

# Chapter 6

## Flow Through Open Channels

### CHAPTER HIGHLIGHTS

- Flow through open channels
- Prismatic channel
- Types of flow in open channels
- Geometrical terminologies
- Uniform flow—Chezy formula
- Energy depth relationships
- Hydraulic gradient lines and energy gradient lines
- Specific energy
- Mathematical expressions
- Hydraulic jump
- Non-uniform flow equation
- Gradually varied flow (GVF)
- Channel bed slopes and flow profiles
- Classification of surface profiles
- Gauging flumes
- Non-modular flume or venturi flume
- Modular flume or standing wave flume
- Flow measurements in channel and pipes

### FLOW THROUGH OPEN CHANNELS

Open channel is a passage through which water flows, with its surface open to atmosphere. Pressure on the water surface is atmospheric pressure. A channel with covered or closed top also can be an open channel. In this case the channel should not be running full and air above water surface should be at atmospheric pressure.

In the case of an open channel flow, the flow happens due to gravity. Bed of the channel is provided with a slope in the direction of flow. The longitudinal profile of the water surface is the hydraulic gradient line for the open channel.

Flow of water in a river or in a man-made channel belongs to the class of open channel flow. Flow of water in a large pipe, when the cross-section of it is not fully filled, is an open channel flow. Such situations arise in irrigation conduits, sewage pipes, etc.

### PRISMATIC CHANNEL

A channel with constant bed slope and same cross-section along the length is known a prismatic channel.

If the area of cross-section of the channel is directly proportional to any power of depth of flow, it is called an exponential prismatic channel. Rectangular, Triangular

and parabolic channels are examples of this type of channel. Trapezoidal and circular channels are non-exponential channels.

### TYPES OF FLOW IN OPEN CHANNELS

Depending upon the change in depth of the flow with respect to space and time, open channel flows can be classified as follows.

1. Steady and unsteady flow
2. Uniform and non-uniform or varied flow
3. Laminar flow and turbulent flow
4. Subcritical flow, critical flow and super critical flow

### Steady and Unsteady Flows

When the discharge rate is constant, the flow is a steady flow. The sectional areas at different sections may be different. So mean velocity of flow at different sections may also be different. Mathematically,

$$\frac{\partial y}{\partial t} = 0, \frac{\partial v}{\partial t} = 0 \text{ or } \frac{\partial Q}{\partial t} = 0$$

for a steady flow.

## Uniform and Non-uniform (or Varied Flow)

When depth of flow is same at all sections, the flow is a uniform flow. This means that velocity is same at all sections. Water surface is parallel to bed in this case. In a non-uniform flow, depth of section and mean velocity is different at different sections. Water surface is not parallel to bed in non-uniform flows.

Mathematically,

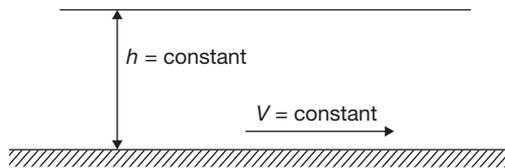
$$\frac{\partial y}{\partial L} = 0, \frac{\partial V}{\partial L} = 0 \text{ in uniform flows.}$$

Uniform flows are possible only in prismatic channels. Depending upon variation of discharge with respect to time, a uniform flow can be steady or unsteady. Unsteady uniform flow is rare in practice.

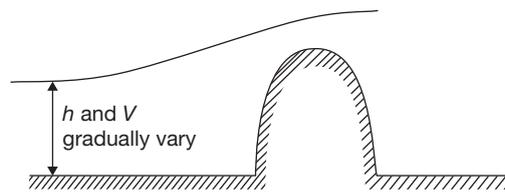
Mathematically,  $\frac{\partial y}{\partial L} \neq 0, \frac{\partial v}{\partial L} \neq 0$  for a non-uniform flow.

A non-uniform or varied flow can be:

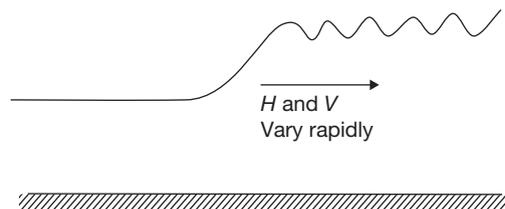
1. Rapidly varied flow (RVF)
2. Gradually varied flow (GVF)



Uniform flow



Gradually varied flow



Gradually varied flow

## GEOMETRICAL TERMINOLOGIES

**Depth of flow ( $h$ ):** Depth of flow ( $h$ ) at any section is the vertical distance of the bed of the channel from the free surface of water at that section.

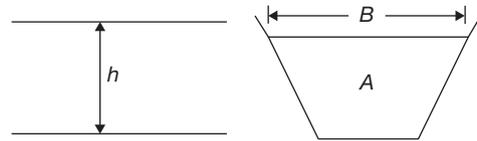
**Top breadth ( $B$ ):** Top breadth ( $B$ ) is the breadth of channel section at the free surface of water.

**Water area ( $A$ ):** It is the cross-sectional area of flow perpendicular to the direction of flow.

**Wetted perimeter ( $P$ ):** It is the perimeter in contact with water at the section.

**Hydraulic radius ( $R_h$ ):** Hydraulic radius ( $R_h$ ) is defined as

$$R_h = \frac{\text{Water area } (A)}{\text{Wetted perimeter } (P)}$$



## UNIFORM FLOW—CHEZY FORMULA

Consider a steady uniform flow of water in a channel whose bed has a slope,  $i$ .

Let  $A$  be the area of flow section,  $P$  the wetted perimeter and  $v$  the mean velocity of flow.

$$\text{Velocity, } v = C\sqrt{mi}$$

Where,  $C = \sqrt{\frac{w}{f'}}$ , a constant depending on the shape and nature of channel.

$$m = \frac{A}{P} = \text{hydraulic mean depth}$$

$C$  is a dimensioned constant having dimension  $L^{1/2}T^{-1}$ .

Value of  $C$  can be found out using following empirical formulae:

### 1. Bazin's formula:

$$C = \frac{157.6}{1.81 + \frac{K}{\sqrt{m}}}$$

Where,  $K$  is a constant depending on the nature of channel surface.

### 2. Kutter's formula:

$$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i}\right) \frac{N}{\sqrt{m}}}$$

Where,  $N$  = Kutter's constant.

### 3. Manning's formula:

$$C = \frac{1}{N} m^{1/6}$$

Where,  $N$  = Roughness coefficient

So,  $v = C\sqrt{mi}$

$$= \frac{1}{N} m^{1/6} \cdot m^{1/2} \cdot i^{1/2}$$

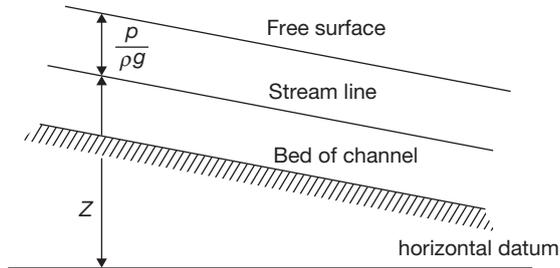
$$= \frac{1}{N} m^{2/3} \cdot i^{1/2}$$

## ENERGY DEPTH RELATIONSHIPS

Bernoulli's equation can be applied to a flow through an open channel. In the case of flow through an open channel, the stream lines are straight and parallel or a little curved. As per Bernoulli's equation, total energy per unit weight,

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{Constant along a stream line.}$$

In the case of an open channel flow height of free surface represents potential head + pressure head, as shown in the figure.



So Bernoulli's equation for an open channel flow is simplified as,

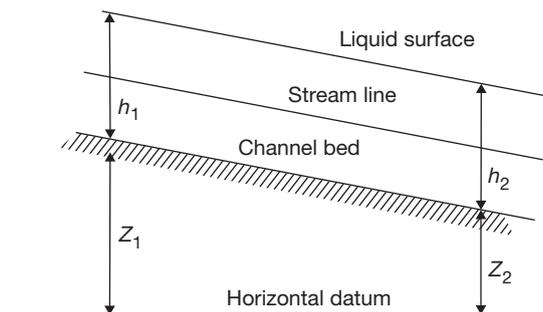
$$\text{Height of liquid surface above datum} + \frac{v^2}{2g} = \text{Constant}$$

(neglecting the frictional losses)

If frictional losses are considered, the Bernoulli's equation for an open channel can be written as,

$$h_1 + Z_1 + \frac{V_1^2}{2g} = h_2 + Z_2 + \frac{V_2^2}{2g} + h_f$$

Where,  $h_f$  = frictional loss,  $V_1$  and  $V_2$  are average flow velocities,  $h_1$  and  $h_2$  are depth of flow and  $Z_1$  and  $Z_2$  are channel bed heights at two section 1 and 2 as shown in the figure.



To account for non-uniformity of velocity over a cross-section, the equation can be modified as,

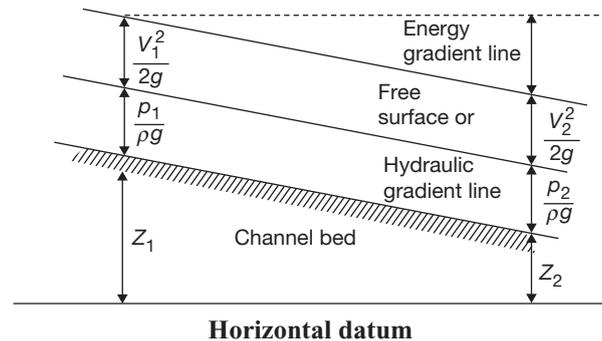
$$\begin{aligned} h_1 + z_1 + \alpha_1 \frac{V_1^2}{2g} \\ = h_2 + Z_2 + \alpha_2 \frac{V_2^2}{2g} + h_f. \end{aligned}$$

## HYDRAULIC GRADIENT LINES AND ENERGY GRADIENT LINES

Hydraulic gradient line is obtained by plotting sum of potential heads and pressure heads  $\left( Z + \frac{P}{\rho g} \right)$  as ordinates

against distance along the flow. Thus hydraulic gradient line is the contour of free surface in an open channel. Energy gradient line is the contour of total mechanical energy per unit weight or the total head. So hydraulic gradient line can be plotted by subtracting velocity head  $\frac{v^2}{2g}$  from the total

head. The concept of hydraulic gradient and energy gradient lines is not restricted to open channels. It can be used for all kinds of flows through closed or open ducts. Figure shows the lines in the case of an open channel flow.



## SPECIFIC ENERGY

Total energy head at a section in an open channel flow is given by:

$$E = z + h + \frac{v^2}{2g}$$

Where,  $z$  is the elevation of the bottom of the channel from an arbitrary datum. If we take the datum as the bottom of the channel itself, the equation becomes.

$$E = h + \frac{v^2}{2g}$$

This represents the specific energy of the channel flow.

Thus the specific energy at any section of a channel is the sum of the depth of flow and the kinetic head at that section.

In the case of a steady uniform flow, depth of flow and velocity is same at all sections. Therefore specific energy head is same at all sections.

Considering a non-uniform flow taking place in an open channel of rectangular cross-section of width  $b$ , at a steady rate  $Q$ .

Discharge per unit width,

$$q = \frac{Q}{b} = \text{Constant.}$$

Velocity at the section,

$$v = \frac{Q}{bh}$$

$$= \frac{q}{h}$$

or

$$q = vh$$

Thus the product of velocity and depth of flow is constant at all sections.

$$E = h + \frac{v^2}{2g}$$

$$= h + \frac{1}{2g} \left( \frac{q}{h} \right)^2$$

$$= h + \frac{q^2}{2g} \times \frac{1}{h^2}$$

$$= h + \frac{K}{h^2}$$

Where,

$$K = \frac{q^2}{2g}$$

Thus, for a given value of  $q$  specific energy head is a function of depth as follows.

$$E = E_1 + E_2$$

Here,  $E_1 = h$

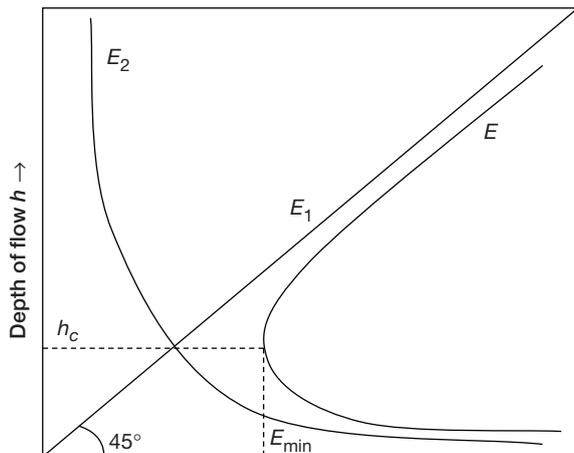
$$\text{and } E_2 = \frac{K}{h^2} = \left( \frac{q^2}{2g} \right) \frac{1}{h^2}$$

Where

$E_1$  = Static energy head

$E_2$  = Kinetic energy head

If we plot static energy head, kinetic energy head and specific energy head against depth flow, the graph obtained is as follows:



Static energy head  $E_1 \rightarrow$   
 Kinetic energy head  $E_2 \rightarrow$   
 Specific energy head  $\bar{E} \rightarrow$

From the above, it follows that specific energy head first becomes lesser and lesser when depth increases and reaches a minimum and then it increases as the depth is increased.

The depth corresponding to minimum specific energy is called the critical depth ( $h_c$ ). When the depth of flow is greater than the critical depth, the flow is called streaming flow or **tranquil flow** or **sub-critical flow**.

**Critical flow:** When the flow is such that specific energy is minimum, the flow is called a critical flow. Therefore at critical flow, depth of flow is equal to the critical depth ( $h_c$ ).

The velocity of flow at critical depth is called the **critical velocity**.

## MATHEMATICAL EXPRESSIONS

We know that,

$$E = h + \frac{K}{h^2}$$

Where,

$$K = \frac{q^2}{2g}$$

$E$  is minimum when,

$$\frac{dE}{dh} = 0$$

$$\frac{dE}{dh} = \frac{d \left[ h + \frac{K}{h^2} \right]}{dh}$$

$$1 - \frac{2K}{h^3}$$

$\therefore E$  is minimum when,

$$1 - \frac{2K}{h^3} = 0$$

or

$$\frac{2K}{h^3} = 1$$

$\therefore$

$$h = \sqrt[3]{2K} = \left( \frac{q^2}{g} \right)^{1/3}$$

or

$$h_c = \left( \frac{q^2}{g} \right)^{1/3}$$

$$\text{Velocity} = \frac{Q}{A} = \frac{Q}{bh} = \frac{q}{h}$$

$\therefore$

$$V_c = \frac{q}{h_c} = \frac{q}{\left( \frac{q^2}{g} \right)^{1/3}}$$

$$= q^{1/3} g^{1/3}$$

or

$$V_c^3 = qg = V_c h_c g$$

or

$$V_c = \sqrt{gh_c}$$

or 
$$\frac{V_c}{\sqrt{gh_c}} = 1$$

But  $\frac{V_c}{\sqrt{gh_c}} = \text{Froude number at critical flow}$

∴ For critical flow, Froude number is 1.  
Specific head is minimum when depth of flow is critical.

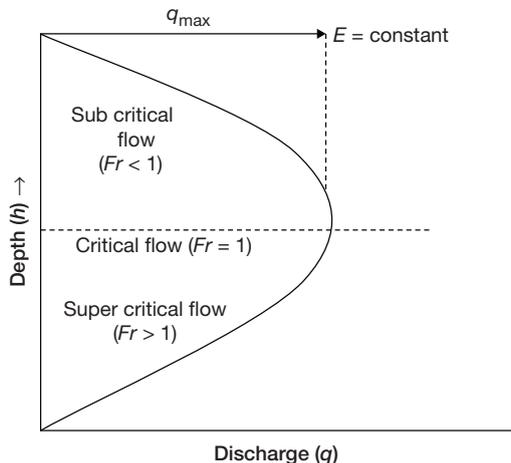
$$\begin{aligned} \therefore E_{\min} &= h_c + \frac{q^2}{2gh_c^2} \\ &= h_c + \frac{1}{2h_c^2} \times \frac{q^2}{g} \\ &= h_c + \frac{1}{2h_c^2} \times h_c^3 \left[ \because \frac{q^2}{g} = h_c^3 \right] \\ &= h_c + \frac{h_c}{2} \\ &= \frac{3}{2} h_c \end{aligned}$$

∴  $E_{\min} = \frac{3}{2} h_c$

or  $h_c = \frac{2}{3} E_{\min}$

When the depth of flow is less than the critical depth, the flow is **super critical** (or shooting or torrential or rapid flow). Froude number is greater than 1 in this case. For a sub-critical flow, Froude number is less than 1.

Depth of Flow	Froude Number $Fr = \frac{V}{\sqrt{gh}}$
$h = h_c$	$Fr = 1$
$h > h_c$	$Fr < 1$
$h < h_c$	$Fr > 1$



## Maximum Flow Rate for a Given Specific Energy

$$E = h + \frac{q^2}{2g} \frac{1}{h^2} = h + \left(\frac{Q}{b}\right)^2 \frac{1}{2gh^2}$$

$$\begin{aligned} \therefore Q^2 &= (E - h)2gb^2h^2 \\ \Rightarrow Q &= bh\sqrt{2g(E - h)}. \end{aligned}$$

The above expression gives flow rate through a uniform rectangular channel of width  $b$  and depth  $h$ . For maximum flow rate,  $\frac{dQ}{dh} = 0$

$$\begin{aligned} \therefore \frac{dQ}{dh} &= \sqrt{2g} b \left[ \sqrt{E - h} - \frac{h}{2\sqrt{E - h}} \right] = 0 \\ \Rightarrow (E - h) &= \frac{h}{2} \\ \Rightarrow 2E - 2h &= h \\ \Rightarrow h &= \frac{2}{3} E \end{aligned}$$

or  $h_{opt} = \frac{2}{3} E.$

The expression is as that of critical depth  $h_c$ . So critical depth  $h_c = \frac{2}{3} E$  and  $Fr = 1$  are also conditions for maximum flow rate for a given specific energy.

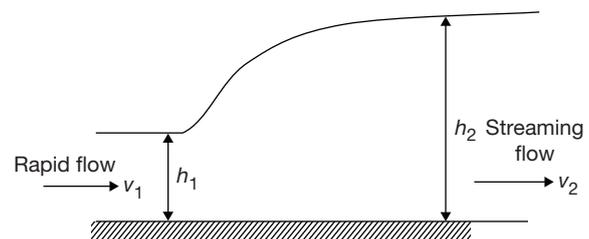
$$\text{From the above, } Q_{\max} = \frac{2}{3} bE \sqrt{\frac{2}{3} gE}.$$

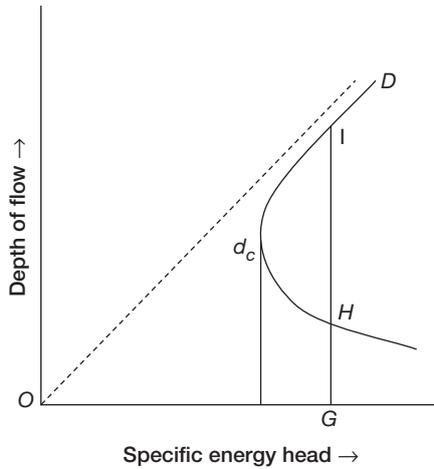
## HYDRAULIC JUMP

In an open channel when rapidly flowing stream suddenly changes to slowly flowing stream a rise or jump in the elevation of liquid surface takes place. This phenomenon is known as hydraulic jump or standing wave. In this process, kinetic energy of the rapidly flowing stream is converted to potential energy.

Due to this there is a loss of kinetic energy. Hydraulic jump occurs at the toe of spillways or below a sluice gate where velocity is very high.

For a hydraulic jump to occur the depth of flow should be less than the critical depth or the flow should be a shooting flow or rapid flow.





Referring to the figures above, corresponding to specific energy head  $OG$ , depth of flow before the jump is  $d_1 = GH$  and after the jump is  $d_2 = GI$ .

### Depth after Hydraulic Jump

Consider a rectangular channel through which discharge is  $Q = bh_1v_1 = bh_2v_2$ .

Applying momentum equation,

$$\begin{aligned} \rho Q(v_2 - v_1) &= p_1 A_1 - p_2 A_2 \\ \Rightarrow \rho b(h_2 v_2^2 - h_1 v_1^2) &= \rho b g \left[ \frac{h_1^2}{2} - \frac{h_2^2}{2} \right] \\ [\because A_1 &= b_1 h_1 \quad A_2 = b_2 h_2] \\ p_1 &= \rho g \frac{h_1}{2} \quad p_2 = \rho g \frac{h_2}{2} \end{aligned}$$

and  $Q = bh_1 v_1 = bh_2 v_2$

$$\begin{aligned} \Rightarrow \frac{Q^2}{b^2} \left( \frac{1}{h_2} - \frac{1}{h_1} \right) &= \frac{g}{2} (h_1^2 - h_2^2) \\ \Rightarrow \frac{Q^2}{b^2} \frac{1}{h_1 h_2} &= \frac{g}{2} (h_1 + h_2) \\ \Rightarrow h_2 (h_1 + h_2) &= \frac{2}{g h_1} \frac{Q^2}{b^2} \\ \Rightarrow h_2^2 + h_1 h_2 - \frac{2}{g h_1} \frac{Q^2}{b^2} &= 0 \\ \Rightarrow h_2 &= \left[ -h_1 + \sqrt{h_1^2 + \frac{8Q^2}{g h_1 b^2}} \right] \times \frac{1}{2} \\ &= \frac{-h_1}{2} + \sqrt{\frac{h_1^2}{4} + \frac{2q^2}{g h_1}} \quad \text{where } q = \frac{Q}{b}. \end{aligned}$$

### Depth as a Function of Froude Number

$$\begin{aligned} \frac{Q^2}{g h_1 b^2} &= \frac{h_1^2 V_1^2}{g h_1} \\ &= (Fr_1)^2 h_1^2 \end{aligned}$$

$$\begin{aligned} \therefore h_2 &= \frac{-h_1 + h_1 \sqrt{1 + 8(Fr_1)^2}}{2} \\ &= \frac{h_1}{2} \left[ \sqrt{1 + 8(Fr_1)^2} - 1 \right] \quad \text{or } \frac{h_2}{h_1} = \frac{\left[ \sqrt{1 + 8(Fr_1)^2} - 1 \right]}{2}. \end{aligned}$$

### Loss of Specific Energy Due to Hydraulic Jump

$$\Delta E_{\text{loss}} = E_1 - E_2$$

$$= (h_1 - h_2) + \frac{1}{2g} (v_1^2 - v_2^2)$$

$$= (h_1 - h_2) + \frac{Q^2}{2g b^2} \left( \frac{h_2^2 - h_1^2}{h_1^2 h_2^2} \right)$$

$$\text{But } \frac{Q^2}{g b^2 h_1 h_2} = \frac{h_1 + h_2}{2}$$

$$\begin{aligned} \therefore \Delta E_{\text{loss}} &= (h_1 - h_2) + \frac{(h_1 + h_2)^2 (h_2 - h_1)}{4 h_1 h_2} \\ &= (h_2 - h_1) \left[ \frac{(h_1 + h_2)^2}{4 h_1 h_2} - 1 \right] \end{aligned}$$

$$\text{i.e., } \Delta E_{\text{loss}} = \frac{(h_2 - h_1)^3}{4 h_1 h_2}$$

Height of standing wave of hydraulic jump  $= h_2 - h_1$ .

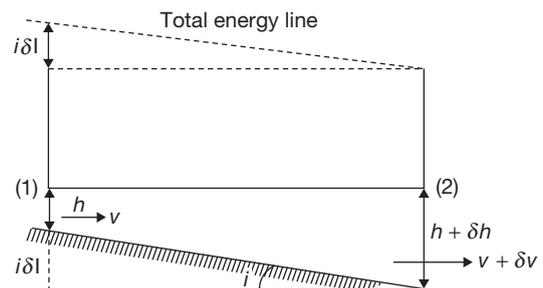
**Length of hydraulic jump:** Exact points of commencement and end of the jump are not well defined. For purposes of analysis, it may be assumed that length of hydraulic jump is 5 to 7 times the height of the jump.

**Relation between pre-jump and post-jump Froude numbers:** It can be shown that,

$$Fr_2 = Fr_1 \left[ \frac{2}{\sqrt{1 + 8Fr_1^2} - 1} \right]^{3/2}.$$

For a hydraulic jump to occur  $Fr_1$  should be greater than 1 and  $Fr_2$  should be less than 1.

### NON-UNIFORM FLOW EQUATION



Consider a steady uniform flow through a channel,

$i$  = Bed slope.

$j$  = Slope of total energy line.

At sections (1) and (2), a distance  $\delta l$  apart  $h$  and  $(h + \delta h)$  are depths  $v$  and  $(v + \delta v)$  are velocities at the sections.

Applying Bernoulli's equation,

$$(i\delta l + h) + \frac{v^2}{2g} = (h + \delta h) + \frac{(v + \delta v)^2}{2g} + j\delta l$$

Ignoring quantities of second order, it can be shown that,

$$\begin{aligned} \frac{\delta h}{\delta l} &= \frac{i - j}{1 - \frac{v^2}{gh}} \\ &= \frac{i - j}{1 - (Fr)^2} \end{aligned}$$

Where,  $Fr$  = Froude number

Since  $j$  is the rate of fall of energy head,

$$v = C\sqrt{mj}$$

or

$$j = \frac{v^2}{mc^2}$$

So,

$$\frac{\delta h}{\delta l} = \frac{i - \frac{v^2}{mc^2}}{1 - \frac{v^2}{gd}}$$

This is called non-uniform flow equation.

By substituting  $v = \frac{Q}{A}$ , the equation becomes

$$\frac{\delta h}{\delta l} = \frac{i - \frac{v^2}{mc^2}}{1 - \frac{bQ^2}{gA^3}}$$

The above equation holds good for channels of all sections approximately.

## GRADUALLY VARIED FLOW (GVF)

When the depth changes gradually over a long distance. The flow is called a gradually varied flow.

A gradually varied flow can occur due to the following reasons:

1. Change in shape and size of the channel cross-section
2. Change in slope of the channel
3. Presence of obstruction
4. Change in frictional forces at the boundaries.

Gradually varied flow is a case of non-uniform flow. So same equation can be applied.

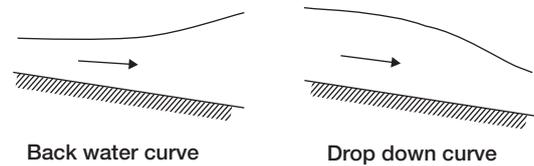
i.e., 
$$\frac{dh}{dl} = \frac{i - j}{1 - (Fr)^2}$$

$\frac{dh}{dl}$  is also called slope of the free water surface.

When  $\frac{dh}{dl} = 0$ ,  $h$  is constant; so free water surface is parallel to the channel bed.

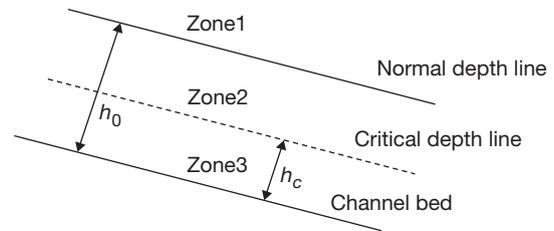
When  $\frac{dh}{dl} > 0$ , depth of water increases in the direction of flow and the profile of water is called **backwater curve**.

When,  $\frac{dh}{dl} < 0$ , depth of water decreases in the direction of flow and the profile is called **drop down curve**.



## CHANNEL BED SLOPES AND FLOW PROFILES

Let  $h_0$  = normal depth of flow,  $h$  = actual depth of flow and  $h_c$  = critical depth of flow.



The space above the channel bed is divided into three zones.

**Zone 1** is the space above the critical and normal depth line.

**Zone 2** is the space between normal depth line and critical depth line.

**Zone 3** is the space below normal depth line and critical depth line.

Based on the relative values of  $h_c$  and  $h_0$  channel bed slopes are classified as follows.

$h_0 > h_c$ : Mild slope or M-slope.

$h_0 = h_c$ : Critical slope or C-slope.

$h_0 < h_c$ : Steep slope or S-slope.

Bed slope = 0, then

$h_0 = \infty$ , horizontal slope or H-slope.

Bed slope negative, then

$h_0 = \infty$ , adverse slope or A-slope

## CLASSIFICATION OF SURFACE PROFILES

We have seen that the rate of change of depth along a channel is,

$$\frac{\delta h}{\delta l} = \frac{i-j}{1-\frac{v^2}{gd}} = \frac{i\left(1-\frac{i}{j}\right)}{\left(1-\frac{v^2}{gd}\right)} \quad (1)$$

Where

$h_0$  = Normal depth of flow

$h$  = Actual depth of flow

Discharge per unit width,

$$q = \frac{h_0 \times 1}{N} m_0^{2/3} i^{1/2}$$

$$= \frac{h \times 1}{N} m^{2/3} j^{1/2}$$

When a channel is very wide,

$m_0 = h_0$  and  $m = h$

So the above relation can be written as

$$h_0^{5/3} i^{1/2} = h^{5/3} j^{1/2}$$

or 
$$\frac{j}{i} = \left(\frac{h_0}{h}\right)^{10/3}$$

$\therefore i\left(1-\frac{j}{i}\right) = i\left[1-\left(\frac{h_0}{h}\right)^{10/3}\right]$

$$1-\frac{v^2}{g} = 1-\frac{q^2}{gh^3}$$

$$= 1-\left(\frac{h_c}{h}\right)^3 \quad \left[\because \frac{q^2}{g} = h_c^3\right]$$

Therefore Eq. (1) becomes,

$$\frac{\delta h}{\delta l} = i \left[ \frac{1-\left(\frac{h_0}{h}\right)^{10/3}}{1-\left(\frac{h_c}{h}\right)^3} \right]$$

If Chezy's formula is adopted, the expression becomes,

$$\frac{dh}{dl} = i \left[ \frac{1-\left(\frac{h_0}{h}\right)^3}{1-\left(\frac{h_c}{h}\right)^3} \right]$$

From the above, it follows that change of depth along the channel depends upon:

1. Slope of the channel bed
2. Ratio of normal depth  $h_0$  to actual depth  $h$ .
3. Ratio of critical depth  $h_c$  to actual depth  $h$ .

Depending upon the relative magnitudes of the actual depth of flow ( $h$ ), normal depth of flow ( $h_0$ ) and critical depth of flow ( $h_c$ ) the slope profiles can be classified as follows.

## Mild Slope Profiles

**$M_1$  profiles:**

$$h > h_0 > h_c$$

$$\frac{\delta h}{\delta L} = \frac{+ve}{+ve} = +ve$$

Back water curve, sub-critical.

**$M_2$  profile:**

$$h_0 > h > h_c$$

$$\frac{\delta h}{\delta L} = \frac{-ve}{+ve} = -ve$$

Draw down curve, sub-critical.

**$M_3$  profile:**

$$h_0 > h_c > h$$

$$\frac{\delta h}{\delta L} = \frac{-ve}{-ve} = +ve$$

Back water curve, sub-critical

## Critical Slope Profiles

$$h_0 = h_c$$

**$C_1$  profile:**

$$h > h_c$$

$$\frac{\delta h}{\delta L} = \frac{+ve}{+ve} = +ve$$

Back water curve, sub-critical.

**$C_2$  profile:**

There is no  $C_2$  profile.

**$C_3$  profile:**

$$h < h_c$$

$$\frac{\delta h}{\delta L} = \frac{-ve}{-ve} = +ve$$

Backwater curve, sub-critical.

## Steep Slope Profiles

**$S_1$  profile:**

$$h > h_c > h_0$$

$$\frac{\delta h}{\delta L} = \frac{+ve}{+ve} = +ve$$

Back water curve, sub-critical.

**$S_2$  profile:**

$$h_c > h > h_0$$

$$\frac{\delta h}{\delta L} = \frac{+ve}{-ve} = -ve$$

Draw down curve, super critical.

**$S_3$  profile:**

$$h_c > h_0 > h$$

$$\frac{\delta h}{\delta L} = \frac{-ve}{-ve} = +ve$$

Backwater curve, super critical.

### Horizontal Slope Profile

$h_0$  does not exist.

$i = 0$  (for analysis it is assumed that  $i$  is very small)

There is no  $H_1$  profile.

$H_2$  profile:

$$h_0 > h > h_c$$

$$\frac{\partial q}{\partial x} = 0$$

Draw down curve, sub-critical.

$H_3$  profile:

$$h_0 > h_c > h$$

$$\frac{\delta h}{\delta L} = \frac{-ve}{-ve} = +ve$$

Back water curve, super critical.

### Adverse Slope Profile

$h_0$  does not exist.

There is no  $A_1$  profile.

$A_2$  profile:

$$h_0 > h > h_c$$

$$\frac{\delta h}{\delta L} = \frac{-ve}{+ve} = -ve$$

Draw down curve, subcritical

$A_3$  profile:

$$h_0 > h_c > h$$

$$\frac{\delta h}{\delta L} = \frac{-ve}{-ve} = +ve$$

Back water curve, super critical.

### SOLVED EXAMPLES

#### Example 1

A rectangular channel of 6 m width and 2 m depth is laid at a slope of 1 in 5000. If Chezy's constant is 60 m<sup>1/2</sup>/s, discharge through the channel is \_\_\_\_\_.

**Solution**

$$b = 6 \text{ m}$$

$$h = 2 \text{ m}$$

$$i = \frac{1}{5000}$$

$$C = 60$$

Hydraulic mean depth,

$$m = \frac{A}{P} = \frac{bh}{b+2h}$$

$$= \frac{6 \times 2}{6 + 2 \times 2}$$

$$= 1.2 \text{ m}$$

Velocity of flow,

$$v = C\sqrt{mi}$$

$$= 60\sqrt{1.2 \times \frac{1}{5000}}$$

$$= 0.9295 \text{ m/s}$$

Discharge,

$$Q = Av$$

$$= bhv$$

$$= 6 \times 2 \times 0.9295$$

$$= 11.154 \text{ m}^3/\text{s}.$$

#### Example 2

A channel of trapezoidal cross-section discharges 50 m<sup>3</sup>/s of water at a velocity of 2 m/s. Side slopes of the channel are 1 vertical to 1 horizontal and bed width is 10 times the depth of water. Determine the value of bed slope.

(Take Mannings constant,  $N = 0.018$ )

**Solution**

$$Q = 50 \text{ m}^3/\text{s}$$

$$v = 2 \text{ m/s}$$

$$b = 10 \text{ hours}$$

Side slope,

$$n = \frac{1}{1} = 1$$

$$N = 0.018$$

Area of flow section,

$$A = \frac{Q}{v} = \frac{50}{2} = 25 \text{ m}^2$$

$$A = (b + nh)h$$

$$\therefore 25 = (10h + 1h)h = 11h^2$$

$$\Rightarrow h = 1.5076 \text{ m}$$

$$\therefore b = 1.5076 \times 10 = 15.076 \text{ m}$$

Wetted perimeter,

$$P = b + 2h\sqrt{n^2 + 1}$$

$$= 15.076 + 2 \times 1.5076\sqrt{1^2 + 1}$$

$$= 19.34 \text{ m}$$

Hydraulic mean depth,

$$M = \frac{A}{P} = \frac{25}{19.34} = 1.293 \text{ m}$$

$$V = \frac{1}{N} m^{2/3} i^{1/2}$$

$$\therefore 2 = \frac{1}{0.018} \times 1.293^{2/3} i^{1/2}$$

$$\Rightarrow i = 6.66 \times 10^{-3} = 0.00666.$$

#### Example 3

Water flows through a rectangular channel of 10 m width at a velocity of 1.8 m/s. If discharge is 18 m<sup>3</sup>/s, determine

- (i) specific energy head.
- (ii) critical depth.
- (iii) critical velocity.
- (iv) minimum value of specific energy head.

**Solution**

$$Q = 18 \text{ m}^3/\text{s}$$

$$b = 10 \text{ m}$$

$$v = 1.8 \text{ m/s}$$

$$\text{Area of flow, } A = \frac{Q}{v} = \frac{18}{1.8} = 10 \text{ m}^2$$

Depth of flow,

$$h = \frac{A}{b} = \frac{10}{10} = 1 \text{ m}$$

(i) Specific energy head,

$$\begin{aligned} E &= h + \frac{v^2}{2g} \\ &= 1 + \frac{1.8^2}{2 \times 9.81} \\ &= 1.165 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{(ii) } q &= \frac{Q}{b} = \frac{18}{10} \\ &= 1.8 \text{ m}^3/\text{s per m width} \end{aligned}$$

Critical depth,

$$\begin{aligned} h_c &= \left( \frac{q^2}{g} \right)^{1/3} \\ &= \left( \frac{1.8^2}{9.81} \right)^{1/3} \\ &= 0.691 \text{ m.} \end{aligned}$$

(iii) Critical velocity,

$$\begin{aligned} v_c &= \sqrt{gh_c} \\ &= \sqrt{9.81 \times 0.691} \\ &= 2.604 \text{ m/s.} \end{aligned}$$

(iv) Minimum specific energy,

$$\begin{aligned} E_{\min} &= \frac{3}{2} h_c \\ &= \frac{3}{2} \times 0.691 \\ &= 1.0365 \text{ m.} \end{aligned}$$

**Example 4**

A 6 m wide rectangular channel with bed slope 0.001 conveys water at a depth of 2.2 m. Determine the width to be provided in the transition to obtain critical depth.

(Take manning's coefficient  $N = 0.018$ )

**Solution**

$$B_1 = 6 \text{ m}$$

$$h_1 = 2.2 \text{ m}$$

$$N = 0.018$$

$$i = 0.001$$

$$A_1 = B_1 h_1 = 6 \times 2.2$$

$$= 13.2 \text{ m}^2$$

$$P = 6 + 2 \times 2.2 = 10.4 \text{ m}$$

Hydraulic mean depth,

$$m = \frac{A}{P} = \frac{13.2}{10.4}$$

$$= 1.2692 \text{ m}$$

$$v_1 = \frac{1}{N} m^{2/3} i^{1/2}$$

$$= \frac{1}{0.018} \times 1.2692^{2/3} \times 0.001^{1/2}$$

$$= 2.0594 \text{ m/s}$$

$$\text{Specific energy, } E_1 = h_1 + \frac{V_1^2}{2g}$$

$$= 2.2 + \frac{(2.0594)^2}{2 \times 9.8}$$

$$= 2.416 \text{ m}$$

Since, bed level is not changed,

$$E_2 = E_1 = 2.416 \text{ m}$$

Since, at the reduced section critical flow is formed.

$$E_2 = E_c = 2.416 \text{ m.}$$

$$h_2 = h_c = \frac{2}{3} E_c$$

$$= \frac{2}{3} \times 2.416 = 1.61 \text{ m}$$

$$v_2 = v_c = \frac{\theta}{\tan \theta}$$

$$= \sqrt{9.81 \times 1.61}$$

$$= 3.975 \text{ m/s}$$

$$= \frac{Q}{A_2} = \frac{A_1 v_1}{B_2 h_2}$$

$$3.975 = \frac{13.2 \times 2.0594}{B_2 \times 1.61}$$

$$\Rightarrow B_2 = 4.2477 \text{ m.}$$

**Example 5**

Water flows through a rectangular channel of uniform cross-section. At inlet the velocity is 6.5 m/s and depth is 65 cm. Determine

- whether the flow is subcritical or super critical.
- specific head at critical condition.

**Solution**

$$\text{(i) } v_1 = 6.5 \text{ m/s}$$

$$h_1 = 65 \text{ cm} = 0.65 \text{ m}$$

Froude number,

$$\begin{aligned} Fr_1 &= \frac{v_1}{\sqrt{gh_1}} = \frac{6.5}{\sqrt{9.81 \times 0.65}} \\ &= 2.574 \end{aligned}$$

Since,  $Fr_1 > 1$  the flow is super critical.

(ii) Since the channel is of uniform cross-section.

$$\begin{aligned} v_1 h_1 &= v_c h_c \\ &= 6.5 \times 0.65 \\ &= 4.225 \end{aligned}$$

At critical condition,

$$\begin{aligned} Fr_c &= 1 = \frac{V_c}{\sqrt{gh_c}} \\ \Rightarrow V_c^2 &= gh_c \\ \Rightarrow V_c^3 &= gh_c v_c \\ &= 9.81 \times 4.225 \\ &= 41.44725 \\ \Rightarrow V_c &= 3.46 \text{ m/s} \\ h_c &= \frac{4.225}{V_c} \\ &= \frac{4.225}{3.96} \\ &= 1.221 \text{ m} \end{aligned}$$

Specific energy,

$$\begin{aligned} E_c &= \frac{3}{2} h_c \\ &= \frac{3}{2} \times 1.221 \\ &= 1.8316 \text{ m.} \end{aligned}$$

### Example 6

A trapezoidal channel has bottom width = 6 m and side slope = 2, horizontal to 1 vertical. When depth of water is 1 m, the flow rate is  $10.4 \text{ m}^3/\text{s}$ . Specific energy of the flow is \_\_\_\_\_.

### Solution

Cross-sectional area of flow,

$$\begin{aligned} A &= 6 \times 1 + 2 \times \left( \frac{1}{2} \times 2 \times 1 \right) \\ &= 6 + 2 = 8 \text{ m}^2 \\ v &= \frac{Q}{A} = \frac{10.4}{8} = 1.3 \text{ m/s} \end{aligned}$$

Specific energy,

$$\begin{aligned} E &= h + \frac{v^2}{2g} \\ &= 1 + \frac{(1.3)^2}{2 \times 9.81} \\ &= 1.086 \text{ m.} \end{aligned}$$

### Example 7

Specific energy of a flow in a 4 m wide rectangular channel is  $3.5 \text{ Nm/N}$ . Value of maximum possible discharge (in  $\text{m}^3/\text{s}$ ) is \_\_\_\_\_.

### Solution

Width of channel  $b = 4 \text{ m}$

Specific energy  $E = 3.5 \text{ Nm/N}$

Discharge will be maximum when the depth of flow is critical,

$$\begin{aligned} h_c &= \frac{2}{3} E \\ &= \frac{2}{3} \times 3.5 = \frac{7}{3} \text{ m} \\ v_c &= \sqrt{gh_c} \\ &= \sqrt{9.81 \times \frac{7}{3}} \\ &= 4.784 \text{ m/s} \\ Q_{\max} &= V_c \times A \\ &= 4.784 \times 4 \times \frac{7}{3} \\ &= 44.65 \text{ m}^3/\text{s.} \end{aligned}$$

### Example 8

Depth and velocity of a stream of water are 90 cm and 6 m/s respectively. Is hydraulic jump possible in this case? If so determine values of depth, velocity and loss of specific energy after the jump.

### Solution

Froude number,

$$\begin{aligned} Fr_1 &= \frac{v_1}{\sqrt{gh_1}} \\ &= \frac{6}{\sqrt{9.81 \times 0.9}} \\ &= 2.0193 \end{aligned}$$

Since  $Fr_1 > 1$ , hydraulic jump is possible.

$$\begin{aligned} h_2 &= \frac{h_1}{2} (\sqrt{1 + 8Fr_1^2} - 1) \\ &= \frac{0.9}{2} [\sqrt{1 + 8 \times (2.0193)^2} - 1] \\ &= 0.45 [5.798 - 1] \\ &= 2.1592 \text{ m} \\ v_2 h_2 &= v_1 h_1 \\ v_2 \times 2.1592 &= 6 \times 0.9 \\ \Rightarrow v_2 &= 2.5009 \text{ m/s} \\ E_1 &= h_1 + \frac{v_1^2}{2g} \\ &= 0.9 + \frac{6^2}{2 \times 9.81} \\ &= 2.735 \text{ m} \\ E_2 &= h_2 + \frac{v_2^2}{2g} \end{aligned}$$

$$= 2.1592 + \frac{(2.5009)^2}{2 \times 9.81}$$

$$= 2.478 \text{ m}$$

Loss of specific energy,

$$E_1 - E_2 = 2.735 - 2.478$$

$$= 0.257 \text{ m of water.}$$

### Example 9

A rectangular channel 6 m wide conveys water at 12 m<sup>3</sup>/s at a depth of 30 cm. If a hydraulic jump occurs, determine the depth of flow after the jump.

### Solution

$$Q = 12 \text{ m}^3/\text{s}$$

$$b = 6 \text{ m}$$

$$h_1 = 30 \text{ cm} = 0.3 \text{ m}$$

$$h_2 = \left[ -h_1 + \sqrt{h_1^2 + \frac{8Q^2}{gh_1b^2}} \right] \times \frac{1}{2}$$

$$= \left[ \frac{-0.3}{2} + \sqrt{(0.3)^2 + \frac{8 \times (12)^2}{9.81 \times 0.3 \times 6^2}} \right] \times \frac{1}{2}$$

$$= [-0.15 + 3.311] \times \frac{1}{2}$$

$$= 1.58 \text{ m.}$$

### Example 10

A sluice spans the entry of a 3.5 m wide rectangular channel and admits 5.5 m<sup>3</sup>/s of water with a uniform velocity of 4.2 m/s. Assuming that conditions for a hydraulic jump exist, determine the depth of flow, height of jump after the jump and rate of dissipation of energy in the jump.

### Solution

$$Q = V_1 \times bh_1$$

or 
$$h_1 = \frac{Q}{V_1 b}$$

$$= \frac{5.5}{4.2 \times 3.5}$$

$$= 0.374 \text{ m}$$

Froude number,

$$Fr_1 = \frac{V}{\sqrt{gh_1}}$$

$$= \frac{4.2}{\sqrt{9.81 \times 0.374}}$$

$$= 2.193$$

For hydraulic jump to occur, depth of flow after jump,

$$h_2 = \frac{h_1}{2} \left[ \sqrt{1 + 8(Fr_1)^2} - 1 \right]$$

$$= \frac{0.374}{2} \left[ \sqrt{1 + 8(2.193)^2} - 1 \right]$$

$$= 0.9879 \text{ m}$$

Height of jump =  $h_2 - h_1$

$$= 0.9879 - 0.374$$

$$= 0.6139 \text{ m}$$

Head loss in the jump,

$$h_j = \frac{(h_2 - h_1)}{4h_1h_2}$$

$$= \frac{(0.6139)^3}{4 \times 0.374 \times 0.9879}$$

$$= 0.15655 \text{ m}$$

Rate of dissipation of energy in the jump

$$= \rho g Q h_j$$

$$= 9810 \times 5.5 \times 0.15655 \text{ W}$$

$$= 8446.6 \text{ W}$$

$$= 8.45 \text{ kW.}$$

### Example 11

In a rectangular channel of 12 m width and 3.7 m depth, water is flowing with a velocity of 1.2 m/s. The channel has a bed slope of 1 in 4000. If the flow of water through the channel is regulated in such a way that slope of the energy line is 0.00004, rate of change of depth of water in the channel is \_\_\_\_\_.

### Solution

Width of channel,

$$b = 12 \text{ m}$$

Depth of channel,

$$h = 3.7 \text{ m}$$

Velocity of flow,

$$V = 1.2 \text{ m/s}$$

Bed slope,

$$i = \frac{1}{4000}$$

$$= 0.00025$$

Slope of energy line  $j = 0.00004$

Rate of change of depth of water,

$$\frac{dh}{dl} = \frac{i - j}{1 - (Fr)^2} = \frac{i - j}{\left(1 - \frac{V^2}{gh}\right)}$$

$$= \frac{0.00025 - 0.00004}{1 - \frac{1.2^2}{9.81 \times 3.7}} = \frac{0.00021}{0.9603}$$

$$= 0.0002187.$$

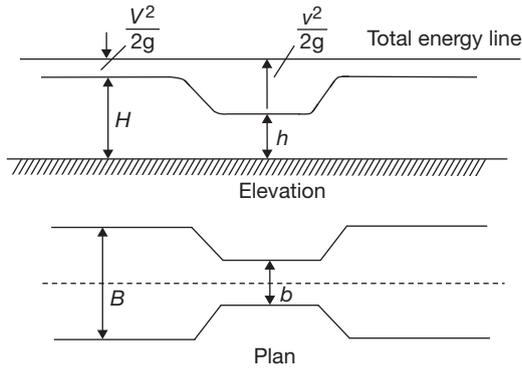
## GAUGING FLUMES

Gauging flume is used for measuring discharge in a channel. A zone with restricted width is provided in a gauging flume

for this purpose. There are two types of gauging flumes. These are:

1. Non-modular flume or venturi flume
2. Modular flume or standing wave flume.

## NON-MODULAR FLUME OR VENTURI FLUME



$B$ ,  $H$  and  $V$  are normal breadth, depth of flow and velocity at entrance to the flume.

$b$ ,  $h$  and  $v$  are breadth, depth of flow and velocity at the throat.

Applying continuity equation,

$$AV = av$$

Where,  $A = BH$  and  $a = bh$

$$\therefore V = \frac{a}{A}v$$

Applying Bernoulli's equation,

$$H + \frac{V^2}{2g} = h + \frac{v^2}{2g}$$

$$\text{or } \frac{v^2 - V^2}{2g} = H - h$$

$$\text{or } v^2 \left[ 1 - \frac{a^2}{A^2} \right] = 2g(H - h)$$

$$\therefore v = \frac{A}{\sqrt{A^2 - a^2}} \sqrt{2g(H - h)}$$

$$\text{Discharge, } Q = av = \frac{Aa}{\sqrt{A^2 - a^2}} \sqrt{2g(H - h)}$$

$$\text{Actual discharge} = C_d \frac{Aa}{\sqrt{A^2 - a^2}} \sqrt{2g(H - h)}$$

Where,  $C_d$  = Coefficient of discharge.

## MODULAR FLUME OR STANDING WAVE FLUME

Modular flume or standing wave flume is similar to non-modular flume. But in this case hydraulic jump or standing

wave is formed in the flume. Depth of flow at the throat region will become less than the critical depth. Hence, at some section in the throat the depth of flow will be equal to critical depth.

Considering upstream section and a throat section.

$$H + \frac{V^2}{2g} = h + \frac{v^2}{2g} = H_1$$

$$\therefore v = \sqrt{2g(H_1 - h)}$$

$$\text{Discharge, } Q = av = bhv = bh\sqrt{2g(H_1 - h)} \quad (1)$$

From the above, maximum discharge occur when,  $h = \frac{2}{3}H_1$ .

For this condition velocity,  $v = \sqrt{gh}$

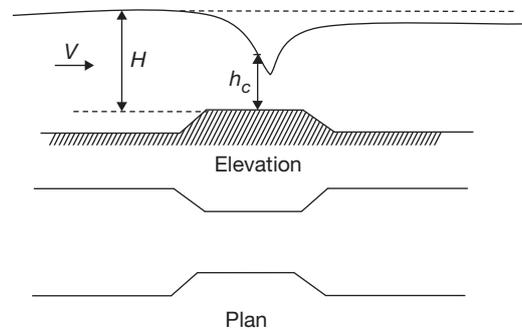
From the above, maximum discharge occur when depth of flow at throat is equal to the critical depth.

Substituting  $h = \frac{2}{3}H_1$  in Eq. (1), we get

$$Q = \frac{2}{3\sqrt{3}} \sqrt{2gb} H_1^{3/2} = 1.705b \left( H + \frac{V^2}{2g} \right)^{3/2}$$

To ensure occurrence of maximum discharge at the throat, throat section is considerably reduced. Generally a hump also is provided at the throat to further reduce the flow section.

Figure shows modular flume provided with a hump.



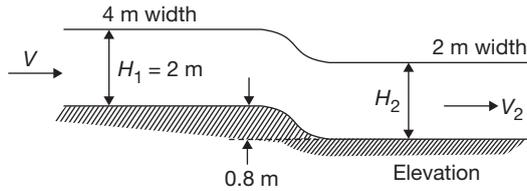
Discharge in this case also is given by,

$$Q = 1.705b \left( H + \frac{V^2}{2g} \right)^{3/2}$$

But  $H$  is measured from the top of hump.

### Example 12

A rectangular channel of width 4 m have a discharge of 12 m<sup>3</sup>/s through it. Depth of flow is 2 m. If the width of channel is reduced 2 m and bed level is lowered by 0.8 m, difference of water level between upstream and downstream sections is \_\_\_\_\_ (loss of energy may be ignored).

**Solution**

**Velocity at upstream section**

Velocity at upstream section,

$$\begin{aligned} V_1 &= \frac{Q}{B_1 H_1} \\ &= \frac{12}{4 \times 2} = 1.5 \text{ m/s} \end{aligned}$$

Velocity at downstream section,

$$\begin{aligned} V_2 &= \frac{Q}{B_2 H_2} \\ &= \frac{12}{2 \times H_2} = \frac{6}{H_2} \end{aligned}$$

Applying Bernoulli's equation,

$$\begin{aligned} 0.8 + 2 + \frac{V_1^2}{2g} &= H_2 + \frac{V_2^2}{2g} \\ \Rightarrow 2.8 + \frac{(1.5)^2}{2 \times 9.81} \\ &= H_2 + \left(\frac{6}{H_2}\right)^2 \times \frac{1}{2 \times 9.81} \\ \Rightarrow H_2 + \frac{1.83486}{H_2^2} &= 2.9147 \\ \Rightarrow H_2 &= 2.654 \text{ m} \end{aligned}$$

 $\therefore$  Water level difference

$$\begin{aligned} &= 0.8 + 2 - 2.654 \\ &= 0.146 \text{ m.} \end{aligned}$$

**Example 13**

A rectangular channel with 2 m width is narrowed to 1 m width to form a throat region. The depths of flow at the upstream and throat section are 1 m and 0.9 m respectively. The channel bed is horizontal.

- Neglecting losses determine discharge through the channel
- If a hump of 0.3 m height is provided at the throat, so that a standing wave is produced, determine the depth of flow at the upstream side for the same discharge.

**Solution**

$$\begin{aligned} \text{(i) } A &= BH = 2 \times 1 = 2 \text{ m}^2 \\ a &= bh = 1 \times 0.9 = 0.9 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \sqrt{A^2 - a^2} &= \sqrt{2^2 - 0.9^2} \\ &= 1.786 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Discharge} &= \frac{Aa}{\sqrt{A^2 - a^2}} \times \sqrt{2g(H-h)} \\ &= \frac{2 \times 0.9}{1.786} \times \sqrt{2 \times 9.81(1 - 0.9)} \\ &= 1.4117 \text{ m}^3/\text{s.} \end{aligned}$$

(ii) The flume is a standing wave flume.

 Discharge,  $Q = 1.4117 \text{ m}^3/\text{s}$ 

Discharge/unit width at throat

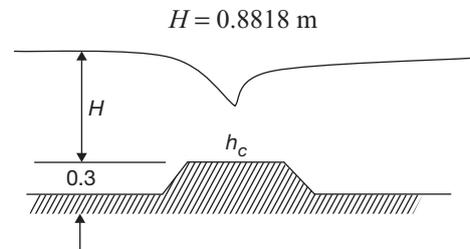
$$q_c = \frac{1.4117}{1} = 1.4117 \text{ m}^3/\text{s per m}$$

Depth of flow at throat,

$$\begin{aligned} h_c &= \sqrt[3]{\frac{q_c^2}{g}} \\ &= \sqrt[3]{\frac{1.4117^2}{9.81}} \\ &= 0.58786 \text{ m} \end{aligned}$$

$$\begin{aligned} H_1 &= \frac{3}{2} \times h_c \\ &= \frac{3}{2} \times 0.58786 \\ &= 0.8818 \text{ m} \end{aligned}$$

$$\therefore H + \frac{V^2}{2g} = 0.8818$$

 Ignoring  $\frac{V^2}{2g}$ ,


$$\begin{aligned} V &= \frac{Q}{B(H + 0.3)} \\ &= \frac{1.4117}{2(1 + 0.3)} \\ &= 0.5430 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \frac{V^2}{2g} &= \frac{0.543^2}{2 \times 9.81} \\ &= 0.015 \text{ m} \end{aligned}$$

 $\therefore$  More correct value of

$$\begin{aligned} H &= 0.8818 - 0.015 \\ &= 0.8668 \end{aligned}$$

$$\begin{aligned}
 V &= \frac{1.4117}{2(0.8668 + 0.3)} \\
 &= 0.6049 \text{ m/s} \\
 \frac{V^2}{2g} &= 0.01865 \text{ m}
 \end{aligned}$$

Still more correct value of

$$\begin{aligned}
 H &= 0.8818 - 0.01865 \\
 &= 0.86315 \text{ m}
 \end{aligned}$$

∴ Depth of flow at upstream

$$\begin{aligned}
 &= 0.86315 + 0.3 \\
 &= 1.16315 \text{ m.}
 \end{aligned}$$

## FLOW MEASUREMENTS IN CHANNEL AND PIPES

Notches and weirs are used for flow measurements in channels. orificemeters, venturimeters, etc., are used for flow measurement in pipes.

### Notches and Weirs

A notch is an opening provided on the side wall of a tank or vessel such that liquid surface is below the top edge of the opening. (In the case of an orifice, the water surface is above the upper edge). It is used to measure the rate of flow (or discharge) from a tank or through a small channel. It is generally made of metallic plate.

A weir is a regular obstruction in an open stream over which the flow takes place. A weir is a notch constructed in large scale. It is made of masonry or concrete. Conditions of flow are generally same as in the case of a notch. Therefore notch is sometimes called as a weir and vice versa.

Weirs are used to measure flow in large channels, rivers or streams. In a dam the surplus quantity of water is discharged through a weir.

The sheet of water discharged through a notch or weir is called **nappe** or **vein**. The upper surface of the notch or weir over which water flows is called crest or sill.

### Classification

Depending upon the shape, important types of notches are:

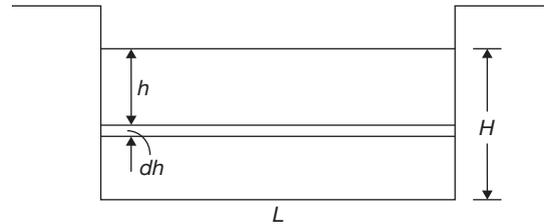
1. Rectangular notch
2. Triangular notch or V-notch
3. Trapezoidal notch
4. Stepped notch

Main types of weirs are:

1. According to shape:
  - (a) Rectangular weir
  - (b) Cippoletti weir
2. According to nature of discharge:
  - (a) Ordinary weir
  - (b) Submerged or drowned weir.

3. According to width of crest:
  - (a) Narrow crested weir.
  - (b) Broad crested weir.
4. According to nature of crest:
  - (a) Sharp crested weir
  - (b) Ogee weir.

### Rectangular Notch



Velocity of water flowing through the elemental strip at a depth  $h$  from the water surface =  $\sqrt{2gh}$ .

Discharge through the elemental strip  $dQ = Ldh \sqrt{2gh}$ .

Total discharge (theoretical)

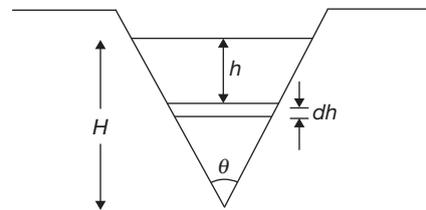
$$\begin{aligned}
 &= \int_0^H L\sqrt{2gh} \, dh \\
 &= \frac{2}{3} L\sqrt{2g} H^{3/2}.
 \end{aligned}$$

Actual discharge,

$$Q = C_d \frac{2}{3} L\sqrt{2g} H^{3/2}.$$

Where,  $C_d$  = Coefficient of discharge.

### Triangular Notch or V-notch



Width of notch at any depth  $h$  from water surface,

$$= 2(H - h) \tan \frac{\theta}{2}$$

Where,  $\theta$  = Angle of the notch.

Theoretical discharge through the elemental strip at a depth  $h = 2(H - h) \tan \frac{\theta}{2} dh \sqrt{2gh}$ .

Actual total discharge,

$$\begin{aligned}
 Q &= C_d 2(H - h) \tan \frac{\theta}{2} \sqrt{2g} \int_0^H \sqrt{h} \, dh \\
 &= C_d \frac{8}{15} \sqrt{2g} \tan \frac{\theta}{2} H^{5/2}.
 \end{aligned}$$

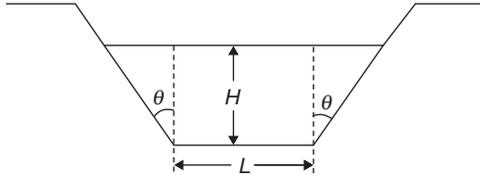
When  $\theta = 90^\circ$ ,

$$Q = C_d \frac{8}{15} \sqrt{2g} H^{5/2}$$

Taking  $C_d = 0.6$ ,

$$Q = 1.417 H^{5/2}, \text{ where } g = 9.81.$$

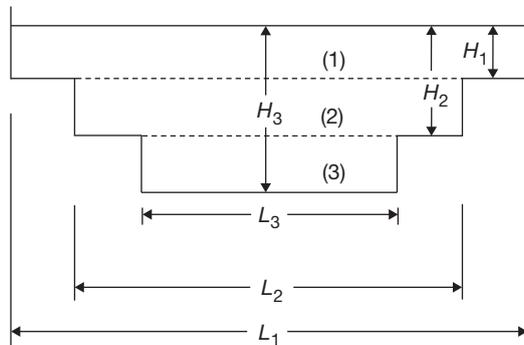
### Trapezoidal Notch



Discharge through a trapezoidal notch can be considered as the sum of a rectangular notch of length  $L$  and a triangular notch with angle of notch  $= 2\theta$ .

$$\begin{aligned} \therefore Q &= \frac{2}{3} C_d L \sqrt{2g} H^{3/2} + \frac{8}{15} C_d \sqrt{2g} \tan \theta \cdot H^{5/2} \\ &= C_d \sqrt{2g} H^{3/2} \left[ \frac{2}{3} L + \frac{8}{15} \tan \theta H \right]. \end{aligned}$$

### Stepped Notch



A stepped notch can be considered as a combination of two or more rectangular notches. For the notch with 3 steps as shown in the figure,

Discharge  $Q =$  Discharge  $Q_1$  through the top rectangular portion + Discharge  $Q_2$  through the middle rectangular portion + Discharge  $Q_3$  through the bottom rectangular portion.

$$\text{That is, } Q = Q_1 + Q_2 + Q_3$$

$$\text{Where, } Q_1 = \frac{2}{3} C_d L_1 \sqrt{2g} H_1^{3/2}$$

$$Q_2 = \frac{2}{3} C_d L_2 \sqrt{2g} [H_1^{3/2} - H_2^{3/2}]$$

$$Q_3 = \frac{2}{3} C_d L_3 \sqrt{2g} [H_3^{3/2} - H_2^{3/2}]$$

### Velocity of Approach

The initial velocity of a stream approaching a weir is called velocity of approach. A weir provided across a river is having a velocity of approach.

Discharge over a weir is increased due to velocity of approach.

Let  $V_a$  be the velocity of approach. The head corresponding to this,

$$h_a = \frac{V_a^2}{2g}$$

When there is a velocity of approach, discharge is given by

$$Q = \frac{2}{3} C_d L \sqrt{2g} [(H + h_a)^{3/2} - h_a^{3/2}]$$

$(H + h_a)$  is called still water head.

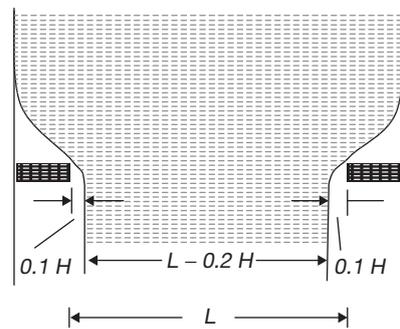
### Empirical Formulae

Francis formula, Bazin's formula and Rehbock formula are 3 important formulae used in measurement of discharge over weirs.

When the length of the weir is less than the length of the stream, effective length of weir is less than the actual length. This is due to the effect of end contraction.

When crest length is equal to width of the channel, the weir is called a **suppressed weir**.

### Francis Formula



Francis on the basis of his experiments found that the crest length is to be reduced by  $0.1 H$  for an end contraction, to get the effective head.

For a rectangular weir, with two end contractions the effective length of the weir

$$= L - 2 \times 0.1 H = L - 0.2 H$$

$$\therefore Q = \frac{2}{3} C_d (L - 0.2H) \sqrt{2g} H^{3/2}$$

If there are obstacles in the stream path, there can be more than 2 end contractions.

$$\therefore Q = \frac{2}{3} C_d (L - n \times 0.1H) \sqrt{2g} H^{3/2}$$

When,  $C_d = 0.623$ ,  $g = 9.81 \text{ m/s}^2$  and end contractions are two, then

$$Q = 1.84 (L - 0.2H) H^{3/2}$$

When end contractions are suppressed, then

$$Q = 1.84 L H^{3/2}$$

When velocity of approach is considered,

$$Q = 1.84(L - n \times 0.1 (H + h_a)) \times [(H + h_a)^{3/2} - h_a^{3/2}].$$

### Bazin's Formula

According to Bazin, discharge over a rectangular weir is given by:

$$Q = mL\sqrt{2g} H^{3/2}$$

$$\text{Where, } m = \frac{2}{3} C_d = 0.405 + \frac{0.003}{H}$$

When velocity of approach is considered, the formula becomes,

$$Q = \left(0.405 + \frac{0.003}{H_1}\right) L\sqrt{2g} H_1^{3/2}$$

Where

$$H_1 = H + \alpha \frac{V_a^2}{2g}$$

$V_a$  = Mean velocity of approach

$\alpha$  = Coefficient equal to 1.6

### Rehbock Formula

As per Rehbock formula, discharge without velocity of approach.

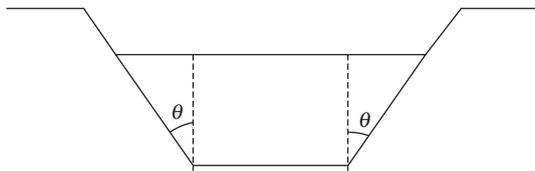
$$Q = \left(1.782 + 0.24 \frac{H_e}{z}\right) LH_e^{3/2}$$

Where

$$H_e = H + 0.0011 \text{ m}$$

$z$  = Height of the crest above the bed of the stream.

### Cippoletti Weir



Cippoletti weir is a trapezoidal weir where  $\tan \theta = \frac{1}{4}$  or  $\theta = 14^\circ 2'$ .

It can be seen that due to two end contractions, the discharge over a rectangular weir is reduced by  $\frac{2}{15} C_d \sqrt{2g} H^{5/2}$

when Francis formula is applied. This loss is compensated by providing slopes of  $14^\circ 2'$  in Cippoletti weir or discharge through the side portions of the Cippoletti weir

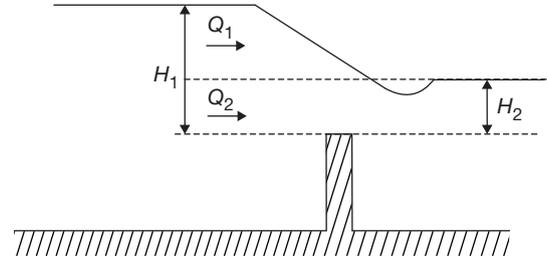
$$= \frac{2}{15} C_d \sqrt{2g} H^{5/2}$$

So avoiding end contraction,

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}.$$

### Submerged Weir

In the case of a submerged weir, water level on the downstream side also is above the crest of the weir.



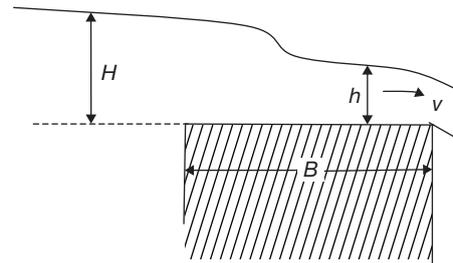
Let  $H_1$  and  $H_2$  be the heights of water level above crest level at upstream and downstream respectively. The total discharge consists of two parts  $Q_1$  and  $Q_2$  where  $Q_1$

$$= \text{Discharge due to a weir action} = \frac{2}{3} C_{d1} L \sqrt{2g} (H_1 - H_2)^{3/2}$$

And  $Q_2$  = discharge through a drowned orifice of area  $(L \times H_2)$  and head  $(H_1 - H_2) = C_{d2} L H_2 \sqrt{2g(H_1 - H_2)}$ .

### Broad Crested Weir

When the crest of the weir is very broad, the flow can be compared to a flow in a channel



Applying Bernoulli's theorem,

$$H = h + \frac{v^2}{2g}$$

$$\therefore v = \sqrt{2g(H - h)}$$

Theoretical discharge = Area  $\times$  Velocity

$$= Lh\sqrt{2g(H - h)}$$

Actual discharge

$$Q = C_d L h \sqrt{2g(H - h)} \\ = C_d L \sqrt{2g} \sqrt{Hh^2 - h^3}$$

Discharge is maximum when  $(Hh^2 - h^3)$  is maximum.

$$\text{or } \frac{d(Hh^2 - h^3)}{dh} = 0$$

That is,  $2Hh - 3h^2 = 0$

or 
$$h = \frac{2}{3}H$$

or 
$$\begin{aligned} Q_{\max} &= C_d L \frac{2}{3} H \sqrt{2g \left( H - \frac{2}{3} H \right)} \\ &= C_d L \frac{2}{3} H \sqrt{2g \frac{H}{3}} \\ &= \frac{2}{3\sqrt{3}} \sqrt{2g} C_d L H^{3/2} \\ &= 1.705 C_d L H^{3/2}. \end{aligned}$$

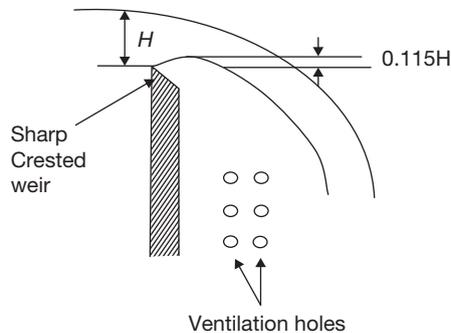
### Narrow Crested Weir

In the case of narrow crested weir  $2B < H$  and discharge is given by rectangular notch formula,

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}.$$

In the case of a sharp crested weir, the nappe rises slightly at the lower surface as it leaves the crest.

A maximum rise of  $0.115 H$  is reached, where  $H$  is the head over the crest.



The nappe should be clear and springing up for the derived equations to be valid. In the case of a suppressed nappe there is tendency of vacuum generation under the nappe and the nappe become depressed or clinging to the wall of the weir. To avoid this ventilation or aeration holes are provided.

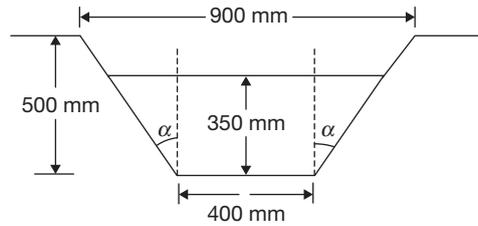
### Ogee Weir

If the space under the free springing up nappe of a sharp crested weir is filled with masonry or concrete, it becomes an ogee weir. Thus the surface of an ogee weir has the profile of the bottom of a free nappe. The discharge is same as in the case of a sharp crested weir.

That is, 
$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}.$$

### Example 14

A sharp edged weir has the shape of a symmetrical trapezium with base width = 400 mm, top width = 900 mm and height 500 mm as shown in the figure.



If head of water over the horizontal crest is 350 mm; discharge over the weir is \_\_\_\_\_. (Assume  $C_d = 0.6$ )

### Solution

$$\tan \alpha = \frac{(900 - 400)}{2(500)} = 0.5$$

$$\theta = 2\alpha$$

$$L = 400 \text{ mm} = 0.4 \text{ m}$$

$$H = 350 \text{ mm} = 0.35 \text{ m}$$

$$C_d = 0.6$$

Discharge,

$$\begin{aligned} Q &= \frac{2}{3} C_d L \sqrt{2g} H^{3/2} + \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} H^{5/2} \\ &= C_d \sqrt{2g} \left[ \frac{2}{3} L H^{3/2} + \frac{8}{15} \tan \alpha H^{5/2} \right] \\ &= 0.6 \times 4.429 \left[ \frac{2}{3} \times 0.4 \times 0.35^{3/2} + \frac{8}{15} \times (0.5) \times 0.35^{5/2} \right] \\ &= 2.6574 [0.0552 + 0.0193] \\ &= 0.198 \text{ m}^3/\text{s}. \end{aligned}$$

### Example 15

Discharge over a rectangular notch is  $0.15 \text{ m}^3/\text{s}$  when the water level is 260 mm above the sill. If coefficient of discharge is 0.62, length of the notch (in m) is \_\_\_\_\_.

### Solution

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

$$0.15 = \frac{2}{3} \times 0.62 \times L \times 4.429 \times 0.26^{1.5}$$

$$\Rightarrow L = 0.618 \text{ m}.$$

### Example 16

Water flows in a rectangular channel at a depth of 0.3 m. Velocity of flow is 0.85 m/s. A sharp crested weir is to be installed in the channel so that the upstream depth is raised to 1.2 m. Neglecting velocity of approach and assuming  $C_d = 0.6$ , height required for the weir (in m) is \_\_\_\_\_.

**Solution**

Length of the weir,

 $L = \text{Width of channel.}$ 

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

 $= L \times \text{Original depth} \times \text{Original velocity}$ 

$$\Rightarrow \frac{2}{3} \times 0.6 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$= 0.3 \times 0.85$$

$$\Rightarrow H^{3/2} = 0.14394$$

$$\Rightarrow H = (0.14394)^{2/3}$$

$$= 0.27466 \text{ m} = 0.275 \text{ m}$$

 Height of the weir =  $1.2 - 0.275$ 

$$= 0.925 \text{ m.}$$

**Example 17**

Discharge measured using a triangular weir at a head of 65 mm is  $0.004 \text{ m}^3/\text{s}$ . When it is discharging  $0.5 \text{ m}^3/\text{s}$ , the width of water surface (in metre) is \_\_\_\_\_. (Assume  $C_d = 0.6$ )

**Solution**

Discharge,

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} H^{5/2}$$

$$0.004 = \frac{8}{15} \times 0.6 \times 4.429 \times \tan \frac{\theta}{2} \times (0.065)^{5/2}$$

$$\Rightarrow \tan \frac{\theta}{2} = 2.62.$$

For the notch,

$$Q \propto H^{5/2}$$

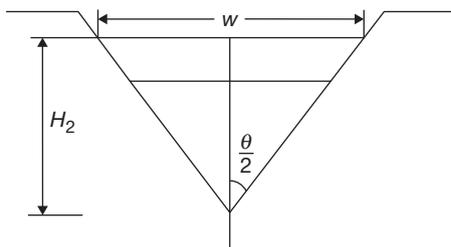
$$\therefore \frac{Q_2}{Q_1} = \left( \frac{H_2}{H_1} \right)^{5/2}$$

 When discharge,  $Q_2 = 0.5 \text{ m}^3/\text{s}$ 

$$\frac{0.5}{0.004} = \left( \frac{H_2}{0.065} \right)^{5/2}$$

$$\Rightarrow \frac{H_2}{0.065} = 125^{0.4}$$

$$\Rightarrow H_2 = 0.4484 \text{ m.}$$

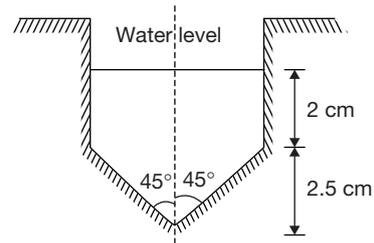

 Let  $w$  be the width of water surface at head  $H_2$ ,

$$\tan \frac{\theta}{2} = \frac{w/2}{H_2}$$

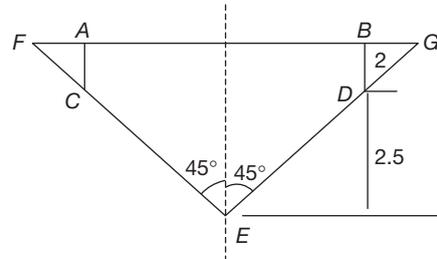
$$\Rightarrow 2.62 = \frac{w}{2 \times 0.4484}$$

$$\Rightarrow w = 2.3496$$

$$= 2.35 \text{ m.}$$

**Example 18**


Assuming coefficient of discharge  $C_d = 0.62$ , determine the discharge over the notch shown in the figure.

**Solution**


Discharge = Discharge through  $FGE - 2 \times$  Discharge through

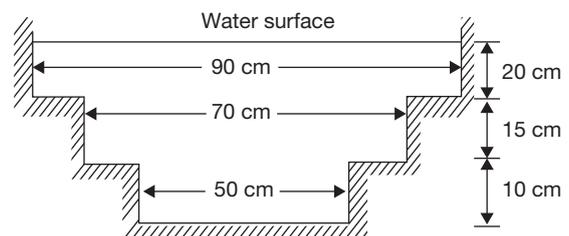
$$BGD = \frac{8}{15} C_d \times \sqrt{2g} \tan \frac{\theta}{2} [H_1^{5/2} - 2 \times H_2^{5/2}]$$

$$= \frac{8}{15} \times 0.62 \times 4.429 \times \tan 45 [4.5^{2.5} - 2 \times 2^{2.5}]$$

$$= 46.34 \text{ m}^3/\text{s.}$$

**Example 19**

For the stepped notch as shown in the figure, determine discharge in  $\text{m}^3/\text{s}$ . (Assume  $C_d = 0.62$ )



**Solution**

Discharge,

$Q$  = Discharge through top portion of dimension 90 cm × 20 cm + Middle portion of dimension 70 cm × 15 cm + Bottom portion of dimension 50 cm × 10 cm

$$\begin{aligned} &= \frac{2}{3} C_d \sqrt{2g} \left[ L_1 \times H_1^{3/2} + L_2 \times \right. \\ &\quad \left. (H_2^{3/2} - H_1^{3/2}) + L_3 \times (H_3^{3/2} - H_2^{3/2}) \right] \\ &= \frac{2}{3} \times 0.62 \times 4.429 \left[ 0.9 \times 0.2^{1.5} \right. \\ &\quad \left. + 0.7 \times (0.35^{1.5} - 0.2^{1.5}) \right. \\ &\quad \left. + 0.5 \times (0.45^{1.5} - 0.35^{1.5}) \right] \\ &= 1.831 [0.0805 + 0.08233 + 0.0474] \\ &= 0.3849 \text{ m}^3/\text{s}. \end{aligned}$$

**Alternative solution:**

$$\begin{aligned} Q &= \frac{2}{3} C_d \sqrt{2g} \left[ 0.5 \times 0.45^{1.5} + (0.7 - 0.5) \times 0.35^{1.5} \right. \\ &\quad \left. + (0.9 - 0.7) \times 0.2^{1.5} \right] \\ &= 1.831 [0.1509 + 0.0414 + 0.0179] \\ &= 0.3849 \text{ m}^3/\text{s}. \end{aligned}$$

**Example 20**

A 2 m long rectangular weir is used to measure discharge of water from a tank.

If head over the weir is 0.7 m, determine the discharge in  $\text{m}^3/\text{s}$  using Francis formula

- when the weir is a suppressed weir.
- when the weir is with 2 end contractions.

**Solution**

- Suppressed weir,

$$\begin{aligned} Q &= 1.84 LH^{3/2} \\ &= 1.84 \times 2 \times 0.7^{3/2} \\ &= 2.155 \text{ m}^3/\text{s}. \end{aligned}$$

- Contracted weir,

$$\begin{aligned} Q &= 1.84 [L - 0.2H] H^{3/2} \\ &= 1.84 \times [2 - 0.2 \times 0.7] 0.7^{3/2} \\ &= 2.0044 \text{ m}^3/\text{s}. \end{aligned}$$

**Example 21**

A suppressed rectangular weir of length is divided into 3 bays, using two vertical posts of width 250 mm each. If head of water over the weir is 0.4 metre, determine the discharge using Francis formula.

**Solution**

Being originally suppressed,  $n = 3 \times 2 - 2 = 4$

$$L = 9.5 - 2 \times 0.25 = 9 \text{ m}$$

$$H = 0.4 \text{ m}.$$

$$\begin{aligned} Q &= 1.84 (L - 0.1 nH) H^{3/2} \\ &= 1.84 (9 - 0.1 \times 4 \times 0.4) \times 0.4^{1.5} \\ &= 4.115 \text{ m}^3/\text{s}. \end{aligned}$$

**Example 22**

Maximum discharge of a stream is 1800  $\text{m}^3/\text{s}$  which has to pass over a weir. A bridge provided over the weir divides the weir into a number of openings of span 6 m each. If head over the crest of the weir is not to exceed 3.5 m and if there is a velocity of approach 3 m/s, the number of openings needed is \_\_\_\_\_.

**Solution**

Considering one opening,

$$L = 6 \text{ m}$$

$$H = 3.5 \text{ m}$$

$$V_a = 3 \text{ m/s}$$

$$h_a = \frac{V_a^2}{2g} = \frac{3^2}{2 \times 9.81}$$

$$= 0.4587 \text{ m}$$

Discharge through one opening,

$$q = 1.84 (L - 0.2H_1) (H_1^{3/2} - h_a^{3/2})$$

Where,

$$\begin{aligned} H_1 &= H + h_a \\ &= 3.5 + 0.4587 \\ &= 3.9587 \text{ m}. \end{aligned}$$

$$H_1^{3/2} = (3.9587)^{1.5} = 7.8764$$

$$h_a^{3/2} = (0.4587)^{1.5} = 0.311$$

$$\begin{aligned} \therefore q &= 1.84 (6 - 0.2 \times 3.9587) \times (7.8764 - 0.311) \\ &= 1.84 \times 5.2083 \times 7.5654 \\ &= 72.5 \text{ m}^3/\text{s}. \end{aligned}$$

Maximum total discharge

$$Q = 1800 \text{ m}^3/\text{s}$$

$$\therefore \text{Number of openings required} = \frac{Q}{q}$$

$$= \frac{1800}{72.5} = 23.84 = 24.$$

**Example 23**

Crest length of a Cippoletti weir is 500 mm. Head of water over the crest is 200 mm. The channel approach is 650 mm wide and 500 mm deep. Assuming coefficient of discharge  $C_d = 0.62$  discharge over the weir is \_\_\_\_\_.

**Solution**

Discharge,

$$Q = \frac{2}{3} C_d L \sqrt{2g} [(H + h_a)^{3/2} - h_a^{3/2}]$$

Where

 $L$  = Crest length $h_a$  = Head corresponding velocity of approach

Ignoring the velocity of approach,

$$Q = \frac{2}{3} \times 0.62 \times 0.5 \times \sqrt{2 \times 9.81} \times 0.2^{1.5}$$

$$= 0.0819 \text{ m}^3/\text{s}$$

Velocity of approach,

$$V_a = \frac{Q}{A} \text{ where } A = \text{Area of approach}$$

$$= \frac{0.0819}{0.65 \times 0.5} = 0.252 \text{ m/s}$$

$$h_a = \frac{V_a^2}{2g} = \frac{(0.252)^2}{2 \times 9.81}$$

$$= 3.237 \times 10^{-3} \text{ m}$$

$$= 0.00324 \text{ m}$$

More accurate value of discharge,

$$Q = \frac{2}{3} \times 0.62 \times 0.5 \times \sqrt{2 \times 9.81}$$

$$[(0.2 + 0.00324)^{3/2} - (0.00324)^{3/2}]$$

$$= 0.9153 \times 0.09144 = 0.0837 \text{ m}^3/\text{s}.$$

**Example 24**

A 50 m long broad crested weir has 0.6 m of water above the crest. If  $C_d = 0.62$ , maximum discharge over the weir is \_\_\_\_\_. (Velocity of approach may be neglected)

**Solution**

Maximum discharge,

$$\begin{aligned} Q_{\max} &= 1.705 C_d \times L \times H^{3/2} \\ &= 1.705 \times 0.62 \times 50 \times 0.6^{1.5} \\ &= 24.565 \text{ m}^3/\text{s}. \end{aligned}$$

**Example 25**

In a submerged weir heights of water above crest level is 0.2 m at the upstream side and 0.1 m at the downstream side respectively.

Discharge coefficients for free and drowned portion are 0.62 and 0.8 respectively. If length of the weir is 3 m, discharge over the weir is \_\_\_\_\_.

**Solution**

$$L = 3 \text{ m}$$

$$H = 0.2 \text{ m}$$

$$h = 0.1 \text{ m}$$

$$C_{d_1} = 0.62$$

$$C_{d_2} = 0.8$$

Discharge = Discharge over free portion + Discharge over drowned portion.

$$= \frac{2}{3} C_{d_1} \times L \times \sqrt{2g} (H - h)^{3/2} + C_{d_2} \times (L \times h) \sqrt{2g(H - h)}$$

$$= \frac{2}{3} \times 0.62 \times 3 \times 4.429 (0.2 - 0.1)^{1.5}$$

$$+ 0.8 \times 3 \times 0.1 \times 4.429 \times \sqrt{0.2 - 0.1}$$

$$= 1.7367 + 0.336$$

$$= 2.0727 \text{ m}^3/\text{s}.$$

**EXERCISES**

- For a given discharge, the critical depth in an open channel depends on
  - channel geometry only.
  - channel geometry and bed slope.
  - channel geometry, bed slope and roughness.
  - channel geometry, bed slope, roughness and Reynolds number.
- The flow in a rectangular channel is sub critical. If the width of the channel is reduced at a certain section, the water surface under no choke condition will
  - drop at a down stream section.
  - rise at a down stream section.
  - rise at an upstream section.
  - not undergo any change.
- Identify the false statement from the following. The specific speed of a pump increases with
  - increase in shaft speed.
  - increase in discharge.
  - decrease in gravitational acceleration.
  - increase in head.
- An air craft is flying in level flight at a speed of 200 km/h through air (density  $\rho = 1.2 \text{ kg/m}^3$  and viscosity  $\mu = 1.6 \times 10^{-5} \text{ Ns/m}^2$ ). The lift coefficient at this speed is 0.4 and drag coefficient is 0.0065. Mass of the air craft is 800 kg. The effective lift area of the air craft is
 

(A) 21.2 m <sup>2</sup>	(B) 10.6 m <sup>2</sup>
(C) 2.2 m <sup>2</sup>	(C) 1.1 m <sup>2</sup>

5. A laboratory model of a river is built to a geometric scale of 1 : 100. The fluid used in the model is oil of density  $900 \text{ kg/m}^3$ . The highest flood in the river is  $10000 \text{ m}^3/\text{s}$ . The corresponding discharge in the model shall be  
 (A)  $0.095 \text{ m}^3/\text{s}$  (B)  $0.100 \text{ m}^3/\text{s}$   
 (C)  $0.105 \text{ m}^3/\text{s}$  (D)  $10.5 \text{ m}^3/\text{s}$
6. A circular pipe has a diameter of 1 m, bed slope of 1 in 1000, and Manning's roughness coefficient equal to 0.01. It may be treated as an open channel flow, when it is flowing just full, i.e., the water level just touches the crest. The discharge in this condition is denoted by  $Q_{\text{full}}$ . Similarly, the discharge when the pipe is flowing half, i.e., with a flow depth of 0.5 m is denoted by  $Q_{\text{half}}$ . The ratio  $Q_{\text{full}}/Q_{\text{half}}$  is  
 (A) 1 (B)  $\sqrt{2}$   
 (C) 2 (D) 4
7. The dimension for kinematic viscosity is  
 (A)  $\frac{L}{MT}$  (B)  $\frac{L}{T^2}$   
 (C)  $\frac{L^2}{T}$  (D)  $\frac{ML}{T}$
8. For sub-critical flow in an open channel, the control section for gradually varied flow profile is  
 (A) at the down stream end.  
 (B) at the upstream end.  
 (C) at both ends.  
 (D) at any intermediate section.
9. List I contains dimensionless parameter and List II contains ratio. Match List I with List II and select the correct answer from given codes:

List I	List II
P. Mach number	1. Ratio of inertial force and gravity force
Q. Reynolds number	2. Ratio of fluid velocity and velocity of sound
R. Weber number	3. Ratio of inertia force and viscous force
S. Froude number	

**Codes:**

P Q R S	P Q R S
(A) 3 2 4 1	(B) 3 4 2 1
(C) 2 3 4 1	(D) 1 3 2 4

10. A trapezoidal channel is 10.0 m wide at the base and has a side slope of 4 horizontal to 3 vertical. The bed slope is 0.002. The channel is lined with smooth concrete (Manning's  $N = 0.012$ ). The hydraulic radius (in m) for a depth of flow of 3 m is \_\_\_\_\_.  
 (A) 20.0 (B) 3.5  
 (C) 3 (D) 2.1
11. A rectangular open channel of width 5.0 m is carrying a discharge of  $100 \text{ m}^3/\text{s}$ . Froude number of the flow is 0.8. Depth of flow (in metre) of the channel is

- (A) 4 (B) 5  
 (C) 16 (D) 20

12. A spillway discharges flood flow at a rate of  $9 \text{ m}^3/\text{s}$  per metre width. If the depth of flow on the horizontal apron at the toe of the spillway is 46 cm, the tail water depth needed to form a hydraulic jump is approximately given by which of the following option?  
 (A) 2.54 m (B) 4.9 m  
 (C) 5.77 m (D) 6.23 m
13. For a rectangular channel section, match List I (Geometrical element) with List II (Proportion for hydraulically efficient section) and select the correct answer using the codes given:

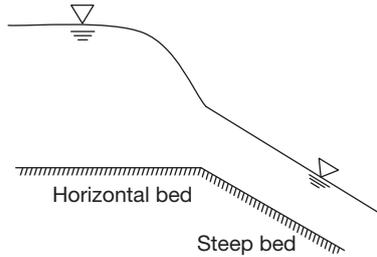
List I	List II
a. Top width	1. $\frac{y_e}{2}$
b. Perimeter	2. $y_e$
c. Hydraulic radius	3. $2y_e$
d. Hydraulic depth	4. $4y_e$

$y_e$  is the flow depth corresponding to hydraulically efficient section.

**Codes:**

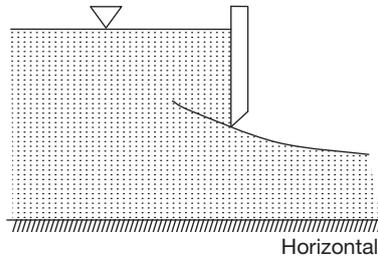
a b c d	a b c d
(A) 2 4 1 3	(B) 3 1 4 2
(C) 3 4 1 2	(D) 3 4 2 1

14. The Froude number of a flow in a rectangular channel is 0.8. If the depth of flow is 1.5 m, the critical depth is 15 cm, the critical depth is  
 (A) 1.80 m (B) 1.56 m  
 (C) 1.36 m (D) 1.29 m
15. The relationship between length scale ratio ( $L_r$ ) and the velocity scale ratio ( $V_r$ ) in hydraulic models, in which Froude dynamic similarity is maintained, is  
 (A)  $V_r = L_r$  (B)  $L_r = \sqrt{V_r}$   
 (C)  $V_r = L_r^{1.5}$  (D)  $V_r = \sqrt{L_r}$
16. There is a free over fall at the end of a free open channel. For a given flow rate, the critical depth is less than the normal depth. What gradually varied flow profile will occur in the channel for this flow rate?  
 (A)  $M_1$  (B)  $M_2$   
 (C)  $M_3$  (D)  $S_1$
17. A channel with a mild slope is followed by a horizontal channel and then by a steep channel. What varied flow profiles will occur?  
 (A)  $M_1, H_1, S_1$  (B)  $M_2, H_2, S_2$   
 (C)  $M_1, H_2, S_3$  (D)  $M_1, H_2, S_2$
18. A horizontal bed channel is followed by a steep bed channel as shown in the figure. The gradually varied flow profiles over the horizontal and steep beds are



- (A)  $H_2$  and  $S_2$  respectively.  
 (B)  $H_2$  and  $S_1$  respectively.  
 (C)  $H_3$  and  $S_2$  respectively.  
 (D)  $H_3$  and  $S_1$  respectively.

19. The flow profile under the gate as shown in the figure is classified as



- (A)  $M_2$  (B)  $H_1$   
 (C)  $H_2$  (D)  $H_3$

20. If  $Fr$  is the Froude number for a flow in a triangular channel, then Froude number for the same velocity and same depth of flow in rectangular channel will be

- (A)  $Fr$  (B)  $\frac{Fr}{\sqrt{2}}$   
 (C)  $\sqrt{2}Fr$  (D)  $\frac{Fr}{\sqrt{3}}$

21. A rectangular open channel of width 4.5 m is carrying a discharge of  $100 \text{ m}^3/\text{s}$ . Critical depth of the channel is  
 (A) 7.09 m (B) 3.69 m  
 (C) 2.16 m (D) 1.31 m

22. A river reach of 2.0 km along with maximum flood discharge of  $10,000 \text{ m}^3/\text{s}$  is to be physically modelled in the laboratory where maximum available discharge is  $0.20 \text{ m}^3/\text{s}$ . For a geometrically similar model based on equality of Froude number, the length of river reach (in m) in the model is  
 (A) 26.4 (B) 25.0  
 (C) 20.5 (D) 18.0

23. A rectangular channel 6.0 m wide carries a discharge of  $16.0 \text{ m}^3/\text{s}$  under uniform flow condition with normal depth of 1.6 m. Manning's coefficient  $N = 0.015$ . Longitudinal slope of the channel is  
 (A) 0.00059  
 (B) 0.00048  
 (C) 0.00038  
 (D) 0.00028

24. For the data given in Question 23, if a hump is to be provided on the channel bed, maximum height of the hump without affecting the upstream flow condition is  
 (A) 0.50 m (B) 0.40 m  
 (C) 0.30 m (D) 0.20 m
25. For the data given in Question 23, if the channel width is to be contracted, the minimum width to which the channel can be contracted without affecting the upstream flow condition is  
 (A) 3.0 m (B) 3.8 m  
 (C) 4.1 m (D) 4.5 m
26. An automobile with projected area  $2.6 \text{ m}^2$  is running on a road with a speed of 120 km/h. The mass density and kinematic viscosity of air are  $1.2 \text{ kg/m}^3$  and  $1.5 \times 10^{-5} \text{ m}^2/\text{s}$  respectively. The drag coefficient is 0.3. The drag force on the automobile is  
 (A) 620 N (B) 600 N  
 (C) 580 N (D) 520 N
27. For the data given in Question 26, metric horse power required over come the drag force is  
 (A) 33.23 (B) 31.55  
 (C) 23.55 (D) 20.23
28. A horizontal water jet with a velocity of 10 m/s and a cross-sectional area of  $10 \text{ mm}^2$  strikes a flat plate held normal to the flow direction. Density of water is  $1000 \text{ kg/m}^3$ . Total force on the plate due to the jet is  
 (A) 100 N (B) 10 N  
 (C) 1 N (D) 0.1 N
29. A 1 : 50 scale model of a spillway is to be tested in the laboratory. The discharge in the prototype is  $1000 \text{ m}^3/\text{s}$ . The discharge to be maintained in the model test is  
 (A)  $0.057 \text{ m}^3/\text{s}$  (B)  $0.08 \text{ m}^3/\text{s}$   
 (C)  $0.57 \text{ m}^3/\text{s}$  (D)  $5.7 \text{ m}^3/\text{s}$
30. The flow rate in a wide rectangular open channel is  $2.0 \text{ m}^3/\text{s}$  per metre width. The channel slope is 0.002. The Manning's roughness coefficient is 0.012. The slope of the channel is classified as  
 (A) critical (B) horizontal  
 (C) mild (D) steep
31. Flow of glycerine (kinematic viscosity,  $\nu_g = 5 \times 10^{-4} \text{ m}^2/\text{s}$ ) in an open channel is to be modeled in laboratory flume using water ( $\nu_w = 10^{-6} \text{ m}^2/\text{s}$ ) as the flowing fluid. If both gravity and viscosity are important, what should be the length scale (i.e., ratio of prototype to model dimensions) for maintaining dynamic similarity?  
 (A) 1 (B) 22  
 (C) 63 (D) 500
32. Critical depth at a section of a rectangular channel is 1.5 m. Specific energy at that section is  
 (A) 0.75 m (B) 1.0 m  
 (C) 1.5 m (D) 2.25 m
33. A partially open sluice gate discharges water into a rectangular channel. The tail water depth in the channel is

3 m and Froude number is  $\frac{1}{2\sqrt{2}}$ . If a free hydraulic jump is to be formed at downstream of the sluice gate after vena contracta of the jet coming out from the sluice gate, the sluice gate opening should be (Coefficient of contraction  $C_c = 0.9$ )

- (A) 0.3 m (B) 0.4 m  
(C) 0.69 m (D) 0.9 m

34. A steep wide rectangular channel takes off from a reservoir having an elevation of 101.2 m. At the entrance, the bottom elevation of the channel is 100 m. If the slope of the channel is increased by 4%, the discharge per unit length in the channel will be

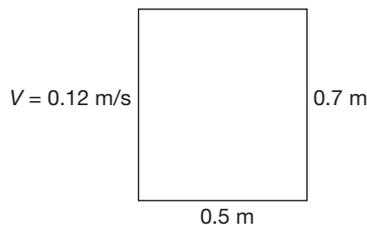
- (A) 2.24 m<sup>2</sup>/s.  
(B) higher than 2.24 m<sup>2</sup>/s by 4%.  
(C) higher than 2.24 m<sup>2</sup>/s by 2%.  
(D) choked.

35. The height of hydraulic jump in the stilling pool of 1:25 scale model was observed to be 10 cm. The corresponding prototype height of the jump is

- (A) Cannot be determined  
(B) 2.5 m  
(C) 0.5 m  
(D) 0.1 m

36. A thin flat plate 0.5 m × 0.7 m in size settles in a large tank of water with a terminal velocity of 0.12 m/s. The coefficient of drag  $C_D = \frac{1.328}{\sqrt{R_L}}$  for a laminar boundary

layer and  $C_D = \frac{0.072}{(R_L)^{1/5}}$  for a turbulent boundary layer, where  $R_L$  is the plate Reynolds number. Assume  $\mu = 10^{-3}$  Ns/m<sup>2</sup> and  $\rho = 1000$  kg/m<sup>3</sup>. The submerged weight of the plate is

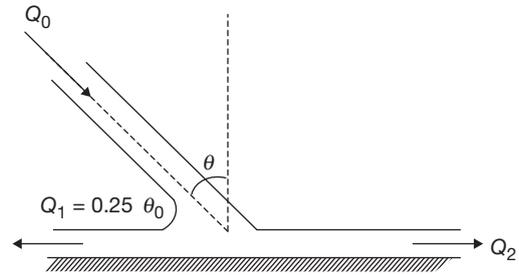


- (A) 0.0115 N (B) 0.0118 N  
(C) 0.0231 N (D) 0.0376 N

37. The net positive suction head (NPSH) required for a pump provided by the manufacturer for a flow of 0.05 m<sup>3</sup>/s is 3.3 m. The temperature of water is 30°C (Vapour pressure head absolute = 0.44 m). Atmospheric pressure is 100 kPa absolute and the head loss from the reservoir to pump is 0.3 Nm/N. The maximum height of the pump above the suction reservoir is

- (A) 10.19 m (B) 6.89 m  
(C) 6.15 m (D) 2.86 m

38. A horizontal jet strikes a frictionless vertical plate (the plan view is shown in the figure). It is then divided into two parts, as shown in the figure. If the impact loss is neglected, what is the value of  $\theta$ ?



- (A) 15° (B) 30°  
(C) 45° (D) 60°

39. A very wide rectangular channel carries a discharge of 8 m<sup>3</sup>/s per metre width. The channel has a bed slope 0.004 and Manning's roughness coefficient  $N = 0.015$ . At a certain section of the channel, the flow depth is 1 m. What gradually varied flow (GVF) profile exists at this section?

- (A)  $M_2$  (B)  $M_3$   
(C)  $S_2$  (D)  $S_3$

40. The prejump Froude Number for a particular flow in a horizontal rectangular channel is 10. The ratio of sequent depths (i.e., post jump depth to pre-jump depth) is \_\_\_\_\_.

41. A 3 m wide rectangular channel carries a flow of 6 m<sup>3</sup>/s. The depth of flow at a section  $P$  is 0.5 m. A flat topped hump is to be placed at the down stream of section  $P$ . Assume negligible energy loss between section  $P$  and hump and consider  $g$  as 9.81 m/s<sup>2</sup>. The maximum height of the hump (expressed in m) which will not change the depth of flow at section  $P$  is \_\_\_\_\_.

42. A penstock of 1 m diameter and 5 km length is used to supply water from a reservoir to an impulse turbine. A nozzle of 15 cm diameter is fixed at the end of the penstock. The elevation difference between the turbine and water level in the reservoir is 500 m. Consider head loss due to friction as 5% of the velocity head available at the jet. Assume unit weight of water = 10 kN/m<sup>3</sup> and acceleration due to gravity ( $g$ ) = 10 m/s<sup>2</sup>. If the overall efficiency is 80%, power generated (expressed in kW and rounded to the nearest integer) is \_\_\_\_\_.

43. Dimensions of kinematic viscosity are  
(A)  $ML^{-1}T^{-2}$  (B)  $ML^{-1}T^{-1}$   
(C)  $L^2 T^{-1}$  (D)  $L T^{-2}$

44. A ship with hull length of 100 m is to run with a speed of 10 m/s. For dynamic similarity, the velocity for a 1 : 25 model of the ship in a towing tank should be

- (A) 2 m/s (B) 10 m/s  
(C) 20 m/s (D) 25 m/s

45. Match List I with List II and select the correct answer using the codes given below:

List I (Flow Regimes for Gradually Varied Flow)		List II (Flow Regimes for Gradually Varied Flow)	
P.	$y_c > y_0 > y$	1.	$C_1$
Q.	$y_c < y_0 < y$	2.	$M_1$
R.	$y_0 > y > y_c$	3.	$S_3$
S.	$y > y_c > y_0$	4.	$M_2$
		5.	$S_1$

( $y_0$  = normal depth,  $y_c$  = critical depth,  $y$  = depth of gradually varied flow)

Codes:

	P	Q	R	S		P	Q	R	S
(A)	1	2	3	4	(B)	3	4	5	2
(C)	3	2	4	5	(D)	5	4	3	1

46. A model reservoir is emptied in 10 minutes. If the model scale is 1 : 25, then the time taken by the prototype to empty itself, would be  
 (A) 250 minutes (B) 50 minutes  
 (C) 6250 minutes (D) 2 minutes
47. When no external energy is imposed; which of the following statements would be true?  
 I. Energy line always falls in the direction of flow.  
 II. Hydraulic gradient line never rises in the direction of flow.  
 III. Specific energy may increase or decrease in the direction of flow.  
 IV. Energy line and hydraulic gradient line can cross each other.  
 Select correct answer using the codes given below  
 (A) I and II (B) II and III  
 (C) III and IV (D) I and III

48. In a gradually varied flow, if  $\frac{dy}{dx}$  is positive, then  $\frac{dE}{dx}$  will be

(A) always zero.	(B) positive if $y > y_c$ .
(C) negative if $y > y_c$ .	(D) always negative.

49. Match List I and List II and select the correct answer using the codes given:

List I (Non-dimensional Numbers)		List II (Application)	
P.	Mach number	1.	Waves in ocean
Q.	Thoma number	2.	Launching of rockets
R.	Reynold's number	3.	Cavitation phenomenon
S.	Weber number	4.	Capillary flow in soil
		5.	Motion of submarine

Codes:

	P	Q	R	S		P	Q	R	S
(A)	1	3	5	2	(B)	2	3	5	4
(C)	3	2	4	1	(D)	2	1	3	4

50. An error of 0.5% in the measurement of head in a V-notch causes an error of  
 (A) 0.5% in the discharge.  
 (B) 1.0% in the discharge.  
 (C) 1.25% in the discharge.  
 (D) 1.5% in the discharge.

51. At a hydraulic jump, the depths at two sides are 0.4 m and 1.4 m. The head loss in the jump is nearly  
 (A) 1.0 m (B) 0.9 m  
 (C) 0.7 m (D) 0.45 m

52. A channel of bed slope 0.0009 carries a discharge of 30 m<sup>3</sup>/s when the depth of flow is 1 m. What is the discharge carried by an exactly similar channel at the same depth of flow if the slope is decreased to 0.0001?  
 (A) 10 m<sup>3</sup>/s (B) 15 m<sup>3</sup>/s  
 (C) 60 m<sup>3</sup>/s (D) 90 m<sup>3</sup>/s

53. The conjugate depths at a location in a horizontal rectangular channel, 4 m wide, are 0.2 m and 1.0 m. The discharge in the channel is \_\_\_\_\_ m<sup>3</sup>/s.

54. The discharge from a spillway on a horizontal floor is 8 cumecs per metre width. If the depth before the jump of 50 cm, the depth after the jump in metres will be

55. A discharge of 1 cumec is flowing in a rectangular channel one metre wide at a depth of 20 cm. The bed slope of the channel is  
 (A) mild (B) critical  
 (C) steep (D) adverse

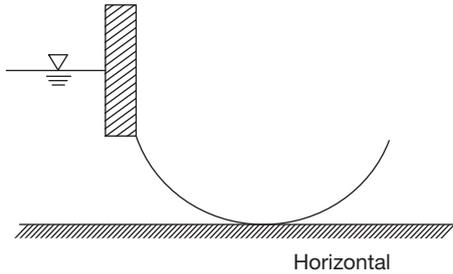
56. Water flows in a rectangular channel at a depth of 1.20 m and a velocity of 2.4 m/s. A local rise in the bed of 0.60 m will cause  
 (A) the surface to rise.  
 (B) the surface to fall.  
 (C) a stationary jump to form.  
 (D) a surge to travel upstream.

57. The hydraulic jump always occurs from  
 (A) a  $M_2$  curve to a  $M_1$  curve.  
 (B) a  $H_3$  curve to a  $H_1$  curve.  
 (C) below normal depth to above normal depth.  
 (D) below critical depth to above critical depth.

58. A steady discharge of 1 cumec flows uniformly in a rectangular channel 1 m wide at a depth of 250 mm. The slope of the channel bed is  
 (A) adverse (B) steep  
 (C) critical (D) mild

59. In deriving the equation for the hydraulic jump in the rectangular channel in terms of the conjugate depths and the initial Froude number,  
 (A) continuity equation and energy equation.  
 (B) continuity equation and momentum equation used.  
 (C) equations of continuity, momentum and energy are used.  
 (D) gradually varied flow equation is used.

60. A section in open channel at which a fixed relationship exists between the discharge and the depth of flow is called as a \_\_\_\_\_ section
61. Flow at critical depth takes place in an open channel when  
 (A) for a given specific energy, discharge is maximum.  
 (B) for a given discharge, specific energy is minimum.  
 (C) discharge is minimum for a given specific force.  
 (D) discharge is maximum for a given specific force.
62. The water surface profile resulting from flow underneath the gate in the given figure is



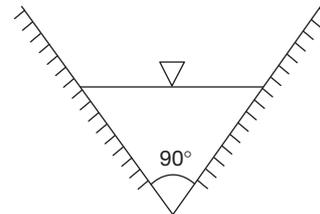
- (A)  $H_3$  (B)  $H_1$   
 (C)  $H_2$  (D)  $S_2$
63. For a 'best' symmetrical trapezoidal section of an open channel with a given area of section and side slopes, one of the following statements holds true:  
 (A) Half the top width is equal to one of the side slope.  
 (B) Half the top width plus the bottom width is equal to both the side slopes put together.  
 (C) Water depths is equal to half bottom width .  
 (D) Hydraulic mean depth is equal to half the top width.
64. The downstream end of long prismatic channel of mild slope ends in a pool created by a dam. The resulting nonuniform water surface profile can be described as one of the following  
 (A)  $M_3$  profile ending in a hydraulic jump.  
 (B)  $M_1$  profile that lies above normal depth line.  
 (C)  $M_2$  profile that lies between critical and normal depth lines.  
 (D)  $M_3$  profile that lies between critical and normal depth lines.
65. The sequent depth ratio of a hydraulic jump in a rectangular horizontal channel is 10.30. The Froude number at the beginning of the jump is  
 (A) 5.64 (B) 7.63  
 (C) 8.05 (D) 13.61
66. Water flows at a depth of 0.1 m with a velocity of 6 m/s in a rectangular channel. The alternate depth is  
 (A) 0.30 m (B) 0.40 m  
 (C) 0.86 m (D) 0.81 m
67. Water flows at a rate of  $10 \text{ m}^3/\text{s}$  in a rectangular channel 3 m wide. The critical depth of flow is  
 (A) 1.13 m (B) 2 m  
 (C) 1.45 m (D) 1.04 m

68. A circular sewer 2 m diameter has to carry a discharge of  $2 \text{ m}^3/\text{s}$  when flowing nearly full. What is the minimum required slope to initiate the flow? Assume Manning's  $N = 0.015$ .  
 (A) 0.00023 (B) 0.000036  
 (C) 0.000091 (D) 0.000014
69. A trapezoidal channel with bottom width of 3 m and side slope of 1V : 1.5H carries a discharge of  $8.0 \text{ m}^3/\text{s}$  with the flow depth of 1.5 m. The Froude number of the flow is  
 (A) 0.066 (B) 0.132  
 (C) 0.265 (D) 0.528
70. In a lined rectangular canal, the Froude number of incoming flow is 3.0. A hydraulic jump forms when it meets the pool of water. The depth of flow after the jump formation is 1.51 m. Froude number of flow after the hydraulic jump is  
 (A) 0.30 (B) 0.71  
 (C) 0.41 (D) None of these

#### Direction for questions 71 and 72:

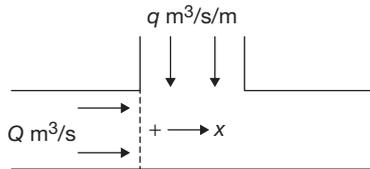
A very wide rectangular channel carries a discharge of  $8 \text{ m}^3/\text{s}$  per metre width. The channel has a bed slope of 0.004 and Manning's roughness coefficient,  $N = 0.015$ . At a certain section of the channel, the flow depth is 1 m.

71. What gradually varied flow profile exists at this section?  
 (A)  $M_2$  (B)  $M_3$   
 (C)  $S_2$  (D)  $S_3$
72. At what distance from this section the flow depth will be 0.9 m? (Use the direct step method employing a single step.)  
 (A) 65 m downstream (B) 50 m downstream  
 (C) 50 m upstream (D) 65 m upstream
73. A hydraulic jump takes place in a triangular channel of vertex angle  $90^\circ$ , as shown in figure. The discharge is  $1 \text{ m}^3/\text{s}$  and the pre-jump depth is 0.5 m. What will be the post-jump depth? (Take  $g = 9.81 \text{ m/s}^2$ )



- (A) 0.57 m (B) 0.91 m  
 (C) 1.02 m (D) 1.57 m
74. For a pipe of radius  $r$ , flowing half-full under the action of gravity, the hydraulic depth is  
 (A)  $r$  (B)  $\frac{\pi r}{4}$   
 (C)  $\frac{r}{2}$  (D)  $0.379r$

75. A wide channel is 1 m deep and has a velocity of flow,  $V$ , as 2.13 m/s. If a disturbance is caused, an elementary wave can travel upstream with a velocity of
- (A) 1.00 m/s  
 (B) 2.13 m/s  
 (C) 3.13 m/s  
 (D) 5.26 m/s
76. A steady flow occurs in an open channel with lateral inflow of  $q$  m<sup>3</sup>/s per unit width as shown in the figure. The mass conservation equation is



- (A)  $\frac{\partial q}{\partial x} = 0$   
 (B)  $\frac{\partial Q}{\partial x} = 0$   
 (C)  $\frac{\partial Q}{\partial x} - q = 0$   
 (D)  $\frac{\partial Q}{\partial x} + q = 0$
77. Critical depth at a section of a rectangular channel is 1.5 m. The specific energy at that section is
- (A) 0.75 m                      (B) 1.0 m  
 (C) 1.5 m                        (D) 2.25 m
78. A partially open sluice gate discharges water into rectangular channel. The tall water depth in the channel is 3 m and Froude number is  $\frac{1}{2\sqrt{2}}$ . If a free hydraulic jump is to be formed at downstream of the sluice gate after the vena contracta of the jet coming out from the sluice gate, the sluice gate opening should be (coefficient of contraction  $C_c = 0.9$ )
- (A) 0.3 m                      (B) 0.4 m  
 (C) 0.69 m                    (D) 0.9 m
79. A triangular irrigation lined canal carries a discharge of 25 m<sup>3</sup>/s at bed slope =  $\frac{1}{6000}$ . If the side slope of the canal are 1 : 1 and Manning's coefficient is 0.018, the central depth of flow is equal to
- (A) 2.98 m                      (B) 3.62 m  
 (C) 4.91 m                      (D) 5.61 m
80. A channel with a mild slope is followed by a horizontal channel and then by a steep channel. What gradually varied flow profiles will occur?
- (A)  $M_1, H_1, S_1$   
 (B)  $M_2, H_2, S_2$   
 (C)  $M_1, H_2, S_3$   
 (D)  $M_1, H_2, S_2$
81. A hydraulic jump occurs in a rectangular, horizontal, frictionless channel. What would be the pre-jump depth if the discharge per unit width is 2 m<sup>3</sup>/s/m and the energy loss
- (A) 0.2 m  
 (B) 0.3 m  
 (C) 0.8 m  
 (D) 0.9 m
82. A very wide rectangular channel is designed to carry a discharge of 5 m<sup>3</sup>/s per metre width. The design is based on the Manning's equation with the roughness coefficient obtained from the grain size using Strickler's equation and results in a normal depth of 1.0 m. By mistake, however, the engineer used the grain diameter in mm in the Strickler's equation instead of in metre. What should be the correct normal depth?
- (A) 0.32 m  
 (B) 0.50 m  
 (C) 2.00 m  
 (D) 3.20 m
83. A rigid boundary rectangular channel having a bed slope of  $\frac{1}{800}$  has its width and depth of flow equal to 2 m and 1 m respectively. If the flow is uniform and the value of Chezy's constant is 60, the discharge through the channel is
- (A) 1.0 m<sup>3</sup>/s  
 (B) 1.5 m<sup>3</sup>/s  
 (C) 2.0 m<sup>3</sup>/s  
 (D) 3.0 m<sup>3</sup>/s
84. Consider the following statements:
- I. In an open channel flow, energy grade lines obtained by adding datum head. Pressure head and velocity head.
  - II. In an open channel, hydraulic grade line is the free surface itself.
  - III. For a pipe and an open channel of same dimension, the hydraulic gradient line is located at the same height above datum.
  - IV. Energy gradient line of an open channel is always horizontal.
- Which of these statements are correct?
- (A) I, II, and III  
 (B) I and IV  
 (C) I, III, and IV  
 (D) II, III, and IV
85. The height of hydraulic jump in a stilling pool was found to be 10 cm in a model with  $L_p/L_m = 36$ . The prototype jump height would be
- (A) 0.6 m  
 (B) 3.6 m  
 (C) 21.6 m  
 (D) Cannot be determined

## PREVIOUS YEARS' QUESTIONS

- There is a free overfall at the end of a long open channel. For a given flow rate, the critical depth is less than the normal depth. What gradually varied flow profile will occur in the channel for this flow rate?  
[GATE, 2007]  
(A)  $M_1$  (B)  $M_2$   
(C)  $M_3$  (D)  $S_1$
- A triangular open channel has a vertex angle of  $90^\circ$  and carries flow at a critical depth of 0.30 m. The discharge in the channel is [GATE, 2007]  
(A)  $0.08 \text{ m}^3/\text{s}$  (B)  $0.11 \text{ m}^3/\text{s}$   
(C)  $0.15 \text{ m}^3/\text{s}$  (D)  $0.2 \text{ m}^3/\text{s}$
- The flow rate in a wide rectangular open channel is  $2.0 \text{ m}^3/\text{s}$  per metre width. The channel bed slope is 0.002. The Manning's roughness coefficient is 0.012. The slope of the channel is classified as [GATE, 2007]  
(A) critical (B) horizontal  
(C) mild (D) steep
- A hump is to be provided on the channel bed. The maximum height of the hump without affecting the upstream flow condition is  
(A) 0.50 m  
(B) 0.40 m  
(C) 0.30 m  
(D) 0.20 m
- The channel width is to be contracted. The minimum width to which the channel can be contracted without affecting the upstream flow condition is [GATE, 2008]  
(A) 3.0 m (B) 3.8 m  
(C) 4.1 m (D) 4.5 m
- Direct step method of computation for gradually varied flow is [GATE, 2009]  
(A) applicable to non-prismatic channels.  
(B) applicable to prismatic channels.  
(C) applicable to both prismatic and non-prismatic channels.  
(D) not applicable to both prismatic and non-prismatic channels.

**Direction for questions 4 and 5:**

A rectangular open channel needs to be designed to carry a flow of  $2.0 \text{ m}^3/\text{s}$  under uniform flow conditions. The Manning's roughness coefficient is 0.018. The channel should be such that the flow depth is equal to half the width, and the Froude number is equal to 0.5.

- The bed slope of the channel to be provided is [GATE, 2007]  
(A) 0.0012 (B) 0.0021  
(C) 0.0025 (D) 0.0052
- Keeping the width, flow depth and roughness the same, if the bed slope of the above channel is doubled, the average boundary shear stress under uniform flow conditions is [GATE, 2007]  
(A)  $5.6 \text{ N/m}^2$  (B)  $10.8 \text{ N/m}^2$   
(C)  $12.3 \text{ N/m}^2$  (D)  $17.2 \text{ N/m}^2$
- A person standing on the bank of a canal drops a stone on the water surface. He notices that the disturbance on the water surface is not travelling upstream. This is because the flow in the canal is [GATE, 2008]  
(A) sub-critical (B) super-critical  
(C) steady (D) uniform
- A rectangular open channel of width 4.5 m is carrying a discharge of  $100 \text{ m}^3/\text{s}$ . The critical depth of the channel is [GATE, 2009]  
(A) 7.09 m (B) 3.69 m  
(C) 2.16 m (D) 1.31 m
- A mild-sloped channel is followed by a steep-sloped channel. The profiles of gradually varied flow in the channel are [GATE, 2010]  
(A)  $M_3, S_2$  (B)  $M_3, S_3$   
(C)  $M_2, S_1$  (D)  $M_2, S_2$
- The flow in a rectangular channel is sub-critical. If width of the channel is reduced at a certain section, the water surface under no-choke condition will [GATE, 2010]  
(A) drop at a downstream section.  
(B) rise at a downstream section.  
(C) rise at an upstream section.  
(D) not undergo any change.
- For a rectangular channel section, Match List I (Geometrical elements) with List II (Proportions for hydraulically efficient section) and select the correct answer using the codes given:

List I	List II
a. Top width	1. $y_e/2$
b. Perimeter	2. $y_e$
c. Hydraulic radius	3. $2 y_e$
d. Hydraulic depth	4. $4 y_e$

$y_e$  is the flow depth corresponding to hydraulically efficient section. [GATE, 2010]

**Direction for questions 7 to 8:**

A rectangular channel 6.0 m wide carries a discharge of  $16.0 \text{ m}^3/\text{s}$  under uniform flow condition with normal depth of 1.60 m. Manning's ' $N$ ' is 0.015. [GATE, 2008]

- The longitudinal slope of the channel is  
(A) 0.000585  
(B) 0.000485  
(C) 0.000385  
(D) 0.000285

**Codes:**

a b c d

- (A) 2 4 1 3  
(B) 3 1 4 2  
(C) 3 4 1 2  
(D) 3 4 2 1
15. The Froude number of flow in a rectangular channel is 0.8. If the depth of flow is 1.5 m, the critical depth is [GATE, 2010]  
(A) 1.80 m (B) 1.56 m  
(C) 1.36 m (D) 1.29 m
16. From a given discharge, the critical flow depth in an open channel depends on [GATE, 2011]  
(A) channel geometry only.  
(B) channel geometry and bed slopes.  
(C) channel geometry, bed slope and roughness.  
(D) channel geometry, bed slope, roughness and Reynolds number.
17. A spillway discharges flood flow at a rate of  $9 \text{ m}^3/\text{s}$  per metre width. If the depth of flow on the horizontal apron at the toe of the spillway is 46 cm, the tail water depth needed to form a hydraulic jump is approximately given by which of the following options? [GATE, 2011]  
(A) 2.54 m (B) 4.90 m  
(C) 5.77 m (D) 6.23 m
18. The flow in a horizontal, frictionless rectangular open channel is super-critical. A smooth hump is built on the channel floor. As the height of hump is increased, choked condition is attained. With further increase in the height of the hump, the water surface will [GATE, 2011]  
(A) rise at a section upstream of the hump.  
(B) drop at a section upstream of the hump.  
(C) drop at the hump.  
(D) rise at the hump.
19. A trapezoidal channel is 10.0 m wide at the base and has a side slope of 4 horizontal to 3 vertical. The bed slope is 0.002. The channel is lined with smooth concrete (Manning's  $n = 0.12$ ). The hydraulic radius (in m) for a depth of flow of 3.0 m is [GATE, 2012]  
(A) 20.0 (B) 3.5  
(C) 3.0 (D) 2.1
20. A rectangular open channel of width 5.0 m is carrying a discharge of  $100 \text{ m}^3/\text{s}$ . The Froude number of the flow is 0.8. The depth of flow (in m) in the channel is [GATE, 2012]  
(A) 4 (B) 5  
(C) 16 (D) 20
21. For sub-critical flow in an open channel, the control section for gradually varied flow profile is [GATE, 2013]  
(A) at the downstream end.  
(B) at the upstream end.  
(C) at both ends.  
(D) at any intermediate section.
22. The normal depth in a wide rectangular channel is increased by 10%. The percentage increase in discharge in the channel is [GATE, 2013]  
(A) 20.1  
(B) 15.4  
(C) 10.5  
(D) 17.2
23. A rectangular channel having a bed slope of 0.0001, width 3.0 m and Manning's coefficient ' $N$ ' 0.015, carries a discharge of  $1.0 \text{ m}^3/\text{s}$ . Given that the normal depth of flow ranges between 0.76 m and 0.8 m. The minimum width of a throat (in m) that is possible at a given section, while ensuring that the prevailing normal depth is not exceeded along the reach upstream of the contraction, is approximately equal to (assume negligible losses) [GATE, 2014]  
(A) 0.64 (B) 0.84  
(C) 1.04 (D) 1.24
24. A rectangular channel of 2.5 m width is carrying a discharge of  $4 \text{ m}^3/\text{s}$ . Considering that acceleration due to gravity as  $9.81 \text{ m/s}^2$ , the velocity of flow (in m/s) corresponding to the critical depth (at which the specific energy is minimum) is \_\_\_\_\_. [GATE, 2014]
25. A hydraulic jump is formed in a 2 m wide rectangular channel which is horizontal and frictionless. The post-jump depth and velocity are 0.8 m and 1 m/s, respectively. The pre-jump velocity is \_\_\_\_\_ m/s. (use  $g = 10 \text{ m/s}^2$ ). [GATE, 2015]
26. A short reach of a 2 m wide rectangular open channel has its bed level rising in the direction of flow at a slope of 1 in 10000. It carries a discharge of  $4 \text{ m}^3/\text{s}$  and its Manning's roughness coefficient is 0.01. The flow in this reach is gradually varying. At a certain section in this reach, the depth of flow was measured as 0.5 m. The rate of change of the water depth with distance  $\frac{dy}{dx}$ , at this section is \_\_\_\_\_ (use  $g = 10 \text{ m/s}^2$ ). [GATE, 2015]
27. A hydraulic jump takes place in a frictionless rectangular channel. The pre-jump depth is  $y_p$ . The alternate and sequent depths corresponding to  $y_p$  are  $y_a$  and  $y_s$  respectively. The correct relationship among  $y_p$ ,  $y_a$  and  $y_s$  is [GATE, 2015]  
(A)  $y_a < y_s < y_p$   
(B)  $y_p < y_s < y_a$   
(C)  $y_p < y_s = y_a$   
(D)  $y_a = y_s = y_p$

28. The pre-jump Froude Number for a particular flow in a horizontal rectangular channel is 10. The ratio of sequent depths (i.e., post-jump depth to pre-jump depth) is \_\_\_\_\_. [GATE, 2016]
29. A 4 m wide rectangular channel, having bed slope of 0.001 carries a discharge of  $16 \text{ m}^3/\text{s}$ . Considering Manning's roughness coefficient = 0.012 and  $g = 10 \text{ m/s}^2$ , the category of the channel slope is \_\_\_\_\_. [GATE, 2016]
- (A) horizontal (B) mild  
(C) critical (D) steep
30. A 3 m wide rectangular channel carries a flow of  $6 \text{ m}^3/\text{s}$ . The depth of flow at a section  $P$  is 0.5 m. A flat-topped hump is to be placed at the downstream of the section  $P$ . Assume negligible energy loss between section  $P$  and hump, and consider  $g$  as  $9.81 \text{ m/s}^2$ . The maximum height of the hump (expressed in m) which will not change the depth of flow at section  $P$  is \_\_\_\_\_. [GATE, 2016]

## ANSWER KEYS

### Exercises

- |            |           |                     |                  |                  |       |       |       |       |       |
|------------|-----------|---------------------|------------------|------------------|-------|-------|-------|-------|-------|
| 1. A       | 2. C      | 3. D                | 4. B             | 5. B             | 6. C  | 7. C  | 8. A  | 9. C  | 10. D |
| 11. A      | 12. C     | 13. C               | 14. D            | 15. D            | 16. B | 17. D | 18. A | 19. D | 20. B |
| 21. B      | 22. A     | 23. A               | 24. B            | 25. C            | 26. D | 27. C | 28. C | 29. A | 30. D |
| 31. C      | 32. D     | 33. C               | 34. C            | 35. B            | 36. C | 37. C | 38. B | 39. D |       |
| 40. 13.651 | 41. 0.203 | 42. 6714.6          | 43. C            | 44. A            | 45. C | 46. B | 47. D | 48. C | 49. B |
| 50. C      | 51. D     | 52. A               | 53. 4.32 to 4.34 | 54. 4.85 to 4.87 | 55. C | 56. D | 57. D |       |       |
| 58. B      | 59. B     | 60. Control section | 61. B            | 62. A            | 63. A | 64. C | 65. B | 66. D |       |
| 67. D      | 68. A     | 69. -               | 70. C            | 71. D            | 72. C | 73. C | 74. B | 75. A | 76. C |
| 77. C      | 78. C     | 79. C               | 80. D            | 81. B            | 82. B | 83. B | 84. A | 85. B |       |

### Previous Years' Questions

- |       |       |       |           |          |            |       |           |       |         |
|-------|-------|-------|-----------|----------|------------|-------|-----------|-------|---------|
| 1. B  | 2. B  | 3. D  | 4. B      | 5. D     | 6. B       | 7. A  | 8. B      | 9. C  | 10. B   |
| 11. B | 12. D | 13. A | 14. C     | 15. D    | 16. A      | 17. C | 18. B     | 19. D | 20. A   |
| 21. A | 22. D | 23. B | 24. 2.504 | 25. 4.83 | 26. 0.0032 | 27. C | 28. 13.65 | 29. B | 30. 0.2 |