800 students in a school and 45% of the students are male. If 30% of the male students and 25% of the female students play varsity sports, how many students play varsity sports?

Chapter 7 Practice Test

1

A chemist mixes x mL of a 34% acid solution with a 10% acid solution. If the resulting solution is 40 mL with 25% acidity, what is the value of x ?

- A) 18.5
- B) 20
- C) 22.5
- D) 25

2

The price of a package of 4 pens is \$8.00. The same pens are sold at \$2.50 each. If Alex bought three packages of pens rather than buying 12 pens individually, the amount he saved on 12 pens is what percent of the amount he paid?

- A) 12%
- B) 20%
- C) 25%
- D) 30%

3

There are 600 bottles of sports drinks in a store. 25% of the bottles are orange flavored drinks. On Monday 30% of the orange flavored drinks in the store were sold and on Tuesday 20% of the remaining orange flavored drinks were sold. How many bottles of orange flavored drinks were sold in the two days?

- A) 52
- B) 58
- C) 66
- D) 75

4

A tablet with a list price of x dollars is discounted by 15% and then discounted an additional 12%. What is the final sale price of the tablet, in terms of x ?

- A) $0.73x$
- B) $0.748x$
- C) $0.75x$
- D) $0.765x$

5

There is a total of n pairs of shoes in a store, all of which are either black or brown. If there are m pairs of brown shoes in the store, then in terms of m and n , what percent of the shoes in the store are black?

- A) $\frac{m}{n}\%$
- B) $\frac{n-m}{n}\%$
- C) $(1 - \frac{100m}{n})\%$
- D) $100(1 - \frac{m}{n})\%$

6

The numbers a , b , and c are positive and a equals $3.2bc$. If b is increased by 150% and c is decreased by 60%, then a is

- A) increased by 90%
- B) increased by 10%
- C) unchanged
- D) decreased by 10%

7

There are 10 history books in a bookcase. When the number of books increases by x percent, the new number of history books is 24. What is the value of x ?

- A) 58
- B) 70
- C) 120
- D) 140

8

Number n is 25 less than 120 percent of itself. What is the value of n ?

- A) 125
- B) 120
- C) 105
- D) 90

9

Of the 500 cars displayed in a certain car dealer, 7 percent are blue and 4 percent are red. The number of blue cars in the car dealer are what percent greater than the number of red cars?

- A) 30%
- B) 50%
- C) 75%
- D) 125%

10

If 300% of 0.18 is equivalent to 20% of b , then b is equivalent to what number?

11

Five people contributed \$9,000 each toward the purchase of a sailboat. If they ended up paying \$38,500 plus 8% sales tax for the boat, how much money should be refunded to each person?

12

A store used to sell an MP3 for \$72, which is 50% more than the wholesale cost. At a special holiday sale, the price of the MP3 was 20% less than the wholesale cost. What was the special sale price of the MP3?

Answer Key

Section 7-1

1. C 2. A 3. B 4. B 5. C
6. 5000 7. 16

Section 7-2

1. 25 2. 1.5 3. 0.9 4. 75 5. 4800
6. B 7. D 8. D

Section 7-3

1. D 2. A 3. C 4. C 5. 80
6. 218

Chapter 7 Practice Test

1. D 2. C 3. C 4. B 5. D
6. C 7. D 8. A 9. C 10. 2.7
11. 684 12. 38.4

Answers and Explanations

Section 7-1

1. C

$$0.03\% \text{ of } 4 = 0.03 \times \frac{1}{100} \times 4 = 0.0012$$

2. A

$$\frac{1}{400} = \frac{1}{400} \times 100\% = \frac{1}{4}\% = 0.25\%$$

3. B

$$\begin{aligned} x - 0.2x & \quad x \text{ is decreased by 20 percent.} \\ = 0.8x & \quad \text{Simplify.} \\ y + 0.2y & \quad y \text{ is increased by 20 percent.} \\ = 1.2y & \quad \text{Simplify.} \end{aligned}$$

The product of decreased x and increased y is $0.8x \times 1.2y = 0.96xy$. So, the product is decreased by 4 percent.

4. B

Divide 4.5×10^5 by 9×10^4 .

$$\frac{4.5 \times 10^5}{9 \times 10^4} = 5$$

$$\text{So, } 4.5 \times 10^5 = (9 \times 10^4) \times 5 = 9 \times 10^4 + 4(9 \times 10^4)$$

$$= 9 \times 10^4 + 400\%(9 \times 10^4).$$

Therefore, 4.5×10^5 is 400% greater than 9×10^4 .

5. C

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$= \frac{72 - 60}{60} = \frac{12}{60} = \frac{1}{5} = 0.2 = 20\%$$

6. 5000

Let x = last year's enrollment in Mesa School District.

$$\underbrace{6000}_{\text{this year's enrollment}} = \underbrace{x + 0.2x}_{20\% \text{ more than last year's enrollment}}$$

$$6000 = 1.2x$$

$$x = \frac{6000}{1.2} = 5000$$

7. 16

$$1.25x = 80$$

125% of x is 80.

$$x = \frac{80}{1.25} = 64$$

Solve for x .

$$x = n\% \times 400$$

x is $n\%$ of 400.

$$x = n \times \frac{1}{100} \times 400$$

Percent means $\frac{1}{100}$.

$$x = n \times 4$$

Simplify.

$$64 = n \times 4$$

Substitute 64 for x .

$$16 = n$$

Divide each side by 4.

Section 7-2

1. 25

$$\frac{28}{100} \times n = 7$$

28% of a number is 7.

$$n = 7 \times \frac{100}{28}$$

Multiply each side by $\frac{100}{28}$.

$$n = 25$$

Simplify.

2. 1.5

$$3.6 = 2.4 \times n$$

3.6 is 240% of a number.

$$\frac{3.6}{2.4} = n$$

Divide each side by 2.4.

$$1.5 = n$$

Simplify.

3. 0.9

$$\frac{1}{2} \times \frac{1}{100} \times 180 = n \quad \frac{1}{2}\% \text{ is } \frac{1}{2} \times \frac{1}{100}.$$

$$\frac{180}{200} = n \quad \text{Simplify.}$$

$$0.9 = n \quad \text{Simplify.}$$

4. 75

$$3\frac{1}{3} \times \frac{1}{100} \times n = 2.5 \quad 3\frac{1}{3}\% \text{ is } 3\frac{1}{3} \times \frac{1}{100}.$$

$$\frac{10}{3} \times \frac{1}{100} \times n = 2.5 \quad \text{Simplify.}$$

$$\frac{1}{30} n = 2.5 \quad \text{Simplify.}$$

$$n = 2.5 \times 30 = 75 \quad \text{Multiply each side by 30.}$$

5. 4800

$$26.4 = 0.55 \times \frac{1}{100} \times n \quad 0.55\% \text{ is } 0.55 \times \frac{1}{100}.$$

$$26.4 = 0.0055n \quad \text{Simplify.}$$

$$\frac{26.4}{0.0055} = \frac{0.0055n}{0.0055} \quad \text{Divide each side by 0.0055.}$$

$$4800 = n \quad \text{Simplify.}$$

6. B

$$\underbrace{\frac{n}{100}}_{\text{what percent}} \times \underbrace{12}_{\text{of}} = 8$$

$$n = 8 \cdot \frac{100}{12} \Rightarrow n = 66\frac{2}{3}$$

8 is $66\frac{2}{3}\%$ of 12.

7. D

54 is 120% of k .

The above expression can be written as the equation $54 = 1.2 \times k$. Or it can be written as

$$\text{the proportion } \frac{120}{100} = \frac{54}{k}.$$

Choice D is correct.

8. D

Let x = Paul's monthly salary.

$$\underbrace{4500}_{\text{Kevin's monthly salary}} = \underbrace{0.72}_{72 \text{ percent}} \times \underbrace{x}_{\text{Paul's monthly salary}}$$

$$4500 = 0.72x$$

$$x = \frac{4500}{0.72} = 6250$$

Section 7-3

1. D

There are n candies in a jar and one candy is removed. So, $n - 1$ candies are left in the jar.

The fraction of candies left in the jar is $\frac{n-1}{n}$.

Thus, the percent of candies left in the jar is $(\frac{n-1}{n})100\%$.

2. A

Let x = the original price of the cellphone.
The discounted price is 25% off the original price, so $x - 0.25x$, or $0.75x$, is the discounted price. After an additional discount of 20% off the first discounted price, the new price is $0.75x - 0.2(0.75x)$, or $0.6x$, which is the final price of \$348. Therefore, $0.6x = 348$. Solving the equation for x yields $x = 580$.

3. C

Let x = the amount of 40% solution to be added.
Let $50 - x$ = the amount of 30% solution to be added.

x liters of 40 % acid + $(50 - x)$ liters of 30 % acid
= 50 liters of 36 % acid

$$0.4x + 0.3(50 - x) = 0.36(50)$$

$$0.4x + 15 - 0.3x = 18$$

$$0.1x + 15 = 18$$

$$0.1x = 3$$

$$x = 30$$

30 liters of 40% acid solution should be added.

4. C

If s is the amount invested in savings and r is the amount invested in bonds, $s + r$ represents the total amount invested, which is equal to \$5,000. Therefore, $s + r = 5000$.

If the amount invested in savings pays 4.5% interest and the amount invested in bonds pays 8% interest, $0.045s + 0.08r$ represents the total income from investment, which is equal to \$305.50. Therefore, $0.045s + 0.08r = 305.50$.

Choice C is correct.

5. 80

Let x = the price of the backpack before adding profit and tax.

After 50% profit the price of the backpack will be $x + 0.5x$, or $1.5x$.

After 8% tax the price of the backpack will be $1.5x + .08(1.5x)$, or $1.62x$, which is equal to \$129.60. Therefore, $1.62x = 129.60$. Solving for x yields $x = 80$.

The price of the backpack before adding profit and tax was \$80.

6. 218

The number of male students = $800 \times 0.45 = 360$.

The number of female students = $800 - 360 = 440$.

30% of male students = $360 \times 0.3 = 108$.

25% of female students = $440 \times 0.25 = 110$.

The number of students who play varsity sports = $108 + 110 = 218$

Chapter 7 Practice Test

1. D

If x mL of a 34% acid solution is added to a 10% acid solution and the resulting solution is 40 mL of a 25% solution, then the amount of the 10% acid solution should be $40 - x$ mL.

x mL of 34 % acid + $(40 - x)$ mL of 10% acid
= 40 mL of 25 % acid

$$0.34x + 0.1(40 - x) = 0.25(40)$$

$$0.34x + 4 - 0.1x = 10$$

$$0.24x = 6$$

$$x = 25$$

2. C

The cost of 3 packages of pens is $3 \times \$8.00$, or \$24 and the cost of 12 pens bought individually is $12 \times \$2.50$, or \$30. The amount saved is $30 - 24$ dollars, or \$6. The percent of savings he saved on 12 pens of the amount he paid is

$$\frac{6}{24} \cdot 100\%, \text{ or } 25\%.$$

3. C

The number of orange flavored drinks in the store = $600 \times 0.25 = 150$.

The number of orange flavored drinks sold on Monday = $150 \times 0.3 = 45$.

Remaining orange flavored drinks = $150 - 45 = 105$.

The number of orange flavored drinks sold on Tuesday is 20% of the remaining orange flavored drinks, which is 105×0.2 , or 21. Therefore, the number of bottles of orange flavored drinks sold in the two days is $45 + 21$, or 66.

4. B

After 15% discount, the price of the tablet is $x - 0.15x$, or $0.85x$. After an additional 12% discount, the price of the tablet is $0.85x - 0.12(0.85x)$, or $0.748x$.

5. D

n = total number of shoes m = the number of brown shoes. So the number of black shoes is $n - m$. The fraction of black shoes in the store

is $\frac{n - m}{n}$, so the percent of black shoes in the

store is $(\frac{n - m}{n}) \times 100\%$. This is equivalent to

$$(\frac{n}{n} - \frac{m}{n}) \times 100\%, \text{ or } (1 - \frac{m}{n}) \times 100\%.$$

6. C

If b is increased by 150%, it becomes $b + 1.5b$, or $2.5b$. If c is decreased by 60%, it becomes $c - 0.6c$, or $0.4c$. Multiplying these new values gives $a = 3.2(2.5b \times 0.4c) = 3.2(bc)$.

Therefore, the value is unchanged.

7. D

If 10 books are increased by x percent, then there will be $10 + 10 \times \frac{x}{100}$ books, which is equal to 24.

$$10 + 10 \times \frac{x}{100} = 24$$

$$\Rightarrow 10 \times \frac{x}{100} = 14 \Rightarrow \frac{x}{10} = 14$$

$$\Rightarrow x = 140$$

8. A

Number n is 25 less than 120 percent of itself.

$$n = 1.2n - 25$$

$$-0.2n = -25$$

$$n = \frac{-25}{-0.2} = 125$$

9. C

The number of blue cars = $500 \times 0.07 = 35$

The number of red cars = $500 \times 0.04 = 20$

Let 35 is n percent greater than 20.

$$\text{Then } 35 = 20 + 20 \cdot \frac{n}{100}.$$

$$35 - 20 = 20 + 20 \cdot \frac{n}{100} - 20$$

$$15 = \frac{1}{5}n$$

$$75 = n$$

The number of blue cars is 75% greater than the number of red cars.

10. 2.7

300% of 0.18 is equivalent to 20% of b .

$$3 \times 0.18 = 0.2b$$

$$300\% = 3, \quad 20\% = 0.2$$

$$0.54 = 0.2b$$

Simplify.

$$\frac{0.54}{0.2} = \frac{0.2}{0.2}b$$

Divide each side by 0.2.

$$2.7 = b$$

Simplify.

11. 684

Total amount contributed by five people

$$= \$9,000 \times 5 = \$45,000.$$

The price of the sailboat after 8% tax

$$= \$38,500 + 0.08 \times \$38,500 = \$41,580.$$

The amount that should be refunded

$$= \$45,000 - \$41,580 = \$3,420.$$

Dividing \$3,420 by 5 yields \$684.

Thus \$684 should be refunded to each person.

12. 38.4

Let m = the wholesale cost of MP3.

The selling price of \$72 is 50% more than the wholesale cost.

$$72 = m + 0.5m$$

$$72 = 1.5m$$

$$48 = m$$

The special holiday sale of the MP3 was 20% less than the wholesale cost. Therefore,

The special price of MP3

$$= m - 0.2m$$

$$= 48 - 0.2 \times 48$$

$$m = 48$$

$$= 38.4$$

The special sale price of the MP3 was \$38.4.