Sample Question Paper - 4
CLASS: XII
Session: 2021-22
Mathematics (Code-041)
Term - 1

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 20 MCQs, attempt any 16 out of 20. 3
3. . Section - B has 20 MCQs, attempt any 16 out of 20
4. Section - C has 10 MCQs, attempt any 8 out of 10.
5. There is no negative marking.
6. All questions carry equal marks.

SECTION – A

Attempt any 16 questions

1. Let f : R $\rightarrow$ R be defined by $f(x)=\begin{cases}2x : x>3\\ x^2 : 1<x\leq 3\\ 3x : x\leq 1\end{cases}$ **[1]**

Then f (–1) + f (2) + f (4) is

a) 5 b) 9

c) none of these d) 14

2. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let F = 4x + 6y be the objective function. Maximum of F – Minimum of F = **[1]**

a) 48 b) 60

c) 42 d) 18

3. Find the value of b for which the function $f(x)=\begin{cases}5x-4 & ,0<x\leq 1\\ 4x^2+3bx & ,1<x<2\end{cases}$ is continuous at every point of its domain, is **[1]**

a) $\frac{13}{3}$ b) -1

c) 1 d) 0

4. Let A be a non-singular square matrix of order 3 $\times$ 3. Then $|\text{adj A}|$ is equal to **[1]**

a) | A | b) 3| A |

c) $|\ A\ |^3$ d) $|\ A\ |^2$

5. A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in **[1]**

the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

Kg per bag		
	Brand P	Brand Q
Nitrogen Phosphoric acid Potash Chlorine	3 1 3 1.5	3.5 2 1.5 2

If the grower wants to maximise the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximum amount of nitrogen added?

a) 150 bags of brand P and 50 bags of brand Q; Maximum amount of nitrogen = 625 kg

b) 140 bags of brand P and 50 bags of brand Q; Maximum amount of nitrogen = 595 kg

c) 160 bags of brand P and 52 bags of brand Q; Maximum amount of nitrogen = 635 kg

d) 145 bags of brand P and 55 bags of brand Q; Maximum amount of nitrogen = 555 kg

6. The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line x + 3y = 8 is **[1]**

a) 3x + y + 8 = 0

b) x + 3y = 0

c) 3x - y = 8

d) x + 3y $\pm$ 8 = 0

7. If ω is a complex cube root of unity then the value of $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$. **[1]**

a) 2

b) 0

c) 4

d) -3

8. If $y = x^2 \sin \frac{1}{x}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $-\cos \frac{1}{x} + 2x \sin \frac{1}{x}$

b) $-x \sin \frac{1}{x} + \cos \frac{1}{x}$

c) $-\cos \frac{1}{x} + x \sin \frac{1}{x}$

d) none of these

9. Determine the maximum value of Z = 11x + 7y subject to the constraints :2x + y ≤ 6, x ≤ 2, x ≥ 0, y ≥ 0. **[1]**

a) 47

b) 43

c) 42

d) 45

10. If $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$ then A is a **[1]**

a) skew-symmetric matrix

b) symmetric matrix

c) none of these

d) diagonal matrix

[1]

11. If $f(x) = \begin{cases} mx+1, & \text{if } x \le \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ then

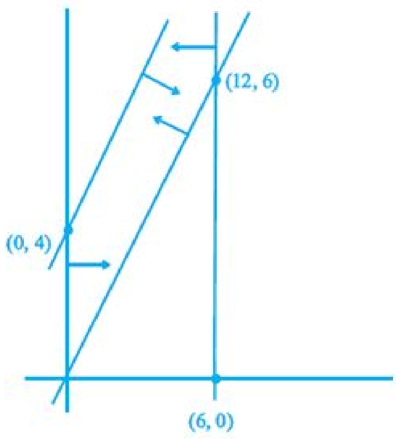
a) $m = n = \frac{\pi}{2}$

b) $n = \frac{m\pi}{2}$

c) m = 1, n = 0

d) $m = \frac{n\pi}{2} + 1$

12. The feasible region for an LPP is shown in the Figure. Let F = 3x - 4y be the objective function. Maximum value of F is. **[1]**



a) - 18

b) 0

c) 8

d) 12

13. The value of k for which f(x) $= \begin{cases} \frac{\sin 5x}{3x}, & \text{if } x \ne 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at x = 0 is **[1]**

a) $\frac{5}{3}$

b) $\frac{3}{5}$

c) 0

d) $\frac{1}{3}$

14. The function $f(x) = \frac{4-x^2}{4x-x^3}$ is **[1]**

a) none of these

b) discontinuous at only one point

c) discontinuous at exactly two points

d) discontinuous at exactly three points

15. If $y = x^{n-1} \log x$ then $x^2 y_2 + (3 - 2n) xy_1$ is equal to **[1]**

a) n^2y

b) $(n - 1)^2 y$

c) $-n^2y$

d) $-(n - 1)^2 y$

16. The function f(x) = tanx - x **[1]**

a) always increases

b) never increases

c) always decreases

d) sometimes increases and sometimes decreases.

17. The point on the curve $y^2 = 4x$ which is nearest to the point (2,1) is **[1]**

a) $(1, 2\sqrt{2})$

b) (-2, 1)

c) (1, -2)

d) (1, 2)

18. $\sin^{-1}(\frac{-1}{2}) + 2\cos^{-1}(\frac{-\sqrt{3}}{2}) = ?$ **[1]**

a) $\frac{3\pi}{2}$

b) π

c) None of these

d) $\frac{\pi}{2}$

19. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is **[1]**

a) $\frac{1-\log x}{1+\log x}$

b) $\frac{1+x}{1+\log x}$

c) $\frac{\log x}{(1+\log x)^2}$

d) not defined

20. The curves $x = y^2$ and $xy = k$ cut orthogonally when **[1]**

a) $6k^2 = 1$

b) None of these

c) $4k^2 = 1$

d) $8k^2 = 1$

SECTION – B

Attempt any 16 questions

21. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b a,b $\in$ T. Then R is **[1]**

a) an equivalence relation

b) neither reflexive nor symmetric

c) transitive but not symmetric

d) reflexive but not transitive

22. f(x) = sin x $\sqrt{3}$ cos x is maximum when x = **[1]**

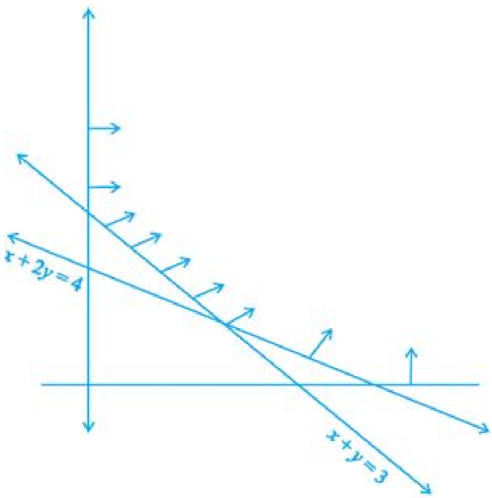
a) $\frac{\pi}{6}$

b) $\frac{\pi}{4}$

c) 0

d) $\frac{\pi}{3}$

23. The feasible region for a LPP is shown in Figure. Evaluate Z = 4x + y at each of the corner points of this region. Find the minimum value of Z, if it exists **[1]**



a) Minimum value = 2

b) Minimum value = 5

c) Minimum value = 4

d) Minimum value = 3

24. The slope of the tangent to the curve $x = 3t^2 + 1$, $y = t^3 - 1$ at x = 1 is **[1]**

a) $\frac{1}{2}$

b) ∞

c) 0

d) -2

25. If, $y = \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} + \frac{1}{1+x^{b-a}+x^{c-a}}$, then $\frac{dy}{dx}$ is equal to **[1]**

a) 1

b) $(a+b+c)^{x^{a+b+c-1}}$

c) none of these

d) 0

26. cot($\tan^{-1}$x + $\cot^{-1}$x). **[1]**

a) 1

b) 1/2

c) 0

d) None of these

27. If the set A contains 5 elements and the set B contains 6 elements, then the number of one – one and onto mappings from A to B is **[1]**

a) none of these

b) 720

c) 120

d) 0

28. tan [2 $\tan^{-1} \frac{1}{5} - \frac{\pi}{4}$] = ? **[1]**

a) $\frac{7}{12}$

b) $\frac{7}{17}$

c) $\frac{-7}{12}$

d) $\frac{-7}{17}$

29. It y = $\tan^{-1} \left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{2}{\sqrt{x}(1+x)}$

b) $\frac{1}{(1+x)}$

c) $\frac{1}{2\sqrt{x}(1+x)}$

d) $\frac{1}{\sqrt{x}(1+x)}$

30. If A’ is the transpose of a square matrix A, then **[1]**

a) |A| + |A'| = 0

b) |A| = |A'|

c) |A| $\neq$ |A'|

d) None of these

31. If $\sqrt{1-x^6} + \sqrt{1-y^6}$ = $a^3(x^3 - y^3)$,then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{y^2}{x^2}\sqrt{\frac{1-y^6}{1-x^6}}$

b) $\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

c) $\frac{x^2}{y^2}\sqrt{\frac{1-x^6}{1-y^6}}$

d) none of these

32. If $y = \frac{e^x-e^{-x}}{e^x+e^{-x}}$, then $\frac{dy}{dx}$ is equal to **[1]**

a) $1 + y^2$

b) None of these

c) $1 - y^2$

d) $y^2 + 1$

33. If the function f(x) = $2x^2$ - kx +5 is increasing on (1, 2), then k lies in the interval **[1]**

a) (4 ,∞)

b) (-∞, 8)

c) (8, ∞)

d) (-∞, 4)

34. Sin ($\tan^{-1}$ x), | x | < 1 is equal to **[1]**

a) $\frac{1}{\sqrt{1+x^2}}$

b) $\frac{x}{\sqrt{1+x^2}}$

c) $\frac{x}{\sqrt{1-x^2}}$

d) $\frac{1}{\sqrt{1-x^2}}$

[1]

35. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = ?$

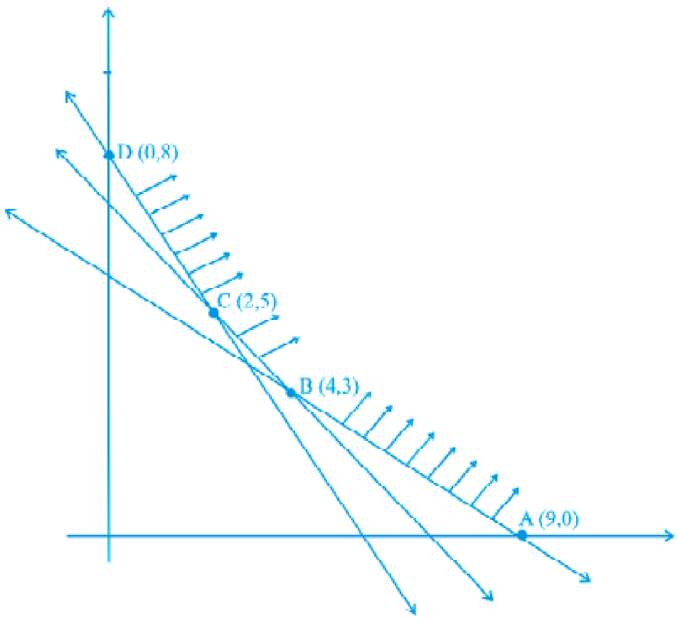
a) 2(a + b+ c)

b) 4abc

c) (ab + be + ca)

d) None of these

36. Feasible region (shaded) for a LPP is shown in the Figure. Minimum of Z = 4x + 3y occurs at the point **[1]**



a) (4, 3)

b) (9, 0)

c) (0, 8)

d) (2, 5)

37. If A is an invertible matrix of order 2, then det (A^{-1}) is equal to **[1]**

a) 0

b) $\frac{1}{\det(A)}$

c) det (A)

d) 1

38. Let f (x) =x^3, then f (x) has a **[1]**

a) point of inflexion at x = 0

b) local maxima at x = 0

c) none of these

d) local minima at x = 0

39. If $f(x) = \sqrt{1-\sqrt{1-x^2}}$, then f(x) is **[1]**

a) none of these

b) continuous on [–1, 1] and differentiable on (–1, 1)

c) continuous on [–1, 1] and differentiable on (–1, 0) ∪ (0, 1)

d) continuous and differentiable on [–1, 1]

40. Which of the following functions from Z into Z are bijections? **[1]**

a) f (x) = x^3

b) f (x) = 2x + 1

c) f (x) = x + 2

d) f (x) = x^2 + 1

SECTION – C

Attempt any 8 questions

41. If $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}=\alpha$, then x^2 = **[1]**

a) cos α
b) sin 2α
c) cos 2α
d) sin α

42. Maximize Z = – x + 2y, subject to the constraints: x ≥ 3, x + y ≥ 5, x + 2y ≥ 6, y ≥ 0. **[1]**

a) Z has no maximum value
b) Maximum Z = 14 at (2, 6)
c) Maximum Z = 12 at (2, 6)
d) Maximum Z = 10 at (2, 6)

43. If f is derivable at x = a , then $\underset{x\to a}{Lt}\ \frac{xf(a)-af(x)}{x-a}$ is equal to **[1]**

a) af'(9a) – f(a)
b) $f(a) - a\ f'(a)$
c) f '(a)
d) None of these

44. If A, B are two n $\times$ n non - singular matrices, then what can you infer about AB? **[1]**

a) AB is singular
b) $(AB)^{-1}$ does not exist
c) AB is non-singular
d) $(AB)^{-1} = A^{-1}B^{-1}$

45. Let S be the set of all real numbers and let R be a relation on S, defined by a Rb ⇔ (1 + ab) > 0. Then, R is **[1]**

a) None of these
b) Reflexive and transitive but not symmetric
c) Symmetric and transitive but not reflexive
d) reflexive and symmetric but not transitive

Question No. 46 to 50 are based on the given text. Read the text carefully and answer the questions:

Consider 2 families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommended daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children.

46. The requirement of calories and proteins for each person in matrix form can be represented as **[1]**

a)
b)

$$\begin{array}{l} \\ Man \\ Woman \\ Children \end{array} \begin{array}{c} \begin{array}{cc} Calories & Proteins \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$

$$\begin{array}{l} \\ Man \\ Woman \\ Children \end{array} \begin{array}{c} \begin{array}{cc} Calories & Proteins \end{array} \\ \begin{bmatrix} 1900 & 55 \\ 2400 & 45 \\ 1800 & 33 \end{bmatrix} \end{array}$$

c) $$\begin{array}{l} \\ Man \\ Woman \\ Children \end{array} \begin{array}{c} \begin{array}{cc} Calories & Proteins \end{array} \\ \begin{bmatrix} 1800 & 33 \\ 1900 & 55 \\ 2400 & 45 \end{bmatrix} \end{array}$$

d) $$\begin{array}{l} \\ Man \\ Woman \\ Children \end{array} \begin{array}{c} \begin{array}{cc} Calories & Proteins \end{array} \\ \begin{bmatrix} 2400 & 33 \\ 1900 & 55 \\ 1800 & 45 \end{bmatrix} \end{array}$$

47. The requirement of calories of family A is **[1]**

a) 15800
b) 15000
c) 24000
d) 24400

48. The requirement of proteins for family B is **[1]**

a) 266 grams
b) 300 grams
c) 332 grams
d) 560 grams

49. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ equals **[1]**

a) A + B
b) 2BA
c) 2AB
d) AB

50. If $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{n \times p}$ and $C = (c_{ij})_{p \times q}$, then the product (BC)A is possible only when **[1]**

a) p = q
b) m = q
c) n = q
d) m = p

Solution

SECTION – A

1. **(b)** 9

Explanation: Given that,

f(x) = $\begin{cases} 2x : x > 3 \\ x^2 : 1 < x \leq 3 \\ 3x^2 : x \leq 1 \end{cases}$

Now,

f(-1) = 3(-1) = -3 [since -1<1 and f(x) = 3x for x $\leq$ 1]

f(2) = 2^2 = 4 [since 2 < 3 and f(x) = x^2 for 1< x $\leq$ 3]

f(4) = 2(4) = 8 [since 4 > 3 and f(x) = 2x for x > 3]

∴ f (– 1) + f (2) + f (4) = -3 + 4 + 8 = 9

2. **(b)** 60

Explanation: Here the objective function is given by : F = 4x +6y .

Corner points	Z = 4x +6 y
(0, 2)	12(Min.)
(3,0)	12.(Min.)
(6,0)	24
(6 , 8)	72
(0 , 5)	30

Maximum of F – Minimum of F = 72 – 12 = 30 .

3. **(b)** -1

Explanation: $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

$\lim\limits_{x\to 1} 5x - 4 = \lim\limits_{x\to 1} 4x^2 + 36x$

5 - 4 = 4 + 3b

1 = 4 + 3b

b = -1

4. **(d)** | A $|^2$

Explanation: For a square matrix of order n×n,

We know that $A.adjA = |A|I$

Here, n=3

$\therefore \quad |A.adjA| = |A|^n$

$|adjA| = |A|^{n-1}$

So, $|AdjA| = |A|^{3-1} = |A|^2$

5. **(b)** 140 bags of brand P and 50 bags of brand Q; Maximum amount of nitrogen = 595 kg

Explanation: Let the number of bags used for fertilizer of brand P = x And the number of bags used for fertilizer of brand Q = y . Here , Z = 3x + 3.5y subject to constraints : :1.5 x +2 y ≤ 310, x + 2y ≥ 240, 3x + 1.5y ≥ 270 , x,y ≥ 0

Corner points	Z =3x + 3.5 y
C(40 ,100)	470..................(Min.)
B (140,50)	595.........................(Max.)
D(20,140)	550

Here Z = 595 is maximum i.e. 140 bags of brand P and 50 bags of brand Q; Maximum amount of nitrogen = 595 kg .

6. **(d)** x + 3y $\pm$ 8 = 0

Explanation: Given equation of the curve is

$3x^2 - y^2 = 5$ .... (i)

Differentiating both sides w.r.t, we get

$6x - 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{3x}{y}$, which is slope of tangent at any point on the curve

$\Rightarrow$ slope of normal at any point on the curve is $-\frac{dx}{dy} = \frac{-y}{3x}$

$\therefore \quad -\frac{y}{3x} = -\frac{1}{3}$

$\Rightarrow$ y = x .... (ii)

From (i) and (ii), we get

$3x^2 - x^2 = 8$

$\Rightarrow x^2 = 4$

$\Rightarrow \quad x = \pm 2$

For x = 2, y = 2 [using (iii)]

and for x = -2, y = -2 [using (iii)]

Thus, the points on the curve at which normal to the curve are parallel to the line x + 3y are (2, 2) and (-2, -2).

$\therefore$ Required equations of normal are

$y - 2 = -\frac{1}{3}(x - 2)$ and y + 2 $= -\frac{1}{3}(x + 2)$

or 3y + x = 8 and 3y + x = -8

7. **(c)** 4

Explanation: $1 + \omega + \omega^2 = 0 \Rightarrow (1 + \omega)$ = $-\omega^2$. $\text{Put}(1 + \omega) = -\omega^2$ and expand.

8. **(a)** $-\cos\frac{1}{x} + 2x\sin\frac{1}{x}$

Explanation: Given that y = $x^2 \sin\frac{1}{x}$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = x^2\cos\frac{1}{x} \times -\frac{1}{x^2} + 2x\sin\frac{1}{x} = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$

9. **(c)** 42

Explanation: Here , maximize Z = 11x + 7y , subject to the constraints :2x + y ≤ 6, x ≤ 2, x ≥ 0, y ≥ 0.

Corner points	Z = 11x +7 y
C(0, 0)	0
B (2,0)	22
D(2,2)	36
A(0,6)	42

Hence the maximum value is 42

10. **(a)** skew-symmetric matrix

Explanation: The diagonal elements of a skew – symmetric matrix is always zero and the elements $a_{ij} = -a_{ji}$.

11. **(b)** $n = \frac{m\pi}{2}$

Explanation: We have, $f(x) = \begin{cases} mx + 1, & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$

$\therefore LHL = \lim\limits_{x \to \frac{\pi}{2}} (mx + 1) = \lim\limits_{h \to 0} \left[m\left(\frac{\pi}{2} - h\right) + 1\right] = \frac{m\pi}{2} + 1$

and $RHL = \lim_{x \to \frac{\pi'}{2}} (\sin x + n) = \lim_{h \to \infty} \left[\sin\left(\frac{\pi}{2} + h\right) + n\right]$

$= \lim_{n \to 0} \cos h + n = 1 + n$

Since the function is continuous, we have

$LHL = RHL$

$\Rightarrow m \cdot \frac{\pi}{2} + 1 = n + 1$

$\therefore\ n = m \cdot \frac{\pi}{2}$

12. **(d)** 12

Explanation:

Corner points	Z = 3x - 4y
(0, 0)	0
(0,4)	-16
(12,6)	12.........................(Max.)

13. **(a)** $\frac{5}{3}$

Explanation: Since f(x) is continuous on 0,then we

$\Rightarrow \lim_{x \to 0} \frac{\sin 5x}{3x} = f(0)$

$\Rightarrow \lim_{x \to 0} \frac{\sin 5x}{3x} \times \frac{5x}{5x} = f(0)$

$\Rightarrow \lim_{x \to 0} \frac{\sin 5x}{5x} \times \frac{5x}{3x} = f(0)$

$\Rightarrow \text{f}(0) = \frac{5}{3}$

$\Rightarrow \text{k} = \frac{5}{3}$

14. **(d)** discontinuous at exactly three points

Explanation: We have, $f(x) = \frac{4-x^2}{4x-x^3} = \frac{(4-x^2)}{x(4-x^2)}$

$= \frac{(4-x^2)}{x(2^2-x^2)} = \frac{4-x^2}{x(2+x)(2-x)}$

Clearly, f(x) is discontinuous at exactly three points x = 0, x = -2 and x = 2.

15. **(a)** n^2y

Explanation: $y = x^{n-1} \log x$

Differentiating both sides w.r.t. to x we get,

$y_1 = x^{n-2} + (n - 1) x^{n-2} \log x$

$xy_1 = x^{n-1} + (n - 1) x^{n-1} \log x$

$= x^{n-1} + (n - 1) y$

Again differentiating both sides w.r.t. to x we get,

$xy_2 + y_1 = (n - 1) x^{n-2} + (n - 1) y_1$

$\Rightarrow x^2y_2 + xy_1 - x (n - 1) y_1 = (n - 1) x^{n-1}$

$\Rightarrow x^2y_2 + xy_1 (1 + 1 - n) = (n - 1) (xy_1 - (n - 1) y)$

$\Rightarrow x^2y_2 + xy_1 (2 - n + 1 - n) = -(n - 1)^2 y$

$\Rightarrow x^2y_2 + xy_1 (3 - 2n) = -(n - 1)^2 y$

16. **(a)** always increases

Explanation: We have, f(x) = tan x - x

$\therefore$ f'(x) = $\sec^2 x - 1$

$\Rightarrow f'(x) \geq 0, \forall x \in R$

So, f(x) always increases

17. **(d)** (1, 2)

Explanation: $y^2 = 4x \Rightarrow x = \frac{y^2}{4}$

$\Rightarrow$ d = $\sqrt{(x-2)^2+(y-1)^2}$

$\Rightarrow$ $d^2 = (x - 2)^2 + (y - 1)^2$

$\Rightarrow$ $d^2 = \left(\frac{y^2}{4}-2\right)^2 + (y - 1)^2$

Let $u = \left(\frac{y^2}{4}-2\right)^2 + (y - 1)^2$

$\Rightarrow \frac{du}{dy} = 2\left(\frac{y^2}{4}-2\right)\frac{y}{2} + 2(y - 1)$

To find minima

$\frac{du}{dy} = 0$

$2\left(\frac{y^2}{4}-2\right)\frac{y}{2} + 2(y - 1) = 0$

$\Rightarrow$ y = 2 $\Rightarrow$ x = 1 $\left(x=\frac{y^2}{4}\right)$

$\frac{d^2u}{dy^2} = \frac{3y^2}{4}$

$\Rightarrow \left(\frac{d^2u}{dy^2}\right)_{(1,2)} = 3 > 0$

Hence, nearest point is (1, 2).

18. **(a)** $\frac{3\pi}{2}$

Explanation: Given: $\sin^{-1}\left(\frac{-1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

Let, $x = \sin^{-1}\left(\frac{-1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

$\Rightarrow x=-\sin^{-1}\left(\frac{1}{2}\right)+2\left[\pi-\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$ ($\because \sin^{-1}(-\theta) = -\sin(\theta)$ and $\cos^{-1}(-\theta) = -\cos^{-1}(\theta)$)

$\Rightarrow x = -\left(\frac{\pi}{6}\right)+2\left[\pi-\frac{\pi}{6}\right]$

$\Rightarrow x = -\left(\frac{\pi}{6}\right)+2\left[\frac{5\pi}{6}\right]$

$\Rightarrow x = -\frac{\pi}{6}+\frac{5\pi}{3}$

$\Rightarrow x=\frac{3\pi}{2}$

19. **(c)** $\frac{\log x}{(1+\log x)^2}$

Explanation: $x^y = e^{x-y}$

Taking log on both sides,

$\log x^y = \log e^{x-y}$

$y \log x = x - y$

$y \log x + y = x$

$y=\frac{x}{\log x+1}$

Differentiate with respect to x,

$\frac{dy}{dx}=\frac{(\log x+1)-x\times\frac{1}{x}}{(\log x+1)^2}$

$\frac{dy}{dx}=\frac{(\log x+1)-1}{(\log x+1)^2}$

$\frac{dy}{dx}=\frac{\log x}{(\log x+1)^2}$

20. **(d)** $8k^2 = 1$

Explanation: Let (α,β) be the point of intersection of the given curves

Now, $x=y^2\Rightarrow 2y\frac{dy}{dx}=1\Rightarrow\frac{dy}{dx}=\frac{1}{2y}\ldots..(i)$

$xy=k\Rightarrow x.\frac{dy}{dx}+y=0\Rightarrow\frac{dy}{dx}=\frac{-y}{x}\ldots(ii)$

$m_1 = \left(\frac{dy}{dx}\right)_{(\alpha,\beta)}=\left(\frac{1}{2y}\right)_{(\alpha,\beta)}=\frac{1}{2\beta}, m_2 = \left(\frac{dy}{dx}\right)_{(\alpha,\beta)}=\left(\frac{-y}{x}\right)_{(\alpha,\beta)}=\frac{-\beta}{\alpha}$

Two curves cut orthogonallly means $m_1.m_2 = -1$

$\Rightarrow\frac{1}{2\beta}.\frac{-\beta}{\alpha}=-1\Rightarrow 2\alpha=1\Rightarrow\alpha=\frac{1}{2}\ldots.(iii)$

Since (α,β) lies on $x=y^2$ we have $\alpha=\beta^2 \Rightarrow \beta^2=\frac{1}{2}\dots(iv)$

Also, Since,(α,β) lies on $xy=k,$ we get $\alpha.\beta=k \Rightarrow k^2=\alpha^2.\beta^2$

$\Rightarrow k^2=\frac{1}{4}.\frac{1}{2}=\frac{1}{8}$ from (iii) and (iv)

$\Rightarrow 8k^2=1$

SECTION – B

21. **(a)** an equivalence relation

Explanation: Let T be the set of all triangles in the Euclidean plane with R, a relation in T is given by R = $\{(T_1,T_2): T_1$ is congruent to $T_2\}$

$(T_1,T_2) \in R$ if T_1 is congruent to T_2.

Reflexivity: $T_1 \cong T_1 \Rightarrow (T_1,T_1) \in R$.

Symmetry: $(T_1,T_2) \in R \Rightarrow T_1 \cong T_2 \Rightarrow T_2 \cong T_1 \Rightarrow (T_2,T_1) \in R$.

Transitivity: $(T_1,T_2) \in R$ and $(T_2,T_3) \in R$.

$\Rightarrow T_1 \cong T_2$ and $T_2 \cong T_3 \Rightarrow T_1 \cong T_3 \Rightarrow (T_2,T_3) \in R$.

Therefore, R is an equivalence relation.

22. **(a)** $\frac{\pi}{6}$

Explanation: f(x) = sin x + $\sqrt{3}$ cos x

$\Rightarrow$ f'(x) = cos x - $\sqrt{3}$ sin x

for maxima or minima

f'(x) = 0

cos x - $\sqrt{3}$ sin x = 0

$\Rightarrow$ tan x = $\frac{1}{\sqrt{3}} \Rightarrow x=\frac{\pi}{6}$

f"(x) = - sin x - $\sqrt{3}$ cos x

$\Rightarrow$ f"$\left(\frac{\pi}{6}\right)$ = - sin$\frac{\pi}{6}-\sqrt{3}$ cos $\frac{\pi}{6}=\frac{-1-\sqrt{3}}{2}<0$

function has local maxima at x = $\frac{\pi}{6}$

23. **(a)** Minimum value = 2

Explanation:

Corner points	**Z = 4x + y**
(0, 2)	2
(0,3)	3
(2,1)	9

Hence the minimum value is 2

24. **(c)** 0

Explanation: x = $3t^2$ + 1 and y = t^3 - 1

$\frac{dx}{dt}=6t$ and $\frac{dy}{dt}=3t^2$

$\Rightarrow \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{t}{2}$ .... (i)

But,

x = 1

$\Rightarrow 3t^2 + 1 = 1$

$\Rightarrow$ t = 0

$\frac{dy}{dx}=\frac{t}{2}=0$ ($\because$ From (i))

25. **(d)** 0

Explanation: $y=\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}+\frac{1}{1+x^{b-a}+x^{c-a}}$

$y=\frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}}+\frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}+\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}}$

$y = \frac{x^b}{x^a+x^b+x^c} + \frac{x^c}{x^a+x^b+x^c} + \frac{x^a}{x^a+x^b+x^c}$

$y = \frac{x^a+x^b+x^c}{x^a+x^b+x^c}$

y = 1

$\frac{dy}{dx} = 0$

26. **(c)** 0

Explanation: Given: $\cot(\tan^{-1}x + \cot^{-1}x)$

Let, $x = \cot(\tan^{-1}x + \cot^{-1}x)$

$x = \cot\left(\frac{\pi}{2}\right)$ $\left(\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right)$

x = 0

27. **(d)** 0

Explanation: Because the no. of elements in domain i.e. in A is less than the no. of elements in co-domain i.e. in B. Therefore, no bijection mapping is possible.

28. **(d)** $\frac{-7}{17}$

Explanation: The given equation is of $\tan\left[2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right]$

Let $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) = \tan\left(\tan^{-1}\left(\frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2}\right) - \frac{\pi}{4}\right)$ $\left(\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{\frac{2}{5}}{1-\frac{1}{25}}\right) - \frac{\pi}{4}\right)$

$= \tan\left(\tan^{-1}\left(\frac{5}{12}\right) - \frac{\pi}{4}\right)$

$= \tan\left(\tan^{-1}\left(\frac{5}{12}\right) - \frac{\pi}{4}\right)$

$= \tan\left(\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1}(1)\right)$ $\left(\because \tan\left(\frac{\pi}{4}\right) = 1\right)$

$= \tan\left(\tan^{-1}\left(\frac{\frac{5}{12}-1}{1+\frac{5}{12}}\right)\right)$ $\left(\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{-7}{17}\right)\right)$

$\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) = \frac{-7}{17}$

29. **(c)** $\frac{1}{2\sqrt{x}(1+x)}$

Explanation: Given that $y = \tan^{-1}\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}$

Let $\sqrt{a} = \tan A$ and $\sqrt{\mathrm{x}} = \tan \mathrm{B}$, then $A = \tan^{-1}\sqrt{a}$ and $B = \tan^{-1}\sqrt{x}$

Hence, $y = \tan^{-1}\frac{\tan A+\tan B}{1-\tan A\tan B}$

Using $\tan(A + B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$, we obtain

$y = \tan^{-1}\tan(A + B) = A + B$

$= \tan^{-1}\sqrt{a} + \tan^{-1}\sqrt{x}$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = 0 + \frac{1}{1+(\sqrt{x})^2} \times \frac{1}{2\sqrt{\mathrm{x}}} = \frac{1}{2\sqrt{\mathrm{x}}(1+\mathrm{x})}$

30. **(b)** |A| = |A'|

Explanation: The determinant of a matrix A and its transpose always same. Because if we interchange the rows into column in a determinant the value of determinant remains unaltered.

31. **(b)** $\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

Explanation: $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3(x^3 - y^3)$

Put $x^3 = \sin u$, $y^3 = \sin v$

$\Rightarrow \cos u + \cos v = a^3(\sin u - \sin v)$

$\Rightarrow 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right) = a^3 \times 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$

$\Rightarrow \cos\left(\frac{u-v}{2}\right) = \sin\left(\frac{u-v}{2}\right)$

$\Rightarrow \frac{u-v}{2} = \tan^{-1}\frac{\pi}{4}$

$\Rightarrow \mathrm{u} - \mathrm{v} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x^3 + \sin^{-1}y^3 = \frac{\pi}{2}$

Differentiating with respect to x,

$\frac{3x^2}{\sqrt{1-x^6}} - \frac{3y^2}{\sqrt{1-y^6}}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

32. **(c)** $1 - y^2$

Explanation: Solution. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)$

$= \frac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2} = \frac{(e^x+e^{-x})^2}{(e^x+e^{-x})^2} - \frac{(e^x-e^{-x})^2}{(e^x+e^{-x})^2} = 1 - y^2.$

Which is the required solution.

33. **(d)** $(-\infty, 4)$

Explanation: $f(x) = 2x^2 - kx + 5$

$f'(x) = 4x - k$

for f(x) to be increasing, we must have

$f(x) > 0$

$4x - k > 0$

$K < 4x$

since $x \in [1,2], 4x \in [4,8]$

so, the minimum value of 4 x is 4.

since $K < 4x$, $K < 4$.

$k \in (-\infty, 4)$

34. **(b)** $\frac{x}{\sqrt{1+x^2}}$

Explanation: Let $\tan^{-1} x = y$, then $\tan y = x \Rightarrow \sin y = \frac{x}{\sqrt{1+x^2}}$

$\therefore\ y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$\Rightarrow \tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$\Rightarrow \sin(\tan^{-1}x) = \sin\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)$

$= \frac{x}{\sqrt{1+x^2}}$

35. **(b)** 4abc

Explanation: Apply $R^1 \to R^1 - (R^2 + R^3)$

Take (-2) common from R_1. Apply $R^2 \to (R^2 - R^1)$ and $R^3 \to (R^3 - R^1)$

36. **(d)** (2, 5)

Explanation: Z=4x+3y

1. (0,8)=24

2.(2,5)=8+15=23

3.(4,3)=16+9=25

4. (9,0)=36+0=36

The correct answer is (2, 5) as it gives the minimum value.

37. **(b)** $\frac{1}{\det(A)}$

Explanation: We know that, $A^{-1} = \frac{1}{|A|}$ Adj (A)

So, $\left|A^{-1}\right| = \left|\frac{1}{|A|}\mathrm{Adj}(A)\right|$

$= \frac{1}{|A|^n}|\text{Adj}(A)|$

$= \frac{1}{|A|^n}|A|^{n-1} = \frac{1}{|A|^1}$

$= \frac{1}{|A|^1}$

{since adj(A) is of order n and |Adj(A)| = $|A|^{n-1}$}

38. **(a)** point of inflexion at x = 0

Explanation: Given f(x) = x^3

f'(x) = $3x^2$

For point of inflexion, we have f'(x) = 0

$f'(x) = 0 \Rightarrow 3x^2 = 0 \Rightarrow x = 0$

Hence, f(x) has a point of inflexion at x = 0.

But x = 0 is not a local extremum as we cannot find an interval I around x = 0 such that

$f(0) \geq f(x) \quad or \quad f(0) \leq f(x) \qquad for \quad all \quad x \epsilon I$

39. **(c)** continuous on [–1, 1] and differentiable on (–1, 0) ∪ (0, 1)

Explanation: Given that $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$

So, the function will be defined for those values of x for which

$1 - x^2 \geq 0$

$\Rightarrow x^2 \leq 1$

$\Rightarrow |x| \leq 1$

$\Rightarrow -1 \leq x \leq 1$

∴ Function is continuous in [-1, 1].

Now, we will check the differentiability at x = 0

LHD at x =0,

$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{h \to 0} \frac{f(0-h) - f(0)}{0 - h - 0}$

$= \lim_{h \to 0} \frac{\sqrt{1 - \sqrt{1 - (0-h)^2}} - (0)}{-h} = -\infty$

∵ LHD does not exist, so f(x) is not differentiable at x = 0

∴ f(x) is not differentiable at x =0.

40. **(c)** f (x) = x + 2

Explanation: Injectivity: Let x, y ∈ Z, then, f(x) = f(y) ⇒ x + 2 = y + 2 ⇒x = y.⇒f is one-one.

Surjectivity: Let f(x) = y , where y ∈ Z, .⇒ x + 2 = y .⇒ x = y – 2 ∈ Z, ⇒ f is onto.

Therefore, f is a bijective function.

SECTION – C

41. **(b)** sin 2α

Explanation: $\tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right) = \alpha$

$\Rightarrow \frac{\sqrt{1+x^2 - \sqrt{1-x^2}}}{\sqrt{1+x^2} + \sqrt{1-x^2}} = \tan\alpha$

$\Rightarrow \frac{\sqrt{1+x^2 - \sqrt{1-x^2}}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \times \frac{\sqrt{1+x^2 - \sqrt{1-x^2}}}{\sqrt{1+x^2} - \sqrt{1-x^2}} = \tan\alpha$

$\Rightarrow \frac{(\sqrt{1+x^2})^2 + (\sqrt{1-x^2})^2 - 2\sqrt{1+x^2}\sqrt{1-x^2}}{(\sqrt{1+x^2})^2 - (\sqrt{1-x^2})^2} = \tan\alpha$

$\Rightarrow \frac{1 - \sqrt{1-x^4}}{x^2} = \tan\alpha$

$1 - \sqrt{1 - x^4} = x^2 \tan\alpha$

$(1 - x^2 \tan\alpha)^2 = 1 - x^4$

$1 - 2x^2 \tan\alpha + x^4 \tan^2\alpha = 1 - x^4$

$x^4 - 2x^2 \tan\alpha + x^4 \tan^2\alpha = 0$

$x^2 (x^2 - 2\tan\alpha + x^2 \tan^2\alpha) = 0$

$x^2 = \frac{2\tan\alpha}{1+\tan^2\alpha}$

$x^2 = \frac{2\tan\alpha}{\sec^2\alpha}$

$x^2 = 2\tan\alpha \cos^2\alpha$

$x^2 = 2\sin\alpha\cos\alpha = \sin 2\alpha$

42. **(a)** Z has no maximum value

Explanation: Objective function is Z = - x + 2 y ....(1).

The given constraints are : x ≥ 3, x + y ≥ 5, x + 2y ≥ 6, y ≥ 0.

Corner points	Z = - x + 2y
D(6,0)	-6
A(4,1)	-2
B(3,2)	1

Here, the open half plane has points in common with the feasible region.

Therefore, Z has no maximum value.

43. **(b)** $f(a) - a\ f'(a)$

Explanation: $\lim\limits_{x \to a} \frac{xf(a) - af(x)}{x-a}$

$= \lim\limits_{h \to 0} \frac{(a+h)f(a) - af(a+h)}{h} = \lim\limits_{h \to 0} \left\{ \frac{hf(a)}{h} - \frac{af(a+h) - af(a)}{h} \right\} = f(a) - af'(a)$

44. **(c)** AB is non-singular

Explanation: If A and B are non - singular then |AB| $\neq$ 0

= AB is non - singular matrix,

As |AB = |A||B|

45. **(d)** reflexive and symmetric but not transitive

Explanation: Let S denote the set of all real numbers. Let R be a relation in S defined as a R b iff 1 + ab > 0.

i. R is reflexive, Let a be any real number.

   Then 1 + aa = 1 + a^2 > 0, since $a^2 \geq 0$.

   Thus a R a ∀ a $\in$ S. Therefore R is reflexive.

ii. R is symmetric. Let a, b be any two real numbers.

   Then a R b $\Rightarrow$ 1 + ab > 0 $\Rightarrow$ 1 + ba > 0 [$\because$ ab = ba]

   $\therefore$ R is symmetric.

iii. R is not transitive. Consider three real number 1, -$-\frac{1}{2}$, -4.

   We have

   $1 + 1\left(-\frac{1}{2}\right) = \frac{1}{2} > 0$

   $\therefore 1R - \frac{1}{2}$

   Further $1 + \left(-\frac{1}{2}\right)(-4) = 3 > 0$

   $\therefore -\frac{1}{2}R - 4$

   But 1 + 1(-4) = -3 Which is not greater than 0. Therefore 1 is not R-related to -4.

   Thus 1 $R - \frac{1}{2}, -\frac{1}{2}R - 4$ and 1 is not R-related to -4.

   $\therefore$ R is not transitive.

46. **(a)** $\begin{matrix} & \begin{matrix} \textit{Calories} & \textit{Proteins} \end{matrix} \\ \begin{matrix} Man \\ Woman \\ Children \end{matrix} & \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{matrix}$

Explanation: Let F be the matrix representing the number of family members and R be the matrix representing the requirement of calories and proteins for each person. Then

$$F = \begin{matrix} Family\ A \\ Family\ B \end{matrix} \overset{\begin{matrix} Men\ Women\ Children \end{matrix}}{\begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix}}$$

$$R = \begin{matrix} Man \\ woman \\ Children \end{matrix} \overset{\begin{matrix} Calories & Proteins \end{matrix}}{\begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}}$$

47. **(d)** 24400

Explanation: The requirement of calories and proteins for each of the two families is given by the product matrix FR.

$$FR = \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$= \begin{bmatrix} 4(2400+1900+1800) & 4(45+55+33) \\ 2(2400+1900+1800) & 2(45+55+33) \end{bmatrix}$$

$$FR = \overset{\begin{matrix} Calories\ Proteins \end{matrix}}{\begin{bmatrix} 24400 & 532 \\ 12200 & 266 \end{bmatrix}} \begin{matrix} Family\ A \\ Family\ B \end{matrix}$$

48. **(a)** 266 grams

Explanation: 266 grams

49. **(a)** A + B

Explanation: Since, AB = B ...(i) and BA = A ..(ii)

$\therefore A^2 + B^2 = A \cdot A + B \cdot B$

= A(BA) + B(AB) [using (i) and (ii)]

- (AB)A + (BA)B [Associative law]

= BA + AB [using (i) and (ii)]

= A + B

50. **(b)** m = q

Explanation: $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{n \times p}$, $C = (c_{ij})_{p \times q}$

$BC = (b_{ij})_{n \times p} \times (C_{ij})_{p \times q} = (d_{ij})_{n \times q}$

$(BC)A = (d_{ij})_{n \times q} \times (a_{ij})_{m \times M}$

Hence, (BC)A is possible only when m = q