CBSE SAMPLE PAPER - 01

Class 11 - Mathematics

Time Allowed: 3 hours   **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. When a clock shows the time 7 : 20, what is the angle between its minute hand and the hour hand?   **[1]**

   a) 120°   b) 100°

   c) 60°   d) 80°

2. Quartile deviation is nearly equal to   **[1]**

   a) $\frac{2}{5}\sigma$ b) $\frac{2}{3}\sigma$

   c) 2σ d) $\frac{3}{2}\sigma$

3. A fair coin is tossed a fixed number of times. If the probability of getting 4 heads is equal to the probability of getting 7 heads, then the probability of getting 2 heads is   **[1]**

   a) $\frac{1}{1024}$ b) $\frac{55}{2048}$

   c) $\frac{3}{4096}$ d) $\frac{3}{1024}$

4. If f (x) = x sin $\left(\frac{1}{x}\right), x \neq 0$, then $\lim\limits_{x \to 0}$ f(x) =   **[1]**

   a) None of these   b) 1

   c) 0   d) -1

5. The equation of the line passing through the point (1, 2) and perpendicular to the line x + y + 1 = 0 is   **[1]**

   a) y – x + 2 = 0   b) y – x + 1 = 0

   c) y – x – 2 = 0   d) y – x – 1 = 0

6. If Q = {x : x = $\frac{1}{y}$, where y ∈ N}, then   **[1]**

   a) $\frac{2}{3}$ ∈ Q   b) 2 ∈ Q

c) $0 \in Q$ d) $1 \in Q$

7. Which of following statements is correct? [1]

     a) (2+ 3i) > (2 - 3i)   b) (5 + 4i) > (-5 - 4i)

     c) (3 + 2i) > (-3 + 2i)   d) None of these

8. If A = {(x, y) : $x^2 + y^2 = 5$} and B = {(x, y) : 2x = 5y}, then A ∩ B contains [1]

     a) two points   b) one-point

     c) infinite points   d) no point

9. If x belongs to set of integers, A is the solution set of 2(x - 1) < 3x - 1 and B is the solution set of 4x - 3 ≤ 8 + x, [1] find A ∩ B

     a) {0, 2, 4}   b) {1, 2, 3}

     c) {0, 1, 2}   d) {0, 1, 2, 3}

10. If $\tan^2 2^o + \tan^2 4^o + \ldots + \tan^2 88^o = a$, then the value of $\sum_{\theta=1^\circ}^{89^\circ} \frac{\sin^4 \theta + \cos^4 \theta}{\sin^2 \theta \cos^2 \theta}$ is : [1]

     a) 2 (89 - 4a)   b) 2 (89 - 2a)

     c) 2 (89 + 4a)   d) 2 (89 + 2a)

11. If the four-letter words (need not be meaningful) are to be formed using the letters from the word MEDITERRANEAN such that the first letter is R and the fourth letter is E, then the total number of all such words is: [1]

     a) $\frac{11!}{(2!)^3}$ b) 59

     c) 110   d) 56

12. If three distinct numbers a, b and c are in GP and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct? [1]

     a) $\frac{d}{a}, \frac{e}{b}$ and $\frac{f}{c}$ are in AP   b) d, e and f are in GP

     c) d, e and f are in AP   d) $\frac{d}{a}, \frac{e}{b}$ and $\frac{f}{c}$ are in GP

13. If the number of integral terms in the expansion of $(3^{\frac{1}{2}} + 5^{\frac{1}{8}})^n$ is exactly 33, then the least value of n is: [1]

     a) 248   b) 128

     c) 256   d) 264

14. If -3x + 17 < -3, then [1]

     a) $x \in (-\infty, 10]$ b) $x \in [10, \infty)$

     c) none of these   d) $x \in (10, \infty)$

15. Let A and B be two sets. Then [1]

     a) $A \cap B \subseteq A \cup B$ b) $A \cup B \subseteq A \cap B$

     c) None of these   d) A ∩ B = A ∪ B

16. The value of cos (36°-A) cos(36° + A) + cos (54° + A) cos(54° - A) is [1]

     a) sin 3A   b) cos 2A

c) sin 2A

d) cos 3A

17. If $z = \frac{(1-i\sqrt{3})}{2(1-i)}$ then $|z| = ?$ [1]

     a) None of these

     b) $\frac{1}{2\sqrt{2}}$

     c) $\frac{1}{\sqrt{2}}$

     d) $\frac{1}{\sqrt{3}}$

18. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such   [1]
arrangements are possible?

     a) 720

     b) 24

     c) 120

     d) 2880

19. **Assertion(A):** In the expansion $(x + x^{-2})^n$ the coefficient of eighth term and nineteenth term are equal, then n =   [1]
25.

     Reason (R): Middle term in the expansion of $(x + a)^n$ has the greatest binomial coefficient.

     a) Both A and R are true and R is the correct explanation of A.

     b) Both A and R are true but R is not the correct explanation of A.

     c) A is true but R is false.

     d) A is false but R is true.

20. **Assertion (A):** If f : R $\rightarrow$ R be defined by f (x) = $\frac{1}{x}$ $\forall x \in R$. Then f is not defined.   [1]

     Reason (R): This function does not exist for every value of the domain.

     a) Both A and R are true and R is the correct explanation of A.

     b) Both A and R are true but R is not the correct explanation of A.

     c) A is true but R is false.

     d) A is false but R is true.

Section B

21. Find the domain and the range of the real function: $f(x) = \frac{1}{\sqrt{x^2-1}}$ [2]

22. Evaluate: $\lim\limits_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$.   [2]

23. Determine whether $x^2 + y^2 + 2x + 10y + 26 = 0$ represent a circle or point.   [2]

OR

Find the eccentricity, coordinates of foci, length of the latus-rectum of the ellipse: $5x^2 + 4y^2 = 1$.

24. There are 210 members in a club. 100 of them drink tea and 65 drink tea but not coffee, each member drinks tea   [2]
or coffee. Find how many drink coffee? How many drink coffee but not tea?

25. Find the equations of the lines which pass through the point (3, -2) and are inclined at 60° to the line   [2]
$\sqrt{3}x + y = 1$.

Section C

26. For any sets A, B and C, prove that: $A \times (B \cap C) = (A \times B) \cap (A \times C)$ [3]

27. Using distance formula prove that the points are collinear: A (4, -3, -1), B (5, -7, 6) and C (3, 1, -8).   [3]

OR

Find the distance between the point (-1, -5, -10) and the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the
plane x - y + z = 5.

28. Using binomial theorem, expand: $\left(x^2 - \frac{2}{x}\right)^7$.   [3]

OR

Show that $2^{4n+4} - 15n - 16$ where $n \in \mathbf{N}$ is divisible by 225

29. If $(a + ib) = \frac{c+i}{c-i}$, where c is real, prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2-1}$ .. [3]

<div align="center">OR</div>

Find the square root of 1 - i.

30. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23. [3]

31. If $^nC_4, ^nC_5$ and $^nC_6$ are in A.P., then find n. [3]

<div align="center">Section D</div>

32. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that all the three balls are red. [5]

33. Find the derivative of (sinx + cosx) from first principle. [5]

<div align="center">OR</div>

Differentiate log sin x from first principles.

34. Prove that cos 12° + cos 60° + cos 84° = cos 24° + cos 48° [5]

<div align="center">OR</div>

Find the value of $\cos^2 76° + \cos^2 16° - \cos 76° \cos 16°$.

35. Find the mean deviation about the mean for the data: [5]

Height (in cm)	95 - 105	105 - 115	115 - 125	125 - 135	135 - 145	145 - 155
Number of boys	9	16	23	30	12	10

<div align="center">Section E</div>

36. **Read the text carefully and answer the questions:** [4]

A farmer wishes to install 2 handpumps in his field for watering.

The farmer moves in the field while watering in such a way that the sum of distances between the farmer and each handpump is always 26m. Also, the distance between the hand pumps is 10 m.
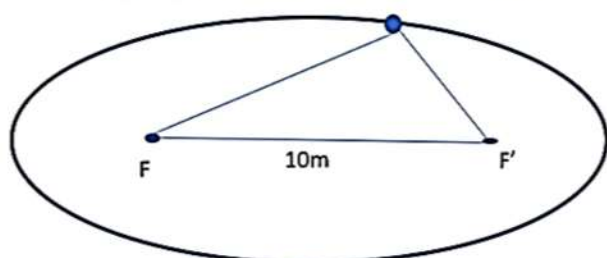
    (i)   Name the curve traced by farmer and hence find the foci of curve.

    (ii)   Find the equation of curve traced by farmer.

    (iii)   Find the length of major axis, minor axis and eccentricity of curve along which farmer moves.

<div align="center">**OR**</div>

    Find the length of latus rectum.

37. **Read the text carefully and answer the questions:** [4]

A sequence whose terms increases or decreases by a fixed number, is called an Arithmetic Progression (AP).

In other words, we can say that, a sequence is called an arithmetic progression if the difference of a term and the previous term is always same i.e. $a_{n+1} - a_n$ = constant for all n.

This constant or same difference is called the common difference of an AP and it is denoted by d.

In an AP, we usually denote the first term by a, common difference by d and the nth term by a_n or T_n defined as

$T_n = a_n = a + (n - 1)d$

Also, $l = a + (n - 1)d$, where l is the last term of the sequence.

The sum of n terms, S_n of this AP is given by $S_n = \frac{n}{2}[2a + (n - 1)d]$.

Also, if l be the last term, then the sum of n terms of this AP is $S_n = \frac{n}{2}(a + l)$.

(i)   Let S_n denote the sum of the first n terms of an AP, if $S_{2n} = 3S_n$, then $S_{3n} : S_n$ is equal to

        a) 10   b) 4

        c) 6   d) 8

(ii)   If 9 times the 9th term of an AP is equal to 13 times the 13th term, then the 22nd term of the AP is

        a) 220   b) 22

        c) 198   d) 0

(iii)   If the sum of n terms of an AP is given by $S_n = 3n + 2n^2$, then the common difference of the AP is

        a) 4   b) 2

        c) 3   d) 6

OR

11th term of an AP 11, 18, 25, ... is equal to

        a) 71   b) 80

        c) 81   d) 70

38.   **Read the text carefully and answer the questions:**   [4]

A survey is conducted by a career counsellor in a college to find career choice of students after the Intermediate. There are 100 students that goes for Engineering Courses, 50 wants to make their career in Medical, 100 students continue their further study in Arts. There are 10 students that go for both Engineering and Medical,

and 3 goes for Medical and Arts. There are 3 students that do not go for any further studies.

(i)   Find the number of students that goes for Engineering or Art.

(ii)   Find the number of students that goes for both Medical and Art.

Section A

1. **(b)** 100°

   Explanation: Angle traced by the hour hand in 12 hours = 360°.

   Angle traced by the hour hand in $\frac{22}{3}$ hours $= \left(\frac{360}{12} \times \frac{22}{3}\right)^{\circ} = 220°$.

   Angle traced by the minute hand in 60 min = 360°.

   Angle traced by the minute hand in 20 min $= \left(\frac{360}{60} \times 20\right)^{\circ} = 120°$.

   Angle between the two hands = (220° - 120°) = 100°

2. **(b)** $\frac{2}{3}\sigma$

   Explanation: Quartile deviation $\frac{Q_3 - Q_1}{2}$ is approximately $\frac{2}{3}$ times the standard deviation.

3. **(b)** $\frac{55}{2048}$

   Explanation: Let a coin be tossed n number of times and X be a random variable denoting the no. of heads then, X follows the binomial distribution.

   Now, P(X = 4) = P(X = 7)

   $\Rightarrow {}^nC_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{n-4} = {}^nC_7\left(\frac{1}{2}\right)^{n-7}\left(\frac{1}{2}\right)^7$

   $\Rightarrow {}^nC_4 = {}^nC_7 = {}^nC_{7-4}$

   $\Rightarrow 4 = 7 - n \Rightarrow n = 11$

   $\therefore$ P(X = 2) $= {}^{11}C_2\left(\frac{1}{2}\right)^{11-2} \cdot \left(\frac{1}{2}\right)^2$

   $= {}^{11}C_2\left(\frac{1}{2}\right)^{11} = \frac{11}{2} \times \frac{10}{1} \times \frac{1}{2^{11}}$

   $= \frac{55}{2^{11}}$

4. **(c)** 0

   Explanation: $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} x \sin\frac{1}{x}, x \neq 0$

   LHL at x = 0

   $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0 - h)$

   $\Rightarrow \lim\limits_{h \to 0} -h \sin\left(\frac{1}{-h}\right)$

   = 0

   RHL at x = 0

   $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0 + h)$

   $\Rightarrow \lim\limits_{h \to 0} h \sin\left(\frac{1}{h}\right)$

   = 0

   $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x)$

   Hence, $\lim\limits_{x \to 0} f(x)^2$ exists and is equal to 0.

5. **(d)** $y - x - 1 = 0$

   Explanation: Here it is given that the lineline passing through the point (1, 2)

   and perpendicular to the line x + y + 1 = 0

   Let the equation of line 'L' is

   x – y + k = 0 …(i)

   Since, L is passing through the point (1, 2)

   $\therefore$ 1 – 2 + k = 0

   $\Rightarrow$ k = 1

   Substituting, the value of k in eq. (i), we obtain

   x – y + 1 = 0

   or y – x – 1 = 0

6. **(d)** $1 \in Q$

    Explanation: $1 \in Q$

7. **(d)** None of these

    Explanation: Two complex numbers cannot be compared

8. **(a)** two points

    Explanation: From A, $x^2 + y^2 = 5$ and from B, $2x = 5y$

    Now, $2x = 5y \Rightarrow x = \frac{5}{2y}$

    $\therefore x^2 + y^2 = 5 \Rightarrow \left(\frac{5}{2y}\right)^2 + y^2 = 5$

    $\Rightarrow 29y^2 = 20 \Rightarrow y = \pm\sqrt{\frac{20}{29}}$

    $\Rightarrow 29y^2 = 20 \Rightarrow y = \pm\sqrt{\frac{20}{29}}$

    $\therefore x = \frac{5}{2}\left(\pm\sqrt{\frac{20}{29}}\right)$

    $\therefore$ Possible ordered pairs = four

    But two ordered pair in which c is positive and y is negative will be rejected as it will not be satisifed by the equation in B. Therefore,

    $A \cap B$ contains 2 elements.

9. **(d)** $\{0, 1, 2, 3\}$

    Explanation: Given $2(x - 1) < 3x - 1$

    $\Rightarrow 2x - 2 < 3x - 1$

    $\Rightarrow 2x - 2 + 2 < 3x - 1 + 2$

    $\Rightarrow 2x < 3x + 1$

    $\Rightarrow 2x - 3x < 3x + 1 - 3x$

    $\Rightarrow -x < +1$

    $\Rightarrow x > -1$ but $x \in Z$

    Hence $A = \{0, 1, 2, 3, 4, ...\}$

    Now $4x - 3 \leq 8 + x$

    $\Rightarrow 4x - 3 + 3 \leq 8 + x + 3$

    $\Rightarrow 4x \leq 11 + x$

    $\Rightarrow 4x - x \leq 11 + x - x$

    $\Rightarrow 3x \leq 11$

    $\Rightarrow \frac{3x}{3} \leq \frac{11}{3}$

    $\Rightarrow x \leq \frac{11}{3}$

    $\Rightarrow x \leq 3\frac{2}{3}$, but $x \in Z$

    Therefore $B = \{....., -2, -1, 0, 1, 2, 3\}$

    Hence $A \cap B = \{0, 1, 2, 3\}$

10. **(c)** $2(89 + 4a)$

    Explanation: $2(89 + 4a)$

11. **(b)** 59

    Explanation: M, EEE, D.I, T, RR, AA, NN

    R - - E

    Two empty places can be filled with identical letters

    [EE, AA, NN] $\Rightarrow$ 3 way

    Two empty places can be filled with distinct letters [M, E, D, I, T, R, A, N] $\Rightarrow {}^8P_2$

    $\therefore$ Number of words $3 + {}^8P_2 = 59$

12. **(a)** $\frac{d}{a}, \frac{e}{b}$ and $\frac{f}{c}$ are in AP

    Explanation: Given, three distinct numbers a, b and c are in GP.

    $\therefore b^2 = ac$ ....(i)

    and the given quadratic equations

    $ax^2 + 2bx + c = 0$ ...(ii)

$dx^2 + 2ex + f = 0$ .....(iii)

For quadratic Eq. (ii),

the discriminant $D = (2b)^2 - 4ac$

$= 4(b^2 - ac) = 0$ [From Eq. (i)]

$\Rightarrow$ Quadratic Eq. (ii) have equal roots, and it is equal to $x = -\frac{b}{a}$ and it is given that quadratic Eqs. (ii) and (iii) have a common root, so

$d\left(-\frac{b}{a}\right)^2 + 2e\left(-\frac{b}{a}\right) + f = 0$

$\Rightarrow db^2 - 2eba + a^2f - 0$

$\Rightarrow d(ac) - 2eab + a^2f = 0$ [$\because b^2 = ac$]

$\Rightarrow dc - 2eb + af = 0$ [$\because a \neq 0$]

$\Rightarrow 2eb = dc + af$

$\Rightarrow 2\frac{e}{b} = \frac{dc}{b^2} + \frac{af}{b^2}$ [dividing each term by b]

$\Rightarrow 2\left(\frac{e}{b}\right) = \frac{d}{a} + \frac{f}{c}$ [$\because b^2 = ac$]

So, $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in AP.

13. **(c)** 256

Explanation: Here, $(3^{\frac{1}{2}} + 5^{\frac{1}{8}})^n$

$T_{r+1} = {}^nC_r(3)^{\frac{n-r}{2}}(5)^{\frac{r}{8}}$

$\because \frac{n-r}{2}$ and $\frac{r}{8}$ are integer

So, r must be 0, 8, 16, 24 ...

Now $n = t_{33} = a + (n - 1)d = 0 + 32 \times 8 = 256$

$\Rightarrow n = 256$

14. **(d)** $x \in (10, \infty)$

Explanation: $-3x + 17 < -13$

$\Rightarrow -3x + 17 - 17 < -13 - 17$

$\Rightarrow -3x < -30$

$\Rightarrow \frac{-3x}{-3} > \frac{-30}{-3}$

$\Rightarrow x > 10$

$\Rightarrow x \in (10, \infty)$

15. **(a)** $A \cap B \subseteq A \cup B$

Explanation: $A \cap B \subseteq A \subseteq A \cup B$, $A \cap B \subseteq A \cup B$

16. **(b)** cos 2A

Explanation: $\cos(36^o - A) \cos(36^o + A) + \cos(54^o + A) \cos(54^o - A)$

$= \cos(36^o - A) \cos(36^o + A) + \sin[90^o - (54^o + A)] \sin[90^o - (54^o - A)]$ [Since $\sin(90° - \theta) = \cos\theta$]

$= \cos(36^o - A) \cos(36^o + A) + \sin(36^o - A) \sin(36^o + A)$

$= \cos(36^o + A - 36^o + A)$ [Using cos(A - B) formula]

$= \cos 2A$

17. **(c)** $\frac{1}{\sqrt{2}}$

Explanation: $|z|^2 = \frac{|1 - i\sqrt{3}|^2}{|(2 - 2i)|^2} = \frac{\left(1^2 + (\sqrt{3})^2\right)}{|2^2 + 2^2|} = \frac{(1+3)}{(4+4)} = \frac{4}{8} = \frac{1}{2}$

$\Rightarrow |z| = \frac{1}{\sqrt{2}}$

18. **(d)** 2880

Explanation: In a row of 9 seats, the 2nd, 4th, 6th and 8th are the even places.

These 4 places can be occupied by 4 women in ${}^4P_4$ ways = 24 ways

Remaining 5 places can be occupied by 5 men in ${}^5P_5$ ways = 120 ways.

$\therefore$ total number of seating arrangements $= (24 \times 120) = 2880$

19. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: It is given that coefficients of $T_8 = T_{19}$

$T_{r+1} = {}^nC_r a^{n-r} b^r$

Applying to the above question we get

${}^nC_7 = {}^nC_{18}$

Now ${}^nC_r = {}^nC_{n-r}$

Using the formula we get

n - 7 = 18

n = 25

It is also true that the middle term has the greatest coefficient in the expansion of x + a)^n since in pascal's triangle the middle term has the largest value.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both assertion and reason are true because in the given function at x = 0, $f(x) = \frac{1}{0} = \infty$. So, the function is not define

Section B

21. Here we have, $f(x) = \frac{1}{\sqrt{x^2-1}}$

we need to find where the function is defined

The condition for the function to be defined

$x^2 - 1 > 0$

$\Rightarrow x^2 > 1$

$\Rightarrow x > 1$

So, the domain of the function is the set of all the real numbers greater than 1

The domain of the function, $D_{\{f(x)\}} = (1, \infty)$

Now put any value of x within the domain set we get the value of the function always a fraction whose denominator is not equalled to 0

The range of the function, $R_{f(x)} = (0, 1)$.

22. When x = 1, the expression $\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$ takes the indeterminate form $\frac{0}{0}$.

Rationalising the denominator, we get,

$\lim\limits_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} = \lim\limits_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)(\sqrt{x}+1)}{(\sqrt{x}+1)(2x+3)(x-1)}$ (form $\frac{0}{0}$)

$= \lim\limits_{x \to 1} \frac{(2x-3)(x-1)}{(\sqrt{x}+1)(2x+3)(x-1)}$ (form $\frac{0}{0}$)

$= \lim\limits_{x \to 1} \frac{2x-3}{(\sqrt{x}+1)(2x+3)} = -\frac{1}{10}$

23. We have, $x^2 + y^2 + 2x + 10y + 26 = 0$

On adding 1 and 25 both sides to make perfect squares, we get

$(x^2 + 2x + 1) + (y^2 + 10y + 25) = -26 + 1 + 25$

$\Rightarrow (x + 1)^2 + (y + 5)^2 =$

$\Rightarrow [x - (-1)]^2 + [y - (-5)]^2 = 0^2$

Hence, it represents a point circle, because it has zero radius.

OR

Given that, $5x^2 + 4y^2 = 1$

$\Rightarrow \frac{x^2}{\frac{1}{5}} + \frac{y^2}{\frac{1}{4}} = 1$

Which is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

where $a^2 = \frac{1}{5}$ and $b^2 = \frac{1}{4}$, i.e. $a = \frac{1}{\sqrt{5}}$ and $b = \frac{1}{2}$

Clearly b > a

Now, $e = \sqrt{1 - \frac{a^2}{b^2}}$

$$\Rightarrow e = \sqrt{1 - \frac{\frac{1}{5}}{\frac{1}{4}}}$$

$$\Rightarrow e = \sqrt{1 - \frac{4}{5}}$$

$$\Rightarrow e = \frac{1}{\sqrt{5}}$$

Coordinates of the foci $= (0, \pm be) = \left(0, \pm \frac{1}{2\sqrt{5}}\right)$

Length of the latus rectum $= \frac{2a^2}{b}$

$$= \frac{2 \times \frac{1}{5}}{\frac{1}{2}}$$

$$= \frac{4}{5}$$

24. n (T) = 100

n (T - C) = 65

n (T ∪ C) = 210

n (T - C) = n (T) - n (T ∩ C)

65 = 100 - n (T ∩ C)

n (T ∩ C) = 35

n (T ∪ C) = n (T) + n (C) - n (T ∩ C)

210 = 100 + n (C) - 35

n (C) = 145.

Now,

n(C - T) = n(C) - n(C ∩ T)

n(C - T) = 145 - 35

n(C - T) = 110

25. Equation of a line having slopes m and tanα is

$$y - y_1 = \left(\frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}\right)(x - x_1)$$

$$\Rightarrow \quad y + 2 = \left(\frac{-\sqrt{3} \pm \tan 60°}{1 \mp (-\sqrt{3})\tan 60°}\right)(x - 3) \quad \text{[put m} = \sqrt{3} \text{ and } \alpha = 60°, \text{ given]}$$

$$\Rightarrow \quad y + 2 = \frac{-\sqrt{3} \pm \sqrt{3}}{1 \mp (-3)}(x - 3)$$

$$\Rightarrow \quad y + 2 = \frac{-\sqrt{3} + \sqrt{3}}{1 - (-3)}(x - 3)$$

and $y + 2 = \frac{-\sqrt{3} - \sqrt{3}}{1 + (-3)}(x - 3)$

$\Rightarrow$ y + 2 = 0

and $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

Section C

26. Here we are given that A, B and C three sets.

To prove: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let us consider, $(x, y) \in A \times (B \cap C)$

$\Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow (x \in A$ and $y \in B)$ $(x \in A$ and $y \in C)$

$\Rightarrow (x, y) \in (A \times B)$ and $(x, y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

From above, we can say that,

$\Rightarrow A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ ....(i)

Let us consider again, $(a, b) \in (A \times B) \cap (A \times C)$

$\Rightarrow (a, b) \in (A \times B)$ and $(a, b) \in (A \times C)$

$\Rightarrow (a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C)$

$\Rightarrow a \in A$ and $(b \in B$ and $b \in C)$

$\Rightarrow a \in A$ and $b \in (B \cap C)$

$\Rightarrow (a, b) \in A \times (B \cap C)$

From above, we can say that,

$\Rightarrow (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ ....(ii)

From (i) and (ii).

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence proved.

27. $AB = \sqrt{(5-4)^2 + (-7+3)^2 + (6+1)^2}$

$= \sqrt{(1)^2 + (-4)^2 + (7)^2}$

$= \sqrt{1 + 16 + 49}$

$= \sqrt{66}$

$BC = \sqrt{(3-5)^2 + (1+7)^2 + (-8-6)^2}$

$= \sqrt{(-2)^2 + (8)^2 + (-14)^2}$

$= \sqrt{4 + 64 + 196}$

$= \sqrt{264}$

$= 2\sqrt{66}$

$AC = \sqrt{(3-4)^2 + (1+3)^2 + (-8+1)^2}$

$= \sqrt{(-1)^2 + (4)^2 + (-7)^2}$

$= \sqrt{1 + 16 + 49}$

$= \sqrt{66}$

Here, AB + AC = $\sqrt{66} + \sqrt{66} = 2\sqrt{66}$ =BC

AB + AC= BC

Hence, the points are collinear.

<div align="center">OR</div>

According to the question, $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ ....(i)

and $x - y + z = 5$ ....(ii)

$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda(\text{say})$

$\Rightarrow x = 3\lambda + 2, \ y = 4\lambda - 1, \ z = 12\lambda + 2$

Point on the line is

$P(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$....(iii)

P lies on the plane, so point P satisfy the plane.

Substitute (iii) in (ii) , we get

$\therefore (3\lambda + 2) - (4\lambda - 1) + (12\lambda + 2) = 5$

$\Rightarrow 3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 = 5$

$\Rightarrow 11\lambda = 0 \Rightarrow \lambda = 0$

Put $\lambda = 0$ in Eq.(iii), we get point of intersection p (2,-1,2).

Distance between points $(-1, -5, -10) \ and \ (2, -1, 2)$

$= \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$

$= \sqrt{9 + 16 + 144} = \sqrt{169}$

=13 units.

28. To find: Expansion of $\left(x^2 - \frac{3x}{7} \right)^7$

Formula used: $^n C_r = \frac{n!}{(n-r)!(r)!}$

We know that ,$(a + b)^n = \ ^nC_0 a^n + \ ^nC_1 a^{n-1}b + \ ^nC_2 a^{n-2}b^2 + \ldots + \ ^nC_{n-1}ab^{n-1} + \ ^nC_n b^n$

Here We have, $(x^2 - \frac{3x}{7})^7$

$\Rightarrow \left[^7C_0 (x^2)^{7-0} \right] + \left[7C_1 (x^2)^{7-1} \left(-\frac{3x}{7}\right)^1 \right] + \left[7c_2 (x^2)^{7-2} \left(-\frac{3x}{7}\right)^2 \right] + \left[7C_3 (x^2)^{7-3} \left(-\frac{3x}{7}\right)^3 \right] + \left[7C_4 (x^2)^{7-4} \left(-\frac{3x}{7}\right)^4 \right]$

$+ \left[7C_5 (x^2)^{7-5} \left(-\frac{3x}{7}\right)^5 \right] + \left[7C_6 (x^2)^{7-6} \left(-\frac{3x}{7}\right)^6 \right] + \left[7C_7 \left(-\frac{3x}{7}\right)^7 \right]$

$\Rightarrow \left[\frac{7!}{0!(7-0)!} (x^2)^7 \right] - \left[\frac{7!}{1!(7-1)!} (x^2)^6 \left(\frac{3x}{7}\right) \right] + \left[\frac{7!}{2!(7-2)!} (x^2)^5 \left(\frac{9x^2}{49}\right) \right] - \left[\frac{7!}{3!(7-3)!} (x^2)^4 \left(\frac{27x^3}{343}\right) \right]$

$+ \left[\frac{7!}{4!(7-4)!} (x^2)^3 \left(\frac{81x^4}{2401}\right) \right] - \left[\frac{7!}{5!(7-5)!} (x^2)^2 \left(\frac{243x^5}{16807}\right) \right] + \left[\frac{7!}{6!(7-6)!} (x^2)^1 \left(\frac{729x^6}{117649}\right) \right] - \left[\frac{7!}{7!(7-7)!} \left(\frac{2187x^7}{823543}\right) \right]$

$- \left[\frac{7!}{7!(7-7)!} \left(\frac{2187x^7}{823543}\right) \right] + \left[21 (x^{10}) \left(\frac{9x^2}{49}\right) \right] - \left[35 (x^8) \left(\frac{27x^3}{343}\right) \right]$

$+ \left[35 (x^6) \left(\frac{81x^4}{2401}\right) \right] - \left[21 (x^4) \left(\frac{243x^5}{16807}\right) \right] + \left[7 (x^2) \left(\frac{729x^6}{117649}\right) \right] - \left[1 \left(\frac{2187x^7}{823543}\right) \right]$

$\Rightarrow x^{24} - 3x^{13} + \left(\frac{27}{7}\right) x^{12} - \left(\frac{135}{49}\right) x^{11} + \left(\frac{405}{343}\right) x^{10} - \left(\frac{729}{2401}\right) x^9 + \left(\frac{729}{16807}\right) x^8 - \left(\frac{2187}{823543}\right) x^7$

$x^{14} - 3x^{13} + \left(\frac{27}{7}\right) x^{12} - \left(\frac{135}{49}\right) x^{11} + \left(\frac{405}{343}\right) x^{10} - \left(\frac{729}{2401}\right) x^9 + \left(\frac{729}{16807}\right) x^8 - \left(\frac{2187}{823543}\right) x^7$

<div align="center">OR</div>

From the given equation we have $2^{4n+4} - 15n - 16 = 2^{4(n+1)} - 15n - 16$

$= 16^{n+1} - 15n - 16$

$= (1 + 15)^{n+1} - 15n - 16$

Using binomial expression we have

$= {}^{n+1}C_0 15^0 + {}^{n+1}C_1 15^1 + {}^{n+1}C_2 15^2 + {}^{n+1}C_3 15^3$

$+ \ldots + x + [C], (15)^{n+1} - 15n - 16$

$= 1 + (n+1)15 + {}^{n+1}C_2 15^2 + {}^{n+1}C_3 15^3$

$+ \ldots + n + 1 C_{n+1}(15)^{n+1} - 15n - 16$

$= 1 + 15n + 15 + {}^{n+1}C_2 15^2 + {}^{n+1}C_3 15$

$+ \ldots + {}^{n+1}C_{n+1}(15)^{n+1} - 15n - 16$

$= 15^2 \left[{}^{n+1}C_2 + {}^{n+1}C_3 15 + \ldots \text{ so on} \right]$

Thus, $2^{4n+4} - 15n - 16$ is divisible 225.

29. Here $a + ib = \frac{c+i}{c-i}$

$= \frac{c+i}{c-i} \times \frac{c+i}{c+i} = \frac{(c+i)^2}{c^2 - i^2}$

$= \frac{c^2 + 2ci + i^2}{c^2 + 1}$

$= \frac{c^2 - 1}{c^2 + 1} + \frac{2c}{c^2 + 1} i$

Comparing real and imaginary parts on both sides, we have

$a = \frac{c^2 - 1}{c^2 + 1}$ and $b = \frac{2c}{c^2 + 1}$

Now $a^2 + b^2 = \left(\frac{c^2 - 1}{c^2 + 1} \right)^2 + \left(\frac{2c}{c^2 + 1} \right)^2$

$= \frac{(c^2 - 1)^2 + 4c^2}{(c^2 + 1)^2} = \frac{(c^2 + 1)^2}{(c^2 + 1)^2} = 1$

Also $\frac{b}{a} = \frac{\frac{2c}{c^2 + 1}}{\frac{c^2 - 1}{c^2 + 1}} = \frac{2c}{c^2 - 1}$

OR

Let $x + yi = \sqrt{1 - i}$

Squaring both sides, we get

$x^2 - y^2 + 2xyi = 1 - i$

Equating the real and imaginary parts

$x^2 - y^2 = 1$ and $2xy = -1 \ldots$ (i)

$\therefore xy = \frac{-1}{2}$

Using the identity

$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$= (1)^2 + 44\left(-\frac{1}{2} \right)^2$

$= 1 + 1$

$= 2$

$\therefore x^2 + y^2 = \sqrt{2} \ldots$ (ii) [Neglecting (-) sign as $x^2 + y^2 > 0$]

Solving (i) and (ii) we get

$x^2 = \frac{\sqrt{2} + 1}{2}$ and $y = \frac{\sqrt{2} - 1}{2}$

$\therefore x = \pm \sqrt{\frac{\sqrt{2} + 1}{2}}$ and $y = \pm \sqrt{\frac{\sqrt{2} - 1}{2}}$

Since the sign of xy is negative.

$\therefore$ if $x = \sqrt{\frac{\sqrt{2} + 1}{2}}$ then $y = -\sqrt{\frac{\sqrt{2} - 1}{2}}$

and if $x = -\sqrt{\frac{\sqrt{2} + 1}{2}}$ then $y = \sqrt{\frac{\sqrt{2} - 1}{2}}$

$\therefore \sqrt{1 - i} = \pm \left(\sqrt{\frac{\sqrt{2} + 1}{2}} - \sqrt{\frac{\sqrt{2} - 1}{2}} i \right)$

30. Let x and x + 2 be two consecutive even positive integers

Then x > 5 and x + x + 2 < 23

$\Rightarrow$ x > 5 and 2x + 2 < 23

$\Rightarrow$ x > 5 and 2 x < 23 - 2

$\Rightarrow$ x > 5 and 2 x < 21

$\Rightarrow$ x > 5 and $x < \frac{21}{2}$

$\Rightarrow 5 < x < \frac{21}{2}$

$\Rightarrow 5 < x < \frac{21}{2}$

$\Rightarrow$ x = 6, 8, 10

Thus required pairs of even positive integers are (6, 8), (8, 10), and (10, 12).

31. Since $^{n}C_4, ^{n}C_5$ and $^{n}C_6$ are in A.P.

$\therefore 2 \cdot ^{n}C_5 = ^{n}C_4 + ^{n}C_6$

$\Rightarrow 2 \times \frac{n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$

$\Rightarrow \frac{2}{5 \times 4!(n-5)(n-6)!} = \frac{1}{4!(n-5)(n-4)(n-6)!} + \frac{1}{6 \times 5 \times 4!(n-6)!}$

$\Rightarrow \frac{2}{5(n-5)} = \frac{1}{(n-5)(n-4)} + \frac{1}{30}$

$\Rightarrow \frac{2}{5(n-5)} - \frac{1}{(n-5)(n-4)} = \frac{1}{30}$

$\Rightarrow \frac{2n-8-5}{5(n-5)(n-4)} = \frac{1}{6}$

$\Rightarrow$ 12n - 78 = n^2 - 9n + 20

$\Rightarrow$ n^2 - 21n + 98 = 0

$\Rightarrow$ (n - 7)(n - 14) = 0

$\therefore$ n = 7 and 14

Section D

32. Given that the bag contains 13 balls and three balls are drawn from the bag

So, the total number of ways of drawing three balls = number of total outcomes = $n(S) = {}^{13}C_3$

Now, we have to find the probability that all three balls drawn are red,

Let A be the event that all drawn balls are red

There are 8 red balls in the bag

So, number of favourable outcomes i.e. all three balls are red = n(A) = $^{8}C_3$

We know that,

$P(A) = \frac{\mathbf{n(A)}}{\mathbf{n(S)}} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

P(all the three balls are red) = $\frac{^{8}C_3}{^{13}C_3}$

$= \frac{\frac{8!}{3!(8-3)!}}{\frac{13!}{3!(13-3)!}}$ $\left[\because ^{n}C_r = \frac{n!}{(n-r)!r!} \right]$

$= \frac{\frac{8 \times 7 \times 6 \times 5!}{3!5!}}{\frac{13 \times 12 \times 11 \times 10!}{3 \times 2 \times 1 \times 10!}}$

$= \frac{\frac{8 \times 7 \times 6}{3 \times 2}}{\frac{13 \times 2 \times 11}{}}$

$= \frac{28}{143}$

33. We have, f(x) = sinx + cosx

By using first principle of derivative

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\therefore f'(x) = \lim_{h \to 0} \frac{\sin(x+h)+\cos(x+h)-\sin x-\cos x}{h}$

$= \lim_{h \to 0} \frac{[\sin x \cdot \cos h + \cos x \cdot \sin h + \cos x \cdot \cos h - \sin x \cdot \sin h - \sin x - \cos x]}{h}$ [$\because$ sin (x + y) = sin x cos y + cos x sin y and cos (x + y) = cos x cos

y - sin x sin y]

$= \lim_{h \to 0} \frac{[(\cos x \cdot \sin h - \sin x \cdot \sin h) + (\sin x \cdot \cos h - \sin x) + (\cos x \cdot \cos h - \cos x)]}{h}$

$= \lim_{h \to 0} \frac{\sin h(\cos x - \sin x) + \sin x(\cos h - 1) + \cos x(\cos h - 1)}{h}$

$= \lim_{h \to 0} \frac{\sin h}{h}(\cos x - \sin x) + \lim_{h \to 0} \frac{\sin x(\cos h - 1)}{h} + \lim_{h \to 0} \frac{\cos x(\cos h - 1)}{h}$

$= 1 \cdot (\cos x - \sin x) + \lim_{h \to 0} \sin x \left[\frac{-(1-\cos h)}{h} \right] + \lim_{h \to 0} \cos x \left[\frac{-(1-\cos h)}{h} \right]$ $\left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$

$= (\cos x - \sin x) - \sin x \cdot \lim_{h \to 0} \left(\frac{1-\cos h}{h} \right) - \cos x \cdot \lim_{h \to 0} \left(\frac{1-\cos h}{h} \right)$

$= (\cos x - \sin x) - \sin x \cdot \lim_{h \to 0} \dfrac{2\sin^2 \frac{h}{2}}{h \times \frac{h}{4}} \times \dfrac{h}{4} - \cos x \cdot \lim_{h \to 0} \dfrac{2\sin^2 \frac{h}{2}}{h \times \frac{h}{4}} \times \dfrac{h}{4}$

$= (\cos x - \sin x) - \sin x \cdot 2 \cdot \dfrac{1}{4} \lim_{\frac{h}{2} \to 0} \left(\dfrac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 \times h - \cos x \cdot 2 \cdot \dfrac{1}{4} \lim_{\frac{h}{2} \to 0} \left(\dfrac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 h$

$= (\cos x - \sin x) - \dfrac{1}{2} \cdot \sin x \cdot (1) \times 0 - \cos x \cdot \dfrac{1}{2} \cdot (1) \times 0 \left[\because \lim_{x \to 0} \dfrac{\sin x}{x} = 1 \right]$

$= (\cos x - \sin x) - 0 - 0$

$= \cos x - \sin x$

<div align="center">OR</div>

Let f (x) = log sin x. Then, f (x + h) = log sin (x + h)

$\therefore \dfrac{d}{dx}(f(x)) = \lim_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$

$\Rightarrow \dfrac{d}{dx}$ (f (x)) $= \lim_{h \to 0} \dfrac{\log \sin(x+h) - \log \sin x}{h}$

$\Rightarrow \dfrac{d}{dx}$ (f (x)) $= \lim_{h \to 0} \dfrac{\log\left\{ \frac{\sin(x+h)}{\sin x} \right\}}{h}$

$\Rightarrow \dfrac{d}{dx}$ (f (x)) $= \lim_{h \to 0} \dfrac{\log\left\{ 1 + \frac{\sin(x+h)}{\sin x} - 1 \right\}}{h}$

$\Rightarrow \dfrac{d}{dx}$ (f (x)) $= \lim_{h \to 0} \dfrac{\log\left\{ 1 + \frac{\sin(x+h)-\sin x}{\sin x} \right\}}{h}$

$\Rightarrow \dfrac{d}{dx}$ (f (x)) $= \lim_{h \to 0} \dfrac{\log\left\{ 1 + \frac{\sin(x+h)-\sin x}{\sin x} \right\}}{h\left\{ \frac{\sin(x+h)-\sin x}{\sin x} \right\}} \times \left\{ \dfrac{\sin(x+h)-\sin x}{\sin x} \right\}$

$\Rightarrow \dfrac{d}{dx}$ (f (x)) $= \lim_{h \to 0} \dfrac{\log\left\{ 1 + \frac{\sin(x+h)-\sin x}{h} \right\}}{\left\{ \frac{\sin(x+h)-\sin x}{h} \right\}} \times \dfrac{\sin(x+h)-\sin x}{h} \times \dfrac{1}{\sin x}$

$\Rightarrow \dfrac{d}{dx}$ (f (x)) $= \lim_{h \to 0} \dfrac{\log\left\{ 1 + \frac{\sin(x+h)-\sin x}{h} \right\}}{\left\{ \frac{\sin(x+h)-\sin x}{h} \right\}} \times \lim_{h \to 0} \dfrac{2\sin \frac{h}{2}\cos\left(x+\frac{h}{2}\right)}{h} \times \dfrac{1}{\sin x}$

$\Rightarrow \dfrac{d}{dx}$ (f (x)) $= \lim_{h \to 0} \dfrac{\log\left\{ 1 + \frac{\sin(x+h)-\sin x}{h} \right\}}{\left\{ \frac{\sin(x+h)-\sin x}{h} \right\}} \times \lim_{h \to 0} \dfrac{\sin\left(\frac{h}{2}\right)\cos\left(x+\frac{h}{2}\right)}{\frac{h}{2}} \times \dfrac{1}{\sin x}$

$\Rightarrow \dfrac{d}{dx}$ (f (x)) $= 1 \times \cos x \times \dfrac{1}{\sin x} = \cot x.$

34. **LHS** $= \cos 12^o + \cos 60^o + \cos 84^o$

$= \cos 12^o + (\cos 84^o + \cos 60^o)$

$= \cos 12^o + \left[2\cos\left(\dfrac{84^\circ + 60^\circ}{2} \right) \times \cos\left(\dfrac{84^\circ - 60^\circ}{2} \right) \right]$

$\left[\because \cos x + \cos y = 2\cos\left(\dfrac{x+y}{2} \right) \cos\left(\dfrac{x-y}{2} \right) \right]$

$= \cos 12^o + \left[2\cos \dfrac{144^\circ}{2} \times \cos \dfrac{24^\circ}{2} \right]$

$= \cos 12^o + \left[2\cos 72^o \times \cos 12^o \right] = \cos 12^o [1 + 2\cos 72^o]$

$= \cos 12^o [1 + 2\cos(90^o - 18^o)]$

$= \cos 12^0 [1 + 2 \sin 18^0] \ [\because \cos (90^0 - \theta) = \sin \theta]$

$= \cos 12^0 \left[1 + 2 \left(\dfrac{\sqrt{5}-1}{4} \right) \right] \ [\because \sin 18^0 = \dfrac{\sqrt{5}-1}{4}]$

$= \left(1 + \dfrac{\sqrt{5}-1}{2} \right) \cos 12^0 = \left(\dfrac{\sqrt{5}+1}{2} \right) \cos 12^0$

RHS $= \cos 24^0 + \cos 48^0$

$= 2\cos\left(\dfrac{24^\circ + 48^\circ}{2} \right) \cos\left(\dfrac{24^\circ - 48^\circ}{2} \right) \ [\because \cos x + \cos y = 2\cos\left(\dfrac{x+y}{2} \right) \cos\left(\dfrac{x-y}{2} \right)]$

$= 2\cos 36^o \cos(-12^o)$

$= 2\cos 36^0 \times \cos 12^0 \ [\because \cos(-\theta) = \cos\theta]$

$= 2 \times \dfrac{\sqrt{5}+1}{4} \times \cos 12^0 = \dfrac{\sqrt{5}+1}{2} \times \cos 12^0 \ [\because \cos 36^0 = \dfrac{\sqrt{5}+1}{4}]$

$\therefore$ **LHS = RHS**

Hence proved.

According to the question, we can write ,

$\cos^2 76^o + \cos^2 16^o - \cos 76^o \cos 16^o$

In the above expression consider $\cos 76^o \cos 16^o$

[By using the trigonometric sum formula, we can say that,

$\cos(C + D) + \cos (C - D) = 2 \cos C \cos D$]

Now multiply and divide this with 2, we get

$\dfrac{2 \times (\cos 76^\circ \cos 16^\circ)}{2} = \dfrac{\cos(76^\circ + 16^\circ) + \cos(76^\circ - 16^\circ)}{2}$

$= \dfrac{\cos 92^\circ + \cos 60^\circ}{2}$

Consider the full expression,

$= \cos^2 76^o + \cos^2 76^o - \cos 76^o \cos 16^o$

$= \cos^2 76^o + \cos 16^o - \left(\dfrac{\cos 92^\circ + \cos 60^\circ}{2} \right)$

Multiplying and dividing the terms $\cos^2 76° + \cos^2 16°$ with 2

$= \dfrac{2 \cos^2 76^\circ}{2} + \dfrac{2 \cos^2 16^\circ}{2} - \left(\dfrac{\cos 92^\circ + \cos 60^\circ}{2} \right)$

$= \frac{1}{2}[\cos 2(76) + 1] + \frac{1}{2}[\cos 2(16) + 1] - \left(\dfrac{\cos 92^\circ + \cos 60^\circ}{2} \right)$

[by using the formula, $\cos 2\theta = 2\cos^2 \theta - 1 \Rightarrow 2\cos^2 \theta = \cos 2\theta + 1$]

$= \frac{1}{2}[2 + (\cos 152^o + \cos 32^o)] - \left(\dfrac{\cos 92^\circ + \cos 60^\circ}{2} \right)$

[by using the formula, $\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$]

$= 1 + \frac{1}{2}[2 \cos \left(\frac{152^\circ + 32^\circ}{2} \right) \cos \left(\frac{152^\circ - 32^\circ}{2} \right)] - \left(\dfrac{\cos 92^\circ + \cos 60^\circ}{2} \right)$

$= 1 + \frac{1}{2}[2 \cos \left(\frac{184^\circ}{2} \right) \cos \left(\frac{120^\circ}{2} \right)] - \left(\dfrac{\cos 92^\circ + \cos 60^\circ}{2} \right)$

$= 1 + \frac{1}{2}[2 \cos(92^o) \cos(60^o)] - \left(\dfrac{\cos 92^\circ + \cos 60^\circ}{2} \right)$

$= 1 + \dfrac{\cos 92^\circ}{2} - \dfrac{\cos 92^\circ}{2} - \dfrac{\frac{1}{2}}{2}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

Hence, $\cos^2 76^o + \cos^2 16^o - \cos 76^o \cos 16^o = \frac{3}{4}$

Hence, the required value is calculated.

35. Given data:

Height (in cm)	Number of boys
95 - 105	9
105 - 115	16
115 - 125	23
125 - 135	30
135 - 145	12
145 - 155	10

Now, we can get the following table from the given data by adding some more columns as below.

Height (in cm)	Number of boys f_i	Mid-points x_i	$f_i x_i$
95 - 105	9	100	900
105 - 115	16	110	1760
115 - 125	23	120	2760
125 - 135	30	130	3900
135 - 145	12	140	1680
145 - 155	10	150	1500

	100				12500

Therefore,

$$\bar{x} = \frac{\sum_{i=1}^{6} f_1 x_1}{\sum_{i=1}^{6} f_1} = \frac{12500}{100} = 125$$

Height (in cm)	Number of boys f_i	Mid-points x_i	$f_i x_i$	$[x_i - \bar{x}]$	$f_i [x_i - \bar{x}]$
95 - 105	9	100	900	25	225
105 - 115	16	110	1760	15	240
115 - 125	23	120	2760	5	115
125 - 135	30	130	3900	5	150
135 - 145	12	140	1680	15	180
145 - 155	10	150	1500	25	250
	100		12500		1160

Thus, the required mean deviation about the mean is given by

Therefore, Mean Deviation $(\bar{x}) = \frac{\sum_{i=1}^{6} f_i[x_i - M]}{\sum_{i=1}^{6} f_i} = \frac{1160}{100} = 11.6$
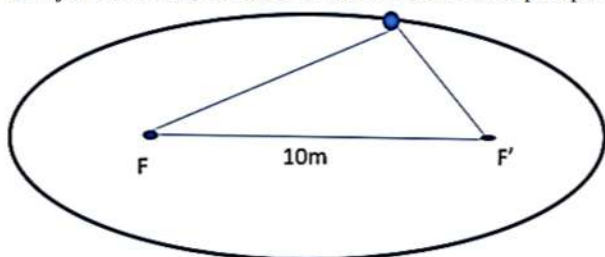
Section E

36. **Read the text carefully and answer the questions:**

A farmer wishes to install 2 handpumps in his field for watering.

The farmer moves in the field while watering in such a way that the sum of distances between the farmer and each handpump is always 26m. Also, the distance between the hand pumps is 10 m.

(i) The curve traced by farmer is ellipse. Because An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

Two positions of hand pumps are foci Distance between two foci = 2c = 10 Hence c = 5 Here foci lie on x axis & coordinates of foci = $(\pm c, 0)$

Hence coordinates of foci = $(\pm 5, 0)$

(ii) $\frac{x^2}{169} + \frac{y^2}{144} = 1$

Sum of distances from the foci = 2a

Sum of distances between the farmer and each hand pump is = 26 = 2a

$\Rightarrow$ 2a = 26 $\Rightarrow$ a = 13 m

Distance between the handpump = 10m = 2c

$\Rightarrow$ c = 5 m

$c^2 = a^2 - b^2$

$\Rightarrow$ 25 = 169 - b^2

$\Rightarrow b^2$ = 144

Equation is $\frac{x^2}{169} + \frac{y^2}{144} = 1$

(iii) Equation of ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$ comparing with standard equation of ellipse a=13, b= 12 and c= 5 (given)

Length of major axis = 2a = 2 × 13 = 26

Length of minor axis = 2b = 2 × 12 = 24

eccentricity $e = \frac{c}{a} = \frac{5}{13}$

OR

Equation of the ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$ hence a = 13 and b = 12

length of latus rectum of ellipse is given by $\frac{2b^2}{a} = \frac{2 \times 144}{13}$

37. Read the text carefully and answer the questions:

A sequence whose terms increases or decreases by a fixed number, is called an Arithmetic Progression (AP).

In other words, we can say that, a sequence is called an arithmetic progression if the difference of a term and the previous term is always same i.e. $a_{n+1} - a_n$ = constant for all n.

This constant or same difference is called the common difference of an AP and it is denoted by d.

In an AP, we usually denote the first term by a, common difference by d and the nth term by a_n or T_n defined as

$T_n = a_n = a + (n - 1)d$

Also, l = a + (n - 1)d, where l is the last term of the sequence.

The sum of n terms, S_n of this AP is given by $S_n = \frac{n}{2}[2a + (n - 1)d]$.

Also, if l be the last term, then the sum of n terms of this AP is $S_n = \frac{n}{2}(a + l)$.

(i) **(c)** 6

Explanation: 6

(ii) **(d)** 0

Explanation: 0

(iii) **(a)** 4

**Explanation:** 4

OR

(c) 81

Explanation: 81

38. Read the text carefully and answer the questions:

A survey is conducted by a career counsellor in a college to find career choice of students after the Intermediate. There are 100 students that goes for Engineering Courses, 50 wants to make their career in Medical, 100 students continue their further study in Arts. There are 10 students that go for both Engineering and Medical, and 3 goes for Medical and Arts. There are 3 students that do not go for any further studies.
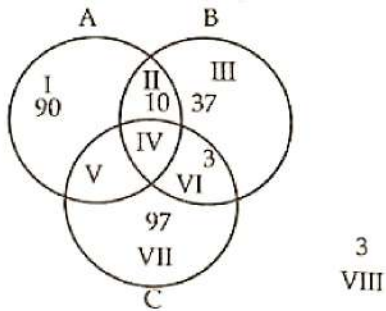

(i) Let A set denotes Engineering, B set denotes Medical and C set denotes Art. Draw the Venn's diagram for the given situation.
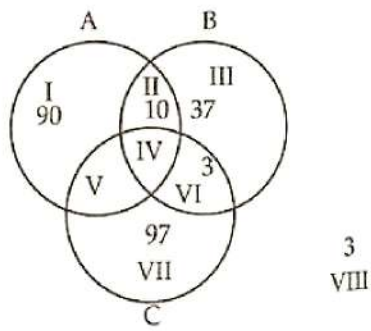
Region I, II, IV, V, and VII shows the student that goes for Engineering or Arts. So, the required number of students is 200.

(ii) Let A set denotes Engineering, B set Medical and C set Art. Draw the Venn's diagram for the given situation.

The region VI shows the student that goes for both Medical and Arts. So, $n(B \cap C) = 3$.