1

Vedic Mathematics

1.01 Introduction

Shankaracharya of Puri Swami Bharati Krishna ji Tirtha was the research scholar and inspiration in the field of Vedic Mathematics. He meditated hard for about eight years in the Sringeri math. After attaining the highest state of perfect siddhi he realized internally the Aphorisms and Mathematical Sutras described in ancient Indian literature Vedas, Brahamans', Sahintas', Vedangas etc. and reconstructed them in Sanskrit the divine language. Vedic Mathematics profounded by Swami ji is based on these Mathematical sixteen Sutras and thirteen Upsutras. These Sutras are very easy, effective, useful and correspond one to many interpretations in general. Problems of any branch of Mathematics can be solved by these Sutras easily.

1.02 Usefulness of Vedic Mathematics

By these Vedic mathematical Sutras calculations become easy and short. Less time is consumed. Students feel less mental stress. By Sutra's based methods the results or answers of the questions can be verified, hence giving the student more confidence. By these formulae the possibility of committing error by students become negligible.

By these Sutras more interest in mathematics is developed in students consequently they attain excellent achievement in subject mathematics. They can mentally solve the hard exercises. That is by Vedic Mathematics is called mental mathematics in the mathematical world. According to Swami ji the capacity and speed of calculations of students increases to five fold by the practice of Vedic Mathematics. By its practice there is an unbelievable development in their intellect and intelligence. Neocortex (mind) of the Vedic Mathematic students evolves very fast.

List of Sutras and Subsutras

Sutras	Subsutras
1. Ekadhikena Purvena (also a corollary)	1. Anurupyena
2. Nikhilam Navata-charamam Dasatah	2. Sisyate Sesamjnah
3. Urdhva-tiryagbhyam	3. Adyamadyenantya-mantyena
4. Paravartya Yojayet	4. Kevalaih Saptakam Gunyat
5. Sunyma Samyasamuchaye	5. Vestanam
6. (Anurupye) Sunyamanyat	6. Yavadunam Tavadunam
7. Sankalana-vyavakalanabhyam (also a corollary)	7. Yavadunam Tavadunikrtya Varganca Yojayet
8. Puranapuranabhyam	8. Antyayoradaskae'pi
9. Chalana-Kalanabhyam	9. Antyayoreva
10. Yavadunam	10. Samuccayagunitha
11. Vyastisamastih	11. Lopanasthapanabhyam
12. Sesanyankena Caramena	12. Vilokanam
13. Sopantyadvayamantyam	13. Gunitasamuccayah Samuccayagunitah
14. Ekanyunena Purvena	
15. Gunitasamuccayah	
16. Gunakasamuccayah	

Meaning and Applications of special Sutras

1.03 Sutra Ekadhikeana Purvena :

(a) Meaning : Sutra is composed of two words 'Ekadhika' and 'Purva', its meaning being "By one more than the previous one". Doing Ekadhika on a number means adding one to that number or marking Ekadhika dot (·) on its unit digit i.e..

Ekadhika of $1\dot{2} = 12 + 1 = 13$

In case if a digit of a number is marked the Ekadhika dot, its new value is to be found out e.g. Ekadhika of digit 3 of a number 1534

yields the new number $= 15\dot{3}4 = 1544$

Purvena (Purva) means 'By the previous one'. In case of digits of a number it points out the previous digit and in case of two numbers it points out the previous number e.g. In a number 685 the previous digit of five is 8 and in a multiplication 62 × 99 the previous number is 62. All the above operations can be mentally done.

(b) Applications :

(i) Addition : Sutra based method is applicable in all types of questions of addition.

Method : Write the numbers of a question columnwise. Start adding from top unit digit. At a digit where the sum equals to ten or more than ten. mark an Ekadhika dot on its previous digit. Start adding again with the unit digit of that sum. Repeat the process. Write the end sum at answer's place. Add other columns in the same way. The method is clarified by the following examples.

Example 1. Add the following :

3 7 9 9 5 ↓		**Steps**
$\dot{0}\ \dot{6}\ \dot{8}\ \dot{9}\ \dot{8}\ 6$	(i)	Column, I $5+6=11$
$\dot{7}\ \dot{5}\ \dot{4}\ 3\ 8$	(ii)	Mark Ekadhika dot (.) on 8
$\dot{0}\ \dot{5}\ \dot{8}\ \dot{9}\ \dot{0}\ 9$	(iii)	Start adding again with 1
2 4 1 3 2 8	(iv)	1+8=9, Again 9+9=18
	(v)	Mark Ekadhika dot on 8
	(vi)	Wrike 8 below at the answer's place
	(vii)	Add other columns in the same way.

Example 2. Add the following:

km.			m.		
	2	8	0	8	4
	$\dot{3}$	2	$\dot{3}$	6	5
$\dot{0}$	6	$\dot{5}$	7	$\dot{2}$	5
	$\dot{3}$	8	$\dot{2}$	5	0
1	6	4	4	2	4

Steps

(i) Three columns in a metre.

(ii) No columns should remain vacant hence write 084 for 84.

(iii) Now add columnwise by the above method.

(ii) Subtraction :

In Vedic Mathematics subtraction can be performed by four or five methods. The best method is based on Sutra Ekadhikena Purvena and Param mitra unka (the best friend). The two digits or numbers are said to be the best friend of each other if their sum is equal to ten i.e. 2 is the best friend of 8, 6 the best friend of 4 and 0 the best friend of $\dot{9}$.

Method : When lower digit does not substract from the upper digit. Then add the param mitr unka of lower digit to the upper digit and write the sum in place of answer. Also write the mark of Ekadhikena on the previous digit of lower digit. By the repeatation the process we get the remainder. If upper digit is greater than or equal to the lower digit, then there is no need to add the param mitra unka. Substract by normal process.

The method is clarified by the following examples

Example 3. Subtract

	5	7	6	2	5
–	$\dot{2}$	$\dot{9}$	$\dot{8}$	4	3
	2	7	7	8	2

Steps

(i) $5 - 3 = 2$ write it on the answer's place

(ii) 4 is not subtracted form 2. So the best friend of 4 is 6 is to be added to 2. Write the sum = 8 at answer's place.

(iii) Also mark an Ekadhikena dot at digit 8 prior to digit 4.

(iv) $\dot{8} = 9$ is not subtracted from 6 so add the best friend of 9 i.e. 1 to 6 i.e. 1+6=7 Write it on aswers' place,

(v) Mark Ekadhika dot on 9.

(vi) Similarly 0+7=7 and $5-\dot{2}=2$ will give the answer.

Example 4. Subtract

Steps

(i) It time unit columnwise bases are different.

(ii) In unit column of second and minute Base = 10

hour	min.	sec.
24	12	15
$\dot{0}\,\dot{6}$	$\dot{2}\,\dot{4}$	30
17	47	45

(iii) In tens's place column of second and minute, Base = 6

(iv) In Hour's column Base = 10.

(v) In all columns best friend of a digit is calculated by these bases.

Note : In Decimal Number System the base is taken as 10.

(iii) Multiplication :

In Vedic Mathematics for multiplication there are variance methods based on Sutrats. Sutra Ekadhikena Purvena based method is universal and effective. Some of its special sub methods are very easy and attractive. Here with let us define some new item . The unit digit of a number is known as Charamam digit and all other digits of a number are called Nikhilam digits. e.g. In a number 723 the Charamam digit is 3 and all the digits 7, 2, are called Nikhilam digits.

By the sutra 'Ekadhikena Purvena' when the sum of the Charamam digits is equal to 10 or power of 10 and the remainder Nikhilam digits are equal to each other, we can find the multiplication of two numbers easily.

Method :

(1) There are two sides of product one is L.H.S. and another is R.H.S.

(2) Write the product of charamam or last digits in L.H.S.

(3) In R.H.S. write the product of nikhilam digit and its Ekadhikena.

(4) In the R.H.S. keep the number of digits double of the number of zeroes in the sum of charamam digit i.e. two digits on R.H.S. if sum = 10.

(5) If number of digit in R.H.S. is less or more then we adjus the digit.

The method is explained by the following examples :

Example 5. Multiply : (Sum = 10)

83×87

$= 8 \times 9 / 3 \times 7$

$= 7221$

Steps

(i) Sum of Charamam digits 3 + 7 = 10.

(ii) remainder Nikhilam digits are equal = 8.

(iii) Two digits inR.H.S. = 21

Example 6. Multiply : (Sum = 100)

586×514
$= 5 \times 6 \ / \ 86 \times 14$
$= 30 \ / \ 1204$
$= 301204$

Steps

(i) Sum of last two digits $= 86 + 14 = 100$

(ii) remainder Nikhilam digits are equal = 5

(iii) Four digits in R.H.S. = 1204

Example 7. Multiply : (Sum = 1000).

3993×3007
$= 3 \times 4 \ / \ 993 \times 007$
$= 12 \ / \ 006951$
$= 12006951$

Steps

(i) Sum of last three digits $= 993 + 007 = 1000$

(ii) Six digits in R.H.S. = 006951

(add two zeros before 6 by adjustment)

Example 8. Multiply : (Sum = 1)

$9\frac{5}{11} \times 9\frac{6}{11}$
$= 9 \times 10 \ / \ \frac{5}{11} \times \frac{6}{11}$
$= 90\frac{30}{121}$

Steps

(i) Sum of fractios $= \frac{5}{11} + \frac{6}{11} = 1$

(ii) remaider nikhilam digits equal to each other =9

Example 9. Multiply : (Sum = 1)

$11 \cdot 7 \times 11 \cdot 3$
$= 11 \times 12 \ / \ \cdot 7 \times \cdot 3$
$= 132 \cdot 21$

Steps

(i) Sum of decimal fractions $= \cdot 7 + \cdot 3 = 1$

(ii) remainder Nikhilam digits equal to each other = 11.

Note: By Sutra based method all questions can be solved orally. Their products can be written in a line.

1.04 Sutra Ekanyunena Purvena :

(a) Meaning : Sutra is composed of two words - 'Eka nyunena' and 'Purvena'. Its meaning is "By one less than the previous one". Place a dot beneath the unit digit of a number which is to be less by one. This dot is known as Eka nyunena mark e.g. Eka nyunena of 7 in 57 = 5$\dot{7}$ = 57 – 1 = 56.

Like the previous Sutra Ekadhikena Purvena in this sutra we can find the value of new number by substract one from any digit of the number. e.g. In a number 124. We get a new number 024 by taking the nyunena of digit 1. i.e. new number = 1$\dot{2}$4 = 024 = 24.

(b) Applications:

(i) Subtraction (Sutra Ekanyunena Purvena + Param mitra unka)

By this sutra every question of subtraction can be solved easily.

Method:

When the lower digit is greater than the upper number digit add the best friend of lower number digit and keep this sum digit at the answer's place. Also place an Eka nyunena dot beneath the digit which is prior to the upper number digit. The repetition of this process will yield the remainder result. The method is clearified by the following examples :

Example 10. Subtract :

$$\begin{array}{r} 5\ \dot{6}\ 0 \\ -\ 3\ 7\ 4 \\ \hline 1\ 8\ 6 \\ \hline \end{array}$$

Steps

(i) The whole process is same as Ekadhikena Purvena method.

(ii) Difference is only one that in this process we place an Ekanyunena dot beneath the digit which is prior to the upper digit instead of Ekadhikena mark.

Example 11. Subtract:

kg.			g.		
1	2	5	0	9	5
	7	8	2	2	8
0	4	6	8	6	7

Steps

(i) 95 g is written as 095.

(ii) 5 + 2 (best freind of 8) = 7, write 7 at the answer's place. Also mark Eka nyunena dot beneath the digit 9.

(iii) 8 - 2 = 6

(iv) 0 + 8 (best freind of 2) = 8, write 8 at the answer's place. Also mark Eka nyunena dot beneath the digit 5.

(v) 4 + 2 (best freind of 8) = 6, write 6 at the answer's place. Also mark Eka nyunena dot beneath the digit 2.

(vi) 1 + 3 (best freind of 7) = 4, write 4 at the answer's place. Also mark Eka nyunena dot beneath the digit 1.

(viii) 0

(ii) Multiplication :

By Sutra Eka nyunena Purvena multiplication of two numbers can be easily performed if one of these numbers consists of digit nine only. For convenience the number having every digit 9 will be termed as multiplier and the other number as multiplicand.

Method :

There are two sides of the product L.H.S. = multiplicand –1

R.H.S. = multiplier – L.H.S.

Hence Multiplier × Multiplicand = Multiplicand –1 / Multiplier – L.H.S.

Three situations arieses in this multiplication :

(1) No. of digits of multiplier = No. of digits of multiplicand

(2) No. of digit of multiplier > No. of digits of multiplicand

(3) No. of digit of multiplier < No. of digits of multiplicand

I Situation :

(No. of digits of multiplier = No. of digits of multiplicand)

Let us see the following examples :

1. 8×9

L.H.S. $= 8 - 1 = 7$

R.H.S. $= 9 - 7 = 2$

$\therefore \quad 8 \times 9 \quad = 8 - 1 \ / \ 9 - 7 \ = 72$

2. 8567×9999

$= 8567 - 1 \ / \ 9999 - 8566$

$= 85661433$

II Situation :

(No. of digits of multiplier > No. of digits of multiplicand)

Let us see the following examples :

3. 68×999

$= 068 \times 999$

$= 067 \ / \ 999 - 067$

$= 67932$

4. 4523×999999

$= 004523 \times 999999$

$= 004522/995477$

$= 4522995477$

Note: (1) Greater the no. of digits of multiplier so many 9 digits appear in the middle of the answer e.g. see question no. 3 and 4.

(2) Sum of respective digits of L.H.S. and R.H.S. of the product will always be equal to 9, i.e. first digit of LHS + first digit of RHS = 9.

III Situation :

(No. of digits of multiplier < No. of digits of multiplicand)

The method is explained by the following examples:

5. 43×9

$= 42/9 - 42$

$= 429 - 42 = 387$

6. 512×99

$= 511/99 - 511$

$= 51199 - 511 = 50688$

Exercise 1.1

By Sutra Ekadhikena Purvena add the following:

1.
$$\begin{array}{r} 98765 \\ 63217 \\ 89522 \\ 60543 \\ \hline \\ \hline \end{array}$$

2.
$$\begin{array}{r} 89789 \\ 97686 \\ 76978 \\ 86798 \\ \hline \\ \hline \end{array}$$

3.
$$\begin{array}{rr} \text{kg.} & \text{g.} \\ 178 & 45 \\ 246 & 725 \\ 569 & 188 \\ 45 & 894 \\ \hline \\ \hline \end{array}$$

4.
$$\begin{array}{rrr} \text{km.} & \text{m.} & \text{cm.} \\ 25 & 510 & 36 \\ 47 & 85 & 52 \\ 18 & 123 & 75 \\ 53 & 805 & 28 \\ \hline \\ \hline \end{array}$$

By vedic method subtract the following:

5.
$$\begin{array}{r} 746 \\ -389 \\ \hline \\ \hline \end{array}$$

6.
$$\begin{array}{r} 4032 \\ -3543 \\ \hline \\ \hline \end{array}$$

7.
$$\begin{array}{r} 6007 \\ -1852 \\ \hline \\ \hline \end{array}$$

8.
$$\begin{array}{r} 8317 \\ -6454 \\ \hline \\ \hline \end{array}$$

By Sutra Ekadhikena Purvena multiply the following.

9. 42×48 10. 103×107

11. 294×206 12. 413×487

Multiply with the help of Sutra Ekanyunena Purvena

13. 54×99 14. 214×999

15. 47×999 16. 342×99999

17. 73×9 18. 467×99

By Vedic method multiply the following:

19. $15\frac{5}{7} \times 15\frac{2}{7}$ 20. $24\frac{10}{13} \times 24\frac{3}{13}$

21. $4 \cdot 5 \times 4 \cdot 5$ 22. $9 \cdot 85 \times 9 \cdot 15$

1.05 Vinculum (Negative) Numbers :

Concept of Vinculum operation is the contribution of Vedic mathematics. By Vinculum operation calculations become easy, short and sometimes oral. By this process numbers of big digits like (6,7,8,9) can be converted into numbers of small digits like (0,1,2,3,4,5). These days Vinculum operation has been adopted by computer science also. Digits with bars $\overline{2}$, $\overline{4}$ etc. are called Vinculum digits. Their respective values are –2and –4. This bar (line) is called Vinculum line or Vinculum mark. Positive and Vinculum digits can be placed together to represent a number. e.g. $2\ \overline{3}$ or $\overline{2}\ \overline{4}$. The number $1\ \overline{2}\ \overline{4}$ is spoken as one Vinculum two Vinculum four. It will not be called one Vinculum twenty four.

1.06 Base, Sub Base, Deviation :

Base : Here Base means a Number Base. Any number greater than 1 can be a base. In Vedic mathematics to make the calculation easy and convenient bases are usually chosen as 10 or 100 or any power of 10. In metric system of numbers base is always taken as 10.

Sub-Base : Sub-Base is the multiple of base. Mostly this number always ends at zero. If Base = 10, then Sub Base .$10 \times x$, x being a whole number, if Base = 100, then Sub Base =$100 \times x$, x being a whole number. In case of Sub Base calculations become easy if Sub-Base are used in place of base but necessary adjustment is to be done, in the left hand side of the answer. The reasons will be explained in the coming examples.

Deviation : When Base or Sub-Base is subtracted from the given number the remainder is called the deviation.

Deviation = Number – Base

or Deviation = Number – Sub-Base

If the number is greater than the Base or Sub-Base, the deviation is positive. If number is smaller than the Base or Sub-base, the deviation is negative. The deviation consists so many digits as there are the number of zero to the Base.

Deviation of 18 w.r.t. Base 10 = +8 (Number of Zero in the Base is one)

and Deviation of 94 w.r.t. Base 100 = –06. (Number of Zero in the Base is two)

1.07 Sutra Nikhilam Navtah Charamam dastah

(a) Meaning : Sutra Nikhilam means "All from nine and last from ten". In ancient Indian Mathematics digit nine is supposed to be very creative and an apex number. The number 10 is taken to be the whole number but here the sutra indicates simply the subtraction process.Applications like Vinculum, Subtraction, Multiplication, Squaring, Cubing, Divisions etc. are based on this sutra.

(b) Applications:

(i) To convert ordinary number into Vinculum number

(Sutra Ekadhikena Purvena + Sutra Nikhilam)

When an ordinary number consists of digit 5 or more than 5, it can be converted into Vinuclum number by Nikhilam method.

Method: (1) Subtract Charmam (unit) digit from 10.

(2) Subtract the remainder digits from the number 9.

(3) Draw Vinculum line on each digit so obtained.

(4) Mark the Ekadhikena dot on the previous digit 0 or less than 5 of the remainder.

The method is explained by the following examples:

Example 12 : Convert the following ordinary number into Vinculum number:

1. $8\ 9\ 8$
$= 0\ 8\ 9\ 8$
$= \dot{0}\ \bar{1}\ 0\ \bar{2}$
$= 1\ \bar{1}\ 0\ \bar{2}$

Steps

(i) Subtract unit digit 8 from 10.

(ii) Subtract remaining digits 9 and 8 from 9.

(iii) Draw Vinculum Bar on 2 and 1.

(iv) Place an Ekadhikena dot on 0 prior to $\bar{1}$

2. $1\ 8\ 4\ 6\ 9$
$= \dot{1}\ \bar{2}\ \dot{4}\ \bar{3}\ \bar{1}$
$= 2\ \bar{2}\ 5\ \bar{3}\ \bar{1}$

Steps

(i) Subtract 9 from 10 and 6 from 9 respectively.

(ii) Draw Vinculum bar on 1 and 3.

(iii) Place Ekadhikena dot on 4.

(iv) Restart the process from digit 8.

Note : (1) When in between the bigger digit of a normal number occures a digit equal or lessthan 5 than we repeat this process.

(2) No vinculum bar is drawn on zero.

(ii) To convert Vinculum number into ordinary number

(Sutra Ekanyunena Purvena + Sutra Nikhilam)

Method:
(1) Subtract the positive value of Charmam digit from 10.
(2) Subtract the positive vlaues of remaining Nikhilam digits from 9 respectively.
(3) Place an Ekanyunena dot beneath the non Vinculum digit.
(4) Repeat the process if necessary.

The method is clarified by the following examples:

Example 13 : Change the following into ordinary number:

1. $2\bar{4}\bar{3}$
$= 2\dot{5}7$
$= 157$

2. $6\bar{2}\bar{4}5\bar{3}\bar{2}$
$= 6\dot{7}6\dot{5}68$
$= 576468$

(iii) Multiplication of two numbers :

(Sutra Nikhilam – Base)

When two numbers are nearer to the base = 10 or 100 or power of 10, their multiplication can be find out easily by Sutra Nikhilam - Base.

Method:
(1) Choose the base = 10 or 100 etc. nearer to the numbers.
(2) Find the deviations of the numbers with refrence to base and write them before their numbers.
(3) Divide the product place in two parts by an oblique line.
(4) Write the product of deviations on R.H.S.
(5) On L.H.S. write one number plus the deviation of the other number.
(6) Adjust the number of digits of R.H.S. according to the number of zeroes of the base.
(7) The deficiency of digit number is fulfilled by writing zero. If the product digit number is more, take it to the left hand side.
(8) If the product of deviations is negative, change it into positive by taking one or two etc. from L.H.S. Kindly note that the vlaue of one from L.H.S. is equivalent to the base value.

The method is explained by the following examples:

Example 14 : Multiply the following by Nikhilam – Base method:

1. 12×14, Base $= 10$

$$\begin{array}{ll} = 12 & +2 \\ 14 & +4 \\ \hline = 14 + 2/2 \times 4 & \end{array}$$

$$= 168$$

Steps

(i) Deviations = +2, +4

(ii) One digit in R.H.S.

2. 92×87, Base $= 100$

$$\begin{array}{ll} = 92 & -08 \\ 87 & -13 \\ \hline = 92 - 13/(-08)(-13) & \end{array}$$

$$= 79/_{1}04 = 8004$$

Steps

(i) Deviations = –08, –13

(ii) Two digits in R.H.S. so digit 1 or 104 to L.H.S.

3. 7×18, Base $= 10$

$$\begin{array}{ll} = 7 & -3 \\ 18 & +8 \\ \hline = 7 + 8/(-3) \times 8 & \end{array}$$

$$= 15/-24$$

$$= 15 - 3/30 - 24$$

$$= 12/30 - 24$$

$$= 126$$

Steps

(i) Product = 15/–24

(ii) Bring 3 from L.H.S. to R.H.S.

(iii) In R.H.S. value of 3 will be equla = 30

4. 1007×1012

$$\begin{array}{ll} = 1007 & +007 \\ 1012 & +012 \\ \hline = 1012 + 7/084 & \end{array}$$

$$= 1019084$$

Steps

(i) Base = 1000

(ii) d =+007, +012

(iii) Three digits in R.H.S. so write zero before 84.

(iv) Multiplication of two numbers

(Sutra Nikhilam – Sub-Base)

Sometimes it becomes difficult to multiply two big deviations. In this case concept of Sub-Base is helpful. The method is similar to the previous Sutra Nikhilam Base method except that for adjustment in L.H.S. product is multiplied by the Sub-Base digit (Upadhar Unka) and R.H.S. remains as usual. The method is explained by the following examples:

Example 15 : Multiply the following by Sutra Nikhilam – Sub-Base.

1. 32×33

$= 32 \quad +2$
$\quad 33 \quad +3$

$= 35 \times 3/6$

$= 1056$

Steps

(i) Base = 10, Sub-Base = $10 \times 3 = 30$.
(ii) Upadhar unka =3
(iii) deviation from Sub-Base = +2 , +3
(iv) Adjustment in L.H.S. = 35×3

2. 54×56

$= 54 \quad +4$
$\quad 56 \quad +6$

$= 60 \times 5/_{2}4$

$= 300/_{2}4$
$= 3024$

Steps

(i) Base =10, Sub-Base = $10 \times 5 = 50$
(ii) Upadhar unka = 3
(iii) deviation from Sub-Base = +4, +6
(iv) L.H.S. = 60×5
(v) Adjustment in R.H.S. will be done in the end.

3. 54×56

$= 54 \quad +4$
$\quad 56 \quad +6$

$= 60 \times \frac{1}{2}/24$

$= 3024$

Steps

(i) Base = 100, Sub-Base = $100 \times \frac{1}{2} = 50$
(ii) Updhar unka = ½
(iii) d =+04 and 06
(iv) Two digits in the R.H.S.

4. 206×212

$= 206 \quad +06$
$\quad 212 \quad +12$

$= 218 \times 2/72$

$= 43672$

Steps

(i) Base = 100, Sub-Base = 100×2
(ii) Upadhar unka = 2
(iii) d =+06, +12
(iv) Two digits in R.H.S.

(v) Multiplication of three numbers:

(Sutra Nikhilam – Base)

Multiplication process is divided into three steps :

Step I : Any number + deviations of other two numbers.
Step II : Sum of the products of two deviations.
Step III : Product of three deviations.

The method is explained by the following examples:

Example 16 : Multiply the following by Sutra Nikhilam – Base.

1. $91 \times 93 \times 96$, Base $= 100$

Number	Deviation
91	-09
93	-07
96	-04

$= 93 - 09 - 04 / 36 + 28 + 63 / (-9)(-4)(-7)$

or 91 – 07 – 04

or 96 – 09 – 07

$= 80/_{1}27/(-252)$

$= 81/27 - 3/300 - 252$

$= 81/24/48$

$= 812448$

Steps

(i) deviation $= -09, -07, -04$

(ii) In the third part $(-09)(-07)(-04) = -252$

(iii) In the middle part $(-09)(-04) + (-07)(-04) + (-09)(-07) = 127$

(iv) 3 taken from II part to III part place value being 300

(v) $300 - 252 = 48$

2. $103 \times 105 \times 106$,

$= 106 + 03 + 05 / 15 + 30 + 18 / 90$

$= 114/63/90$

$= 1146390$

Steps

(i) Base $= 100$

(ii) deviation $= +06, +03, 05.$

(iii) rest process as above.

3. $12 \times 13 \times 15$

$= 12 + 3 + 5 / 6 + 15 + 10 / 30$

$= 20/_{3}1/_{3}0$

$= 2340$

Steps

(i) Base $= 10$

(ii) deviation $= 2, 3, 5.$

Note: So as many digits are to be kept in II and III parts as there are number of zeroes in the Base.

(vi) Multiplication of three numbers:

(Sutra Nikhilam – Sub-Base)

In Nikhilam Sub-Base method the first part (from the left) and middle part of the product is multiplied by (Sub-Base digit)^2 or (Upadhar Unka)^2 and (Sub Base digit) or (Upadhar Unka) respectively. This is the only difference between Nikhilam Base and Sub-Base methods.

The method is explained by the following examples:

Example 17 : Multiply the following Nikhilam Sub-Base method.

1. $21 \times 24 \times 25$

21	+1
24	+4
25	+5

Steps

(i) Base $= 10$, Sub-Base $= 10 \times 2 = 20$.

(ii) Upadhar Unka $= 2$

(iii) deviation $= +1, +4, +5$

(iv) One digit each in II and III part.

$= 2^2 (21 + 4 + 5) / 2(4 + 20 + 5) / 1 \times 4 \times 5$

$= 4 \times 30 / 2 \times 29 / 20$

$= 120 / {}_{5}8 / {}_{2}0$

$= 12600$

2. $502 \times 503 \times 504$

$= 5^2 (502 + 03 + 04) / 5(6 + 12 + 8) / 2 \times 3 \times 4$

$= 25 \times 509 / 5 \times 26 / 24$

$= 12725 / 130 / 24$

$= 127263024$

Steps

(i) Base $= 100$,
Sub-Base $= 100 \times 5$

(ii) Upadhar Unka $= 5$

Exercise 1.2

Change into Vinculum numbers :

1. 89
2. 878
3. 9687
4. 6578

Change into ordinary numbers :

5. $3\bar{2}1$
6. $2\bar{4}\bar{3}\bar{2}$
7. $4\bar{3}0\bar{2}$
8. $450\bar{4}\bar{9}$

Multiply the followng by Sutra Nikhilam

9. 102×107
10. 94×92
11. 72×73
12. 203×204
13. $11 \times 12 \times 13$
14. $97 \times 98 \times 99$
15. $102 \times 103 \times 104$
16. $99 \times 101 \times 103$

Methods of checking the results

There are two methods of checking the results received from any operation.

(a) Navanka method

(b) Ekadashanka Method

(a) Navanka method :

In the navanka method we find the beejanka of any number by taking base as digit 9. After subtracting 9 from the digits of a number or sum of the digits of a number remaining single digit is know as Beejanka of that number e.g. Beejanka of 947 = 2.

In the different operation the application of navanka method is explained by the following example.

Example 18 :

(1) Checking of Addition results:

	Beejanka
5 3 8 9	7
6 4 7 2	1
5 9 3 6	5
4 1 6 8	1
2 1 9 6 5	5

Steps

(i) Sum of Beejanka (row-wise)

$= 7 + 1 + 5 + 1 = 5$

(ii) Beejanka of the sum

$= 2 + 1 + 9 + 6 + 5 = 5$

Both are equal, hence answer is correct.

(2) Checking of the Subtraction results:

$$\begin{array}{r} 8134 \\ -\,5678 \\ \hline 2456 \end{array}$$

Steps

(i) Beejanka of the minuend $= 7 \quad = 16$

(ii) Beejanka of the subtrahend $= 8$ or -8

(iii) Beejanka of the remainder $= 8$

(3) Checking of the Multiplication results:

$$73 \times 77 = 5621$$

(i) Beejanka of the multiplicand $= 7 + 3 = 10 = 1$

(ii) Beejanka of the multiplier $= 7 + 7 = 14 = 5$

(iii) Beejanka of the multiplicand $\times$ multiplier $= 1 \times 5 = 5$

(iv) Beejanka of the product $= 5+6+2+1 = 5$

Beejanka of L.H.S. =Beejanka of R.H.S.

Note: 1. If two digits of any row or two digits of any column interchange their places the error cannot be spotted by Navanka method.

2. In Vedic Mathematics there are several methods to solve a question. The result can be verified by Ekadashanka Method.

(b) Ekadashanka Method or Difference Method:

In this method the difference of the sumof digits at odd places and the sum of digits at even places is called the Beejanka of the number e.g. Beejanka of 63254

$$= 4-5+2-3+6 = 4$$

The use of the Ekadashanka method for different operations is explained here with by the following examples:

(i) Checking of Addition results :

	Row-wise Beejanka
63254	$4-5+2-3+6=4$
54327	$7-2+3-4+5=9$
89325	$5-2+3-9+8=5$
206906	Sum = 18

Beejanka of the sum $18 = 8-1 = 7$

Beejanka of the sum 206906 $= 6-0+9-6+0-2 = 7$

Both are equal, hence answer is correct.

(ii) Checking of the Subtraction results:

	Row-wise Beejanka
7348	$8-4+3-7=0$
−5249	$9-4+2-5=2$
2099	

Beejanka of differene = (i) – (ii) = 0 – 2 = –2

Beejanka of the Remainder = 9 – 9 + 0 – 2 = – 2

Since both are equal, the answer is correct.

(iii) Checking of the Multiplication results:

$54 \times 56 = 3024$

Beejanka of $54 = 4 - 5 = -1$

Beejanka of $56 = 6 - 5 = +1$

Multiplicationof Beejanka $(-1) \times (+1) = -1$

Beejanka of the product $= 4 - 2 + 0 - 3$
$= -1$

Both are equal, hence answer is correct.

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Answers

Exercise 1.1

1. 312047
2. 351251
3. 1039 kg., 852 gm.
4. 144 Km. 524 m. 91 cm.
5. 357
6. 489
7. 4155
8. 1863
9. 2016
10. 11021
11. 60564
12. 201131
13. 5346
14. 213786
15. 46953
16. 34199658
17. 657
18. 46233
19. $240\frac{10}{49}$
20. $600\frac{30}{169}$
21. $20 \cdot 25$
22. $90 \cdot 1275$

Exercise 1.2

1. $1\bar{1}\bar{1}$
2. $1\bar{1}\bar{2}\bar{2}$
3. $10\bar{3}\bar{1}\bar{3}$
4. $1\bar{3}\bar{4}\bar{2}\bar{2}$
5. 281
6. 1568
7. 3698
8. 44951
9. 10914
10. 8648
11. 5256
12. 41412
13. 1716
14. 941094
15. 1092624
16. 1029897

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