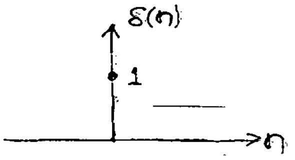
Chapter - 08
Discrete time signal

1) Unit-impulse signal $\delta(n) \rightarrow$

$$\delta(n) = \begin{cases} 1 \; ; \; n=0 \\ 0 \; ; \; n \neq 0 \end{cases}$$



properties $\rightarrow$

(1.) $\delta(n)$ is an even signal.

(2.) $\delta(n)$ is an energy signal.

$\delta(t)$ is NENP sig.

(3.) $\delta(an) = \delta(n)$

$\delta(at) = \frac{1}{|a|}\delta(t)$

(4.) $x(n)\,\delta(n-n_1) = x(n_1)\,\delta(n-n_1)$

(5.) $x(n) * \delta(n-n_1) = x(n-n_1)$

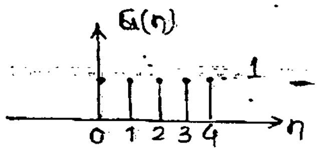
(6.) $\int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$

$\int = \Sigma, \; t = n, \; \tau = k$

$$\sum_{k=-\infty}^{n} \delta(k) = u(n)$$

(2.) Unit-step sig. $\rightarrow$

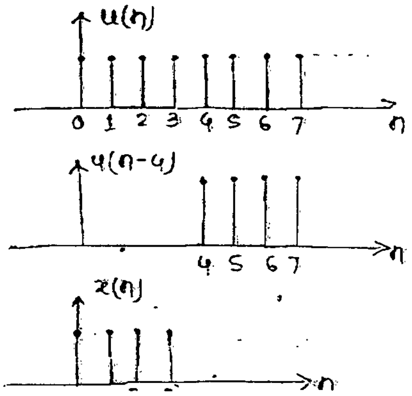
$$u(n) = \begin{cases} 1 \; , \; n \geq 0 \\ 0 \; , \; n < 0 \end{cases}$$



Q. $\rightarrow$ Draw sig. $x(n)$

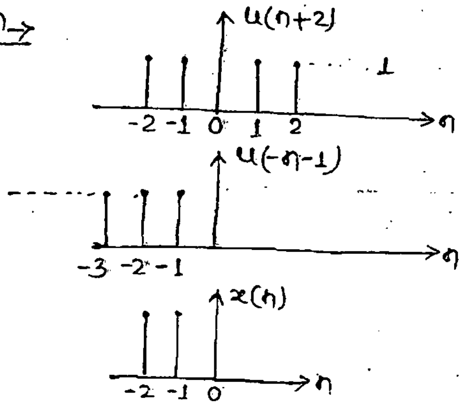
$$x(n) = u(n) - u(n-4)$$

Sol^n $\rightarrow$



Que→ Draw the signal $x(n) = u(n+2)\cdot u(-n-1)$

Soln→



$-n-1 = 0$

↑ $n = -1$ → starting point

Because LS.

Operations on signal→

(1) Time-shifting→

$x(n) = \{\overset{n=-2}{5}\ 3\ \underset{\uparrow}{7}\ 4\ 8\ \overset{n=3}{9}\}$

axis not shifts only signal shifts

$x(n-1) = \{5\ \underset{\uparrow}{3}\ 7\ 4\ 8\ 9\}$ (RS)

$x(n+2) = \{5\ 3\ 7\ 4\ \underset{\uparrow}{8}\ 9\}$ (LS)

(2) Time Compression→(Decimation)→

$x(n) = \{\overset{n=-3}{5}, 3, 7, \underset{\uparrow}{8}, -2, 4, \overset{n=3}{9}\}$

$f(n) = x(2n) = \{3, \underset{\uparrow}{8}, 4\}$

$f(n) = x(3n) = \{5, \underset{\uparrow}{8}, 9\}$

$f(-2) = x(-4) = 0$

$f(-1) = x(-2) = 3$

$f(0) = x(0) = 8$

$f(1) = x(2) = 4$

$f(2) = x(4) = 0$

(3) Time expansion→(Interpolation)

$x(n) = \{\overset{n=-1}{4}, \underset{\uparrow}{3}, \overset{n=1}{5}\}$

$f(n) = x\left(\frac{n}{2}\right) = \{4, 0, 3, 0, 5\}$

$1 = 2-1$

$x\left(\frac{n}{3}\right) = \{4, 0, 0, 3, 0, 0, 5\}$

$2 = 3-1$

$f(-3) = x(-3/2) = 0$

$f(-2) = x(-1) = 4$

$f(-1) = x(-1/2) = 0$

$f(0) = 0 = 3$

$f(1) = x(1/2) = 0$

$f(2) = x(1) = 5$

$f(3) = x(3/2) = 0$

Que→ $x(n)=\{1,2,\underset{\uparrow}{3},4,5\}$ Find $y(n)$

(i) $y(n)=x\left(\frac{2n}{3}\right)$

Sol^n→ $x(n)\xrightarrow{\text{Dec.}}x(2n)\xrightarrow{\text{Int}}x\left(\frac{2n}{3}\right)$

$\{1,2,\underset{\uparrow}{3},4,5\}\quad\{1,\underset{\uparrow}{3},5\}\quad\{1,0,0,\underset{\uparrow}{3},0,0,5\}$

(ii) $y(n)=x(-2n)$

Sol^n→ $x(n)\longrightarrow x(2n)\longrightarrow x(-2n)$ (-ve - folding about arrow)

$\{1,2,\underset{\uparrow}{3},4,5\}\quad\{1,\underset{\uparrow}{3},5\}\quad(5,\underset{\uparrow}{3},1)$

(iii) $y(n)=x(-n-1)$

Sol^n→ $x(n)\longrightarrow x(n-1)\longrightarrow x(-n-1)$

$(1,2,\underset{\uparrow}{3},4,5)\quad(1,\underset{\uparrow}{2},3,4,5)\quad(5,4,3,\underset{\uparrow}{2},1)$

(iv) $y(n)=x(2n-1)$

Sol^n→ $x(n)\longrightarrow x(n-1)\longrightarrow x(2n-1)$

$(1,2,\underset{\uparrow}{3},4,5)\quad(1,\underset{\uparrow}{2},3,4,5)\quad(\underset{\uparrow}{2},4)$

(4) Convolution→

$$y(n)=x_1(n)*x_2(n)$$
$$=\sum_{k=-\infty}^{\infty}x_1(k)\,x_2(n-k)$$

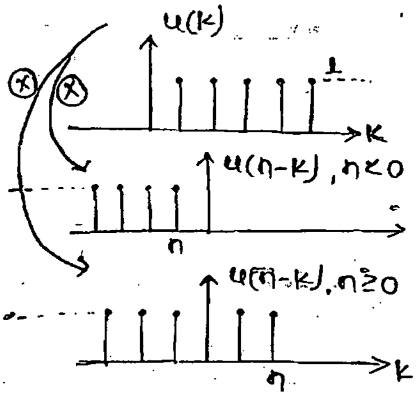
signal	Extension	Length
$x_1(n)$	$n_1\le n\le n_2$	L_1
$x_2(n)$	$n_3\le n\le n_4$	L_2
$y(n)$	$n_1+n_3\le n\le n_2+n_4$	L_1+L_2-1

length = no. of samples

Que→ $y(n)=u(n)*u(n)$

Sol^n→

$$y(n)=u(n)*u(n)$$
$$=\sum_{k=-\infty}^{\infty}u(k)\cdot u(n-k)$$



$n-k=0 \rightarrow$ -ve (LS)
$n=k$

$$y(n) = u(n) * u(n)$$
$$= \sum_{-\infty}^{\infty} u(k)\cdot u(n-k)$$
$$= \begin{cases} 0 & , n<0 \\ \sum_{k=0}^{n} 1 & ; n \geq 0 \end{cases}$$
$$= \begin{cases} 0 & , n<0 \\ n+1 & , n \geq 0 \end{cases}$$
$$\boxed{y(n) = (n+1)\, u(n)}$$

Que.→ $x_1(n) = (1, \underset{\uparrow}{2}, -2)$; $x_2(n) = (2, 0, \underset{\uparrow}{1})$
$y(n) = x_1(n) * x_2(n) = ?$

Sol^n→ Tabular method →

$x_2(n)$ \ $x_1(n)$	1	2	-2
2	2	4	-4
0	0	0	0
→ 1	1	2	-2

$x_1(n) = (1, \underset{\uparrow}{2}, -2)$ = 2nd element
$x_2(n) = (2, 0, \underset{\uparrow}{1})$ = 3rd element
$y(n) = (2, 4, -3, \underset{\uparrow}{2}, -2)$

2+3=5
5−1=
(arrow
point

Que.→ $x_1(n) = (-1, \underset{\uparrow}{2}, 0, 1)$ $x_2(n) = (3, 1, \underset{\uparrow}{0}, -1)$
$y(n) = x_1(n) * x_2(n)$

Sol^n→

$x_2(n)$ \ $x_1(n)$	-1	2	0	1
3	-3	6	0	3
1	-1	2	0	1
→ 0	0	0	0	0
-1	1	-2	0	-1

$y(n) = (-3, 5, 2, \underset{\uparrow}{4}, -1, 0, -1)$

Que →

$$x(n) = (\underset{\uparrow}{1}, -1) \longrightarrow \boxed{\text{LTI sys.}}_{h(n)} \longrightarrow y(n) = (\underset{\uparrow}{1}, 0, 0, 0, -1)$$

Find $h(n) = ?$

(a.) $(\underset{\uparrow}{1}, 0, 0, 1)$ ~~(b)~~ $\cancel{(\underset{\uparrow}{1}, 0, 1)}$

(c.) $(\underset{\uparrow}{1}, 1, 1, 1)$ (d) $(\underset{\uparrow}{1}, 1, 1)$

sol^n → $y(n) = x(n) * h(n)$

$x(n) = 2$

$y(n) = 5$

$5 = 2 + L_2 - 1$

$L_2 = 4$ {ans (c) or (a)}

Tabular method →

$h(n)$ \ $x(n)$	1	−1
1	1	−1
1	1	−1
1	1	−1
1	1	−1

Que → $y(n) = h(n) * g(n)$, $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$g(n)$ is causal sequence.

If $y(0) = 1$, $y(1) = 1/2$ then $g(1)$ is equal to

(a) 0 (b) 1/2 (c) 1 (d) 3/2

sol^n →

$g(n)$ \ $h(n)$	1	1/2	1/4	1/8 ....
1	1	1/2	1/4	1/8 ....
0	0	0	0	0 ....

ans. (a)

* Energy & power signal→

* Energy signal→

* $E = \text{Finite}$; $P = 0$
* This are absolutely summable signal i.e.

$$\boxed{\sum_{n=-\infty}^{\infty} |x(n)| < \infty} \qquad \boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$$

Que→ Calculate energy of signal:-

(i) $x(n) = \delta(n)$

Sol^n→ $x(n) = \delta(n)$

$$= (\cdots 0, \underset{\uparrow}{1}, 0 \cdots)$$

$$E = \sum |x(n)|^2 = 1$$

(ii) $x(n) = \left(\frac{1}{3}\right)^n u(n)$

Sol^n→ $x(n) = \left(\frac{1}{3}\right)^n u(n)$

$$E = \sum_{-\infty}^{\infty} |x(n)|^2 = \sum_{0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$= 1 + \frac{1}{9} + \left(\frac{1}{9}\right)^n + \cdots$$

$$= \frac{1}{1 - 1/9} = \frac{9}{8}$$

(iii) $x(n) = (\underset{\uparrow}{1+j}, 1-j, -2, 2)$

Sol^n→ $E = \sum |x(n)|^2$

$$= (\sqrt{2})^2 + (\sqrt{2})^2 + (2)^2 + (2)^2$$

$$= 12$$

Que→ calculate energy of $y(n)$

$$x(n) = (\underset{\uparrow}{1}, 2, 3, 4, 5)$$

(i) $y(n) = x(-n)$ (ii) $y(n) = x(n-1)$ (iii) $y(n) = x\left(\frac{n}{2}\right)$ (iv) $y(n) = -x(n)$ (v) $y(n) = x(3n)$

Sol^n→ (i) $y(n) = x(-n)$

$x(n) = E[x(n)] = 55$

$x(-n) = 55$

(ii) $y(n) = x(n-1)$

$E_{x(n-1)} = 55$

(iii) $y(n) = x\left(\frac{n}{2}\right)$

$= (\underset{\uparrow}{1}, 0, 2, 0, 3, 0, 4, 0, 5) \ldots$

$E_{x(n/2)} = 55.$

(iv) $y(n) = -x(n)$

$E_{y(n)} = 55$

(v) $y(n) = x(3n)$

$= (1, 4)$

$E_{y(n)} = 1^2 + 4^2 = 17$

Note:- Energy calculation is independant of time shifting, time reversal, amp. reversal & interpolation.

* Power signal →

(1) P = Finite, $E = \infty$

(2) $P = \begin{cases} \frac{1}{N}\sum_{n=N}|x(n)|^2 \text{; for periodic signal.} \\ \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x(n)|^2 \text{; For Non-periodic signal.} \end{cases}$

Que.→ Calculate power of signal.

(i) $x(n) = A_0 u(n)$

Sol^n→ $x(n) = A_0 u(n)$

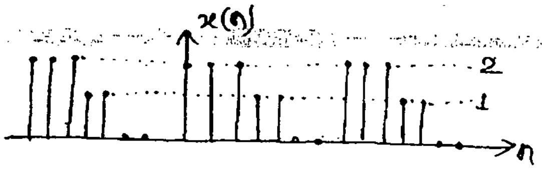
$$P = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x(n)|^2$$

$$= \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=0}^{N}A_0^2(N+1)$$

$$= \lim_{N\to\infty}\frac{A_0^2(N+1)}{2N+1}$$

$$\boxed{P = \frac{A_0^2}{2}}$$

(ii)



Sol^n→

$$P = \frac{1}{N}\sum|x(n)|^2$$

$$= \frac{2^2+2^2+2^2+1^2+1^2+0^2+0^2}{7}$$

$$= \frac{14}{7}$$

$$\boxed{P = 2}$$

Que→ Find even, odd, CS & CAS for sig. $x(n)$

$$x(n) = (-4-5j,\ \underset{\uparrow}{1+2j},\ 4)$$

Sol^n→ * Even part → $\frac{x(n)+x(-n)}{2}$

$x(-n) = (4\ \ \underset{\uparrow}{1+2j}\ \ -4-5j)$

$$= \left(\frac{-5j}{2}\ \ \underset{\uparrow}{1+2j}\ \ \frac{-5j}{2}\right)$$

* Odd part → $\frac{x(n)-x(-n)}{2}$

$$= \left(-4-\frac{5j}{2},\ 0,\ 4+\frac{5j}{2}\right)$$

* CAS part → $\frac{x(n)-x^*(-n)}{2}$

$x^*(-n) = (4,\ 1-2j,\ -4+5j)$

* Periodic Signal →

$$\boxed{x(n) = x(n \pm kN)}$$

where; k = an-integer

N = FTP = integer

$x(n) = x_1(n) + x_2(n)$

$\downarrow N_1$ $\quad$ $\downarrow N_2$

$\rightarrow \frac{N_1}{N_2} = \frac{\text{integer}}{\text{integer}}$ = Rational no.(always)

Note:- The sum of 2 (or) more 2 periodic signals in case of discrete time system will be always periodic.

Complex-exponantial → Complex exponantial & sinusoidal signals are always periodic in case of contineous time signal

eg:- $x(n) = A_0 e^{j\omega_0 n}$

Let N be the FTP of $x(n)$ i.e.

$x(n) = x(n+N)$

$A_0 e^{j\omega_0 n} = A_0 e^{j\omega_0 (n+N)}$

$e^{j\omega_0 N} = 1 = e^{j2\pi k}$ (k = an integer)

$\omega_0 N = 2\pi k$

$$\boxed{\frac{2\pi}{\omega_0} = \frac{N}{k} = \text{Rational no.}}$$

In case of discrete time sys; complex exponantials & sinusoidal sig. will be periodic only if ratio $2\pi/\omega_0$ is rational no.

$$\boxed{N = \frac{2\pi}{\omega_0} k}$$

k is a least int. for which N is an integer.

Que. → Calculate FTP of sig. if it is periodic

(i) $x(n) = e^{j2n}$.

$\frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi$ = irrational no.

(ii) $x(n) = \cos \frac{3\pi}{4} n$.

$\frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi/4} = \frac{8}{3}$ (R.no.)

(iii) $x(n) = \sin\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{5\pi}{4}n\right)$

Sol^n →

$$x(n) = \sin\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{5\pi}{4}n\right)$$

$N_1 = 8$ $\qquad$ N_2

$$N_2 = \frac{2\pi}{\omega_2}k_2 = \frac{2\pi}{5\pi/7}k_2 = \frac{14}{5}k_2 = 14$$

$$N = LCM(N_1, N_2)$$

$$= LCM(8, 14)$$

$$= 56.$$

$e^{j2t} \longrightarrow$ Periodic

$e^{j2n} \longrightarrow NP$

$\sin 4t \longrightarrow P$

$\sin 4n \longrightarrow NP$

$\underset{(P)}{\sin 2t} + \underset{(P)}{\cos 4t} \longrightarrow P$

$\underset{(NP)}{\sin 3n} + \underset{(NP)}{\cos 4n} \longrightarrow NP$