CBSE Test Paper 04

Chapter 1 Relations and Functions

1. The range of the function f(x) =$|x-1|$ is
   a. $(-\infty, \infty)$
   b. $[0, \infty)$
   c. $(-\infty, 0)$
   d. $(0, \infty)$
2. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Then R is
   a. An equivalence relation.
   b. Symmetric and transitive but not reflexive.
   c. Reflexive and symmetric but not transitive
   d. Reflexive and transitive but not symmetric
3. The identity element for the binary operation * defined on Q - {0} as a * b = $\frac{ab}{2}$ $\forall$ a, b $\in$ Q - {0} is
   a. 1
   b. 0
   c. 2
   d. $\frac{1}{2}$
4. A relation R on a non-empty set A is said to be an equivalence relation if
   a. R is reflexive and symmetric
   b. R is reflexive, symmetric and transitive
   c. R is symmetric and transitive
   d. R is reflexive and transitive
5. Let f : {1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3} be given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Then gof is
   a. {(1, 3), (3, 1), (4, 1)}
   b. {(1, 3), (3, 1), (4, 3)}
   c. {(1, 3), (3, 3), (4, 3)}
   d. {(1, 3), (3, 3), (4, 1)}
6. A relation R defined on a set A is said to be ________, if (x, y)$\in$ R and (y, z)$\in$ R$\Rightarrow$ (x, z)

$\in$ R, where x, y, z $\in$A.

7. The set of second elements of all ordered pairs in R, i.e. {y : (x, y) $\in$R} is called the ________ of relation R.
8. The inverse of a bijective function is also a ________ function.
9. Show that a one – one function f: {1, 2, 3} $\rightarrow$ {1, 2, 3} must be onto.
10. Prove that the function f: R $\rightarrow$ R, given by f(x) = 2x, is one – one.
11. Let f : R $\rightarrow$ R be the function defined by f(x) = 4x - 3 $\forall$ x $\in$ R. Then write f^{-1}
12. Define universal relation. Give example.
13. Let f : R $\rightarrow$ R be defined as f(x) = 10x + 7. Find the function g : R $\rightarrow$ R such that gof = fog = I_R.
14. Let T be the set of all triangles in a plane with R a relation in T given by R = {(T_1, T_2): T_1 is congruent to T_2}.

    Show that R is an equivalence relation.
15. Let f : X $\rightarrow$ Y be an invertible function. Show that the inverse of f^{-1} is f, i.e., $(f^{-1})^{-1}$ = f.
16. Let * be the binary operation on N given by a * b = L.C.M. of a and b, Find:
    i. 5 * 7, 20 * 16
    ii. Is * commutative?
    iii. Is * associative?
    iv. Find the identity of * in N.
17. Give an example of a relation, which is:
    i. Symmetric but neither reflexive nor transitive.
    ii. Transitive but neither reflexive nor symmetric.
    iii. Reflexive and symmetric but not transitive.
    iv. Reflexive and transitive but not symmetric.
18. Show that the relation R defined in the set A of all triangles as R = {$(T_1, T_2) : T_1$ is similar to T_2 }, is an equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5. T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

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Solution

1. b. $[0, \infty)$

   Explanation: We have, f(x) =$|x-1|$, which always gives non-negative values of f(x) for all x∈R. Therefore range of the given function is all non-negative real numbers i.e. $[0, \infty)$.

2. d. Reflexive and transitive but not symmetric

   Explanation: The relation R is not symmetric, (1,2) ∈ R , but (2,1) ∉ R , (1,3) ∈R ,but (3,1) ∉ R , (3,2) ∈ R, but (2,3) ∉ R.

3. c. 2

   Explanation: If the identity element is e, then, a*e = a = e * a

   $= \frac{ae}{2} \Rightarrow a = \frac{ea}{2} \Rightarrow e = 2.$

4. b. R is reflexive, symmetric and transitive

   Explanation: A relation R on a non empty set A is said to be reflexive if xRx for all x ∈ A .

   A relation R on a non empty set A is said to be symmetric if xRy ⇔ yRx, for all x, y ∈A.

   A relation R on a non empty set A is said to be transitive if xRy and yRz ⇒ x Rz, for all x,y,z ∈ A.

   An equivalence relation satisfies all these three properties.

5. b. {(1, 3), (3, 1), (4, 3)}

   Explanation: (gof)(1)=g(f(1))=g(2)=3, (gof)(3)=g(f(3))=g(5)=1, (gof)(4)=g(f(4))=g(1)=3,

6. transitive
7. range
8. bijective
9. Since, f is one - one three elements of {1, 2, 3} must be taken to 3 different elements of the co - domain {1, 2, 3} under f. Hence, f has to be onto.
10. x_1 ,x_2 are two different elements of R

Let $f(x_1) = f(x_2)$

$2x_1 = 2x_2$

$x_1 = x_2$, hence f is one-one.

11. Given that f(x) = 4x - 3 = y (say), then 4x = y + 3

$\Rightarrow x = \frac{y+3}{4}$

Hence $f^{-1}(y) = \frac{y+3}{4} \Rightarrow f^{-1}(x) = \frac{x+3}{4}$

12. A Relation R in a set A called universal relation if each element of A is related to every element of A.

Eg. Let A = {2,3,4}

R = (A $\times$ A) = {(2,2),(2,3) (2,4) (3,2) (3,3) (3,4) (4,2) (4,3) (4,4)}

13. Given: f(x) = 10x + 7

Now gof = g[f(x)] = g(10x+7) fog = f[g(x)] = 10g(x) + 7

$\Rightarrow$ 10g(x) + 7 = I_R(x) = x

$\Rightarrow g(x) = \frac{x-7}{10}$

14. R is reflextive, since every triangle is congruent to itself.

Now

$(T_1, T_2) \in R$ means T_1 is congruent to T_2. So we can say that T_2 is also congruent to T_1.

$\Rightarrow T_2 \cong T_1$

Therefore $(T_2, T_1) \in R$

So R is symmetric.

Now

$(T_1, T_2) \in R$ and $(T_2, T_3) \in R$

That means T_1 is congruent to T_2 and T_2 is congruent to T_3

so we can say that T_1 is congruent to T_3

$\Rightarrow (T_1, T_3) \in R$ Since three triangles are congruent to each other.

Therefore, R is transitive

Now since R is reflexive, symmetric and transitive

Therefore, R is an equivalence relation

15. Let f: X $\rightarrow$ Y be an invertible function.

Then f is one-one and onto

$\Rightarrow$ g : y $\rightarrow$ X where g is also one-one and onto such that

$gof(x) = I_x$ and $fog(y) = I_y$

$\Rightarrow g = f^{-1}$

Now, $f^{-1}o(f^{-1})^{-1} = I$ and $fo[f^{-1}o(f^{-1})^{-1}] = foI$

$\Rightarrow [fof^{-1}]o(f^{-1})^{-1} = f$

$\Rightarrow Io(f-1)^{-1} = f$

$\Rightarrow (f^{-1})^{-1} = f$

16. a * b = L.C.M. of a and b

i. 5 * 7 = L.C.M. of 5 and 7 = 35
   20 * 16 = L.C.M. of 20 and 16 = 80
ii. a * b = L.C.M. of a and b = L.C.M. of b and a = b*a
   Therefore, operation * is commutative.
iii. a*(b*c) = a*(L.C.M. of b and c) = L.C.M. of (a and L.C.M. of b and c)
   = L.C.M. of a, b and c
   Similarly, (a * b)*c = L.C.M. of a, b and c
   Thus, a*(b*c) = (a*b)*c
   Therefore, the operation is associative.
iv. Identity of * in N = 1 because a*1 = L.C.M. of a and 1 = a

17. i. The relation "is perpendicular to". l_1 is not perpendicular to l_1, so R is not reflexive. If $l_1 \perp l_2$ then $l_2 \perp l_1$, however if $l_1 \perp l_2$ and $l_2 \perp l_3$ then l_1 is not perpendicular to l_3.
ii. So it is clear that R "is perpendicular to" is a symmetric but neither reflexive nor transitive.
iii. Relation R = {(x, y) : x > y}
   We know that x > x is false. Also x > y but y > x is false and if x > y, y > z this implies x > z.
   Therefore, R is transitive, but neither reflexive nor symmetric.
iv. "is friend of" R = {(x, y) : x is a friend of y}
   It is clear that x is friend of x,
   $\therefore$ R is reflexive.
   Also x is friend of y and y is friend of x.

$\therefore$ R is symmetric.

Also if x is friend of y and y is friend of z then x cannot be friend of z

$\therefore$ R is not transitive.

Therefore, R is reflexive and symmetric but not transitive.

v. "is greater or equal to" R = {(x, y) : x $\geqslant$ y}

It is clear that x $\geqslant$ x

$\therefore$ R is reflexive.

And x $\geqslant$ y does not imply y $\geqslant$ x

$\therefore$ R is not symmetric.

But x $\geqslant$ y, y $\geqslant$ z $\Rightarrow$ x $\geqslant$ z $\therefore$ R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

18. i. Each triangle is similar to itself thus $(T_1, T_1) \in R$

$\therefore$ R is reflexive.

ii. $(T_1, T_2) \in R$

$\Rightarrow T_1$ is similar to T_2

$\Rightarrow T_2$ is similar to T_1

$(T_2, T_1) \in R$

R is symmetric

iii. T_1 is similar to T_2 and T_2 is similar to T_3

$\Rightarrow T_1$ is similar to T_3

$\Rightarrow (T_1, T_3) \in R$

$\therefore$ R is transitive.

Hence R is the equivalence

(II) part T_1 = 3, 4, 5

T_2 = 5, 12, 13

T_3 = 6, 8, 10

$\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$ T_1 is relative to T_3.

as T_1 IS SIMILAR TO T_3